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Interactions in a nonlocal isotropic thermoelastic solid with diffusion due to thermal source in frequency domain

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Abstract. This study investigates the dynamic behaviour of nonlocal thermoelastic solid with diffusion subjected to thermal source, with a focus on the influence of angular frequency on the material. To examine this we consider nonlocal thermoelastic solid with diffusion. The governing equations are solved in the frequency domain to analyse the frequency-dependent response of the nonlocal thermoelastic solids with diffusion. Our findings are that angular frequency significantly effects the normal stress, shear stress, mass concentration and temperature change. The study highlights the importance of angular frequency in determining the stress, temperature and concentration fields in nonlocal thermoelastic diffusive solids under concentrated load. The results offer valuable insights for applications in advanced materials science, microscale and nanoscale engineering and dynamic load analysis, where it is crucial to understand the combined effects of nonlocality, thermoelasticity and diffusion.

Keywords: angular frequency; diffusion; Fourier transformation; nonlocal; stress; thermal; thermoelastic

1. Introduction

Research on nonlocal thermoelastic materials with diffusive solids has been the attention in recent years due to its relevance in advanced materials science. The elliptic equation theory was utilized to establish the existence and uniqueness of generalized solutions for boundary value problems in the elasticity of dipolar materials containing voids (Marin 1997). The frequency-dependent behavior of materials under various loading is a significant area of research, this research outlines important contributions to the field highlighting theoretical and computational developments (Srinivasa and Reddy 2017).

The two-dimensional problem involving a generalized thermoelastic diffusion material with both thermal and diffusion relaxation times was analyzed using the finite element method within the frame- work of the Lord-Shulman theory (Abbas *et al.* 2012). The propagation of plane waves in a thermo- microstretch elastic solid half-space has been investigated using the finite element method. The analysis is conducted based on both the Lord-Shulman model and the classical coupled dynamical thermoelasticity theory (Abbas and Othman 2012).

Abbas and Youssef (2015) studied the two-dimensional thermoelastic problem in a porous half-

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space using fractional-order generalized thermoelasticity with one relaxation time. The surface is traction free with constant heat flux, and the solution is obtained using normal mode analysis and the eigenvalue method. The influence of dual temperature effects on an axially moving microbeam exposed to ramp-type thermal loading has been investigated. The analysis is carried out within the framework of the generalized thermoelasticity theory incorporating a single relaxation time model (Carrera *et al.* 2015).

Abbas (2015a) investigated the solution of thermoelastic interactions induced by thermal shock in a functionally graded material using the fractional-order three-phase lag model. To obtain the solution, they employed the eigenvalue approach method. Abbas (2015b) analyzed thermoelastic interactions in a microscale beam influenced by a moving heat source, based on Green and Naghdi's type III theory using the eigenvalue method.

Abbas *et al.* (2021) investigated the photo-thermalelastic interactions in an unbounded semi-conductor medium with a cylindrical cavity, employing the hyperbolic two-temperature model. The analysis is carried out within the framework of the coupled thermoelasticity and plasma wave theories. The photo-thermoelastic interaction in an unbounded semiconductor medium with a spherical cavity is analysed using a newly developed hyperbolic two-temperature model (Alzahrani and Abbas 2020).

Diffusion is defined as the movement of molecules from high concentration to low concentration. Fick's law explained it but does not say much by itself because Fick's law explains only mass transfer flux with respect to concentration gradient but does not express the interaction between substance and the matrix together with the coupling effect which is generated due to more than one field like electric, magnetic, thermal and elastic. Because of these aforementioned drawbacks, thermoelastic diffusion theory presumes that regardless of the mass and heat transfer mechanism, it has to satisfy the classical Fourier and Fick's laws (Li *et al.* 2022).

The study of time-harmonic interactions in nonlocal thermoelastic diffusion solids integrates Multiphysics phenomena critical for advanced materials in nanotechnology and aerospace engineering (Marin 2010). Nonlocal elasticity theory, pioneered by (Eringen 1972), accounts for long-range atomic interactions, essential at nanoscales. Coupling thermoelasticity with diffusion introduced by (Nowacki 1974) enables analysis of materials under simultaneous thermal, mechanical and mass transport loads. Recent advances focus on dynamic responses to time-harmonic thermal sources, vital for optimizing high-frequency devices.

Thermoelasticity combines thermal and elastic behaviours. Thermoelasticity deals with how materials deform under temperature changes and how mechanical changes affect temperature. Material's behaviour is also affected by diffusion processes. The material's response involves mechanical deformation, thermal effects and diffusion, all considered in a nonlocal framework. The focus is on how a thermal source (like a heat input) influences these interactions under time-harmonic conditions (Sharma and Grover 2011).

Eringen (1972) proposed the idea of nonlocal elasticity, which has been critical in understanding material behaviour at tiny scales, where traditional continuum mechanics fails to describe size dependent phenomena. Eringen's nonlocal theory states that stress at a given place is determined by the strain field at all other points in the body, making it appropriate for modelling nanomaterials and structures with intrinsic heterogeneity. Recent research has expanded this theory to incorporate thermal and diffusive effects, resulting in a more complete framework for studying coupled phenomena (Karami *et al.* 2018).

Nonlocal effects, thermal diffusion, and mechanical loads all have an effect on the frequency-dependent behaviour of nonlocal thermoelastic-diffusive materials. Abbas *et al.* (2022) examined the frequency response of nonlocal thermoelastic materials under harmonic loading and found that nonlocal effects have a substantial impact on the dispersion and attenuation properties of stress waves. Their findings emphasize the need of taking nonlocal characteristics into account when designing high-frequency applications (Marin *et al.* 2014).

Nonlocal thermoelasticity delves into exploring how mechanical and thermal deformation interact in materials where the traditional notion of interaction doesn't apply anymore. This perspective takes into consideration size effects that becomes notable in materials with characteristics. In the realm of thermoelasticity, the behaviour of both the mechanical and thermal domains are dictated by constitutive equations that encompass a global perspective. The correlation between stress and strain, in a context usually involves a length scale factor that determines the extent of connectivity among adjacent material locations (Zarei and Pilla 2023).

Abouelregal *et al.* (2022) developed a mathematical model encompassing generalized thermoelastic diffusion characterized by four lags and incorporating higher-order time fractional derivatives. Alterations were made to the heat equations and Fick's law, with particular adjustments made to the Fourier law to accommodate a higher time-fractional order of heat conduction. Singh and Lata (2023) studied the effect of two temperature parameters on the axisymmetric deformation in a two- dimensional nonlocal homogeneous isotropic thick circular plate without energy dissipation. Abbas *et al.* (2023) examined the impact of three thermal relaxation times on living tissue within the framework of the Three-Phase-Lag (TPL) bioheat model. By applying Laplace transforms, analytical solutions for tissue temperature and the associated thermal damage are derived.

Bhatti *et al.* (2023) explored the flow characteristics of a hybrid nanofluid modeled using third-grade sodium alginate. This material shows promise for applications in geothermal energy extraction. The researchers considered two types of nanoparticles in their analysis: zinc oxide (ZnO) and copper oxide (CuO). Yadav *et al.* (2024) analyzed the propagation of plane waves in a hygrothermal medium and identified three distinct types of waves: a longitudinal displacement wave (P-wave), a thermal diffusion wave (TD-wave) and a moisture diffusion wave (MD-wave). In addition, a shear vertical wave (SV-wave) was also found to propagate through the medium. The velocities of all four wave types-P-wave, TD-wave, MD-wave, and SV-wave-were calculated based on the properties of the hygrothermal material.

Lata and Himansh (2022) investigated the impact of the fractional-order parameter on a two-dimensional orthotropic magnetothermoelastic solid within the framework of generalized thermoelasticity without energy dissipation. The study considered fractional-order heat transfer in the presence of Hall current, rotation, and a two-temperature model under the influence of a normal force. The solution was derived using Laplace and Fourier transform techniques. Kumar *et al.* (2022) developed a mathematical model that incorporated nonlocal, phase-lag and temperature dependent features. Employing this model, they investigated the nonlocal phase-lag and temperature dependent behaviours within a modified pair stress thermoelastic diffusive medium under the influence of thermomechanical sources.

Lata and Heena (2024) analysed two-dimensional deformation in a transversely isotropic thermoelastic diffusion medium, exploring the influence of diffusion and thermal effects on such solids under an inclined load. The study employed Laplace and Fourier transformation techniques for the investigation. Singh and Lata (2024) had studied the thermomechanical deformations that appear in a nonlocal homogeneous isotropic thick circular plate through the frequency domain without energy dissipation and solved the problem through Hanker transformation techniques.

Researches we have described on nonlocal thermoelastic media with diffusion above are limited, with many studies focusing on transient or time-domain formulations rather than fully coupled field equations. This lack of thorough analysis in the frequency domain is crucial for understanding wave propagation, resonance phenomena and system stability. Most studies neglect energy dissipation or focus solely on mechanical and thermal interactions without fully integrating diffusive effects. This gap highlights the need for advanced mathematical modelling and analytical techniques to bridge the gap between classical and modern nonlocal thermoelastic diffusion theories in the frequency domain.

2. Basic equations

Following Eringen (2002) the stress tensor at arbitrary point x of a nano material body not only depends up on the stress tensor at x, but also depend on all points of the body. The nonlocal stress tensor for a homogeneous isotropic elastic material in the absence of body force can be expressed as

$$\sigma_{ij}(x) = \int_v \alpha(|x\text{-}x'|,\xi)t_{ij}(x)dV\,(x'),$$

By employing Eringen's nonlocal formulation, the nonlocal stress tensor $\sigma_{ij}(x)$ can be expressed as

$$(1\text{-}\xi^2\nabla^2)\sigma_{ij}(x) = t_{ij}(x).$$

The constitutive equation for coupled thermoelastic diffusion medium while neglecting the body forces can be expressed as

$$(1\text{-}\xi^2\nabla^2)\sigma_{ij}(x) = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk}\text{-}\gamma_1 T\text{-}\gamma_2 C),$$

$$P = \text{-}\gamma_2 e_{kk}\text{-}aT + bC.$$

Following Eringen (1974), Eringen (1977), Ram *et al.* (2008) and Malik *et al.* (2023) the basic equations in isotropic nonlocal thermoelastic media with diffusion can be given by

$$(\lambda + \mu)\,\nabla.\nabla\boldsymbol{u} + \mu\nabla^2\boldsymbol{u}\text{-}\gamma_1\nabla T\text{-}\gamma_2\nabla C + \rho(1 - \xi^2\nabla^2)F = \rho(1\text{-}\xi^2\nabla^2)\ddot{\boldsymbol{u}}, \tag{1}$$

$$\left(K\frac{\partial}{\partial t} + K^*\right)\nabla^2 T = \rho\,(1 - \xi^2\nabla^2)\frac{\partial^2}{\partial t^2}\,\left(\rho CET + \gamma_1 T_0\boldsymbol{u}_{i,i} + aT_0 C\right) \tag{2}$$

$$\begin{aligned}(1 - \xi^2\nabla^2)\,\dot{C} &= d\nabla^2(bC\text{-}\gamma_2 e_{kk}\text{-}aT),\\ \gamma_1 &= (3\lambda + 2\mu)\alpha_t,\\ \gamma_2 &= (3\lambda + 2\mu)\alpha c.\end{aligned} \tag{3}$$

Where in Eqs. (1)-(3) T is temperature, ρ is density, α_t is the coefficient of linear thermal expansion, α_c is the coefficient of diffusion expansion, K is coefficient of thermal conductivity, K^* is the materialistic constant, a is coefficients of thermoelastic diffusive effects, b is coefficients of diffusive effects, σ_{ij} are components of stress tensor, C is concentration distribution, ρ is density, u_i is displacement vector, λ and μ are Lame's constants, C_E is specific heat at constant strain, T_0 is temperature of the medium in its natural state assumed, that $|T/T_0| < 1$, d is diffusion constant, ξ are nonlocal parametrs, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$; i, j are $1, 2, 3$, ∇ is gradient and ∇^2 is laplacian operator.

3. Formulation of problem

Considering a two dimentional homogeneous nonlocal isotropic thermoelastic body initialy at undeformed state at initial temperature T_0. We take a rectangular coordinate system (x, y, z) having origin on the surface $z = 0$ with $z\text{-}axis$ pointing vertically downward into the medium is introduced. The surface of half space is subjected to normal force. For two-dimensional problem in xz plane we take,

$$U = (u, 0, w), u = u(x, z, t), w = w(x, z, t),$$
$$T = T(x, z, t) \text{ and } C = C(x, z, t).$$

From Eqs. (1)-(3) the component form of the equations are derived as

$$(\lambda + \mu)\frac{\partial y}{\partial x} + \mu\nabla^2 u\text{-}\gamma_1\frac{\partial y}{\partial x}\text{-}\gamma_2\frac{\partial y}{\partial x} + \rho(1\text{-}\xi^2\nabla^2)F_1 = \rho(1\text{-}\xi^2\nabla^2)\ddot{u}, \tag{5}$$

$$(\lambda + \mu)\frac{\partial y}{\partial x} + \mu\nabla^2 w - \gamma_1\frac{\partial y}{\partial x}\text{-}\gamma_2\frac{\partial y}{\partial x} + \rho(1 - \xi^2\nabla^2)F_3 = \rho(1\text{-}\xi^2\nabla^2)\ddot{w}, \tag{6}$$

$$(K\frac{\partial}{\partial t} + K^*)\nabla^2 T = \rho(1\text{-}\xi^2\nabla^2)\frac{\partial^2}{\partial t^2}(\rho C_E\frac{\partial T}{\partial t} + \gamma_1 T_0 e + aT_0 C), \tag{7}$$

$$(1 - \xi^2\nabla^2)\frac{\partial C}{\partial t}\, d\left(b\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\right) - \gamma_2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\text{-}a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right)\right). \tag{8}$$

The condition of the variables at initial are given by the folloeing forms

$$\begin{gathered} u(x, z, 0) = 0 = \dot{u}(x, z, 0), \\ w(x, z, 0) = 0 = \dot{w}(x, z, 0), \\ T(x, z, 0) = 0 = \dot{T}(x, z, 0), \\ C(x, z, 0) = 0 = \dot{C}(x, z, 0), for\, z \geq 0, \text{-}\infty < x < \infty, \\ u(x, z, t) = w(x, z, t) = T(x, z, t) = C(x, z, t) = 0, for\, t > 0, \quad when, z \to \infty. \end{gathered} \tag{9}$$

In our calculation we used the following dimensionless quantities for simplicity

$$\begin{gathered} x' = \frac{{\omega_1}^*}{c_1}x,\ z' = \frac{{\omega_1}^*}{c_1}z,\ t' = {\omega_1}^* t,\ u' = \frac{{\omega_1}^*}{c_1}u, a' = \frac{{\omega_1}^*}{c_1} \\ w' = \frac{{\omega_1}^*}{c_1}w,\quad C' = \frac{\gamma_2 C}{\rho {c_1}^2},\ T' = \frac{\gamma_1 C}{\rho {c_1}^2}T,\ F_1' = \frac{F_1}{\gamma_1 T_0},\ F_3' = \frac{F_3}{\gamma_1 T_0},\ e' = e,\ c_1^2 = \frac{\lambda + 2\mu}{\rho}, \\ \omega_1^* = \frac{\rho C_E {c_1}^2}{K}. \end{gathered} \tag{10}$$

We express the displacement variables $u(x, z, t)$ and $w(x, z, t)$ in terms of dimensionless potential functions $\Psi 1$ and $\Psi 2$ as

$$u = \frac{\partial \Psi_1}{\partial x}\text{-}\frac{\partial \Psi_2}{\partial z},\quad w = \frac{\partial \Psi_1}{\partial z} + \frac{\partial \Psi_2}{\partial x}. \tag{11}$$

The general solution of the problem is given in the following form

$$(\Psi_1, \Psi_2, T, C)(x, z, t) = (\Psi_1, \Psi_2, T, C)(x, z)e^{i\omega t}. \tag{12}$$

The Fourier transformation is given as follow

$$f(\zeta, z, t) = \int_{-\infty}^{\infty}(\Psi_1, \Psi_2, T, C)(x, z, t)e^{-i\zeta x}dx. \tag{13}$$

By using the concept of Fourier transformation; from Eqs. (5)-(8) after substitute Eq. (11) in them, we get the following

$$(A_1(D^2-\zeta^2)+\omega^2(1\text{-}\xi^2\nabla^2))\widehat{\Psi_1}\text{-}\hat{T}\text{-}\hat{C}=0, \tag{14}$$

$$((A_2(D^2-\zeta^2)+\omega^2(1\text{-}\xi^2\nabla^2))\widehat{\Psi_2}=0, \tag{15}$$

$$(A_3(D^2\text{-}\zeta^2)(1\text{-}\xi^2\nabla^2)\widehat{\Psi_1}+A_4(1\text{-}\xi^2\nabla^2)\text{-}A_5(D^2\text{-}\zeta^2)\hat{T}+A_6(1\text{-}\xi^2\nabla^2)\hat{C}=0, \tag{16}$$

$$A_9(D^2\text{-}\zeta^2)^2\widehat{\Psi_1}+A_{10}((D^2\text{-}\zeta^2)\hat{T}+(A_7(1\text{-}\xi^2\nabla^2)\text{-}A_8(D^2\text{-}\zeta^2))\hat{C}, \tag{17}$$

Where: $A_1=\frac{(\lambda+2\mu)}{\rho {c_1}^2}$, $A_2=\frac{\mu}{\rho {c_1}^2}$, $A_3=\text{-}\rho\omega^2\gamma_1 T_0$, $A_4=i\omega^2\frac{\rho^3 {C_1}^2{\omega_1}^*}{\gamma_1}C_E$,
$A_5=\frac{\rho c1}{\gamma_1}(K{\omega_1}^* i\omega+K^*)$, $A_6=\text{-}\omega^2\frac{\rho^2 {C_1}^3}{{\omega_1}^*\gamma_1}T_0 a'$, $A_7=i\omega\frac{\rho {C_1}^2{\omega_1}^*}{\gamma_2}$,
$A_8=db\frac{\rho{\omega_1}^{*2}}{\gamma_2}$, $A_9=d\gamma_2\frac{{\omega_1}^{*2}}{{C_1}^2}$, $A_{10}=ad\frac{\rho{\omega_1}^{*2}}{\gamma_1}$, $A_{11}=d\gamma_2\frac{{\omega_1}^{*2}\varepsilon^2}{{C_1}^2}$,
$A_{12}=\gamma_1 T_0=A_{14}$, $A_{13}=\frac{\rho{\omega_1}^*}{\gamma_1}c_1$, $A_{15}=\frac{\rho{\omega_1}^*}{\gamma_2}c_1$, $A_{16}=\frac{A_{12}}{A_{13}}$, $A_{17}=\frac{A_{14}}{A_{13}}$.

The Eqs. (14)-(17) posess a non-trivial solution if determinant of their coefficients vanishes. By simplifying the equation formed from them, we have the following polynomial equation with respect to $(\widehat{\psi_1},\widehat{\psi_2},\ \hat{T},\hat{C})$

$$(D^6R_1+D^4R_2+D^2R_3+R_4)(\widehat{\psi_1},\hat{T},\hat{C})=0, \tag{18}$$

$$(D^2+r)\,\widehat{\psi_2}=0,\ r=\frac{R_6}{R_5}. \tag{19}$$

Where

$$\begin{gathered}M_1=-A_4A_7\omega^2,\ M_2=-A_1A_4A_7+A_4A_8\omega^2-A_5A_7\omega^2-A_6A_{10}\omega^2+A_3A_7,\ M_3=\\ \text{-}A_1A_4A_8+A_5A_8\omega^2,\ M4=A_1A_5A_7+A_1A_6A_{10}+A_3A_8+A_9A_6+A_3A_{10}\text{-}A_4A_9,\\ M_5=A_1A_5A_8\text{-}A_5A_9,\ M_6=M_2{A_{11}}^2\text{-}{A_{11}}^3M_1+M_4A_{11}+M_5,\ M_7=\\ {A_{11}}^2M_1\text{-}2M_2A_{11}\text{-}M_3A_{11}\text{-}M_4,\ M_8=M_2+M_3\text{-}3A_{11}M_1,\ M_9=M_1,\ R_1=M_6,\ R_2=\\ \text{-}3M_6\zeta^2+M_7,\ R_3=3M_6\zeta^4\text{-}2M_7\zeta^2+M_8,\ R4=M_7\zeta_4+M_9\text{-}M_6\zeta^6\text{-}M_8\zeta^2,\ R_5=\\ A_2\text{-}A_{11}\omega^2,\ R_6=\text{-}A_{11}\zeta^2\omega^2\text{-}A_2\zeta^2\text{-}1.\end{gathered} \tag{20}$$

When we solve Eqs. (18)-(19) we get their roots $\pm r_i$, $(i=1,2,3)$. We have also at infinity the solutions are vanishes or as $z\rightarrow\infty$, the solutions of these equations tend to zero, therefore general solutions of each variable are given in the following form.

$$\begin{gathered}\widehat{\Psi_1}=B_1e^{\text{-}r_1z}+B_2e^{\text{-}r_2z}+B_3e^{\text{-}r_3z},\\ \hat{T}=d_1B_1e^{\text{-}r_1z}+d_2B_2e^{\text{-}r_2z}+d_3B_3e^{\text{-}r_3z},\\ \hat{C}=l_1B_1e^{\text{-}r_1z}+l_2B_2e^{\text{-}r_2z}+l_3B_3e^{\text{-}r_3z},\\ \widehat{\Psi_2}=B_4e^{\text{-}r_4z}.\end{gathered} \tag{21}$$

Where

$$\begin{gathered}d_i=\frac{{r_i}^6M_{13}+{r_i}^4M_{14}+{r_i}^2M_{15}+M_{16}}{{r_i}^4M_{10}+{r_i}^2M_{11}+M_{12}},\ i=(1,2,3),\\ l_i=\frac{{r_i}^6M_{17}+{r_i}^4M_{18}+{r_i}^2M_{19}+M_{20}}{{r_i}^4M_{10}+{r_i}^2M_{11}+M_{12}},i=(1,2,3),\end{gathered} \tag{22}$$

where:

$$N_1 = A_1 - \omega^2 A_{11},\ N_2 = (A_1 - \omega^2 A_{11})\zeta^2 + \omega^2,\ N_3 = -A_3 A_{11},\ N_4 = A_3 + 2A_3 A_{11}\zeta^2,\ N_5 = -A_3 A_{11}\zeta^4 + A_3\zeta^2,\ N_6 = -A_4 A_{11} - A_5,\ N_7 = A_4 + A_4 A_{11}\zeta^2 + A_5\zeta^2,\ N_8 = A_6 A_{11},\ N_9 = A_6 A_{11}\zeta^2 + A_6,\ N_{10} = -A_9,\ N_{11} = 2A_9\zeta^2,\ N_{12} = A_9\zeta^4\ N_{13} = A_{10},\ N_{14} = A_{10}\zeta^2, N_{15} = -A_8 - A_7 A_{11},\ N_{16} = A_8 + A_7 A_{11} + A_7,\ N_{17} = A_2 + \omega^2 2A_{11},\ N_{18} = (A_2 - \omega^2 2A_{11})\zeta^2,\ M_{10} = N_6 N_{15} + N_8 N_{13},\ M_{11} = N_6 N_{16} + N_7 N_{15} - N_9 N_{13} - N_8 N_{14},\ M_{12} = N_7 N_{16} + N_9 N_{14}\ M_{13} = N_3 N_{15} - N_8 N_{10},\ M_{14} = 2 N_9 N_{10} - N_3 N_6 - N_4 N_{15} + N_8 N_{11},\ M_{15} = N_4 N_{16} - N_5 N_{15} - N_9 N_{11} + N_8 N_{12},\ M_{16} = N_5 N_{16} + N_9 N_{12},\ M_{17} = -N_3 N_{13} - N_6 N_{10},\ M_{11} = N_6 N_{11}, -N_7 N_{10} + N_4 N_{13} - N_3 N_{14},\ M_{19} = N_7 N_{11} - N_6 N_{12} - N_5 N_{13} - N_4 N_{14},\ M_{20} = N_5 N_{14} - N_7 N_{12}$$

4. Boundary conditions

On the half-space *z*=0 normal force is applied. we consider the following boundary conditions.

$$(1 - \xi^2 \nabla^2)\sigma_{zz} = 0, \tag{25}$$

$$(1 - \xi^2 \nabla^2)\sigma_{zx} = 0, \tag{26}$$

$$\frac{\partial}{\partial z} T((x, y, t) = F_2 \Psi_2(x) e^{i\omega t}, \tag{27}$$

$$\frac{\partial}{\partial z} C\ (x, z, t) = 0. \tag{28}$$

Apply 12 and (13) on the set of Eqs. (25)-(28) and we obtain

$$M_{21}B_1 + M_{22}B_2 + M_{23}B_3 + M_{24}B_4 = 0, \tag{29}$$

$$M_{25}B_1 + M_{26}B_2 + M_{27}B_3 + M_{28}B_4 = 0, \tag{30}$$

$$M_{29}B_1 + M_{30}B_2 + M_{31}B_3 = A_{16}\, F_2 \Psi_2(\zeta) e^{i\omega t}, \tag{31}$$

$$M_{32}B_1 + M_{33}B_2 + M_{34}B_3 = 0. \tag{32}$$

where

$$M_{21} = A_{11}\lambda({r_1}^2 - \zeta^2) - \lambda {A_{11}}^2({r_1}^4 - 2\zeta^2 {r_1}^2) + \zeta^4) - 2\mu r_1 + 2\mu A_{11}{r_1}^3 - 2\mu A_{11}\zeta^2 r_1 - A_3 d_1 - A_{11}A_3 d_1 {r_1}^2 + A_{11}A_3\zeta^2 d_1 - A_3 l_1 - A_{11}A_3 l_1 {r_1}^2 + A_{11}A_3\zeta^2 l_1,\ M_{22} = A_{11}\lambda({r_2}^2 - \zeta^2) - \lambda {A_{11}}^2({r_2}^4 - 2\zeta^2 {r_2}^2) + \zeta^4) - 2\mu r_2 + 2\mu A_{11}{r_2}^3 - 2\mu A_{11}\zeta^2 r_2 - A_3 d_2 - A_{11}A_3 d_2 {r_2}^2 + A_{11}A_3\zeta^2 d_2 - A_3 l_2 - A_{11}A_3 l_2 {r_2}^2 + A_{11}A_3\zeta^2 l_2,\ M_{23} = A_{11}\lambda({r_3}^2 - \zeta^2) - \lambda {A_{11}}^2({r_3}^4 - 2\zeta^2 {r_3}^2) + \zeta^4) - 2\mu r_3 + 2\mu A_{11}{r_3}^3 - 2\mu A_{11}\zeta^2 r_3 - A_3 d_3 - A_{11}A_3 d_1 {r_3}^2 + A_{11}A_3\zeta^2 d_3 - A_3 l_3 - A_{11}A_3 l_3 {r_3}^2 + A_{11}A_3\zeta^2 l_3,\ M24 = i\zeta - A_{11}i\zeta {r_4}^2 + A_{11}i\zeta^3,\ M25 = \mu(-r_1 + A_{11}({r_1}^3 - \zeta^2 r_1)),\ M26 = \mu\big(-r_2 + A_{11}({r_2}^3 - \zeta^2 r_2)\big),\ M27 = \mu(-r_3 + A_{11}({r_3}^3 - \zeta^2 r_3)),\ M28 = \mu(i\zeta - A_{11}(i\zeta {r_4}^2 - i\zeta^3)),\ M_{29} = -r_1\, d_1,\ M_{30} = -r_2\, d_2,\ M_{31} - r_3\, d_3,\ M_{32} = -r_1\, l_1,\ M_{33} = -r_2\, l_2,\ M_{34} = -r_3\, l_3 \tag{33}$$

By solving system of Eqs. (29)-(32), we calculate the nontrivial values of $B_i,\ i = 1,2,3,4$ given by

$$B_1 = \frac{A_{16}F_2\,\Psi_2(\zeta)e^{i\omega t}\Delta_{12}}{\Delta}, \tag{34}$$

$$B_2 = \frac{A_{16}F_2\,\Psi_2(\zeta)e^{i\omega t}\Delta_{22}}{\Delta}, \tag{35}$$

$$B_3 = \frac{A_{16}F_2\,\Psi_2(\zeta)e^{i\omega t}\Delta_{32}}{\Delta}, \tag{36}$$

$$B_4 = \frac{A_{16}F_2\,\Psi_2(\zeta)e^{i\omega t}\Delta_{42}}{\Delta}. \tag{37}$$

where

$$\begin{aligned}\Delta_{12} &= M_{28}M_{22}M_{34} - M_{28}M_{23}M_{33} + M_{24}M_{27}M_{33} - M_{24}M_{26}M_{34},\\ \Delta_{22} &= M_{24}M_{25}M_{34} - M_{24}M_{27}M_{32} + M_{28}M_{23}M_{32} - M_{21}M_{28}M_{34},\\ \Delta_{32} &= M_{24}M_{26}M_{32} - M_{24}M_{25}M_{33} + M_{28}M_{21}M_{33} - M_{22}M_{28}M_{32},\\ \Delta_{42} &= M_{23}M_{26}M_{32} - M_{22}M_{27}M_{32} + M_{23}M_{25}M_{33} - M_{21}M_{27}M_{33} - M_{22}M_{25}M_{34} +\\ &\quad M_{21}M_{26}M_{34}\end{aligned} \tag{38}$$

By applying Fourier transform on Eq. (11), subistitute from Eq. (21) using Eqs. (34)-(37) we get the components of displacement, stress, temperature and concentration as follow.

$$\hat{u} = \frac{\widehat{F_2}A_{16}\,\widehat{\Psi_2}(\zeta)e^{i\omega t}}{\Delta}\left(i\zeta\sum_{n=1}^{3}\Delta_{n2}e^{-r_n z} + r_4\Delta_{42}e^{-r_4 z}\right), \tag{39}$$

$$\widehat{w} = \frac{\widehat{F_2}A_{16}\,\widehat{\Psi_2}(\zeta)e^{i\omega t}}{\Delta}\left(\sum_{n=1}^{3} r_n\Delta_{n2}e^{-r_n z} - \Delta_{42}e^{-r_4 z}\right), \tag{40}$$

$$\hat{T} = \frac{\widehat{F_2}A_{16}\,\widehat{\Psi_2}(\zeta)e^{i\omega t}}{\Delta}\left(\sum_{n=1}^{3} \Delta_{n1}d_n e^{-r_n z}\right), \tag{41}$$

$$\hat{C} = \frac{\widehat{F_2}A_{16}\,\widehat{\Psi_2}(\zeta)e^{i\omega t}}{\Delta}\left(\sum_{n=1}^{3} \Delta_{n1}l_n e^{-r_n z}\right), \tag{42}$$

$$\hat{\sigma}_{zz} = \frac{\widehat{F_2}A_{16}\,\widehat{\Psi_2}(\zeta)e^{i\omega t}}{\Delta}\left(\sum_{n=1}^{3} \Delta_{n2}(\lambda {r_n}^2 - \lambda\zeta^2 - 2\mu r_n - A_3 d_n - A_3 l_n) + \Delta_{42}\right), \tag{43}$$

$$\hat{\sigma}_{xz} = \frac{\widehat{F_2}A_{16}\,\widehat{\Psi_2}(\zeta)e^{i\omega t}}{\Delta}\left(-2\sum_{n=1}^{3} \Delta_{n2}r_n\mu i\zeta - \Delta_{42}(\mu {r_4}^2 + \mu\zeta^2\right), \tag{44}$$

5. Applications

5.1 Concentrated load

A concentrated mechanical load is a force or pressure applied at a single point, or over an area that is very small. In mechanical analysis, this kind of load is commonly modeled as a point force (such as a load on the center of a beam or a pin). In case a concentrated load is applied on a plate

Table 1 Copper parameter and their values

Constant notation	Values	Constant notation	Values
λ	7.76×10^{10} NM^{-2}	μ	3.86×10^{10} NM^{-2}
α_t	1.78×10^{-5} K^{-1}	α_c	$2.65 \times 10^{(-4)}$ K^{-1}
ρ	8954	γ_1	0.02 s
γ_2^8	0.2 s	K	386 J(msk)$^{-4}$
α	1.2×10^4 m^{2}KS^2	b	0.9×10^6 Kgm^{5}s^2
T_0	293K	C_E	383.1 J(KgK)$^{-1}$
d	8.5×10^{-9}	ξ	3.95×10^{-10}
ζ	2×10^{-10}	K^*	1.2
s	1		

of copper material, we consider $\psi_1 = \delta(x)$ and $\psi_2 = \delta(x)$. By Fourier transformation on both of these equations, we get the potential function aŝ $\psi_1(\zeta) = 1$ and $\psi_2(\zeta) = 1$, $\delta(x)$ is called the Dirac delta function

6. Inversion of the transforms

The inverse Laplace transform converts a function in the transformed domain back to a function in the time domain and the inverse Fourier transform formula also converts a function from the transformed domain back to the time domain.

The inversion transforms from Eqs. (39)-(44) are given as follow.

$$f(x,z,t) = \frac{1}{\pi}\int_0^\infty (cos(\omega x)f_e - sin(\omega x)f_0)d\omega, \tag{45}$$

where f_e and f_0 are even function and odd function of $f(\omega, x, s)$ respectively. The inverse transforms are calculated by using Honig and Hirdes (1984) methods and we get equations in physical domain.

7. Numerical solution

In order to investigate angular frequency effects on nonlocal thermoelastic media with diffusion, the material considered is copper. As mentioned in the studies of Malik *et al.* (2023), the material constants of copper metal are given by the following table.

The values of constants in copper metal parameters with their standard units.

8. Discussion

8.1 Concentrated load

These Figs. 1-4 above are shown the impact of angular frequency on the normal stress, shear

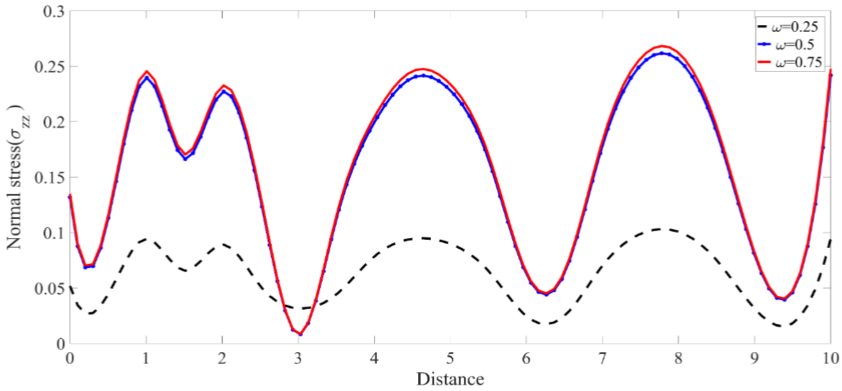

Fig. 1 Variation in normal stress due to cngular frequency (concentrated load)

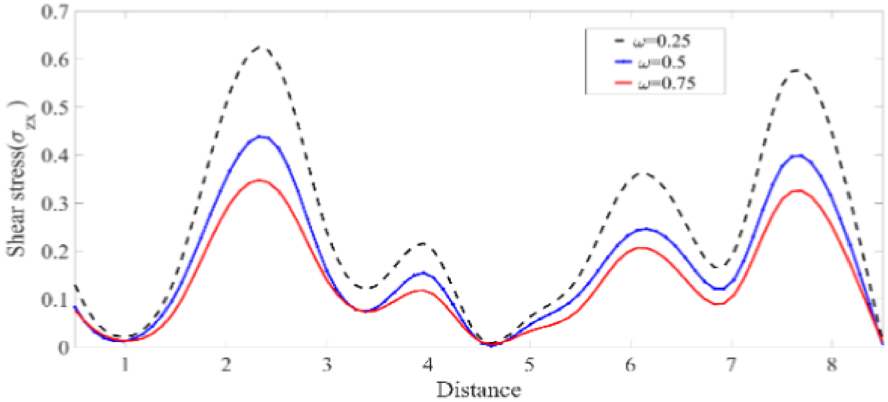

Fig. 2 Variation in shear stress due to angular frequency (concentrated load)

stress, mass concentration and temperature change reveals how they influence the overall behaviour of the graphs of nonlocal thermoelastic media with diffusion subjected to concentrated load under thermal force. Fig. 1 illustrates that for various values of the angular frequency; the normal stress begins by decreasing and continuous slightly decreasing with different values of angular frequency as the distance increase.

From Fig. 2, the graph of shear stress concerning thermoelastic media with diffusion starts from a different point and oscillates. The variation among graphs of different values of angular frequency are in the proportional. This indicates that changes in angular frequency affect the properties of thermoelastic media with diffusion.

Fig. 3 illustrates the deviations of mass concentration C under the frame work of nonlocal thermoelasticity subjected to concentrated load. From the graph, it can be observed that mass concentration (C) started by increasing and oscillates uniformly for different values of angular frequency. From Fig. 4, it is observed that the temperature change of the material exhibits a slight decrease for different values of the angular frequency. This variation in the temperature profile indicates that the angular frequency significantly influences the conductive temperature behaviour of the thermoelastic media with diffusion.

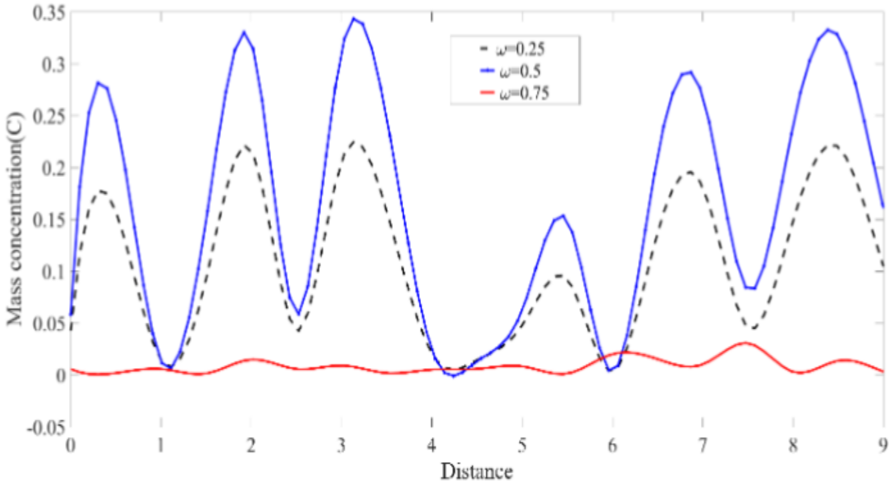

Fig. 3 Mass concentration due to angular frequency (concentrated load)

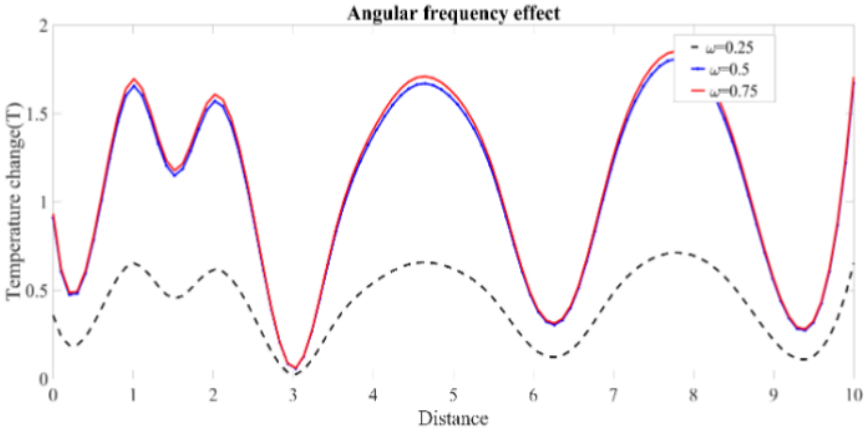

Fig. 4 Vatiation in temperature change due to angular frequency (concentrated load)

9. Conclusions

In this paper, we analyzed the effects of angular frequency on normal stress, shear stress, mass concentration and temperature change in thermoelastic media with diffusion subjected to concentrated load. In thermoelastic media with diffusion under the angular frequency domain, the coupled governing equations for normal stress, shear stress, mass concentration and temperature change are solved in the frequency domain, providing insights into the dynamic behaviour of the material. The interaction between thermal, elastic and diffusive fields significantly influences the material's response. From all the graphs change of angular frequency all ways make changes in nonlocal thermoelastic media with diffusion. This result has wide-ranging of applications in fields such as geophysics, materials science and engineering. For instance, it is essential for designing materials and structures subjected to dynamic loads, optimizing thermal management systems in complex media.

Data availability

All data generated or analysed during this study are included in this published article.

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