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A novel integral parabolic plate theory incorporating stretching effects for the bending analysis of advanced FG plates resting on Winkler–Pasternak foundations

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Abstract. This paper presents a novel parabolic shear deformation plate theory incorporating the stretching effect for bending analysis of simply supported advanced functionally graded plates resting on a Winkler-Pasternak elastic foundation. The theory considers a parabolic distribution of transverse shear strains and satisfies zero traction boundary conditions on the plate surfaces without requiring shear correction factors. This theory involves only five unknowns, fewer than those in other shear and normal deformation theories. The originality of the present work lies in the introduction of a new displacement field based on undetermined integral variables that simultaneously incorporates both shear and stretching effects, providing a more accurate yet simple formulation compared to existing theories. Material properties are assumed to vary through the thickness direction following a simple power law distribution based on the volume fractions of the constituents. The accuracy of the proposed theory is validated by comparing its results with those available in the literature. The effects of the volume fraction index of the functionally graded material, the side-to-thickness ratio, and the Winkler-Pasternak elastic foundation on the bending responses of functionally graded plates are investigated. The study concludes that the proposed theory is both accurate and straightforward in predicting the bending responses of functionally graded plates, considering the stretching effect on an elastic foundation.

Keywords: analytical modeling; bending; functionally graded (FG) plates; new plate theory; shear and normal deformation; Winkler-Pasternak elastic foundation

1. Introduction

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Functionally graded materials (FGMs) are an advanced class of composites gaining considerable attention across various engineering fields, particularly in high-temperature applications such as thermo-mechanical load-bearing structures, aerospace, spacecraft, and plasma coatings for fusion reactors (Sui *et al.* 2023, Li *et al.* 2008, Taleb *et al.* 2018, Hissaria *et al.* 2023, Tu *et al.* 2019, Kannaiyan *et al.* 2025). FGMs offer distinct advantages over conventional composite materials by mitigating stress distribution and eliminating interface issues (Bennoun *et al.* 2016). These materials achieve their superior properties through a gradual variation in the volume fraction of their constituent materials, resulting in a smooth and continuous change in material properties from one surface to another. Typically composed of ceramics and metals, FGMs leverage the low thermal conductivity of ceramics for high-temperature resistance and the toughness of metals to prevent fracture, thus maintaining structural integrity under intense thermal gradients (Huang and Shen 2004). The unique combination of high specific stiffness and strength inherent in advanced composite materials makes FGMs particularly valuable in aerospace, marine, civil, and mechanical industries. These engineering structures are subjected to a variety of defined and undefined external and internal excitations under operational conditions, resulting in significant vibration and dynamic responses. Therefore, it is imperative to have a comprehensive understanding of the free vibration response of FGM plates during the design process. Consequently, examining their responses under various types of loading using accurate structural models is crucial. As a result, many researchers have conducted numerous studies in recent years to analyze the bending, vibration, thermomechanical, and buckling behaviors of functionally graded structural members (Rezaiee-Pajand *et al.* 2018, Chauhan *et al.* 2022, Lee *et al.* 2015, Uymaz and Uymaz 2024, Raza *et al.* 2024, Singha and Bandyopadhyay 2024, Mouffoki *et al.* 2017, Merzouki and Houari 2024, Smain *et al.* 2022, Rezaiee-Pajand *et al.* 2018, Al-Toki *et al.* 2020, Al-Maliki *et al.* 2021, Heydari *et al.* 2021, Bennai *et al.* 2022, Debbaghi *et al.* 2024, Nebab *et al.* 2024, Ellali *et al.* 2024, Zohra *et al.* 2024, Mokhefi *et al.* 2024, Lahdiri *et al.* 2025, Kannaiyan *et al.* 2025, Muthanna *et al.* 2025, Menasria *et al.* 2025, Benadouda *et al.* 2025, Ghalem *et al.* 2025), and an extensive range of plate theories have been developed to provide more accurate mechanical responses. Plates embedded on elastic foundations are prevalent in various mechanical and civil engineering applications, including foundations for tanks, railroad tracks, nuclear reactors, impact-type machines, and turbo generators (Kerr 1964). These structures are also commonly encountered in engineering scenarios such as the bottom plates of hydraulic structures and surface plates of airports (Ke-rang 1990). Various foundation models have been proposed to characterize the interaction between the plate and the elastic foundation. The simplest model is the Winkler model (Winkler 1867), a one-parameter mathematical representation where normal pressure estimates the mechanical response of the plate embedded on the elastic foundation. The Winkler foundation is equivalent to continuously assigned linear springs without coupling effects between them. Pasternak later improved this model by introducing two dependent parameters: normal pressure and a shear layer to account for shear interaction in the spring system (Pasternak 1954). The Winkler and Pasternak models have been widely used in applications such as railroad tracks, airports, motorways, concrete slabs, rigid concrete pavement for highways, and raft foundations. The literature reveals extensive studies on plates supported by elastic foundations, highlighting their significant attention from researchers. For instance, Al-Hosani *et al.* (1999) proposed fundamental solutions and boundary integral equations for thick Reissner plates on a Winkler elastic foundation, considering the effect of transverse normal stresses due to foundation reaction on the plate surface. Similarly, Darilmaz (2009) introduced a four-node hybrid stress element for analyzing arbitrarily shaped plates on a two-parameter elastic foundation.

Malekzadeh *et al.* (2004) used the differential quadrature method (DQM) based on first-order shear deformation plate theory (FSDT) to investigate the free vibration response on an elastic basis. Huang *et al.* (2008) reported a benchmark three-dimensional precise solution for thick functionally graded plates sitting on an elastic foundation and discovered that the elastic support has a substantial influence on the mechanical behavior of the thick FGM plate. Building on the theory of elasticity, Malekzadeh (2009) utilized the differential quadrature method (DQM) along with three-dimensional elasticity theory to analyze the free vibration of functionally graded material (FGM) plates embedded in an elastic medium. Ait Atmane *et al.* (2010) employed the Navier method to calculate the eigenvalues using the higher-order deformation theory for a simply supported functionally graded material (FGM) plate embedded on Winkler and Pasternak elastic foundations. Fallah *et al.* (2013) examined the free vibration frequency of fairly thick functionally graded material (FGM) plates sitting on a Winkler elastic foundation under various boundary conditions by utilizing Mindlin plate theory in combination with the Kantorovich technique. Jung *et al.* (2014) investigated the bending and vibration of a FGM micro-plate on a Pasternak elastic basis by combining the first-order shear deformation theory and the modified couple stress theory. The variations in the bending deflection and natural frequency of the FGM micro-plate with the foundation parameters, material gradient parameters, and scale effect parameters under uniform load are investigated. Gupta *et al.* (2016) investigated the natural frequency of FG plates resting on elastic foundations using the finite element method, with the objective to determine the effect of volume fraction index and foundation parameters on the vibration behavior of FG materials plates resting on a two-parameter Pasternak foundation. Parida and Mohanty (2018) have utilized various analytical methods to calculate the natural frequency of a plate made of functionally graded materials (FGM) embedded in different elastic mediums. They have employed different plate theories and considered various boundary conditions in their analysis. Singh and Harsha (2020) investigated the nonlinear vibration properties of a functionally graded material (FGM) sandwich plate placed on a Pasternak elastic basis, including both free and forced vibrations. Parida *et al.* (2023) analyzed the performance characteristics of laminated composite plates using the delamination model and fifth-order shear deformation theory. Their main emphasis was on investigating the correlation between the strength of the laminated composite plate and the filler material used. In recent research, Kehli *et al.* (2024) introduced a novel four-unknown quasi-3D shear deformation theory for analyzing the vibration responses of functionally graded (FG) beams with open-edge cracks, supported by three-parameter viscoelastic foundations (VEFs). This new theory simplifies the number of unknowns and governing equations, enhancing its usability compared to conventional theories. Zenkour and Alghanmi (2022) also studied the bending problem of sandwich plates with the refined quasi-3D shear and normal deformation theory by considering the shear and normal strains of sandwich plates. A novel modified quasi-3D theory was employed by Rachid *et al.* (2022) to examine the bending and free vibration of simply supported cylindrical and spherical FG shells resting on elastic foundations.

In their study, Gao *et al.* (2024) extend the simple refined plate theory (S-RPT) to analyze the bending and buckling behaviors of functionally graded graphene platelet-reinforced composite (FG-GPLRC) plates on local elastic foundations. This novel analytical method simplifies the determination of elastic foundation distribution and improves the accuracy of displacement patterns under various boundary conditions. Their findings detail the effects of material properties, boundary conditions, and foundation parameters. Esen (2019) examined the response of functionally graded Timoshenko beams resting on two-parameter elastic foundations under moving masses with variable velocities. Subsequently, Abdelrahman *et al.* (2021) conducted an analytical study on the dynamic performance of CNTRC nanobeams, within the framework of nonlocal strain gradient theory, supported by two-parameter foundations and subjected to moving loads. Building on this line of research, Abdelrahman *et al.* (2023) further investigated the dynamic behavior of CNT-reinforced composite beams resting on two-parameter elastic foundations, also under moving loads, by applying the nonlocal strain gradient theory. More recently, Yıldız *et al.* (2024) proposed a sinusoidal higher-order shear deformation model for magneto-electro-elastic nano-sandwich plates with an auxetic core, showing that modifications in the core geometry, smart layer configuration, and external fields enable the tuning of vibration, thermal resistance, and wave propagation. In the same context, Esen *et al.* (2025) analyzed the dynamic response of symmetric and sigmoid FGM Timoshenko beams supported by flexible foundations, while Yıldız and Esen (2025) focused on flexural wave propagation in biocompatible sandwich nanoplates under thermal and magnetic environments, employing nonlocal strain gradient elasticity in combination with higher-order shear deformation theory. Based on the finite element method, Esen *et al.* (2021) analyzed the dynamic response of a FG Timoshenko beam on an elastic foundation subjected to an accelerating/decelerating mass. The material properties are assumed to have a lateral distribution as an exponential or power function.

Mellal *et al.* (2024) examined the wave propagation in elastically supported PFGM plates via a quasi-3D shear deformation theory. Djebbour *et al.* (2025a) used an enhanced quasi-3D HSDT for free vibration analysis of porous FG-CNT beams on a new concept of orthotropic VE-foundations. Djebbour *et al.* (2025b) studied the vibration and buckling behaviour of a functionally graded carbon nanotube (CNT) reinforced composite beam using quasi-three-dimensional HSDT. Latroch *et al.* (2023) used finite element analysis to analyse how inclined transverse cracking influences the natural frequency of elastically supported FGM beams. Dahmane *et al.* (2024) proposed classical finite element method investigate the dynamic response of Euler-Bernoulli imperfect FG beamscracked on Winkler-elastic foundation. Frahlia *et al.* (2023) studied the analytical vibration response of an FG plate resting on a viscoelastic foundation using HSDT with only four unknowns.

Reviewing the aforementioned research, it is evident that an extensive range of plate theories has been developed to describe the mechanical and dynamic responses of plates. These plate theories can be categorized into three groups: classical plate theory (CPT), first-order shear deformation plate theory (FSDT), and higher-order shear deformation plate theory (HSDT). The classical plate theory (CPT), which disregards the transverse shear deformation effect, provides accurate conclusions only for the examination of thin plates without considering the transverse shear deformation effect. However, the first-order shear deformation plate theory (FSDT) overcomes this issue by accounting for this effect. It is suitable for plates that are both thin and moderately thick. Nevertheless, a suitable shear correction factor is necessary to adjust the distribution of transverse shear stress. To eliminate the need for shear correction factors, researchers have developed higher-order shear deformation theories (HSDTs). These theories can be classified into two types: those based on a three-dimensional approach and those based on a two-dimensional approach with a nonlinear variation of high-order axial displacement. The latter approach results in a parabolic variation of transverse shear strains across the thickness of the plate (Kant 1993).

The novelty of this paper lies in the application of recently developed polynomial and non-polynomial higher-order shear deformation theories (Taleb *et al.* 2018, Daikh *et al.* 2023) to incorporate the stretching effect into the static bending analysis of functionally graded (FG) plates, through a novel displacement field with undetermined integral variables. These four-variable plate

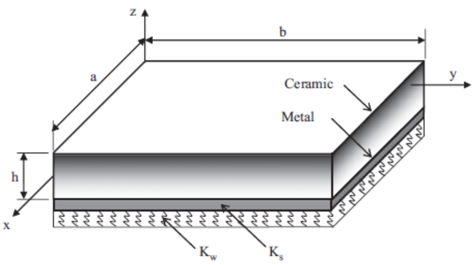

Fig. 1 Schematic representation of a rectangular FG plate resting on elastic foundation

theories neglect the thickness stretching effect and assume a constant transverse displacement across the thickness, akin to Kirchhoff–Love thin plate theories. This assumption is inadequate for FG plates, which exhibit significant variations in material properties through the thickness. A notable feature of this theory is its accommodation of a parabolic variation in transverse shear strains across the thickness, satisfying zero traction boundary conditions on the plate's top and bottom surfaces without requiring shear correction factors. The proposed theory examines the static behavior of FG plates resting on two-parameter elastic foundations, boasting only five unknowns, fewer than other Quasi-3D theories. The effects of various parameters, including gradient index, plate geometry, mode number, and elastic foundations, on the mechanical responses of FG plates are presented. The results demonstrate that these theories are both accurate and straightforward for predicting the bending responses of FG plates on two-parameter elastic foundations.

2. Theoretical formulation

2.1 Material properties

Consider a simply supported rectangular functionally graded plate of length a, width b and uniform thickness h in the unstressed reference configuration. The coordinate system for FG plates is shown in Fig. 1.

The FG plate is made of elastic and isotropic functionally graded material with its material properties vary smoothly through the thickness direction only. The effective material properties of the FG plate such as Young's modulus $E(z)$, thermal conductivity $k(z)$, thermal expansion $\alpha(z)$ and mass density $\rho(z)$ based on the rule of mixture, and are expressed as (Daikh *et al.* 2023)

$$P_{eff}(z) = P_m + \left(P_c - P_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{1}$$

where (E_m, ρ_m) and (E_c, ρ_c) are the corresponding properties of the metal and ceramic, respectively, and p is the volume fraction exponent which takes values greater than or equal to zero. The value of p equals to zero represents a fully ceramic plate. Note that the volume fraction of the ceramic is high near the top surface of the plate, and that of metal is high near the bottom surface. In addition, Eq. (1) indicates that the bottom surface of the plate ($z = -h/2$) is fully metal whereas the top surface of the plate ($z = +h/2$) is fully ceramic.

For simplicity, Poisson's ratio (ν) of plate is assumed to be constant in this study for that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus (Delale and Erdogan 1983).

2.2 Constitutive equations

For elastic and isotropic FGMs, the constitutive relations are given as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 \\ 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix} \text{ and } \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{2}$$

where (σ_x, σ_y, σ_z,τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y,ε_z, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively. Using the material properties defined in Eq. (3), stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11} = Q_{22} = Q_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{3a}$$

$$Q_{12} = Q_{13} = Q_{23} = \frac{\nu\,(1-\nu)\,E(z)}{(1-2\nu)(1+\nu)}, \tag{3b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}, \tag{3c}$$

Based on the thick plate theory and including the effect of transverse normal stress (thickness stretching effect), the basic assumptions for the displacement field of the plate can be described as

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{4a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \tag{4b}$$

$$w(x,y,z) = w_0(x,y) + g(z)\varphi_z(x,y) \tag{4c}$$

The coefficients k_1 and k_2 depends on the geometry and the proposed theory of present study has a parabolic function in the form:

$$f(z) = \frac{3\pi}{2}\left(\frac{\cosh(1/2)^2 - 1}{\cosh(1/2)^2}\right) - \frac{\pi}{2}\left(\frac{z^3}{2h^2}\right) \tag{5}$$

It can be observed that the kinematic in Eq. (4) uses only five unknowns (u_0, v_0, w_0, θ and ϕ_z). Nonzero strains of the five variable plate models are expressed as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{6a}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{6b}$$

$$\varepsilon_z = g'(z)\, \varepsilon_z^0 \tag{6c}$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y} \int \theta\, dx + k_2 \dfrac{\partial}{\partial x} \int \theta\, dy \end{Bmatrix} \tag{7a}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta\, dy + \dfrac{\partial \varphi_z}{\partial y} \\ k_1 \int \theta\, dx + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix}, \quad \varepsilon_{\mathbf{z}}^0 = \varphi_{\mathbf{z}} \tag{7b}$$

$$g'(z) = \frac{dg(z)}{dz} \tag{7c}$$

It can be observed from Eq. (6) that the transverse shear strains $(\gamma_{xz}, \gamma_{yz})$ are equal to zero at the upper $(z = h/2)$ and lower $(z = -h/2)$ surfaces of the plate. A shear correction coefficient is, hence, not required.

The integrals used in the above equations shall be resolved by a Navier type procedure and can be expressed as follows

$$\frac{\partial}{\partial y} \int \theta\, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x} \int \theta\, dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \int \theta\, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta\, dy = B' \frac{\partial \theta}{\partial y} \tag{8}$$

where the coefficients A' and B' are considered according to the type of solution employed, in this case via Navier method. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{9}$$

where α and β are defined in Eq. (21).

3. Governing equations

Considering the displacement components of the present simple quasi-3D theory in Eq. (1) and using the Hamilton's principle, the equations of motion of FG plates resting on elastic foundation can be obtained as

$$\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\
&\delta w_0: \ \frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x \partial y}+\frac{\partial^2 M_y^b}{\partial y^2}-+K_w w-K_s \nabla^2 w+q=0\\
&\delta \theta: -k_1 M_x^s-k_2 M_y^s-(k_1 A'+k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y}+k_1 A'\frac{\partial S_{xz}^s}{\partial x}+k_2 B'\frac{\partial S_{yz}^s}{\partial y}= 0\\
&\delta \varphi_z: \frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y}-N_z+g(z)q=0
\end{aligned} \tag{10}$$

In the above equations dot above each parameter denotes partial differentiating with respect to time. The parameters K_w and K_s are the Winkler and Pasternak parameters for elastic foundation. Also the stress resultants (N, M^b , M^s , S^s and N_z) and the mass inertias ($I_0, J_0, I_1, I_2, J_2, K_2, K_3$) are as follows

$$\left(N_i, M_i^b, M_i^s\right)=\int_{-h/2}^{h/2}(1, z, f)\sigma_i dz \quad (i=x,y,xy); \quad N_z=\int_{-h/2}^{h/2} g'(z)\sigma_z dz \tag{11a}$$

$$\left(S_{xz}^s, S_{yz}^s\right)=\int_{-h/2}^{h/2} g(z)\left(\tau_{xz}, \tau_{yz}\right) \tag{11b}$$

and

$$\left(I_0, J_0, I_1, I_2\right)=\int_{-h/2}^{h/2}\left(1, g(z), z, z^2\right)\rho(z)dz \tag{12a}$$

$$\left(J_1, J_2, K_2, K_3\right)=\int_{-h/2}^{h/2}\left(f(z), z\ f(z), f^2(z), g^2(z)\right)\ \rho(z)dz \tag{12b}$$

Substituting Eqs. (11) and (12) into (10) and using stress–strain relations, the governing equations of motion are obtained as

$$\begin{aligned}
&A_{11}d_{11}u_0+A_{66}\,d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0+X_{13}d_1\varphi_z-B_{11}d_{111}w_0-\left(B_{12}+2B_{66}\right)d_{122}w_0\\
&\quad+\left(B_{66}^s\left(k_1 A'+k_2\,B'\right)\right)d_{122}\theta+\left(B_{11}^s k_1+B_{12}^s\,k_2\right)d_1\theta=I_0\ddot{u}_0-I_1\,d_1\ddot{w}_0+J_1\,A' k_1 d_1\ddot{\theta},
\end{aligned} \tag{13a}$$

$$A_{22}\, d_{22}v_0 + A_{66}\, d_{11}v_0 + (A_{12} + A_{66})\, d_{12}u_0 + X_{23}d_2\varphi_z - B_{22}\, d_{222}w_0 - (B_{12} + 2B_{66})\, d_{112}w_0 + \left(B_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{112}\theta + \left(B_{22}^s k_2 + B_{12}^s k_1\right) d_2\theta = I_0\ddot{v}_0 - I_1\, d_2\ddot{w}_0 + J_1\, B' k_2\, d_2\ddot{\theta}, \tag{13b}$$

$$\begin{aligned} &B_{11}\, d_{111}u_0 + (B_{12} + 2B_{66})d_{122}u_0 + (B_{12} + 2B_{66})d_{112}v_0 + B_{22}\, d_{222}v_0 + Y_{13}\, d_{11}\varphi_z + Y_{23}\, d_{22}\varphi_z \\ &\quad - D_{11}d_{1111}w_0 - 2(D_{12} + 2D_{66})\, d_{1122}w_0 - D_{22}\, d_{2222}w_0 + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11}\theta \\ &\quad + 2\left(D_{66}^s\left(k_1A' + k_2B'\right)\right) d_{1122}\theta + \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22}\theta \\ &+ K_w\, w - K_s\, \nabla^2 w = I_0\ddot{w}_0 + I_1\left(d_1\ddot{u}_0 + d_2\, \ddot{v}_0\right) - I_2\left(d_{11}\ddot{w}_0 + d_{22}\ddot{w}_0\right) + J_2\left(k_1A'\, d_{11}\ddot{\theta} + k_2\, B'\, d_{22}\ddot{\theta}\right) + J_0\, \ddot{\varphi}_z \end{aligned} \tag{13c}$$

$$\begin{aligned} &-\left(B_{11}^s k_1 + B_{12}^s k_2\right)d_1u_0 - \left(B_{66}^s\left(k_1A' + k_2B'\right)\right)d_{122}u_0 - \left(B_{66}^s\left(k_1A' + k_2B'\right)\right)d_{112}v_0 - \left(B_{12}^s k_1 + B_{22}^s k_2\right)d_2v_0 \\ &- k_1Y_{13}^s\theta_z - k_2Y_{23}^s\theta_z + \left(D_{11}^s k_1 + D_{12}^s k_2\right)d_{11}w_0 + 2\left(D_{66}^s\left(k_1A' + k_2B'\right)\right)d_{1122}w_0 + \left(D_{12}^s k_1 + D_{22}^s k_2\right)d_{22}w_0 \\ &- H_{11}^s k_1^2\, \theta - H_{22}^s\, k_2^2\, \theta - 2H_{12}^s\, k_1 k_2\theta - \left(\left(k_1A' + k_2B'\right)^2 H_{66}^s\right)d_{1122}\theta + A_{44}^s\left(k_2B'\right)^2 d_{22}\theta + A_{55}^s\left(k_1\, A'\right)^2 d_{11}\theta \\ &+ A_{44}^s\left(k_2B'\right) d_{22}\varphi_z + A_{55}^s\left(k_1A'\right) d_{11}\varphi_z = -J_1\left(k_1\, A' d_1\ddot{u}_0 + k_2\, B'\, d_2\ddot{v}_0\right) + J_2\left(k_1\, A' d_{11}\ddot{w}_0 + k_2\, B'\, d_{22}\ddot{w}_0\right) \\ &- K_2\left(\left(k_1A'\right)^2 d_{11}\ddot{\theta} + \left(k_2\, B'\right)^2 d_{22}\ddot{\theta}\right) \end{aligned} \tag{13d}$$

$$X_{13}d_1u_0 + X_{23}\, d_2u_0 + Z_{33}\varphi_z + Y_{13}\, d_{11}w_0 + Y_{23}\, d_{22}w_0 + A_{44}^s\left(k_2B'\right)d_{22}\theta + A_{55}^s\left(k_1\, A'\right)d_{11}\theta + A_{44}^s\, d_{22}\varphi_z + A_{55}^s\, d_{11}\varphi_z = K_3\, \ddot{w}_0 + J_0\, \ddot{\varphi}_z, \tag{13e}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operator

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j},\quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},\quad d_i = \frac{\partial}{\partial x_i},\ (i, j, l, m = 1,2). \tag{14}$$

The stiffness coefficients used in Eq. (13) are defined as

$$\begin{Bmatrix} A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s \\ A_{12}, B_{12}, D_{12}, B_{12}^s, D_{12}^s, H_{12}^s \\ A_{66}, B_{66}, D_{66}, B_{66}^s, D_{66}^s, H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right)\begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{15a}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}\left[g^2(z)\right]dz \tag{15b}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) \tag{15c}$$

$$\left(X_{ij}, Y_{ij}, Y_{ij}^s, Z_{ij}\right) = \int_{-h/2}^{h/2}\left(1, z, f(z), g'(z)\right)g'(z)C_{ij}dz \tag{15d}$$

4. Analytical solution of simply supported FG plate

Based on Navier procedure, the following expansions of generalized displacements are chosen to automatically satisfy the simply supported boundary conditions, the following expressions of displacements (u_0, v_0, w_0, θ, and ϕ_z) are taken

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \\ \varphi_z \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}\sin(\alpha\, x)\sin(\beta\, y) \\ Z_{mn}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{16a}$$

where

$$\alpha = m\pi / a\,, \quad \beta = n\pi / b \tag{16b}$$

(U_{mn} , V_{mn} , W_{mn} , X_{mn} , Z_{mn}) are the unknown maximum displacement coefficients. The transverse load $q(x,y)$ is also expanded as

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{17}$$

The coefficients Q_{mn} are given below for some typical loads

$$Q_{mn} = \frac{4}{a\,b}\int_0^a\int_0^b q(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, dxdy \tag{18}$$

For uniformly distributed load

$$Q_{mn} = \begin{cases} \dfrac{16 q_0}{mn\pi^2} & m,n = 1,3,5,... \\ 0 & m,n = 2,4,6,... \end{cases} \tag{19}$$

For sinusoidal distributed load; $Q_{mn} = q_0$ in which q_0 is the intensity of the load.
Substituting Eq. (16) into Eq. (13), the analytical solutions can be determined by

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{12} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{13} & a_{23} & a_{33} & a_{34} & a_{35} \\ a_{14} & a_{24} & a_{34} & a_{44} & a_{45} \\ a_{15} & a_{25} & a_{35} & a_{45} & a_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Z_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix} \tag{20}$$

in which

$$a_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right), \quad a_{12} = -\alpha\beta\,\left(A_{12} + A_{66}\right), \quad a_{13} = \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right),$$

$$a_{14} = \alpha\left(k_1 B_{11}^s + k_2 B_{12}^s - \left(k_1 A^{'} + k_2 B^{'}\right)B_{66}^s\beta^2\right), \quad a_{15} = X_{13}\alpha$$

$$a_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right), \quad a_{23} = \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right),$$

$$a_{24} = \beta\,\left(k_2 B_{22}^s + k_1 B_{12}^s - \left(k_1 A^{'} + k_2 B^{'}\right)B_{66}^s\alpha^2\right)$$

$$a_{25} = X_{23}\beta\,, \quad a_{33} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right)$$

Table 1 Material properties used in the FG plate Benahmed *et al.* (2017)

Properties	Metal	Ceramic
	(Al)	(Al_2O_3)
E (GPa)	70	380
ν	0.3	0.3
ρ (kg/m^3)	2702	3800

$$\begin{gathered} a_{34} = -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1A' + k_2B')\,D_{66}^s\alpha^2\beta^2 - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right), \\ a_{35} = -\left(Y_{13}\,\alpha^2 + Y_{23}\,\beta^2\right) \\ a_{44} = -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1A' + k_2B')^2 H_{66}^s\alpha^2\beta^2 - k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - \left(k_1A'\right)^2 A_{55}^s\alpha^2 - \left(k_2B'\right)^2 A_{44}^s\beta^2 \\ a_{45} = -k_1A'A_{55}^s\alpha^2 - k_2B'A_{44}^s\beta^2 + k_1Y_{13}^s + k_2Y_{23}^s,\; a_{55} = -\left(A_{55}^s\,\alpha^2 + A_{44}^s\,\beta^2 + Z_{33}\right) \end{gathered} \tag{21}$$

5. Numerical results

In this paper, numerical results are presented to illustrate the effects of various parameters, including the volume fraction index of functionally graded materials, the side-to-thickness ratio, and the Winkler-Pasternak elastic foundation on the bending responses of FG plates. These effects are analyzed using a novel quasi-3D parabolic shear deformation plate theory that incorporates the stretching effect. A plate subjected to uniformly distributed loads (UDL) and sinusoidal (SL) loads is considered. Through self-developed Maple code, various examples are provided to verify the accuracy and efficiency of the present theory in predicting the bending responses of simply supported FG plates on a Winkler-Pasternak elastic foundation. The material properties of FG plates are listed in Table 1.

Numerical results are presented in terms of non-dimensional stresses and deflection. The various nondimensional parameters used are

$$\begin{gathered} \hat{w} = \frac{100E}{q_0hS^4}w\left(\frac{a}{2},\frac{b}{2},\bar{z}\right),\; \hat{\sigma}_x = \frac{1}{q_0S^2}\sigma_x\left(\frac{a}{2},\frac{b}{2},\bar{z}\right),\; \hat{\sigma}_y = \frac{1}{q_0S^2}\sigma_y\left(\frac{a}{2},\frac{b}{2},\bar{z}\right),\; \hat{\tau}_{xy} = \frac{1}{q_0S^2}\tau_{xy}\left(0,0,\bar{z}\right), \\ \bar{\tau}_{yz} = \frac{1}{q_0S}\tau_{yz}\left(\frac{a}{2},0,\bar{z}\right),\; \bar{\tau}_{xz} = \frac{1}{q_0S}\tau_{xz}\left(0,\frac{b}{2},\bar{z}\right).\; S = a/h \\ \overline{K_w} = \frac{K_wa^4}{D_m}\; \overline{K_s} = \frac{K_wa^2}{D_m}\; D_m = \frac{E_mh^3}{12\left(1-\nu^2\right)}. \\ \overline{u_x} = \frac{100D}{q_0a^4}u_x\left(0,\frac{b}{2},-\frac{h}{2}\right),\; \overline{u_z} = \frac{100D}{q_0a^4}u_z\left(\frac{a}{2},\frac{b}{2},0\right),\; \overline{\sigma_x} = -\frac{h^2}{q_0a^2}\sigma_x\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right), \\ \overline{\tau_{xy}} = \frac{h^2}{q_0a^2}\tau_{xy}\left(0,0,-\frac{h}{2}\right), \\ \overline{\tau_{xz}} = \frac{1}{q_0}\tau_{xz}\left(0,\frac{b}{2},0\right),\; \tau_{xy}^* = \frac{1}{10q_0}\tau_{xy}\left(0,0,-\frac{h}{3}\right),\; \tau_{xz}^* = \frac{1}{10q_0}\tau_{xz}\left(0,\frac{b}{2},0\right),\; k_w = \frac{K_wa^4}{h^3}, \\ k_s = \frac{K_sa^2}{h^3\nu} = \frac{K_sb^2}{h^3\nu},\; D = Eh^3/12\left(1-\nu^2\right). \end{gathered} \tag{22}$$

Table 2 Effect of normal strain ε_z on the dimensionless stresses and transversal displacement for isotropic square plate (*a*/*h*=10) subjected to a UDL

Theory	$\hat{w}(a/2,b/2,0)$	$\hat{\sigma}_x(h/2)$	$\hat{\sigma}_y(h/2)$	$\hat{\tau}_{xy}(h/2)$	$\bar{\tau}_{xz}(0,b/2,0)$	$\bar{\tau}_{yz}(a/2,0,0)$
Present $\varepsilon_z \neq 0$	4.630	0.301	0.301	0.198	0.482	0.500
Benahmed *et al.* (2017) $\varepsilon_z \neq 0$	4.633	0.302	0.302	0.197	0.481	0.502
Shimpi *et al.* (2003) $\varepsilon_z \neq 0$	4.625	0.307	0.307	0.195	0.505	0.505
Exact 3D (Srinivas *et al.* 1970a)	4.639	0.290	0.290	/	0.488	/

Table 3 Comparison of nondimensional deflection $D10^3\,w(0.5a,0.5b,z=0)/qa^4$ of simply supported isotropic thin square plate under uniformly distributed load (*a*/*h*=100)

k_w	k_s	$D10^3\,w(0.5a,0.5b,z=0)/qa^4$			
		Present $\varepsilon_z \neq 0$	Benyoucef *et al.* (2010)	3D Huang *et al al.* (2008)	3D Lam *et al.* (2000)
1	1	3.8492	3.8550	3.8546	3.853
	3^4	0.7621	0.7630	0.7630	0.763
	5^4	0.1154	0.1153	0.1153	0.115
3^4	1	3.2063	3.2108	3.2105	3.210
	3^4	0.7316	0.7317	0.7317	0.732
	5^4	0.1145	0.1145	0.1145	0.115
5^4	1	1.4759	1.4765	1.4765	1.476
	3^4	0.5711	0.5704	0.5704	0.570
	5^4	0.1095	0.1095	0.1095	0.109

As a preliminary example, the deflections and dimensionless stresses of a square isotropic plate (*a/h*=10) under a uniform distributed load (UDL) are presented in Table 2. These results are compared with the quasi-3D solutions provided by Shimpi *et al.* (2003), the exact solution by Srinivas *et al.* (1970) and Quasi-3D hyperbolic plate theory of developed by Benahmed *et al.* (2017). The table demonstrates that the results are closely aligned.

The second example to validate the present method for plates resting on an elastic foundation, the results for dimensionless deflections of a thick isotropic plate are compared with results published previously. Table 3 presents the center deflections of a uniformly loaded homogeneous square plate simply supported on a Winkler-Pasternak foundation. The results are compared with those of Benyoucef *et al.* (2010), 3D solution of Huang *et al.* (2008), Lam *et al.* (2000). It can be seen that the results agree closely.

In the third example, Table 4 compares the displacement and stress results obtained using the present method with those of Thai and Choi (2011), Zenkour and Sobhy (2012), where the thickness stretching effect ($\varepsilon_z = 0$) is neglected. The comparison is made for an FG Al_2O_3 rectangular plate subjected to a uniformly distributed load, considering the power index k and elastic foundation parameters. The results in Table 6 indicate that the deflections and stresses reported by Thai and Choi (2011), Zenkour and Sobhy (2012) are overestimated due to the omission of the thickness stretching effect in their theories.

Another example is provided in Table 5 for an FG plate subjected to sinusoidal loading. The results from the present method are compared with those from the refined trigonometric shear deformation theory developed by Bouderba *et al.* (2013), considering various values of the power

Table 4 Comparison of the displacements and stresses of simply supported Al/Al_2O_3 rectangular plate under uniformly distributed load (a=10h, b=3a)

p	k_w	k_s	Theory	$\overline{u_x}$	$\overline{u_z}$	$\overline{\sigma_x}$	$\overline{\sigma_{xy}}$	$\overline{\tau_{xz}}$
0.5	0	0	Thai and Choi (2011)	0.3491	1.9345	0.2337	0.0941	—
			Zenkour and Sobhy (2012)	0.34919	1.93441	0.23372	0.09415	7.68354
			Present $\varepsilon_z \neq 0$	0.33500	1.90214	0.23942	0.09008	7.56254
	100	0	Thai and Choi (2011)	0.3358	1.8590	0.2242	0.0916	—
			Zenkour and Sobhy (2012)	0.33586	1.85907	0.22424	0.09167	7.42978
			Present $\varepsilon_z \neq 0$	0.32247	1.82954	0.22990	0.08774	7.31679
	100	100	Thai and Choi (2011)	0.3012	1.6640	0.1999	0.0850	—
			Zenkour and Sobhy (2012)	0.30131	1.66399	0.19989	0.08503	6.76069
			Present $\varepsilon_z \neq 0$	0.28991	1.64138	0.20536	0.08151	6.66745
2	0	0	Thai and Choi (2011)	0.6564	3.2266	0.4395	0.1766	—
			Zenkour and Sobhy (2012)	0.65655	3.22672	0.43961	0.17666	6.91072
			Present $\varepsilon_z \neq 0$	0.60340	3.07560	0.44695	0.16202	6.79513
	100	0	Thai and Choi (2011)	0.6156	3.0218	0.4105	0.1690	—
			Zenkour and Sobhy (2012)	0.61576	3.02190	0.41060	0.16906	6.53895
			Present $\varepsilon_z \neq 0$	0.56771	2.88981	0.41881	0.15538	6.44548
	100	100	Thai and Choi (2011)	0.5186	2.5364	0.3423	0.1501	—
			Zenkour and Sobhy (2012)	0.51872	2.53642	0.34233	0.15020	5.63882
			Present $\varepsilon_z \neq 0$	0.48190	2.44461	0.35187	0.13876	5.59033
5	0	0	Thai and Choi (2011)	0.7802	3.8506	0.5223	0.2103	—
			Zenkour and Sobhy (2012)	0.78046	3.85174	0.52237	0.21044	6.14557
			Present $\varepsilon_z \neq 0$	0.72062	3.69376	0.53104	0.19389	6.03129
	100	0	Thai and Choi (2011)	0.7230	3.5620	0.4816	0.1996	—
			Zenkour and Sobhy (2012)	0.72323	3.56296	0.48167	0.19975	5.75485
			Present $\varepsilon_z \neq 0$	0.66999	3.42858	0.49132	0.18445	5.66241
	100	100	Thai and Choi (2011)	0.5922	2.9046	0.3897	0.1740	—
			Zenkour and Sobhy (2012)	0.59231	2.90518	0.38971	0.17410	4.84302
			Present $\varepsilon_z \neq 0$	0.55294	2.81786	0.40061	0.16159	4.79290

index p and elastic foundation parameters k_w and k_s. The table shows excellent agreement for all values of the power-law index p and the foundation parameters k_w and k_s. Additionally, it is observed that the deflection and stresses decrease in the presence of elastic foundations.

Figs. 2 and 3 present the deflection $\overline{w}$ of the plate centroid as a function of the side-to-thickness ratio a/h for various values of the foundation stiffness of a functionally graded (FG) square plate, with a constant foundation stiffness parameter $p = 2$. It is observed that as the side-to-thickness ratio a/h increases, the center deflection of the FG plate decreases. This implies that thicker plates (with smaller a/h ratios) experience greater deflection at the center compared to thinner plates. Additionally, from Figs. 2 and 3, it is evident that an increase in the foundation modulus parameter results in a further reduction in the center deflection of the FG plate. This indicates that a stiffer foundation provides greater resistance against deflection. However, for thin plates, the influence of foundation stiffness becomes less pronounced, suggesting that the stiffness of the foundation has a diminishing effect on deflection as the plate becomes thinner.

Table 5 Effect of the volume fraction exponent and elastic foundation parameters on the dimensionless and stresses of an FGM rectangular plate under sinusoidal load ($a = 10h, b = 2a, q_0 = 100$)

p	k_w	k_s	theory	$\overline{w}$	$\overline{\sigma_x}$	$\tau_{xy}{}^*$	$-\tau_{xz}{}^*$
0	0	0	Present $\varepsilon_z \neq 0$	0.67669	0.44410	0.85538	-0.38933
			Bouderba *et al*. (2013)	0.68131	0.42424	0.86240	-0.39400
	100	0	Present $\varepsilon_z \neq 0$	0.40481	0.26567	0.51170	-0.23290
			Bouderba *et al*. (2013)	0.40523	0.25233	0.51296	-0.23435
	0	100	Present $\varepsilon_z \neq 0$	0.084133	0.055215	0.10635	-0.048406
			Bouderba *et al*. (2013)	0.083654	0.052093	0.10589	-0.048377
	100	100	Present $\varepsilon_z \neq 0$	0.077649	0.050960	0.098154	-0.044675
			Bouderba *et al*. (2013)	0.077197	0.048071	0.097724	-0.044643
0.5	100	100	Present $\varepsilon_z \neq 0$	0.079180	0.048733	0.080257	-0.038219
			Bouderba *et al*. (2013)	0.078729	0.045788	0.081728	-0.038066
1	100	100	Present $\varepsilon_z \neq 0$	0.079761	0.047891	0.071203	-0.035252
			Bouderba *et al*. (2013)	0.079321	0.044892	0.073054	-0.035023
2	100	100	Present $\varepsilon_z \neq 0$	0.080200	0.047581	0.065302	-0.032442
			Bouderba *et al*. (2013)	0.079758	0.044595	0.067185	-0.032215
5	100	100	Present $\varepsilon_z \neq 0$	0.080628	0.048596	0.062746	-0.030046
			Bouderba *et al*. (2013)	0.080150	0.045736	0.064125	-0.029922
∞	100	100	Present $\varepsilon_z \neq 0$	0.081721	0.030345	0.058449	-0.026603
			Bouderba *et al*. (2013)	0.081190	0.050559	0.058148	-0.026565

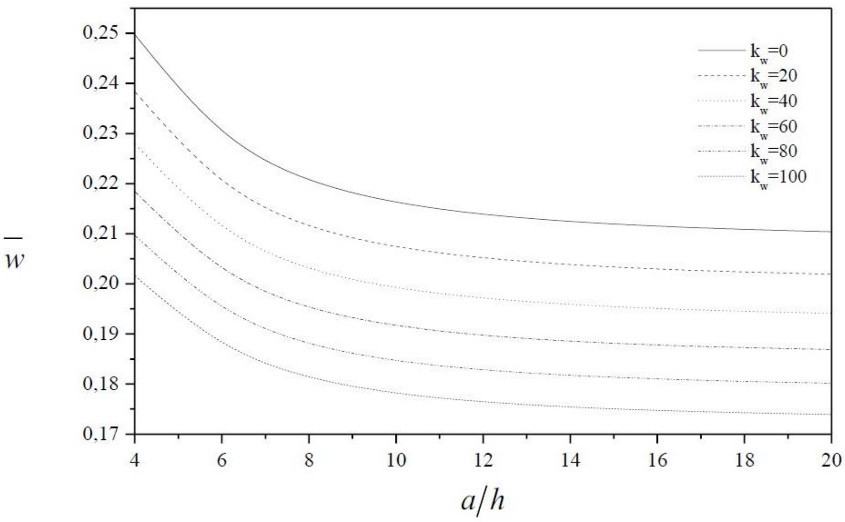

Fig. 2 Effect of Winkler modulus parameter on the dimensionless center deflection of a square FG plate ($p = 2, k_s = 10$) for different side-to-thickness ratio

Variations in the axial stress $\overline{\sigma_x}$ through the thickness of the functionally graded (FG) plate are graphically depicted in Figs. 4 and 5 for different values of the elastic foundation parameter. The graphs reveal that the maximum compressive stresses are located near the top surface, while the maximum tensile stresses are found near the bottom surface of the FG plate. This distribution

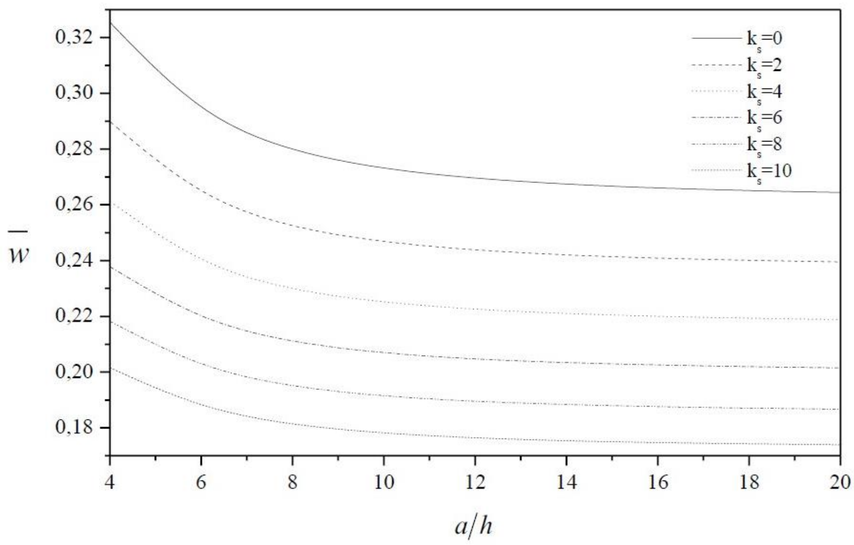

Fig. 3 Effect of Pasternak modulus parameter on the dimensionless center deflection of a square FG plate $(p = 2, k_w = 100)$ for different side-to-thickness ratio

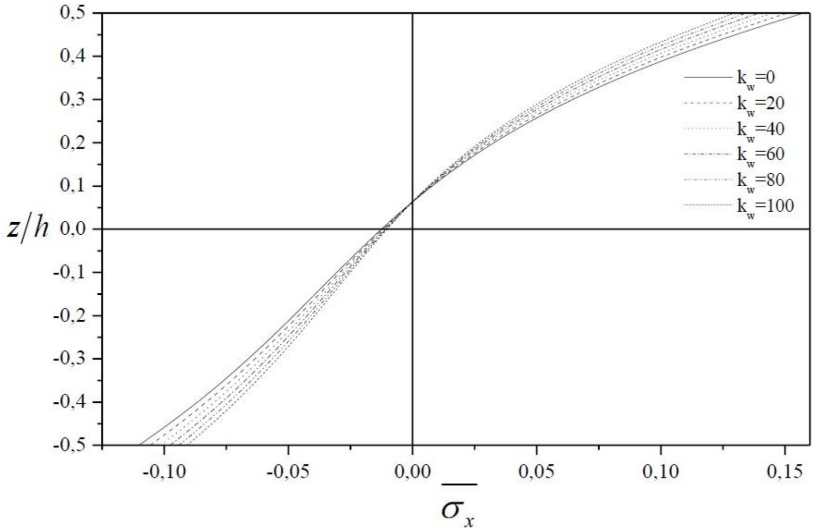

Fig. 4 Variation of dimensionless axial stress $\overline{\sigma_x}$ through-the-thickness of a square FG plate $(p = 2, k_s = 10, a/h = 10)$ for different values of Winkler modulus parameter

indicates that the FG plate experiences compressive forces at the top and tensile forces at the bottom. Additionally, it is observed that the normal stress $\overline{\sigma_x}$ increases progressively with higher values of foundation stiffness, indicating that a stiffer foundation results in greater normal stress throughout the plate thickness. However, the effect of the Pasternak shear modulus parameter is notably more significant than that of the Winkler modulus parameter. This suggests that the Pasternak foundation model, which accounts for shear interactions, has a greater impact on the stress distribution compared to the Winkler foundation model, which considers only the normal

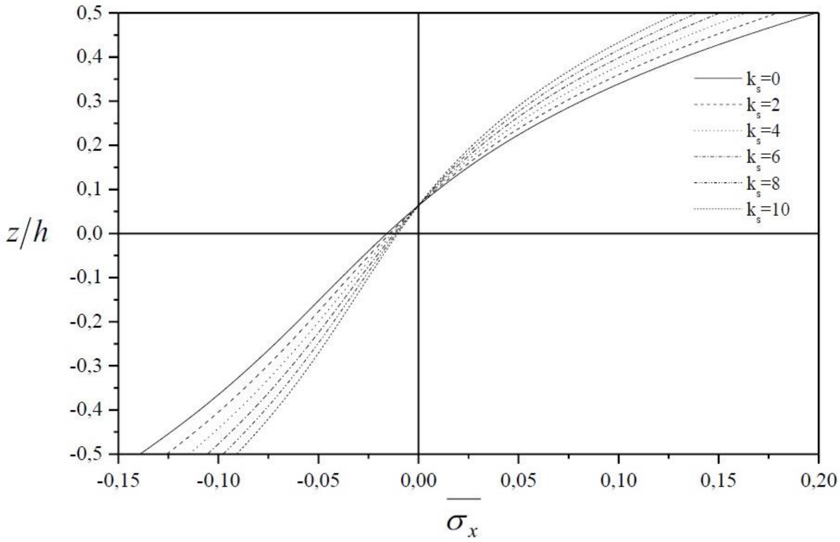

Fig. 5 Variation of dimensionless axial stress $\overline{\sigma_x}$ through-the-thickness of a square FG plate $(p = 2, k_w = 100, a/h = 10)$ for different values of Pasternak modulus parameter

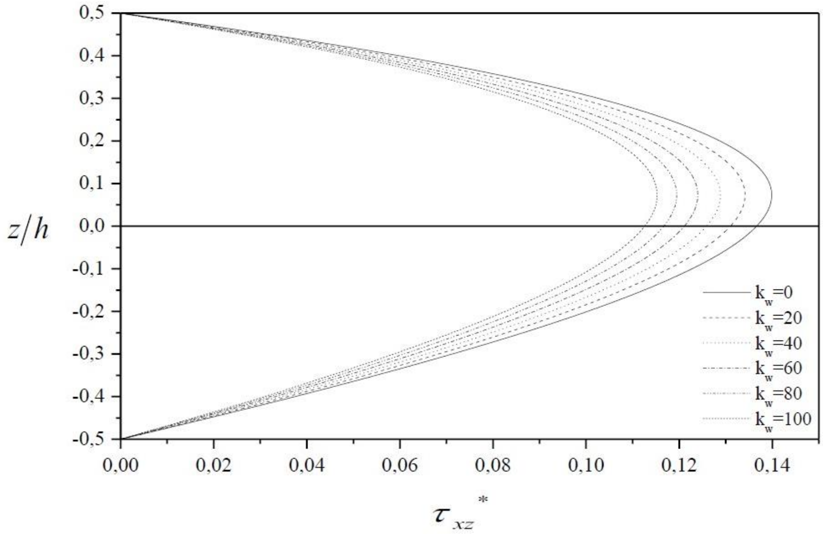

Fig. 6 Variation of dimensionless shear stress $\tau_{xz}{}^*$ through-the-thickness of a square FG plate $(p = 2, k_s = 10, a/h = 10)$ for different values of Winkler modulus parameter

pressure effects.

Figs. 6 and 7 depict across-the-thickness distributions of the shear stress $\tau_{xz}{}^*$ in a square FG plate under a sinusoidally distributed load. The distinction between the curves in Figs. 6 and 7 is obvious. As seen, the transverse shear stress $\tau_{xz}{}^*$ increases gradually with decreasing k_w or k_s, which indicates that increased moduli of the elastic foundation can enhance the bending rigidity of the plate.

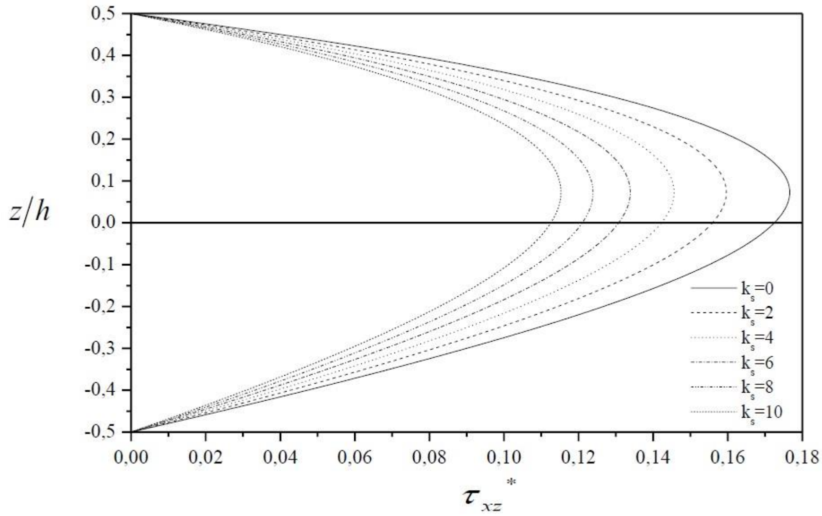

Fig. 7 Variation of dimensionless shear stress $\tau_{xz}{}^{*}$ through-the-thickness of a square FG plate $(p = 2, k_w = 100, a/h = 10)$ for different values of Pasternak modulus parameter

6. Conclusions

This study introduces a novel refined integral quasi-3D parabolic shear deformation theory for analyzing the bending responses of functionally graded (FG) plates resting on Winkler–Pasternak elastic foundations. The proposed theory involves only five unknown displacements and inherently satisfies the zero traction boundary conditions on the plate surfaces, eliminating the need for a shear correction factor and thereby reducing computational cost. The accuracy and efficiency of the formulation have been validated through comparisons with existing benchmark solutions, showing excellent agreement across all tested cases. The main findings demonstrate that incorporating the thickness stretching effect enhances the overall stiffness of FG plates and significantly reduces transverse deflection, while the Pasternak shear layer modulus exerts a more pronounced influence on the structural response compared to the Winkler modulus. These results highlight the contribution of the present theory as a reliable and computationally efficient tool for modeling the bending behavior of FG plates on elastic foundations. Nevertheless, the generalisation of the conclusions is subject to certain limitations, as the present study is restricted to static bending responses, isotropic material gradation laws, and idealised Winkler–Pasternak foundation models. Moreover, factors such as thermal effects, dynamic loading conditions, or more complex boundary constraints may influence the applicability and accuracy of the proposed model. Future investigations should address these aspects to extend the scope and robustness of the present contribution.

References

Abdelrahman, A.A., Esen, I., Daikh, A.A. and Eltaher, M.A. (2023), "Dynamic analysis of FG nanobeam reinforced by carbon nanotubes and resting on elastic foundation under moving load", *Mech. Bas. Des. Struct. Mach.*, **51**(10), 5383-5406. https://doi.org/10.1080/15397734.2021.1999263.

Ait Atmane, H., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2010), "Free vibration analysis of functionally graded plates resting on Winkler–Pasternak elastic foundations using a new shear deformation theory", *Int. J. Mech. Mater. Des.*, **6**, 113-121. https://doi.org/10.1007/s10999-010-9110-x.

Al-Hosani, K., Fadhil, S. and El-Zafrany, A. (1999), "Fundamental solution and boundary element analysis of thick plates on Winkler foundation", *Comput. Struct.*, **70**(3), 325-336. https://doi.org/10.1016/S0045-7949(98)00171-0.

Al-Maliki, A.F., Ahmed, R.A., Moustafa, N.M. and Faleh, N.M. (2020), "Finite element based modeling and thermal dynamic analysis of functionally graded graphene reinforced beams", *Adv. Comput. Des.*, **5**(2), 177-193. https://doi.org/10.12989/acd.2020.5.2.177.

Al-Toki, M.H., Fenjan, R.M., Ahmed, R.A., Faleh, N.M. and Abdullah, W.N. (2021), "Analyzing dynamic characteristics of nonlocal porous graded beams under impulse and thermal loading", *Adv. Comput. Des.*, **6**(4), 301-317. https://doi.org/10.12989/acd.2021.6.4.301.

Benadouda, M., El Amin Bourouis, M., Dahmane, M., Bennai, R., Ait Atmane, H. and Safer, O. (2025), "Dynamic response of wave propagation in functionally graded beams with defects: Effects of porosity and cracks", *Acta Mechanica*, 1-18. https://doi.org/10.1007/s00707-025-04270-4.

Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K. and Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng*, **12**(1), 9-34. https://doi.org/10.12989/gae.2017.12.1.009.

Bennai, R., Mellal, F., Nebab, M., Fourn, H., Benadouda, M., Atmane, H.A., ... & Hussain, M. (2022), "On wave dispersion properties of functionally graded plates resting on elastic foundations using quasi-3D and 2D HSDT", *Earthq. Struct.*, **22**(5), 447-460. https://doi.org/10.12989/eas.2022.22.5.447.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five-variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**, 423-431. https://doi.org/10.1080/15376494.2014.984088.

Benyoucef, S., Mechab, I., Tounsi, A., Fekrar, A., Ait Atmane, H. and Adda Bedia, E.A. (2010), "Bending of thick functionally graded plates resting on Winkler–Pasternak elastic foundations", *Mech. Compos. Mater.*, **46**, 425-434. https://doi.org/10.1007/s11029-010-9159-5.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104. https://doi.org/10.12989/scs.2013.14.1.085.

Chauhan, M., Dwivedi, S., Jha, R., Ranjan, V. and Sathujoda, P. (2022), "Sigmoid functionally graded plates embedded on Winkler-Pasternak foundation: Free vibration analysis by dynamic stiffness method", *Compos. Struct.*, **288**, 115400. https://doi.org/10.1016/j.compstruct.2022.115400.

Dahmane, M., Benadouda, M., Bennai, R., Saimi, A. and Atmane, H.A. (2024), "Effect of crack on the dynamic response of bidirectional porous functionally graded beams on an elastic foundation based on finite element method", *Acta Mechanica*, **235**(6), 3849-3860. https://doi.org/10.1007/s00707-024-03906-1.

Daikh, A.A., Belarbi, M.O., Ahmed, D., Houari, M.S.A., Avcar, M., Tounsi, A. and Eltaher, M.A. (2023), "Static analysis of functionally graded plate structures resting on variable elastic foundation under various boundary conditions", *Acta Mechanica*, **234**(2), 775-806. https://doi.org/10.1007/s00707-022-03405-1.

Darilmaz, K. (2009), "An assumed-stress hybrid element for modeling of plates with shear deformations on elastic foundation", *Struct. Eng. Mech.*, **33**(5), 573-588. https://doi.org/10.12989/sem.2009.33.5.573.

Debbaghi, S., Dahmane, M., Benadouda, M., Ait Atmane, H., Bendenia, N. and Hadji, L. (2024), "Wave propagation of bi-directional porous FG beams using Touratier's higher-order shear deformation beam theory", *Couple. Syst. Mech.*, **13**(1), 43-60. https://doi.org/10.12989/csm.2024.13.1.043.

Delale, F. and Erdogan F. (1983), "The crack problem for a non-homogeneous plane", *ASME J. Appl. Mech.*, **50**(3), 609-614. https://doi.org/10.1115/1.3167098.

Djebbour, K.D., Nebab, M., Avcar, M., Ait Atmane, H., Bernard, F., Bennai, R. and Ikinci, B. (2025b), "Stability and vibration analysis of shear deformable FG-CNTRC beams resting on elastic foundations", *Adv. Nano Res.*, **18**(3), 205-221. https://doi.org/10.12989/anr.2025.18.3.205.

Djilali Djebbour, K., Mokhtar, N., Hassen, A.A., Alghanmi, R.A., Hadji, L. and Riadh, B. (2025a), "An enhanced quasi-3D HSDT for free vibration analysis of porous FG-CNT beams on a new concept of orthotropic VE-foundations", *Mech. Adv. Mater. Struct.*, **32**(5), 893-909. https://doi.org/10.1080/15376494.2024.2356728.

Ellali, M., Alazwari, M.A., Bouazza, M., Eltaher, M.A. and Benseddiq, N. (2024), "Effects of changing materials properties for vibration of FGM beam using integral shear deformation model", *Couple. Syst. Mech.*, **13**(4), 277-291. https://doi.org/10.12989/csm.2024.13.4.277.

Esen, I. (2019), "Dynamic response of a functionally graded Timoshenko beam on two-parameter elastic foundations due to a variable velocity moving mass", *Int. J. Mech. Sci.*, **153**, 21-35. https://doi.org/10.1016/j.ijmecsci.2019.01.033.

Esen, I., Eltaher, M.A. and Abdelrahman, A.A. (2023), "Vibration response of symmetric and sigmoid functionally graded beam rested on elastic foundation under moving point mass", *Mech. Bas. Des. Struct. Mach.*, **51**(5), 2607-2631. https://doi.org/10.1080/15397734.2021.1904255.

Esen, İ., Koç, M.A. and Eroğlu, M. (2021), "Dynamic behaviour of functionally graded Timoshenko beams on a four parameter linear elastic foundation due to a high speed travelling mass with variable velocities", *J. Smart Syst. Res.*, **2**(1), 48-75.

Fallah, A., Aghdam, M.M. and Kargarnovin, M.H. (2013), "Free vibration analysis of moderately thick functionally graded plates on elastic foundation using the extended Kantorovich method", *Arch. Appl. Mech.*, **83**, 177-191. https://doi.org/10.1007/s00419-012-0645-1.

Frahlia, H., Bennai, R., Nebab, M., Ait Atmane, H. and Tounsi, A. (2023), "Assessing effects of parameters of viscoelastic foundation on the dynamic response of functionally graded plates using a novel HSDT theory", *Mech. Adv. Mater. Struct.*, **30**(13), 2765-2779. https://doi.org/10.1080/15376494.2022.2062632.

Gao, X.Y., Wang, Z.Z. and Ma, L.S. (2024), "Bending and buckling analysis of functionally graded graphene platelets reinforced composite plates supported by local elastic foundations based on simple refined plate theory", *Arch. Appl. Mech.*, **94**(8), 2123-2150. https://doi.org/10.1007/s00419-024-02629-y.

Ghalem, M., Zahaf, S., Dahmane, M., Ait Atmane, H. and Bennai, R. (2025), "Natural frequencies of 2D-FG beams with internal crack defects on Winkler–Pasternak elastic foundation using finite element technique", *Mech. Bas. Des. Struct. Mach.*, 1-18. https://doi.org/10.1080/15397734.2025.2458097.

Gupta, A., Talha, M. and Chaudhari, V.K. (2016), "Natural frequency of functionally graded plates resting on elastic foundation using finite element method", *Procedia Technol.*, **23**, 163-170. https://doi.org/10.1016/j.protcy.2016.03.013.

Heydari, A. (2021), "The damped vibration of the annular and rectangular graded beams in the presence of the attached lumped mass", *Adv. Comput. Des.*, **6**(4), 329-338. https://doi.org/10.12989/acd.2021.6.4.329.

Hissaria, P., Ramteke, P.M., Hirwani, C.K., Mahmoud, S.R., Kumar, E.K. and Panda, S.K. (2023), "Numerical investigation of eigenvalue characteristics (vibration and buckling) of damaged porous bidirectional FG panels", *J. Vib. Eng. Technol.*, **11**(4), 1889-1901. https://doi.org/10.1007/s42417-022-00677-8.

Huang, X.L. and Shen, H.S. (2004), "Nonlinear vibration and dynamic response of functionally graded plates in thermal environments", *Int. J. Solid. Struct.*, **41**(9-10), 2403-2427. https://doi.org/10.1016/j.ijsolstr.2003.11.012.

Huang, Z.Y., Lü, C.F. and Chen, W.Q. (2008), "Benchmark solutions for functionally graded thick plates resting on Winkler–Pasternak elastic foundations", *Compos. Struct.*, **85**(2), 95-104. https://doi.org/10.1016/j.compstruct.2007.10.010.

Jung, W.Y., Park, W.T. and Han, S.C. (2014), "Bending and vibration analysis of S-FGM microplates embedded in Pasternak elastic medium using the modified couple stress theory", *Int. J. Mech. Sci.*, **87**, 150-162. https://doi.org/10.1016/j.ijmecsci.2014.05.025.

Kannaiyan, G.N., Balasubramaniam, V., Pappula, B. and Makgato, S. (2025), "Critical buckling analysis of functionally graded porous beam using Karush-Kuhn-Tucker conditions", *Adv. Comput. Des.*, **10**(1), 1. https://doi.org/10.12989/acd.2025.10.1.001.

Kant, T. (1993), "A critical review and some results of recently developed refined theories of fiber-reinforced laminated composites and sandwiches", *Compos. Struct.*, **23**(4), 293-312. https://doi.org/10.1016/0263-8223(93)90230-N.

Kehli, A., Nebab, M., Bennai, R., Ait Atmane, H. and Dahmane, M. (2024), "Dynamic characteristics analysis of functionally graded cracked beams resting on viscoelastic medium using a new quasi-3D HSDT", *Mech. Adv. Mater. Struct.*, **31**(30), 12651-12664. https://doi.org/10.1080/15376494.2024.2326983.

Ke-rang, W. (1990), "Thick rectangular plates with free edges on elastic foundations", *Appl. Math. Mech.*, **11**, 869-879. https://doi.org/10.1007/BF02014353.

Kerr A. (1964), "Elastic and viscoelastic foundation models", *J. Appl. Mech.*, **31**(3), 491-498. https://doi.org/10.1115/1.3629667.

Lahdiri, A., Hadji, L., Ait Atmane, H. and Kadri, M. (2024), "Isogeometric finite element method for the dynamic analysis of functionally graded plates", *Couple. Syst. Mech.*, **13**(6), 509-527. https://doi.org/10.12989/csm.2025.13.6.509

Lam, K.Y., Wang, C.M. and He, X.Q. (2000), "Canonical exact solutions for Levy-plates on two-parameter foundation using Green's functions", *Eng. Struct.*, **22**(4), 364-378. https://doi.org/10.1016/S0141-0296(98)00116-3.

Latroch, N., Dahmane, M., Benosman, A.S., Bennai, R., Atmane, H.A. and Benadouda, M. (2024), "Inclined crack identification in bidirectional FG beams on an elastic foundation using the h-version of the finite element method", *Mech. Adv. Mater. Struct.*, **31**(28), 10477-10483. https://doi.org/10.1080/15376494.2023.2290226.

Lee, W.H., Han, S.C. and Park, W.T. (2015), "A refined higher order shear and normal deformation theory for E-, P-, and S-FGM plates on Pasternak elastic foundation", *Compos. Struct.*, **122**, 330-342. https://doi.org/10.1016/j.compstruct.2014.11.047.

Li, Q., Iu, V.P. and Kou, K.P. (2008), "Three-dimensional vibration analysis of functionally graded material sandwich plates", *J. Sound Vib.*, **311**(1-2), 498-515. https://doi.org/10.1016/j.jsv.2007.09.018.

Malekzadeh, P. (2009), "Three-dimensional free vibration analysis of thick functionally graded plates on elastic foundations", *Compos. Struct.*, **89**(3), 367-373. https://doi.org/10.1016/j.compstruct.2008.08.007.

Malekzadeh, P. and Karami, G. (2004), "Vibration of non-uniform thick plates on elastic foundation by differential quadrature method", *Eng. Struct.*, **26**(10), 1473-1482. https://doi.org/10.1016/j.engstruct.2004.05.008.

Mellal, F., Bennai, R., Nebab, M., Atmane, H.A., Bourada, F., Hussain, M. and Tounsi, A. (2024), "Investigation on the effect of porosity on wave propagation in FGM plates resting on elastic foundations via a quasi-3D HSDT", *Wave. Random Complex Media*, **34**(5), 3727-3753. https://doi.org/10.1080/17455030.2021.1983235.

Menasria, A., Himeur, N., Bouhadra, A., Bazzouzi, M. and Tounsi, A. (2025), "Combined effects of partial loading and partial elastic foundation on the bending response of functionally graded structures", *Couple. Syst. Mech.*, **14**(2), 175-199. https://doi.org/10.12989/csm.2025.14.2.175.

Merzouki, T. and Houari, M.S.A. (2024), "Nonlocal strain gradient theory for free vibration analysis of FG nano-scale beams in thermal environments using an efficient numerical model", *J. Vib. Eng. Technol.*, **12**(7), 8775-8800. https://doi.org/10.1007/s42417-024-01389-x.

Mokhefi, D., Bessaim, A., Houari, M.S.A., Deffane, Z., Houari-Belkadi, H., Ali, B., ... & Merzouki, T. (2024), "A nonlocal strain gradient model for buckling analysis of advanced FG CNT-reinforced composite nanobeams", *Couple. Syst. Mech.*, **13**(5), 375-393. https://doi.org/10.12989/csm.2024.13.5.375.

Mouffoki, A., Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst.*, **20**(3), 369-383. https://doi.org/10.12989/sss.2017.20.3.369.

Muthanna, A., Ali, M., Hasan, H.M., Najim, K.B., Njim, E.K., Madan, R. and Al-Maamori, M.H. (2025), "Vibration investigation of an imperfect FGM cylindrical shell reinforced by various types of stiffeners with temperature-dependent properties resting on an elastic foundation", *Couple. Syst. Mech.*, 105-127. https://doi.org/10.12989/csm.2025.14.2.105.

Nebab, M., Ait Atmane, H., Bennai, R. and Dahmane, M. (2024), "Warping and porosity effects on the mechanical response of FG-Beams on non-homogeneous foundations via a Quasi-3D HSDT", *Struct. Eng. Mech.*, **90**(1), 83-94. https://doi.org/10.12989/sem.2024.90.1.083.

Parida, S. and Mohanty, S.C. (2018), "Free vibration and buckling analysis of functionally graded plates resting on elastic foundation using higher order theory", *Int. J. Struct. Stab. Dyn.*, **18**(04), 1850049. https://doi.org/10.1142/S0219455418500499.

Parida, S.P., Jena, P.C. and Dash, R.R. (2023), "Dynamics of rectangular laminated composite plates with selective layer-wise fillering rested on elastic foundation using higher-order layer-wise theory", *J. Vib. Control*, **29**(23-24), 5598-5615. https://doi.org/10.1177/10775463221138353.

Pasternak, P.L. (1954), "On a new method of analysis of an elastic foundation by means of two foundation constants", *Gos. Izd. Lit. po Strait i Arkh*.

Rachid, A., Ouinas, D., Lousdad, A., Zaoui, F.Z., Achour, B., Gasmi, H., ... & Tounsi, A. (2022), "Mechanical behavior and free vibration analysis of FG doubly curved shells on elastic foundation via a new modified displacements field model of 2D and quasi-3D HSDTs", *Thin Wall. Struct*., **172**, 108783. https://doi.org/10.1016/j.tws.2021.108783.

Raza, A., Dwivedi, K., Pathak, H. and Talha, M. (2024), "Free vibration of porous functionally graded plate with crack using stochastic XFEM approach", *J. Vib. Eng. Technol.*, **12**(4), 5849-5864. https://doi.org/10.1007/s42417-023-01223-w.

Rezaiee-Pajand, M., Masoodi, A.R. and Mokhtari, M. (2018), "Static analysis of functionally graded non-prismatic sandwich beams", *Adv. Comput. Des.*, **3**(2), 165-190. https://doi.org/10.12989/acd.2018.3.2.165.

Shimpi, R.P., Arya, H. and Naik, N.K. (2003), "A higher order displacement model for the plate analysis", *J. Reinf. Plast. Compos.*, **22**(18), 1667-1688. https://doi.org/10.1177/073168403027618.

Singh, S.J. and Harsha, S.P. (2020), "Nonlinear vibration analysis of sigmoid functionally graded sandwich plate with ceramic-FGM-metal layers", *J. Vib. Eng. Technol.*, **8**, 67-84. https://doi.org/10.1007/s42417-018-0058-8

Singha, T.D. and Bandyopadhyay, T. (2024), "Free vibration characteristics of FG-GRC sandwich shallow shells with porous core in thermal environments", *J. Vib. Eng. Technol.*, **12**(4), 6741-6762. https://doi.org/10.1007/s42417-024-01281-8.

Smain, B., Aicha, B., Mohammed Sid Ahmed, H. and Marc, A. (2022), "A new quasi-3D plate theory for free vibration analysis of advanced composite nanoplates", *Steel Compos. Struct.*, **45**(6), 839-850. https://doi.org/10.12989/sss.2022.45.6.839.

Srinivas, S. and Rao, A.K. (1970), "Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates", *Int. J. Solid. Struct.*, **6**(11), 1463-1481. https://doi.org/10.1016/0020-7683(70)90076-4.

Sui, S., Zhu, C., Li, C. and Lei, Z. (2023), "Free vibration of axially traveling moderately thick FG plates resting on elastic foundations", *J. Vib. Eng. Technol.*, **11**(1), 329-341. https://doi.org/10.1007/s42417-022-00582-0.

Taleb, O., Houari, M.S.A., Bessaim, A., Tounsi, A. and Mahmoud, S.R. (2018), "A new plate model for vibration response of advanced composite plates in thermal environment", *Struct. Eng. Mech.*, **67**(4), 369-383. https://doi.org/10.12989/sem.2018.67.4.369.

Thai, H.T. and Choi, D.H. (2011), "A refined plate theory for functionally graded plates resting on elastic foundation", *Compos. Sci. Technol.*, **71**(16), 1850-1858. https://doi.org/10.1016/j.compscitech.2011.08.016.

Tu, T.M., Quoc, T.H. and Van Long, N. (2019), "Vibration analysis of functionally graded plates using the eight-unknown higher order shear deformation theory in thermal environments", *Aerosp. Sci. Technol.*, **84**, 698-711. https://doi.org/10.1016/j.ast.2018.11.010.

Uymaz, B. and Uymaz, G. (2024), "Three-dimensional thermal vibration of CFFF functionally graded carbon nanotube-reinforced composite plates", *J. Vib. Eng. Technol.*, **12**(4), 5345-5368. https://doi.org/10.1007/s42417-023-00957-x.

Winkler, E. (1867), *Theory of Elasticity and Strength*.

Yıldız, T. and Esen, I. (2025), "A comprehensive study on determining the flexural wave propagation properties of biocompatible sandwich nanoplate exposed to thermal and magnetic fields", *Mech. Bas. Des. Struct. Mach.*, 1-27. https://doi.org/10.1080/15397734.2025.2549472.

Yildiz, T., Kabave Kilinçarslan, S. and Esen, İ. (2024), "Vibration simulation of sandwich nano-smart plate with an auxetic core and piezo-electromagnetic face layers based on sinusoidal higher-order theory", *Arab. J. Sci. Eng.*, 1-25. https://doi.org/10.1007/s13369-024-09746-4.

Zenkour, A.M. and Alghanmi, R.A. (2022), "A refined quasi-3D theory for the bending of functionally graded porous sandwich plates resting on elastic foundations", *Thin Wall. Struct.*, **181**, 110047. https://doi.org/10.1016/j.tws.2022.110047.

Zenkour, A.M. and Sobhy, M. (2012), "Elastic foundation analysis of uniformly loaded functionally graded viscoelastic sandwich plates", *J. Mech.*, **28(**3), 439-452. https://doi.org/10.1017/jmech.2012.53.

Zohra, A., Rabia, B. and Tahar, H.D. (2024), "Study and analysis of porosity distribution effects on the buckling behavior of functionally graded plates subjected to diverse thermal loading", *Couple. Syst. Mech.*, **13**(2), 115-132. https://doi.org/10.12989/csm.2024.13.2.115.