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Effects of changing materials properties for vibration of FGM beam using integral shear deformation model

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Abstract. The objective of this work is to study the effects of the modification of material properties on the vibration of the FGM beam using an integral shear strain model. In the present theory, the rotational displacement is replaced by an integral term in the displacement fields. The use of a shear correction factor is not necessary because our model gives a parabolic description of shear stress through the thickness while satisfying the conditions of zero shear stresses on the bottom and top surfaces of the beam. The FGM beam is assumed that the beam is a mixture of metal and ceramic, and that its properties change depending on the power functions of the thickness of the beam such as: linear, quadratic, cubic and inverse quadratic. By applying Hamilton's principle, general formulas were obtained to obtain the frequencies of the FGM beam. The effects of changing compositional characteristics of materials presented by volume fraction of FGM beams with simply supported edges on free vibration and some mode shapes are investigated.

Keywords: effects of changing materials properties; FGM beam; integral shear deformation model; vibration

1. Introduction

Composite materials generally consist of two or more materials of different types and arranged in the form of a laminate, the combination of which results in an assembly with higher performance than those of the components taken separately. However, the interface of these layers

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poses a major problem which is the discontinuity of properties and constraints at this level. This discontinuity causes high concentrations of stresses, a separation between the layers due to a sudden passage in the material component. A technique has been developed by a group of Japanese researchers to overcome these harmful effects; it consists of using an evaluated gradient material, its original meaning is FGM: Functionally Graded Material (Niino 1987, Kieback *et al.* 2003, Bharti *et al.* 2013, Bouazza *et al.* 2011, 2017, 2018, Bouazza and Adda-Bedia 2013).

Over the past three decades, FGM materials have attracted significant attention and interest due to their advantages; therefore their use is additional in several fields such as civil engenering and constructions navels and also as a source of bio-mechanical energy (Amara *et al.* 2016, Ellali *et al.* 2022a, b, 2023a, b, 2024a, b, c, d, Bouazza and Zenkour 2021, Fattahi *et al.* 2021, Feng *et al.* 2023, Safaei *et al.* 2022, 2023, Nuhu and Safaei 2022a, b, Rahmani *et al.* 2022, Pan *et al.* 2022, Ding *et al.* 2023, Alhijazi *et al.* 2023, Boucheta *et al.* 2016, 2024, Tounsi *et al.* 2004, 2005).

Several beam theories have been developed to investigate the behavior of functionally noted (FG) beams through the increased use of composite and FGMs in engineering structures (Derbale *et al.* 2021, Bouazza *et al.* 2015, 2017, 2019, Selmi 2020). The plate and beam theory and is applicable to FG (Dihaj *et al.* 2018, Abdelaziz *et al.* 2017, Sahnoun *et al.* 2013, Atmane, *et al.* 2016). A precise and detailed vibration analysis of FGM beams has attracted the attention of many researchers. For example, the free vibration analysis of TD-FGM nanobeams with small scale effects is presented by Mamen *et al.* (2022) based on the Hamilton principle and the Quasi-3D three variable models. In order to take into account the shear effect, a number of first order shear strain theories have been developed. However, these theories assume a uniform distribution of transverse shear strains across the thickness. For this purpose, shear correction factors are introduced to work around this problem (Timoshenko 1921, Xu *et al.* 2018, Mellouli *et al.* 2019). In order to take into account the shear effect, a number of first order shear strain theories have been developed. However, these theories assume a uniform distribution of transverse shear strains across the thickness. For this purpose, shear correction factors are introduced to work around this problem (Alazwari *et al.* 2022, Abdelrahman *et al.* 2022, Kumar and Srinivas 2017). To avoid the use of a shear correction factor and to have a better response prediction of FG beams, the higher order shear deformity theories have been developed (Reddy 1987, Basha *et al.* 2022, Simsek 2010, Nguyen and Tran 2018, Bouazza *et al.* 2019, Thai and Vo 2012, Ellali *et al.* 2018, Vo *et al.* 2014b, Xin and Kiani 2023, Sayyad and Ghugal 2018). Moreover, recently, many new types of research have been carried out in the field of behavioral studies of porous structures (Safaei 2020, 2021, Ellali 2024).

It appears from the available literature that the effects of changing the characteristics of the beams, the composition of the constituent material present by the volume fraction of the material on the vibration of FGM beams by the integral shear deformation model has not been fully investigated. The paper introduces a new higher-order methodology and a new displacement field that integrates indeterminate integral terms, to explore the vibration behavior of FG beam. It is assumed that the beam is a mixture of metal and ceramic whose properties change with the thickness of the beam with four functions of power such as: linear, quadratic, cubic and inverse quadratic. The results obtained are compared with the results of the literature.

2. Model mathematic

2.1 Material proprieties of FGM beam

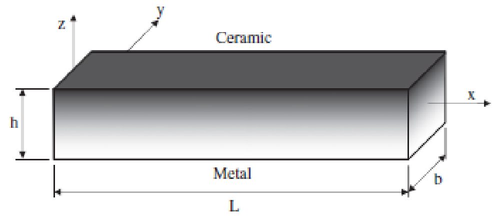

Fig. 1 Geometry and coordinate of a FG beam

Consider a beam as shown in Fig. 1, where h is the thickness of the beam. It is assumed that the FGM beam is made of a mixture of a metal phase (denoted by "m") and a ceramic phase (denoted by "c"), with the material composition varying smoothly along its thickness direction (i.e., in the z-axis) only. Thus, the materials properties of FGM, like Young's modulus E, can be expressed as, Shanab *et al.* (2022)

$$P(z) = P_m + (P_c - P_m)V_c \tag{1}$$

Where P_c and P_m are the material properties of the ceramic and metal. V_c is the ceramic volume fractions and we assume follows a simple power law as

$$\begin{array}{ll} \text{P-FGM:} & V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^k \\ \text{Linear:} & V_c = {}^{z}/_{h} + 0.5 \\ \text{Quadratic:} & V_c = \left({}^{z}/_{h} + 0.5\right)^2 \\ \text{Cubic:} & V_c = 3\left({}^{z}/_{h} + 0.5\right)^2 - 2\left({}^{z}/_{h} + 0.5\right)^3 \\ \text{Inverse quadratic:} & V_c = 1 - \left(0.5 - {}^{z}/_{h}\right)^2 \end{array} \tag{2}$$

2.2 Integral shear deformation model

In this work, a new model of shear strain FGM beam was developed based on the assumptions of the polynomial shear strain theory in which the axial displacements contain an integral component, expressed by

$$\begin{gathered} u(x,z,t) = u_0(x,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,t)dx \\ w(x,z,t) = w_0(x,t) \end{gathered} \tag{3}$$

Note that integrals have no limits. In the present study, we consider the shape function $f(z)$ which defines the distribution of transverse shear stresses through the thickness can be expressed as follows

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{4}$$

The non-zero deformations of this model are

$$\begin{gathered} \varepsilon_x = \varepsilon_x^0 + zk_x^b + f(z)k_x^s \\ \gamma_{xz} = g(z)\gamma_{xz}^0 \end{gathered} \tag{5}$$

Where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\ k_x^b = -\frac{\partial^2 w_0}{\partial x^2},\ k_x^s = k_1\theta$$
$$\gamma_{xz}^0 = k_1 \int \theta\, dx\ ,\ g(z) = \frac{df(z)}{dz} \tag{6}$$

The integral terms used in this model can be solved by the use of Navier procedure and can be given as follows

$$\int \theta\, dx = A' \frac{\partial \theta}{\partial x} \tag{7}$$

Where A', the coefficients is expressed according to the type of solution used. In this case of Navier's method, A' is given by

$$A' = -\frac{1}{\alpha^2}$$
$$k_1 = \alpha^2 \tag{8}$$

Where α is defined in the formulation of Navier method (Eq (21)).

In the elastic domain the constitutive relations can be given as follows

$$\sigma_x = Q_{11}(z)\varepsilon_x$$
$$\tau_{xz} = Q_{55}(z)\gamma_{xz} \tag{9}$$

Where, the elastic constants of elasticity of rigidity Q_{11}, Q_{55} of materials FGM beam expressed by

$$Q_{11} = E(z)$$
$$Q_{55} = \frac{E(z)}{2(1+\nu)} \tag{10}$$

To obtain the equations of motion using Hamilton's principle which express analytically as

$$\int_0^t (\delta U - \delta K) dt = 0 \tag{11}$$

where t is the time, δU is the change in strain energy; and δK is the variation of the work exterior generated by external loads applied to the FGM beam. The variation in the strain energy of the FGM beam is stated by

$$\delta U = \int_V [\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz}] dV \tag{12}$$

By replacing Eqs. (5) and (9) in Eq. (12) and integrating it into the thickness of the FGM beam, Eq. (12) can be rewritten as the following form

$$\delta U = \int_A [N_x \delta\varepsilon_x^0 + M_x^b \delta k_x^b + M_x^s \delta k_x^s + S_{xz}^s \delta\gamma_{xz}^s] dA \tag{13}$$

N, M_x^b, M_x^s and S_{xz}^s can be given as

$$\left(N_x, M_x^b, M_x^s\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(1, z, f(z)\right) \sigma_x dz$$
$$S_{xz}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} g(z) \tau_{xz} dz \tag{14}$$

The change in kinetic energy can be given as

$$\delta K = \int_V [\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}]\rho(z)dV \tag{15}$$

So

$$\delta K = \int_A \left\{ I_0(\dot{u}_0\delta\dot{u}_0 + \dot{w}_0\delta\dot{w}_0) - I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0\right) + J_1\left((k_1A')\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0\right)\right) + \right.$$
$$\left. I_2\left(\frac{\partial\delta\dot{w}_0}{\partial x}\frac{\partial\dot{w}_0}{\partial x}\right) - J_2\left((k_1A')\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right)\right) + K_2\left((k_1A')^2\left(\frac{\partial\delta\dot{\theta}}{\partial x}\frac{\partial\dot{\theta}}{\partial x}\right)\right)\right\} dA \tag{16}$$

The moments of inertia are expressed as a function of the density is given as follows

$$(I_0, I_1, I_1) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, z^2)\rho(z)dz$$
$$(J_1, J_1, K_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (f(z), zf(z), f(z)^2)\rho(z)dz \tag{17}$$

By substituting the expressions for strain displacement as well as strain stress in the Hamilton energy equation. The equations of motion take the following form

$$\delta u_0:\ \frac{\partial N_x}{\partial x} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} + k_1A'J_1\frac{\partial\ddot{\theta}}{\partial x}$$
$$\delta w_0:\ \frac{\partial^2 M_x^b}{\partial x^2} = I_0\ddot{w}_0 + I_1\frac{\partial\ddot{u}_0}{\partial x} - I_2\frac{\partial^2\ddot{w}_0}{\partial x^2} + J_2(k_1A')\frac{\partial^2\ddot{\theta}}{\partial x^2} \tag{18}$$
$$\delta\theta:\ -k_1M_x^s + k_1A'\frac{\partial S_{xz}^s}{\partial x} = -J_1(k_1A')\frac{\partial\ddot{u}_0}{\partial x} + J_2(k_1A')\frac{\partial^2\ddot{w}_0}{\partial x^2} - K_2(k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2}$$

By substituting Eq. (9) in Eq. (14) and integrating along the thickness of the FGM beam, the resulting forces and moments are given by

$$\begin{Bmatrix} N_x \\ M_x^b \\ M_x^s \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} & B_{11}^s \\ B_{11} & D_{11} & D_{11}^s \\ B_{11}^s & D_{11}^s & H_{11}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x^b \\ k_x^s \end{Bmatrix}$$
$$S_{xz}^s = A_{55}^s\gamma_{xz}^0 \tag{19}$$

with

$$(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}(1, z, z^2, f(z), zf(z), f(z)^2)dz$$
$$A_{55}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55}[g(z)]^2 dz \tag{20}$$

In this work, Navier's method is used, and the displacements are expanded in double trigonometric Fourier series in terms of unknown parameters. The choice of the trigonometric function in the series is restricted to those which satisfy the boundary conditions of the problem given by the following displacement functions

$$\begin{Bmatrix} u_0(x,t) \\ w_0(x,t) \\ \theta(x,t) \end{Bmatrix} = \begin{Bmatrix} U_m\cos(\alpha x)e^{i\omega t} \\ W_m\sin(\alpha x)\,e^{i\omega t} \\ X_m\sin(\alpha x)\,e^{i\omega t} \end{Bmatrix} \tag{21}$$

Table 1 Comparison of fundamental frequency with the power-law index for different theories (L/h=5)

Theory	k						
	0	0.5	1	2	5	10	Full metal
PSDBT[a]	5.15274	4.41108	3.99042	3.62643	3.40120	3.28160	2.67732
TSDBT[a]	5.15313	4.41138	3.99067	3.62632	3.39979	3.28107	2.67753
HSDBT[a]	5.15274	4.41108	3.99042	3.62647	3.40136	3.28169	2.67732
ESDBT[a]	5.15422	4.41222	3.99139	3.62671	3.39905	3.28134	2.67810
ASDBT[a]	5.15422	4.41222	3.99139	3.62671	3.39905	3.28134	2.67810
Present	5.15275	4.41062	3.99041	3.62643	3.40118	3.28158	2.67732

[a]Simsek (2010)

With

$$\alpha = m\pi/L \tag{22}$$

By substituting Navier method displacement functions in Eq. (21), the solution to the vibration problem can be obtained as follows

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}\right) \begin{Bmatrix} U_m \\ W_m \\ X_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

With

$$\begin{gathered} S_{11} = -\alpha^2 A_{11}\ ; S_{12} = \alpha^3 B_{11}; S_{13} = \alpha k_1 B_{11}^s; S_{21} = \alpha^3 B_{11}; S_{22} = \alpha^4 D_{11}; S_{23} = -\alpha^2 k_1 D_{11}^s; \\ S_{31} = \alpha k_1 B_{11}^s; S_{32} = -\alpha^2 k_1 D_{11}^s; S_{32} = -{k_1}^2 H_{11}^s - \alpha^2 (k_1 A')^2 A_{55}^s m_{11} = -I_0; m_{12} = \alpha I_1; \\ m_{13} = -\alpha k_1 A' J_1\ ; m_{21} = \alpha\, I_1 m_{22} = -\, I_0 - \alpha^2\, I_2;\ m_{23} = \alpha^2\, k_1 A'\, J_2; m_{31} = \\ -\alpha\, k_1 A'\, J_1 m_{32} = \alpha^2\, k_1 A'\, J_2; m_{33} = -\alpha^2 (\, k_1 A')^2 K_2 \end{gathered} \tag{24}$$

3. Numerical results

In this paper, the behaviors of FGM beams in free vibrations are analyzed for several volume fractions. The FGM beam consists of a mixture of metal (aluminum, Al) and ceramic (alumina, Al2O3) with the following characteristics: Metal (aluminum, Al): E_m =70 GPa, $\nu_m = 0.3, \rho_m = 2702\,\mathrm{kg/m^3}$. Ceramic (alumina, Al2O3): $E_c = 380$ GPa, $\nu_c = 0.3, \rho_c = 3960\,\mathrm{kg/m^3}$

Based on the mathematical formulations of the new FGM beam theory, a computer code is developed which allows the free vibration analysis of FGM beams to be performed. In order to ensure the accuracy of this model for analyzes vibration of FGM simply supported beams was examined by comparing the results with the results in the literature.

First, numerical results of the fundamental frequencies of FGM beams with simple-simple supports (S-S) are given in Tables 1-5 for two different values of the geometric ratio, (L/h=5 and 20). FGM beams consist of aluminum (Al) and alumina (Al2O3) and the properties of FGM beams change through thickness by power law. The bottom surfaces of FGM beams are rich in aluminum, while the upper surfaces of FGM beams are rich in alumina. In this work, non-dimensioned fundamental frequencies are given by: $\lambda^2 = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$.

Table 2 Comparison of non-dimensional fundamental frequency with the power-law index for different theories (L/h=20)

Theory	k						
	0	0.5	1	2	5	10	Full metal
PSDBT[a]	5.46030	4.65159	4.20503	3.83611	3.64850	3.53896	2.83716
TSDBT[a]	5.46036	4.65159	4.20509	3.83611	3.64838	3.53896	2.83716
HSDBT[a]	5.46030	4.65159	4.20503	3.83611	3.64850	3.53902	2.83716
ESDBT[a]	5.46042	4.65165	4.20515	3.83617	3.64832	3.53896	2.83722
ASDBT[a]	5.46042	4.65165	4.20515	3.83617	3.64832	3.53896	2.83722
Present	5.46031	4.65112	4.20506	3.83614	3.64851	3.53901	2.83714

[a]Simsek (2010)

Table 3 Comparison of non-dimensional flexural natural of simply supported functionally graded (P-FGM) beams first three frequency modes (L/h=5)

Modes	Theory	k					
		0	1	2	5	10	Full metal
1	CBT[a]	5.3953	4.1484	3.7793	3.5949	3.4921	2.80336
	TBT[b]	5.15247	3.99023	3.63438	3.43119	3.31343	2.67718
	HSDT[c]	5.15274	3.99042	3.62643	3.40120	3.28160	2.67732
	HSDT[d]	5.15275	3.99042	3.62644	3.40120	3.28160	2.67732
	HSDT[e]	5.15275	3.97160	3.59791	3.37429	3.26534	2.67732
	ESDT[a]	5.15423	3.99140	3.62671	3.40000	3.28135	2.67810
	Present	5.15275	3.99041	3.62643	3.40118	3.28158	2.67732
2	CBT[a]	20.6187	15.7982	14.3260	13.5876	13.2376	-
	HSDT[d]	17.8810	14.0098	12.6407	11.5444	11.0246	-
	ESDT[a]	17.8996	14.0224	12.6466	11.5281	11.0264	9.03573
	Present	17.8812	14.0100	12.6405	11.5432	11.0241	9.29093
3	CBT[a]	43.3483	33.0278	29.7458	28.0850	27.4752	-
	HSDT[d]	34.2085	27.0971	24.3151	21.7187	20.5569	-
	ESDT[a]	33.3835	27.2496	24.6889	21.3661	20.5815	17.3457
	Present	34.2097	27.0980	24.3150	21.7159	20.5561	17.7751

[a]Sayyad and Ghugal (2018); [b]Simsek (2010); [c]Reddy (1984); [d]Thai and Vo (2012); [e]Vo *et al.* (2014)

To demonstrate the precision of the present model, the results obtained are compared with the work existing in the literature. The first natural frequencies of an FGM (S-S) beam consist of ceramic and aluminum are compared with the results of Simsek (2010). In this contribution, employing a new displacement field which includes indeterminate integrated terms.

As shown in Tables 1 and 2, the current frequencies agree well with the results of Simsek (2010) and the difference between the frequencies of the two studies is very low. Comparison of Table 1 with Table 2, the response are very similar, but the non-dimension fundamental frequency increases when the geometric parameter a/h is increased. In addition, the non-dimensional fundamental frequency increases as the volume fraction index k decreases.

Still with the aim of validating the present model to predict the natural bending frequencies of fully supported FGM beams. Natural frequencies of FGM beams were calculated for different

Table 4 Comparison of non-dimensional flexural natural of simply supported functionally graded (P-FGM) beams first three frequency modes (L/h=20)

Modes	Theory	k					
		0	1	2	5	10	Full metal
1	CBT[a]	5.47770	4.21630	3.84720	3.66280	3.55470	2.84618
	TBT[b]	5.46032	4.20505	3.83676	3.65088	3.54156	2.83713
	HSDT[c]	5.46030	4.20503	3.83611	3.64850	3.53896	2.83716
	HSDT[d]	5.46032	4.20505	3.83613	3.64849	3.53899	2.83714
	HSDT[e]	5.46032	4.20387	3.83428	3.64663	3.53787	2.83714
	ESDT[a]	5.46043	4.20513	3.83614	3.64830	3.53895	2.83720
	Present	5.46031	4.20506	3.83614	3.64851	3.53901	2.83714
2	CBT[a]	21.8438	16.8100	15.3334	14.5959	14.1676	-
	HSDT[d]	21.5732	16.6344	15.1619	14.3748	13.9264	-
	ESDT[a]	21.5738	16.6342	15.1620	14.3670	13.9147	11.2096
	Present	21.5732	16.6343	15.1619	14.3746	13.9262	11.2093
3	CBT[a]	48.8999	37.6173	34.2954	32.6357	31.6883	-
	HSDT[d]	47.5930	36.7679	33.4691	31.5789	30.5373	-
	ESDT[a]	47.5913	36.8705	33.6304	31.5655	30.5349	27.5383
	Present	47.5928	36.7675	33.4688	31.5779	30.5366	24.72890

[a]Sayyad and Ghugal (2018); [b]Simsek (2010); [c]Reddy (1984); [d]Thai and Vo (2012); [e]Vo *et al.* (2014).

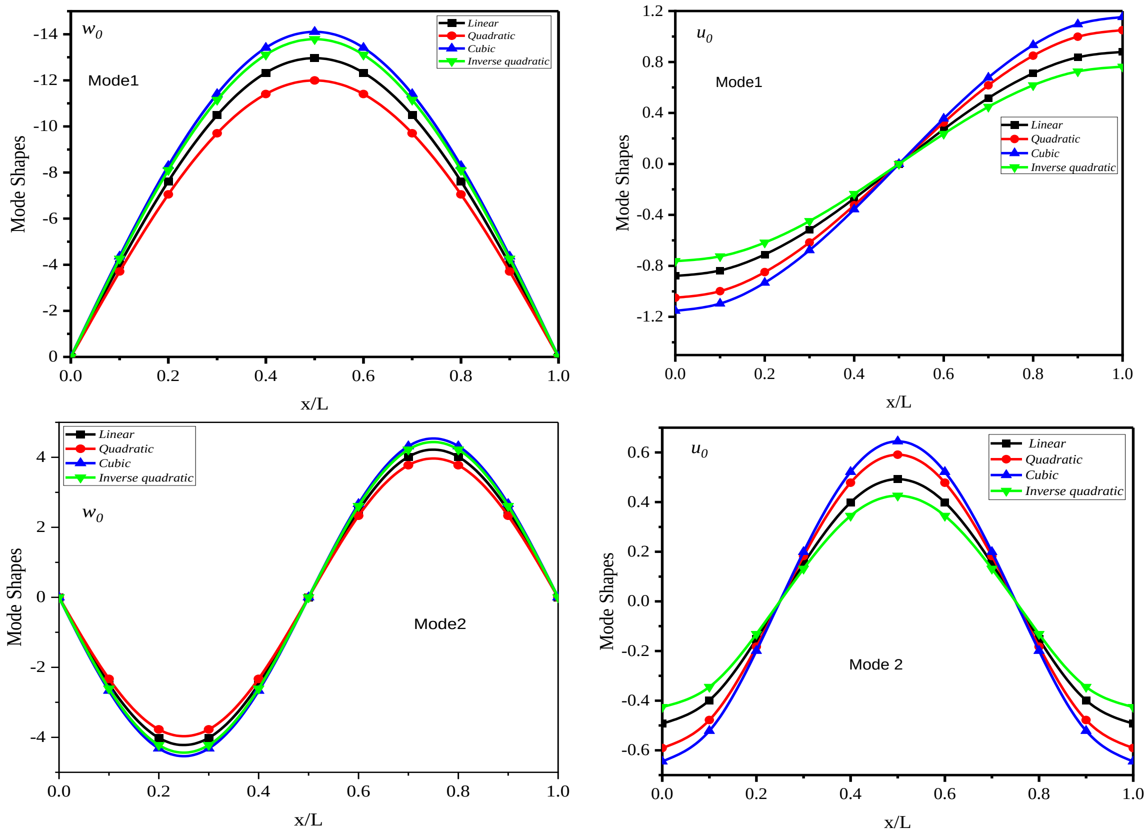

Fig. 2 Five first mode shapes of FG beam (L/h=5)

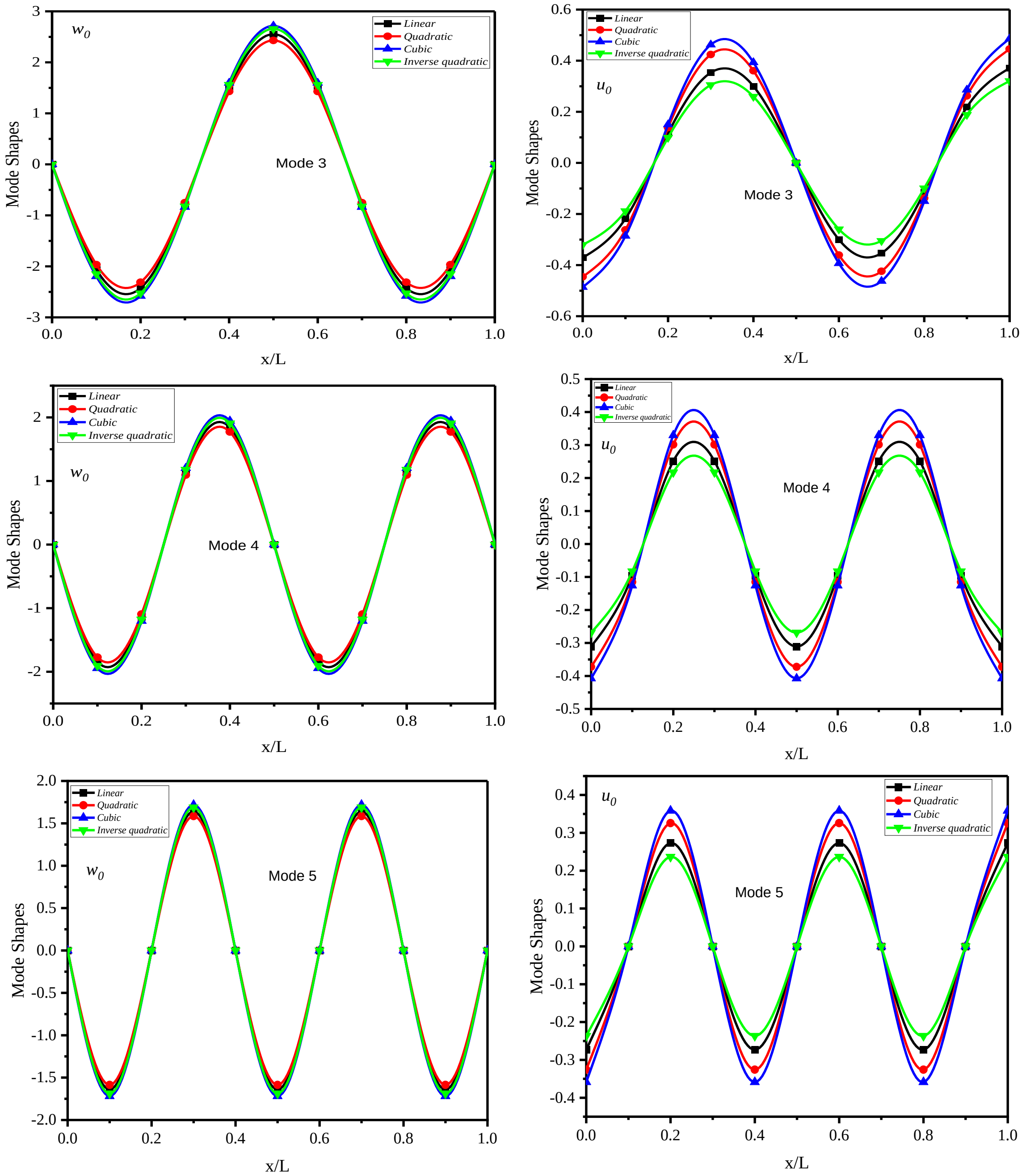

Fig. 2 Continued

values of the power law index are presented in Tables 3 and 4. The results obtained from the present model are compared to those given by Simsek (2010) based on the theory of Timoshenko; Reddy (1984) using higher order beam theory; Thai and Vo (2012) using higher order beam theory based on the following assumptions that axial and transverse displacements are divided into

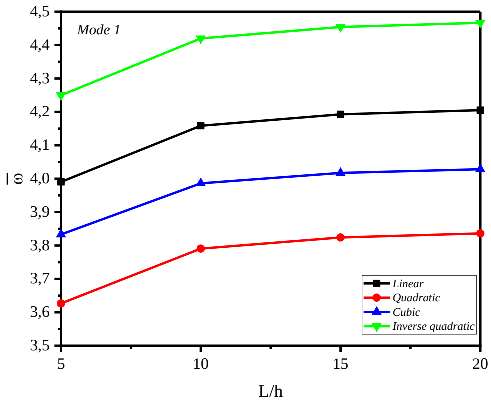

Fig. 3 Variation of first frequency parameter with *L/h* ratio for different material distribution

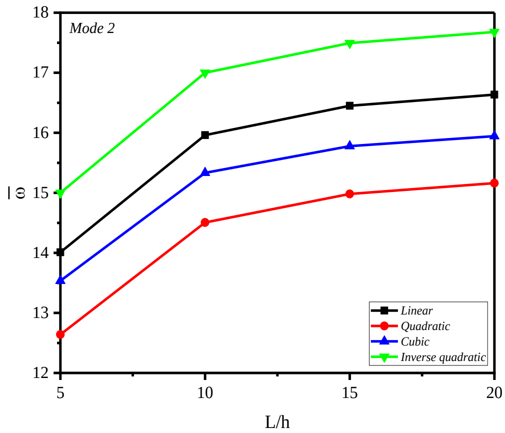

Fig. 4 Variation of second frequency parameter with *L/h* ratio for different material distribution

bending and shear components; Vo *et al.* (2014b) based on a refined shear strain theory and a finite element model; exponential shear deformation theory and the classical Euler-Bernoulli beam theory developed by Sayyad and Ghugal (2018). Also, physically, transverse shear deformation has considerable effect on the natural bending frequencies of FG beam, especially for a thick functionally graded beam.

The variation of the vibration mode of the aluminum-alumina FGM beams for different fractions of volume V_c following simple power laws are plotted in Fig. 2 such that the profile composition have linear, quadratic, cubic, inverse quadratic, respectively. The geometric ratio was chosen equal 5 (*L/h*=5). u_0 and w_0 relative measures of the displacement components of the median surface along the x and z directions, respectively.

In Figs. 3-5 we present the variations of the first three frequencies with geometric ratio (*L/h*) for different material distribution the results were found by this new approach. The composition of the beams graduated by two constituent materials Al and Al2O3 with a variation linear, cubic, quadratic, inverse quadratic. As seen from the figure, the (*L/h*) ratio is more efficient for higher modes. However, we see that the values of the quadratic case are lower than the values of frequency parameter of the cubic, linear and inverse quadratic case; whereas, the values of the

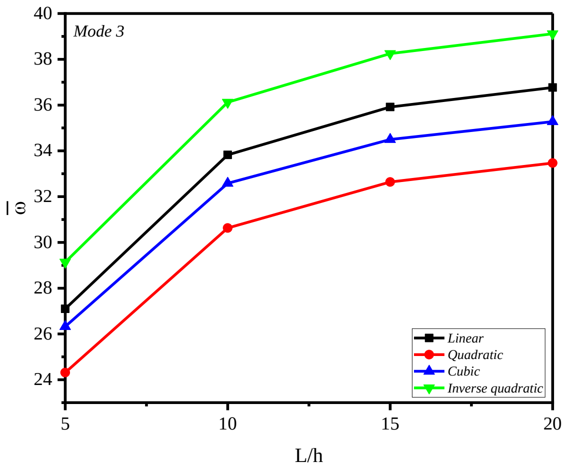

Fig. 5 Variation of third frequency parameter with *L/h* ratio for different material distribution

inverse quadratic case are greater than the values of the frequency parameter of the cubic and linear cases. In addition, the line corresponding to the case of the linear composition profile is greater than the values of the frequency parameter of cubic case. In practical applications the linear composition profile is therefore widely used, unless specific conditions require cubic, quadratic, inverse quadratic composition profiles.

4. Conclusions

In this work we have presented the steps of the analytical vibration analysis of functionally graduated beams using a new model. The novelty of this approach that displacements of rotations are replaced by terms in integral the fields of displacement. In addition, this model does not require a shear correction factor and gives a parabolic description of the shear stresses through the thickness while satisfying the conditions for nullity of the shear stresses on the bottom and top surfaces of the beam. It is assumed that the beam is a mixture of metal and ceramic that its properties changes as the power functions of the beam thickness. The results of this work are compared with the data available in the literature. The numerical results support the following conclusions:

When the ratio length-thickness of the FGM beam increases, the effects of the compositional profile to the value of the dimensionless frequency parameter increase considerably.

For this case, when linear, cubic, quadratic and inverse quadratic profiles are compared with each other the highest effect on the frequency parameter is observed in the inverse quadratic profile. When the compositional profile changes quadratic, the value of the dimensionless frequency parameter is the lowest.

In the application of technology, the study of this phenomenon is very essential to choose suitable material parameters of FGM beams.

Finally, we can say that this modest work presents a contribution of understanding the behavior of composite materials with gradient of properties and precisely in free vibration.

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