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Calibration of a discrete spherical tank model through a mechanical-equivalent mass-spring model

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Abstract. The analysis of fluid-structure interaction (FSI) in storage tanks is a fundamental aspect of structural design, particularly in spherical tanks used for water storage. Numerical methods implemented in finite element software are commonly employed to address this phenomenon. These methods are typically based on complex formulations that integrate fluid dynamics, providing detailed and accurate results. However, in structural design, engineers often prefer simplified models that adequately capture the essence of the problem without requiring complex simulations. This research aims to validate the modeling approach of FSI in spherical storage tanks using a mechanical-equivalent mass-spring model (MEMSM). The impulsive and convective components of the fluid were represented as lumped masses and springs whose stiffness is associated with the oscillation of the convective component. A perfectly fixed base was assumed, neglecting effects of soil-structure interaction (SSI), to remain consistent with experimental conditions used for validation. The model was validated by comparing the natural vibration frequencies, and the results indicate that the MEMSM model predicts these frequencies with a difference of 1.3% for the impulsive component and a maximum difference of 6.7% for the convective component. This result validates the proposed approach and underscores its main contribution: providing an alternative, efficient, and easily implementable tool for the dynamic analysis of spherical tanks, suitable for early stages of structural design without compromising accuracy.

Keywords: fluid-structure interaction; mass-spring system; numerical model; spherical tank

1. Introduction

Spherical storage tanks are unconventional structures that play an important role in various industrial and civil engineering applications, such as the storage of water, fuels, and chemicals. They represent a highly efficient alternative, as their shape allows for the use of minimal construction material while maximizing storage volume due to the uniform distribution of stresses throughout the structure (Fiore *et al.* 2018, Zhang *et al.* 2015, Lyu *et al.* 2022). The structural dynamics of these systems are complex due to the interaction between the tank and the contained fluid, which significantly influences their behavior under seismic loading (Wieschollek *et al.* 2011, Ma *et al.* 2020).

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This interaction between the stored liquid and the tank walls constitutes a coupled system, as neither the fluid nor the structural component can be solved independently due to the unknown internal forces that link them (Zienkiewicz and Taylor 2000). Traditionally, numerical analyses of such coupled systems have relied on complex models that combine structural elements with fluid dynamics formulations, enabling a highly accurate representation of the involved phenomena (Ibrahimbegovic *et al.* 2016). However, the implementation of these models entails high computational costs, which restricts their use in preliminary design stages or routine structural evaluations. In this context, simplified models based on equivalent mass–spring systems have emerged as a useful and efficient tool for representing fluid-structure interaction (FSI). These approaches aim to strike a balance between accuracy and computational efficiency. As highlighted in engineering practice, the analytical or numerical methods that tend to gain broader acceptance are those that not only provide reliable predictions but also remain sufficiently simple to implement within the constraints of daily design tasks (Hadzalic *et al.* 2018).

Many simplified models proposed in design guidelines and prior research have been developed specifically for tanks with regular geometries (mainly cylindrical or rectangular) where the dynamic interaction between fluid and structure has been more extensively studied. In the case of spherical tanks, existing models remain scarce, despite the geometric and mechanical particularities that differentiate their seismic behavior from that of more conventional configurations. The curvature of the vessel and the distribution of mass introduce distinct dynamic responses, requiring specific evaluation to determine whether traditional modeling assumptions remain valid.

Moreover, validating simplified models through comparison with experimental results is a key step toward establishing their reliability. Experimental data serve not only to verify numerical formulations but also to highlight potential discrepancies resulting from idealized assumptions, especially when dealing with structures whose geometry deviates from standard typologies. This type of validation reinforces the applicability of simplified models for use in practical design, particularly when full-scale simulations are not feasible due to resource constraints.

Based on these considerations, the present research aims to validate the fluid–structure interaction model in spherical tanks through a mass–spring representation, by comparing the numerical results with experimental data reported in the literature.

2. Background

Housner (1963) laid the foundation for the dynamic analysis of storage tanks by decoupling the effects of FSI and sloshing. He proposed a mass–spring model that has been widely adopted in current design codes such as ASCE/SEI 7-22 (2022), ACI 350.3-20 (2021), and Eurocode 8 (2006). In this formulation, the impulsive component is represented as a mass rigidly attached to the structure. In contrast, the convective component is modeled as a mass–spring system oscillating independently, as shown in Fig. 1. This approach transformed a problem that was originally continuous and complex into a simpler, discrete system, making it more tractable for practical engineering analysis.

Following Housner's pioneering work, Veletsos and Yang (1976) expanded the impulsive–convective formulation by introducing analytical expressions for hydrodynamic pressures and modal properties in both rigid and flexible cylindrical tanks. Their research offered a solid theoretical foundation for determining the effective masses and stiffnesses associated with fluid

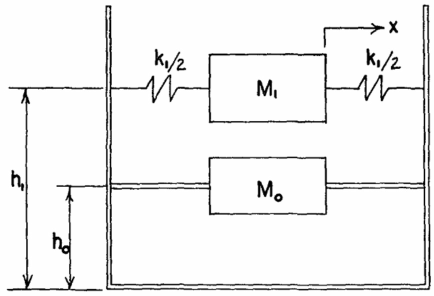


Fig. 1 Housner's simplified model (Housner 1963)

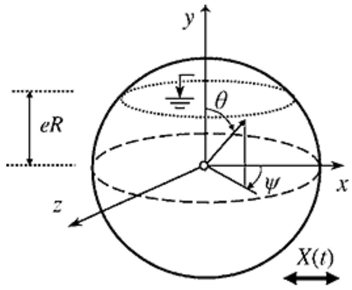


Fig. 2 Spherical storage vessel (Karamanos *et al.* 2006)

motion, while also accounting for the influence of wall flexibility on the dynamic response. Subsequently, Haroun and Housner (1981) refined the mechanical analogy by incorporating multiple convective modes and investigating their relative importance under seismic loading. Their work demonstrated that while the fundamental convective mode often dominates, higher-order modes may become relevant in tanks with certain geometrical proportions or excitation characteristics. This insight underscored the necessity of accurately representing fluid dynamics in seismic design, particularly for slender tanks or those with variable fill levels. Extending this approach, Malhotra *et al.* (2000) proposed an enhanced mechanical-equivalent model that integrates the flexibility of the structure with both impulsive and convective components of the fluid. In their model, each mass is placed at a defined height and connected to the structure through springs representing the stiffness of each mode. This approach improved the accuracy of seismic demand predictions and remains widely cited in modern design guidelines for storage tanks, especially due to its balance between simplicity and predictive capability.

Further studies (Vern *et al.* 2021, Maedeh *et al.* 2017, Sivy and Musil 2017, Vathi and Karamanos 2015) have focused on validating simplified models for storage tanks with regular geometries. However, the dynamic effects of stored liquid in spherical tanks had not been addressed until the work of Karamanos *et al.* (2006), who investigated the sloshing phenomenon in spherical storage vessels subjected to horizontal excitation (Fig. 2). In their study, the authors determined the natural frequencies associated with the convective modes of the stored liquid, as well as the corresponding modal masses. This methodology enables the direct estimation of the fluid's dynamic response, thereby facilitating its integration into simplified structural models.

Later works (Sivy and Musil 2018, Garrido and Fernandez-Davila 2016, 2019) have employed the values proposed by Karamanos *et al.* (2006) to validate their applicability through comparisons with models developed using finite element software. Garrido and Fernandez-Davila (2016) conducted a comparative analysis between a three-dimensional model based on the equivalent mass–spring mechanical system and a finite element model incorporating fluid elements to represent the stored liquid. The study found an 8% difference in the natural vibration frequencies and a 5% discrepancy in the effective participating masses between the two numerical methods. These results support the feasibility of using simplified models for the dynamic analysis of this type of structure. In a similar vein, Chaulagain *et al.* (2019) created a three-dimensional model for the seismic analysis of partially filled spherical storage tanks. The simplified model was validated against a more detailed FSI model that incorporated both wall flexibility and sloshing effects. The comparison included dynamic characteristics and seismic response parameters for various fill levels. The results showed close agreement between both modelling approaches, reinforcing the suitability of mass-spring analogues to reproduce the essential features of FSI and the dynamic behavior of spherical tanks.

However, the studies discussed focused on numerical comparisons, without incorporating experimental results for validation. This limits the scope of validation of the numerical models developed and underscores the need to contrast this numerical approach with laboratory data to ensure the robustness of simplified models in practical applications.

3. Methodology

3.1 Sensitivity analysis

In the simplified models reported in the literature, the convective component of the stored liquid is typically represented by multiple oscillating masses, each associated with a convective mode of the fluid. This approach allows for a more detailed characterization of the fluid's dynamic behavior. However, several studies (Zhang *et al.* 2015, Drosos *et al.* 2019, Karaferis *et al.* 2024) indicate that, for practical purposes in structural design and analysis, it is sufficient to consider only the first convective mode. Together with the impulsive mode, this representation can account for approximately 85% to 98% of the total mass of the stored liquid and provides accurate results.

To validate this premise and adapt it to the specific case of elevated spherical tanks, a sensitivity analysis was conducted to determine the appropriate number of convective modes needed. This assessment was guided by seismic design codes (ASCE/SEI 7-22 2022, E.030 2020), which recommend achieving a minimum cumulative participating modal mass percentage (CPMMP) of 90% to ensure an adequate representation of the system's dynamic behavior. The analysis focused on evaluating the contribution of each convective mode to the dynamic response using the CPMMP as the primary criterion. To achieve this, a simplified model was utilized (Fig. 3), incorporating two lumped masses to represent the convective component and a third mass to account for the impulsive component and the structural system. All masses were located at the geometric center of the spherical vessel, as all resultant forces associated with the fluid pressure act at this point (Drosos *et al.* 2008).

The motion of discrete multi-degree-of-freedom (MDOF) systems is described by a set of N homogeneous differential equations, as represented in Eq. (1). These equations are coupled through the mass matrix $\boldsymbol{m}$ and the stiffness matrix $\boldsymbol{k}$. In this context, N refers to the number of degrees of freedom, while $\boldsymbol{u}$ denotes the displacement vector and $\ddot{\boldsymbol{u}}$ represents the acceleration vector (Chopra 2014).

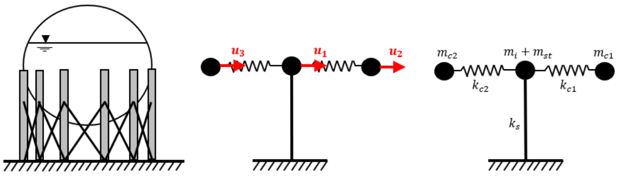


Fig. 3 Simplified model of spherical tank

Table 1 Sloshing factors (Karamanos *et al*. 2006)

H/R	λ_1	λ_2	λ_3
0.10	1.0347	6.5638	13.8911
0.20	1.0723	6.2008	11.8764
0.40	1.1583	5.6742	9.8543
0.60	1.2625	5.3683	8.9418
0.80	1.3924	5.2406	8.5509
1.00	1.5602	5.2756	8.5045
1.20	1.7882	5.4930	8.7793
1.40	2.1232	5.9729	9.4763
1.60	2.6864	6.9574	10.9566
1.80	3.9595	9.4551	14.7598
1.90	5.7615	13.1776	20.4520
1.95	8.3121	18.5527	28.6891

$$\boldsymbol{m} \cdot \ddot{\boldsymbol{u}} + \boldsymbol{k} \cdot \boldsymbol{u} = \boldsymbol{0} \tag{1}$$

Table 1 presents the sloshing factors ($\lambda_n = {\omega_{cn}}^2 \cdot R/g$) for spherical tanks, determined by Karamanos *et al*. (2006), as a function of the parameter H/R, defined as the ratio of the liquid height (H) to the radius of the spherical storage vessel (R).

Additionally, Table 2 provides the corresponding values of convective mass ratios M_{1C}/M_L, M_{2C}/M_L, M_{3C}/M_L, and the impulsive mass fraction M_I/M_L, determined by Karamanos *et al.* (2006).

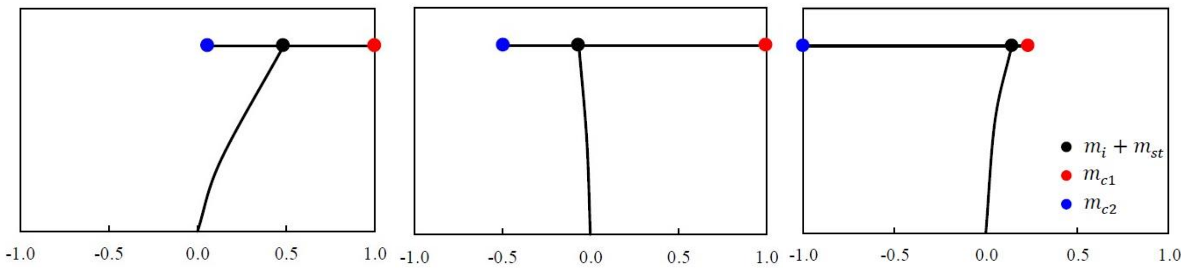
The frequency and mass values provided are consistent with those reported by Karamanos *et al.* (2006), which were mentioned earlier. By assembling the mass and stiffness matrices, the following matrix equation of motion (Eq. (2)) is obtained

$$\begin{pmatrix} m_i + m_{st} & 0 & 0 \\ 0 & m_{c1} & 0 \\ 0 & 0 & m_{c2} \end{pmatrix} \cdot \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{pmatrix} k_s + k_{c1} + k_{c2} & -k_{c1} & -k_{c2} \\ -k_{c1} & k_{c1} & 0 \\ -k_{c2} & 0 & k_{c2} \end{pmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2}$$

Where m_i represents the mass of the impulsive component, m_{st} corresponds to the structural mass, k_s is the stiffness of the supporting structure, m_{cn} is the mass associated with the n-th convective mode, and k_{cn} is the corresponding stiffness. This matrix equation governs the coupled dynamic behavior of the tank-fluid system, where the degrees of freedom u_1, $u_2$and u_3 represent the displacement of the structural mass and the convective masses. The interaction between the structural and fluid components is captured through the off-diagonal terms of the stiffness matrix,

Table 2 Mass ratios (Karamanos *et al*. 2006)

H/R	M_{1C}/M_L	M_{2C}/M_L	M_{3C}/M_L	$\Sigma M_C/M_L$	M_I/M_L
0.10	0.96594	0.000387	0.0000064	0.96634	0.03366
0.20	0.93038	0.001374	0.0000667	0.93184	0.06816
0.40	0.85437	0.004341	0.0005155	0.85947	0.14053
0.60	0.77117	0.007850	0.0013969	0.78136	0.21864
0.80	0.67990	0.011396	0.0025337	0.69619	0.30381
1.00	0.57969	0.014576	0.0037169	0.60594	0.39406
1.20	0.46981	0.016874	0.0047195	0.49844	0.50156
1.40	0.35009	0.017526	0.0052523	0.38440	0.61560
1.60	0.22222	0.015419	0.0048997	0.26162	0.73838
1.80	0.09363	0.009185	0.0031036	0.12608	0.87392
1.90	0.03655	0.004387	0.0015438	0.05586	0.94414
1.95	0.01364	0.001851	0.0006696	0.01810	0.98190



(a) First mode (b) Second mode (c) Third mode

Fig. 4 Vibrations mode of the simplified model

Table 3 Evaluation of modal contribution

Mode	PMMP (%)	CPMMP (%)
1	83.83	83.83
2	13.88	97.71
3	2.29	100.00

reflecting the coupling effects. Considering the geometry of a scaled spherical tank model analyzed by Curadelli *et al.* (2010), and assuming a filling level of 50% (*H/R*=1.00), the following vibration modes (Fig. 4) were obtained:

In the first mode (*a*), a marked interaction between the structure and the fluid is observed, as the components exhibit coupled displacements. This mode is associated with the impulsive behavior of the fluid mass, which moves in unison with the supporting structure.

In contrast, the second and third modes (*b* and *c*) display a response governed by the convective components, which oscillate relative to a virtually rigid structure. In these modes, the structure behaves almost as a rigid body, while the liquid free surface experiences significant dynamic motion, typical of convective behavior.

As shown in Table 3, the first mode alone accounts for approximately 84% of the total modal mass participation, which highlights its dominant role in capturing the global dynamic behavior of

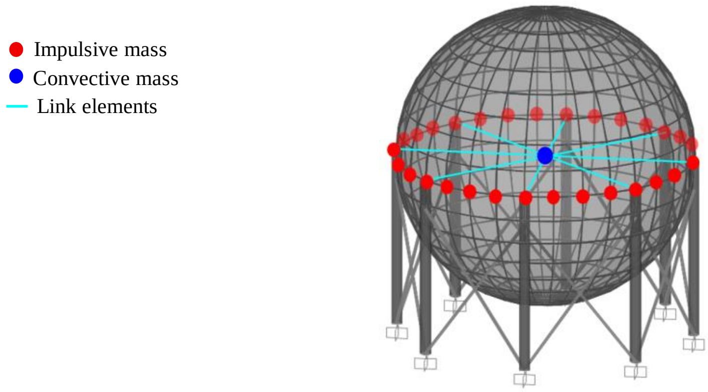


Fig. 5 Numerical model of spherical tank

the system. When the second mode is included, the cumulative contribution rises to nearly 98%, indicating that this mode (associated with the first convective component) also plays a relevant role in the overall response. However, the contribution of the third mode is minimal, adding just over 2%, which confirms that higher-order convective modes have a negligible impact under the conditions analyzed.

The distribution of modal mass participation observed justifies the simplification adopted in the numerical modeling. By including only the first oscillation mode of the convective component, it is possible to capture the essential features of FSI with sufficient accuracy for engineering applications. This approach not only preserves the representativeness of the convective effects but also allows for reduced computational cost while maintaining reliable estimations of convective effects.

This result supports the decision to consider only the first oscillation mode of the convective component for modeling and result comparison, as it provides an accurate representation of these effects in the overall structural response.

3.2 Numerical model

The computational tool SAP2000 (CSI 2016) was used to develop the numerical model (Fig. 5). The spherical vessel was modeled using shell elements, while the supporting columns and braces were represented using frame elements. The simulated stored water used lumped masses, following the method proposed by Wilson and Khalvati (1983). This approach considers that an incompressible confined fluid can be represented as an added mass.

The convective mass was placed at the center of the spherical vessel, as all resultant forces associated with fluid pressure act at this point (Drosos *et al.* 2008), and it was connected to the supporting columns using link elements. On the other hand, the impulsive mass was distributed along the horizontal central diameter of the sphere. The bases of the supports were modeled as perfectly fixed, neglecting soil-structure interaction (SSI) effects, to ensure consistency with the experimental conditions described later in this study. The mass and stiffness values for each component were derived from Tables 1 and 2 in the study by Karamanos *et al.* (2006), and the material properties applied are detailed in subsequent sections.

One of the main advantages of simulating the fluid using lumped masses and springs, instead of

Fig. 6 Spherical tank (Curadelli and Ambrosini 2011)

explicitly modeling it with three-dimensional fluid elements, lies in the significant reduction of computational cost. Fully coupled FSI models, typically implemented in advanced finite element software such as ANSYS or ABAQUS, require detailed meshing of the fluid domain and the solution of complex fluid dynamics equations, often under Lagrangian-Eulerian (ALE) frameworks. These models are accurate but computationally expensive and require advanced modeling techniques, which makes them less accessible for professionals facing practical time and resource constraints. In contrast, the mass–spring formulation offers an efficient alternative that retains the essential dynamic characteristics of fluid–structure interaction, especially for estimating overall structural responses. Moreover, this approach can be implemented in widely used structural engineering platforms, such as SAP2000, which are more familiar to engineers in professional practice. This compatibility facilitates the adoption of more realistic dynamic modeling strategies in the design and assessment of industrial structures without the need for highly specialized software in computational fluid dynamics (CFD).

It is important to note that the mass–spring representation used in this study does not provide a detailed evaluation of the hydrodynamic pressure distribution on the inner surface of the spherical vessel. As a result, local effects on the tank walls, such as stress concentrations or shell buckling due to sloshing pressures, cannot be reliably captured. Nevertheless, this limitation is acceptable when the objective is to evaluate the global dynamic behavior of the structure. In particular, the model is suitable for estimating the natural periods of vibration and, therefore, the seismic response quantities that are most relevant in structural design, such as base shear, overturning moment, and lateral displacements. Several studies have shown that simplified models with equivalent mass–spring systems can provide sufficiently accurate estimates of these global response parameters for practical purposes, especially when the dominant modes are properly accounted for.

3.3 Impulsive response validation

A comprehensive free vibration test on a spherical tank was performed by Curadelli and Ambrosini (2011) to determine the fundamental frequency associated with the structural response of the system. This frequency is primarily affected by the interaction between the tank structure and the impulsive component of the stored liquid. As is well established, the impulsive mass

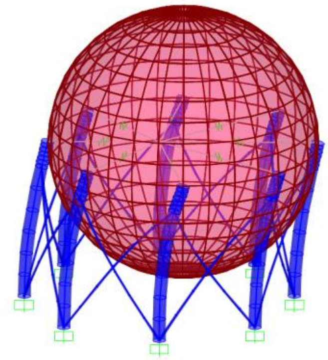


Fig. 7 Impulsive vibration mode of the numerical model

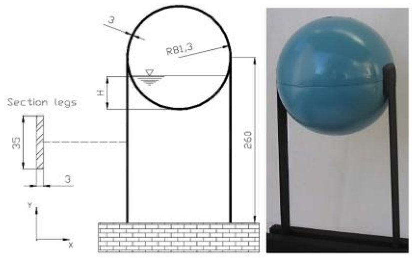


Fig. 8 Structural scale model (Curadelli *et al.* 2010)

behaves as an integral part of the structure during seismic excitation, moving in unison with it and thus vibrating at the same natural frequency. This experimental fundamental frequency was utilized to evaluate the impulsive component representation in the proposed numerical model.

The spherical tank under study (Fig. 6) consists of a spherical storage vessel with a capacity of $1000\ \mathrm{m}^3$, containing polypropylene liquid with a density of $500\ \mathrm{kg/m}^3$ under normal conditions. The spherical container has an external diameter of 12.4 m and is fabricated from steel plates with a uniform shell thickness of 25.4 mm. The material properties of the steel include a Young's modulus of 2.1 GPa, a Poisson's ratio of 0.3 and a mass density of $7850\ \mathrm{kg/m}^3$. The equator of the sphere is elevated 9.225 m above ground level. The vessel is supported by eight equally spaced circular columns, arranged in a circular layout, with an external diameter of 0.786 m and a wall thickness of 12 mm. These columns are braced by pairs of "X"-shaped steel members, each with a cross-section of $300\ \mathrm{mm} \times 25.4\ \mathrm{mm}$.

With a filling level of 33.1%, the tank was excited by the impact of an 8-ton weight dropped from a height of 5 m and located 20 m away. The structural response, recorded by high-sensitive accelerometers, yielded a vibration frequency of 8.68 Hz, as reported by Curadelli and Ambrosini (2011).

According to the fluid-structure interaction representation outlined in Section 3.2, we developed a numerical model of the actual spherical tank and conducted a modal analysis to determine its vibration frequencies. Fig. 7 displays the impulsive vibration mode obtained from this numerical model.

A numerical frequency of 8.57 Hz was obtained, resulting in a difference of only 1.3% for the experimental result reported, thereby validating the representation of the impulsive response by

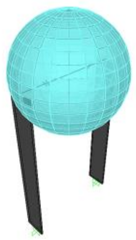


Fig. 9 Numerical model of scaled spherical tank

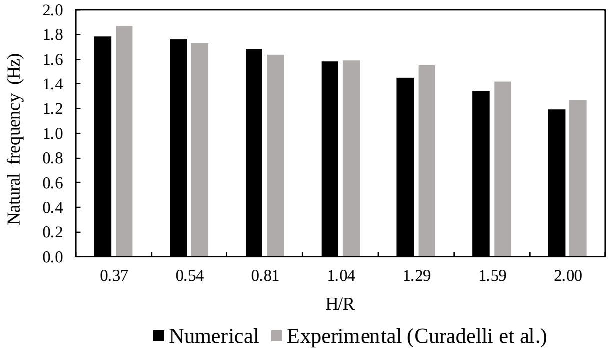


Fig. 10 Numerical and experimental natural frequencies

discrete lumped masses distributed along the horizontal diameter of the spherical shell, simulating the portion of the contained fluid that moves in phase with the structure.

3.4 Convective response validation

To validate the results of the convective response in the numerical model, comparisons were made with the experimental results of a scaled spherical tank model reported by Curadelli *et al.* (2010). The experimental setup used for validation consists of a 1:75 scale spherical tank (Fig. 8). This setup features a plastic sphere with a radius of 81.3 mm, a thickness of 3 mm, and a mass density of 980 kg/m³. The sphere is supported by two columns, each 260 mm long, fixed at the base, with a rectangular cross-section of 3.35 mm². The physical properties of the support material are Young’s modulus E=2.35 GPa, Poisson’s ratio ν=0.3, and mass density ρ_L=980 kg/m^3. The fluid used in the experimental tests was water, which is suitable for the present study.

The experimental results by Curadelli *et al.* focused on the natural vibration frequencies for different filling levels, obtained through free vibration testing. For each filling level, four acceleration time histories were recorded corresponding to the dynamic response in the equatorial plane of the sphere, using a high-sensitivity capacitive accelerometer (700 mV/g) from PCB Piezotronics. This type of accelerometer measures acceleration based on internal variations in capacitance caused by movement and is well-suited for capturing low-frequency vibrations, such as those generated during free vibration tests.

Based on the results of the sensitivity analysis performed, the first vibration mode of the convective component was deemed sufficient to capture the associated dynamic effects. The

Table 4 Comparison of natural frequencies

H/R	f_{exp} [Hz]	f_{num} [Hz]	% dif
0.37	1.782	1.869	4.88
0.54	1.758	1.729	-1.65
0.81	1.685	1.637	-2.85
1.04	1.585	1.593	0.50
1.29	1.450	1.547	6.69
1.59	1.343	1.415	5.36
2.00	1.190	1.270	6.72

corresponding numerical model is shown in Fig. 9.

Fig. 10 displays the natural frequencies obtained from numerical simulations (f_{num}) alongside those reported from experimental testing (f_{exp}) of a scaled spherical water tank. The results were calculated for seven different filling levels, represented by the *H/R* ratio, where *H* indicates the filling height and *R* represents the radius of the spherical tank.

As illustrated in Fig. 10, both experimental and numerical frequencies exhibit a decreasing trend as the H/R ratio increases, which is expected and physically consistent with the behavior of partially filled liquid containers. As the water level rises, the effective mass participating in both the impulsive and convective modes increases, while the structural stiffness remains constant. This results in a decrease in the system's natural frequency. The bar chart indicates that the numerical model closely matches the experimental trend across all values of *H/R*. The differences in natural frequency values are relatively small, and the overall variation of frequency with water level is consistently preserved. In intermediate fill conditions ($H/R \approx 1.0$), the numerical and experimental frequencies are nearly indistinguishable. For lower or higher levels, some deviations are observed, but the magnitude remains limited.

The observations indicate that the simplified numerical model can effectively replicate the system's frequency response across a wide range of fill levels, despite the assumptions made in the modeling.

4. Discussion

In this section, a comprehensive analysis of the numerical results was conducted, highlighting the comparison with experimental data and the calibration of a simplified formula for estimating frequencies in practice.

4.1 Comparison of natural frequencies

The minimal difference observed in the impulsive frequency (1.3%) confirms the suitability of modeling the impulsive component as fully coupled with the tank motion, consistent with the classical Housner model commonly applied to cylindrical and rectangular tanks, where the impulsive mass moves rigidly with the tank shell.

After the qualitative comparison presented in Fig. 10, a quantitative assessment is provided in Table 4, which reports the relative differences between the numerical and experimental natural

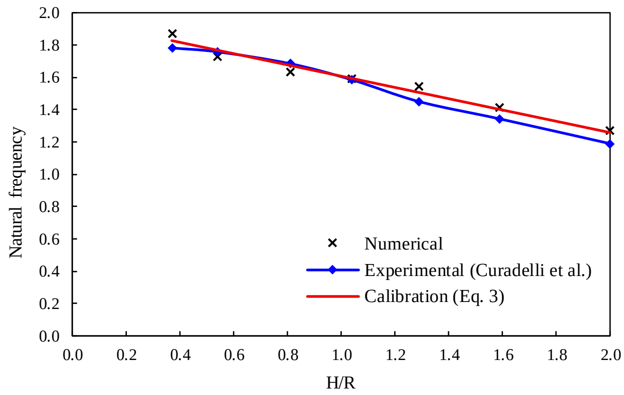


Fig. 11 Calibration of numerical model

frequencies for varying values of *H/R*. The percentage difference was calculated using the following expression $\%\ dif = 100 \cdot (f_{num} - f_{exp})/f_{exp}$ for each pair of results. The maximum difference was found to be 6.7%, and the average difference reached a value of 4.1%. These results are considered acceptable for simplified models that approximate FSI using mechanical-equivalent systems.

In most conditions, the numerical model underestimates the experimental frequencies, particularly at higher water levels ($H/R > 1.0$). This is likely due to idealizations such as neglecting shell deformations or omitting high-order hydrodynamics effects. The best agreement occurs at $H/R = 1.04$, with minimal difference. This suggests that the mass-spring representation is particularly effective for intermediate fill conditions, where the dynamic balance between structural stiffness and fluid inertia is best captured.

The relatively consistent and bounded deviation across all cases demonstrates that the proposed modeling approach is robust and reliable for estimating natural frequencies, even under varying fluid levels. In general, the agreement between experimental and numerical results is reasonable, demonstrating that the methodology used to assess the effect of fluid-structure interaction on the dynamic response was appropriate. This further supports its applicability to the seismic evaluation of elevated spherical tanks, where frequency-dependent effects play a key role.

4.2 Model calibration

Section 4.1 demonstrated that the fill level parameter, defined as the ratio of the liquid height to the radius of the spherical tank (*H/R*), significantly affects the system's dynamic response, especially the natural frequencies of the convective component. As this parameter increases, the distribution of hydrodynamic mass changes, which in turn alters the relative contributions of the convective and impulsive components.

To estimate the natural vibration frequency without complex numerical analysis, an empirical expression is proposed based on the *H/R* parameter. Eq. (3) shows the derivation of results obtained from the numerical model developed in this study, which has been previously validated using experimental data, with a maximum error of 6.7% and an acceptable margin for practical engineering applications. The expression is fitted using linear regression and reasonably estimates the variation of the natural vibration frequency (f) with the fill level parameter (H/R).

$$f = -0.3491 \cdot (H/R) + 1.9567 \quad (3)$$

Fig. 11 displays the numerical results obtained in this study, along with the experimental data from Curadelli *et al.* (2010) and the regression line associated with the proposed expression. The analysis achieved a coefficient of determination of R^2=0.974, indicating a strong correlation between the predicted and observed values.

The calibration performed exhibited acceptable behavior in terms of accuracy, with a maximum error of 5.8% relative to the available experimental data. These results suggest that the simplified expression successfully reflects the trend of natural vibration frequency with the fill level parameter. However, it is important to note that the calibration was performed using a limited number of cases (only seven fill levels). Therefore, the validity of this expression is confined to conditions like those studied. The accuracy of the simplified expression may decrease if applied to tank geometries, support conditions, or fluid properties that vary significantly from those analyzed in this study. Furthermore, the proposed model assumes linear fluid behavior and is only intended for estimating the natural frequency of the first convective mode. It does not capture potential nonlinear effects, such as those associated with large-amplitude sloshing, tank–structure interaction beyond elastic response, or fluid viscosity and damping mechanisms. For critical applications or more extreme conditions, more detailed models should be used, possibly including CFD or advanced multiphysics approaches.

Future work may extend this calibration to other geometric configurations, such as tanks with different slenderness ratios or multi-leg supports. It will also consider nonlinear dynamic regimes and additional experimental datasets at both the model and full scale.

Although it has limitations, the expression functions as a reliable and useful tool for initial analysis and backs the practical application of simplified modeling strategies in engineering design processes.

5. Conclusions

In the present study, the fluid–structure interaction was represented through a mechanical-equivalent mass-spring model, and its validity was assessed by comparing numerical simulations with experimental results available in the literature. Based on the results obtained, the following conclusions were drawn:

- A major limitation identified in this study is the scarcity of experimental research focused on the dynamic response of spherical tanks and associated fluid-structure interaction mechanisms, since most existing investigations focus on cylindrical or rectangular geometries, thereby restricting the calibration and validation of numerical models for spherical tanks.
- The parametric variation of the filling level (H/R) confirmed its direct influence on the dynamic characteristics of the system, emphasizing the importance of accounting for fluid fill conditions when evaluating the behavior of spherical storage tanks.
- The sensitivity analysis performed demonstrated that considering only the first vibrational mode of the convective component proved to be sufficient to achieve a significant dynamic response of the system, allowing for an effective yet simplified representation of fluid-structure interaction.
- For the impulsive component, the numerical analysis yielded a difference of only 1.3% compared to experimental data reported in the literature, confirming the accurate representation

of this component through a distributed lumped mass model.

• The comparison between the natural frequencies obtained through the numerical model and the experimental values reported in the literature demonstrated the consistency of the lumped mass-spring approach, with a maximum difference of 6.7%, supporting its applicability in simulating dynamic effects in spherical tanks.

• Based on the results and through a linear regression fit, an empirical expression was formulated to estimate the natural convective vibration frequencies as a function of the liquid fill height. This expression showed a maximum error of 5.8% relative to the experimental results reported with a coefficient of determination $R^2 = 0.974$.

• The primary results can provide a basis for future studies that focus on more complex dynamics, seismic performance assessments, or the development of design guidelines specifically for elevated spherical tanks.

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