Advances in Aircraft and Spacecraft Science, *Vol. 12, No. 2 (2025) 117-139*
https://doi.org/10.12989/aas.2025.12.2.117

A new higher-order displacement model for laminated composite cylindrical shells

Ali Meksi[1], Kada Draiche*[2,3], Emrah Madenci[4,5] and Abdelouahed Tounsi[3,6,7]

[1]University Mustapha Stambouli of Mascara, BP 305, 29000 Mascara, Algeria
[2]Department of Civil Engineering, University of Tiaret, BP 78 Zaaroura, 14000 Tiaret, Algeria
[3]Material and Hydrology Laboratory, Faculty of Technology, Civil Engineering Department, University of Sidi Bel Abbes, Algeria
[4]Department of Civil Engineering, Necmettin Erbakan University, 42090, Konya, Turkey
[5]Department of Technical Sciences, Western Caspian University, Baku 1001, Azerbaijan
[6]Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia
[7]Department of Civil and Environmental Engineering, Lebanese American University, 309 Bassil Building, Byblos, Lebanon

(Received October 26, 2024, Revised May 6, 2025, Accepted May 9, 2025)

Abstract. This paper presents a unified solution approach to investigate the bending and free vibration behaviors of laminated composite cylindrical shells with varying radii of curvature and simply supported edges, using a new refined shear deformation shell theory (RSDST). The theoretical formulation of the proposed approach is based on a new displacement model that incorporates undetermined integral terms to account for the effects of transverse shear deformation. It also meets the shear stress-free boundary conditions on the upper and lower surfaces of the cylindrical shell. The governing equations are derived from the principle of virtual work and are resolved using Navier-type closed form solutions. The effects of material properties and geometric parameters on the static bending and free vibration of laminated composite cylindrical shells are presented and discussed in detail. Convergence and validation studies clearly indicate that the values for displacements and stresses derived from the present theory are highly consistent with those of previous higher-order shell theories. Furthermore, a satisfactory convergence was observed when compared with 3D elasticity solutions (the percentage errors for transverse shear stresses are a maximum of 1.38%, 3.19% and 21.33% for isotropic, orthotropic and laminated composite cylindrical shells, respectively). It is shown that the present model with only four variables is able to accurately predict the stress distributions and natural frequencies, with less computational effort compared to conventional HSDST models.

Keywords: bending; free vibration; laminated cylindrical shells; new displacement model; RSDST

1. Introduction

Over the past few decades, numerous researchers have taken an active interest in the study of plates and shells, with the aim of developing a holistic theory to adequately describe their mechanical behavior. Indeed, cylindrical shells made up of fiber-reinforced composite materials are widely used as the most important structural components in many modern engineering fields,

*Corresponding author, Professor, E-mail: kdraiche@yahoo.fr

http://www.techno-press.org/?journal=aas&subpage=7 ISSN: 2287-528X (Print), 2287-5271 (Online)

such as aerospace, automotive, marine, civil engineering, petrochemical industries and other applications (Errico *et al.* 2019, Lai *et al.* 2022, Sobhani 2023). This is due to their excellent mechanical properties, especially their high specific modulus, high specific strength, and their good corrosion and fatigue resistance compared to traditional metallic materials. The increasing demand for cylindrical shells in engineering has piqued the curiosity of numerous researchers, and has been the subject of extensive research in order to find appropriate models and simulations that best reflect the real behavior and control the vibration of these structures, and combine simplicity and economy in terms of cost and computation time. In this regard, numerous theories of shells have been developed, including the classical shell theory, commonly known as "CST", the first-order shear deformation shell theory "FSDST", and the higher-order shear deformation shell theory "HSDST". All of these shell theories are known as the class of equivalent single-layer models "ESLM", which are founded on certain assumptions regarding the kinematics of strain/stress throughout the thickness.

The first category has been demonstrated to yield satisfactory results for thin shells, wherein transverse shear strains are omitted without considering rotatory inertia. With this background, Simitses and Chen (1988) studied the buckling behavior of thin delaminated cylindrical panels under uniform pressure using Koiter-Budiansky formulations based on CST. Sivadas and Ganesan (1991) used Love's first approximation thin shell theory in conjunction with a semi-analytical finite element procedure to determine the natural frequencies of laminated conical shells with variable thickness under two types of boundary conditions. Lam and Loy (1994) have presented a theoretical analysis based on the CST to investigate the vibrations of simply supported rotating laminated composite cylindrical shells. In this study, the influence of various parameters, such as thickness-to-radius ratio, length-to-radius ratio, as well as lamination schemes on frequencies is discussed. To obtain the governing differential equations and solve the dynamic problems of multi-stepped circular cylindrical shells with arbitrary boundary conditions, Tang *et al.* (2017) developed a semi-analytical method based on Flügge's thin shell theory. Li *et al.* (2018) employed the CST to examine the resonance parameters of a rotating FG cylindrical thin shell subjected to a thermal environment, taking into consideration the temperature-dependent properties of materials and Hamilton's principle. In the last few years, Zhong *et al.* (2019) proposed a new exact solution using the CST to analyze the free vibration response of cross-ply laminated cylindrical shells. However, the implementation of this theory is imprecise for moderately thick and/or thick composite shells, and can lead to large errors due to neglecting transverse shear deformation effects and rotatory inertia. For these motivations, Reissner (1955) endeavored to build a first-order approximation theory of shells to study the nonlinear dynamic behavior of circular cylindrical shells, where some of the shortcomings of Love's elementary theory were eliminated, rendering the system of equations uncomplicated. This category is called first-order shear deformation shell theory "FSDST", in which the transverse shear deformation effects are included, but it does not satisfy the zero shear conditions at the top and bottom surfaces of the shell. This requires the introduction of a shear correction factor to properly account for the strain energy of shear deformation (Swami and Ghugal 2021). A similar approach was used later by Naghdi (1973) for static bending analysis of moderately thick plates and shells based on some kinematic assumptions of Reissner. Similarly, many studies based on the FSDST concept have been widely adopted by many researchers during the last few years to improve the analysis of orthotropic and multilayered composite shells (Qatu 1999, Jafari *et al.* 2005, Topal 2009, Torkamani *et al.* 2009, Civalek 2013, *et al.* Ghasemi 2013, Sheng and Wang 2018, Satankar *et al.* 2020, Li *et al.* 2024).

To address the poor performance of CST and FSDST for laminated composite shells, with the possibility of including realistic parabolic variation of transverse shear stresses across the thickness of shell structures without resorting to a shear correction coefficient (as opposed to first-order shear deformation theory). This trend has motivated researchers to try to find new modeling strategies to accurately solve the problem of composite shell structures based on higher-order shear deformation shell theories "HSDSTs". In this regard, several of the most important contributions reported in the literature are mentioned here. Reddy and Liu (1985) presented a simple HSDST for the mechanical analysis of cross-ply laminated composite shells. This model involves the same unknown variables as in FSDST, in which the displacements of the middle surface are developed as cubic functions of the thickness coordinate, and the transverse displacement is supposed to be constant across the thickness. An eight-noded isoparametric finite element model based on a new higher-order theory was presented by Bhaskar and Varada (1991) for the bending analysis of laminated anisotropic cylindrical shells of revolution under various boundary conditions. The accuracy of this model is verified for the estimation of all the displacements and stresses. In 1996, Liew and Lim developed a higher-order shear deformation shell theory for the free vibrations of doubly curved shallow shells with arbitrary boundary conditions. The theory accounts for the cubic distribution of transverse shear strains across the shell thickness, contrary to the parabolic shear distribution reported by Reddy and Liu (1985). Matsunaga (2007) used the higher-order shear and normal deformation theory in conjunction with Navier's solution method to obtain the natural frequencies and critical buckling loads of cross-ply laminated composite shallow shells. Furthermore, the nonlinear finite element method, founded on higher-order shear deformation theory in conjunction with Green-Lagrange nonlinear strains, has also been applied. Panda and Singh (2010), Mahapatra and Panda (2014), Kar *et al.* (2015) and Singh *et al.* (2016) have used this approach to investigate the effects of various geometries and material properties on the bending, thermal post-buckling and nonlinear free vibration responses of laminated composite plate/spherical shell panels subjected to mechanical loading, as well as to thermal environments. However, Mantari and his colleagues (2011) utilized a novel HSDST for the static and free vibration analysis of laminated cylindrical and spherical shells. This approach is analogous to that proposed by Reddy and Liu (1985), which fulfils tangential boundary conditions without stress on the shell surfaces. Kumar *et al.* (2013) presented the free vibration of laminated composite cylindrical shells with rectangular cutouts based on the HSDST model and a nine-noded isoparametric finite element formulation, considering different curvatures, thicknesses and boundary conditions. Tornabene *et al.* (2013) employed a generalized differential quadrature method combined with a 2D higher-order ESL theory to evaluate the frequency parameters of doubly-curved laminated composite shells and panels having different curvatures. Sahoo *et al.* (2016) employed two higher-order shear deformation theories in conjunction with finite element methods to analyze the bending, free vibration and transient responses of laminated composite plates. A finite element formulation based on a modified higher-order displacement field has been developed by Thakur *et al.* (2017) to analyze the static and free responses of doubly curved laminated shells. Further, Sharma and their colleagues (2017, 2018) developed a HSDST based on a coupled finite and boundary element formulation to investigate the vibro-acoustic behavior of laminated composite single/doubly curved panels subjected to harmonic excitation. Various support conditions and geometrical parameters were considered in their study. Monge *et al.* (2019) adopted a new hybrid refined shear deformation model to examine the flexural behavior of simply supported laminated composite doubly-curved shells subjected to various types of mechanical loadings. Shinde and Sayyad (2020) have utilized a fifth-order shear and normal deformation theory with the inclusion of a polynomial shape function for the static bending analysis of laminated and sandwich spherical shells. The spline approximation method based on third-order shear deformation theory was used by Saad Al Nuwairan and Javed (2021) to investigate the free vibration of composite cylindrical shells. In other research, Ramteke *et al.* (2021) have evaluated the effect of porosity distribution and grading patterns on the stress response of different types of functionally graded porous structures. They have employed an isoparametric finite element approach within the framework of the HSDT kinematics, a method which has been proven to deliver consistently accurate results. Consequently, Sahu *et al.* (2022) utilized a finite element formulation based on Newmark's time integration method and a higher order kinematic model to examine the dynamic deflection of fibre-reinforced hybrid curved composite shell panels under mechanical loading in the elevated thermal environment. Kumar *et al.* (2023) combined a finite element method with higher-order shear deformation theory to derive the numerical result for the frequencies of variable stiffness composite laminated "VSCL" flat-panel structures. In recent times, Draiche and his colleagues (2024) have developed an enhanced mathematical model for the flexural and free vibration response of doubly-curved shell structures made of advanced composite materials. This model is based on a new refined sinusoidal shear deformation shell theory.

This paper presents a new formulation combining a simple and refined shear deformation shell theory with the fundamental elasticity relations, with the aim of assessing the stress bending and free vibration characteristics of isotropic and laminated fiber-reinforced composite cylindrical shells. The present model, developed in this paper, is improved over the other existing higher-order shear deformation shell theories, in which the displacement field involves an undetermined integral component in order to reduce the number of unknowns, and takes into account the transverse shear deformation effect. The strain-displacement relations are induced from linear theory of elasticity. The governing equations are determined by applying the dynamic version of the principle of virtual work and subsequently solved for simply supported boundary conditions by using the Navier solution methodology. The results emerging from this theory show excellent agreement with those found in the literature. The impact of some parameters such as the curvature-radius-to-side ratio, the material properties, as well as the lamination schemes on the stress distributions and natural frequencies are explored in this analysis.

2. Theoretical formulation

2.1 Laminated cylindrical shells

We consider an elastic laminated cylindrical shell with width a, length b, a constant thickness h in the z-direction, and the principal radius of curvature R as shown in Fig. 1. The cylindrical shell consists of N number of orthotropic layers, which are perfectly bonded together and subjected to transverse mechanical loads $q(x,y)$ acting on the top surface (i.e., $z=-h/2$) of the cylindrical shell.

2.2 Displacement and constitutive equations

In this paper, we modify the conventional higher-order shear deformation shell theory developed by Sayyad and Ghugal (2022), which includes five unknowns, by introducing some simplifying assumptions in order to reduce the number of unknown variables. The generalized displacement field of the conventional HSDST located at any point (x,y,z) of the cylindrical shell

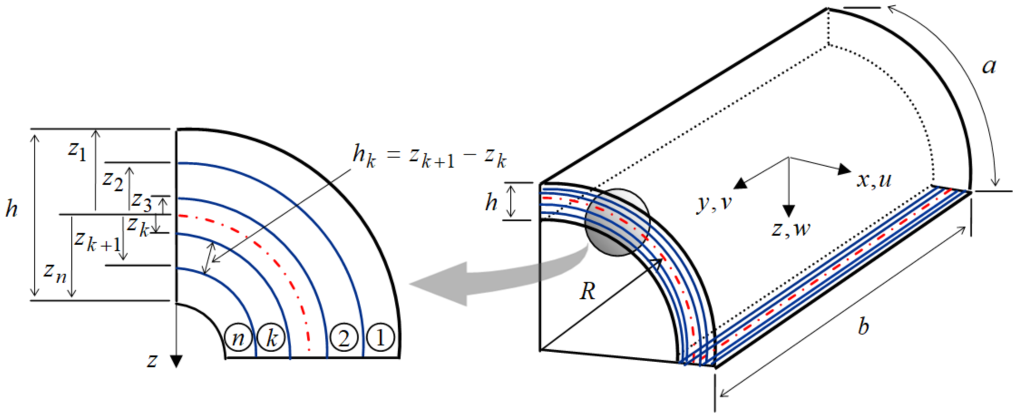


Fig. 1 Geometry and coordinate system of laminated composite cylindrical shell

can be defined as

$$\begin{aligned} u(x,y,z,t) &= \left(1+\frac{z}{R}\right)u_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial x} + f(z)\,\varphi(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial y} + f(z)\,\psi(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

Where u, v, w and u_0, v_0, w_0 represent the global and mid-plane displacement components along the x, y and z directions, respectively; φ and ψ are the unknown slopes of the transverse normal around y and x axes, respectively. Besides, $f(z)$ denotes the shape function that controls the distribution of transverse shear strains and stresses through the shell thickness. By assuming $\varphi(x,y,t) = \int \theta\,(x,y,t)dx$ and $\psi(x,y,t) = \int \theta\,(x,y,t)dy$, the new displacement field of the proposed shell theory can be rewritten with only four unknowns as follows

$$\begin{aligned} u(x,y,z,t) &= \left(1+\frac{z}{R}\right)u_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial x} + k_1 f(z)\int \theta(x,y,t)\,dx \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial y} + k_2 f(z)\int \theta(x,y,t)\,dy \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{2}$$

The integral terms defined in the previous equations can be solved by using the Navier-type procedure, and then the displacement field can be rewritten as follows

$$\begin{aligned} u(x,y,z,t) &= \left(1+\frac{z}{R}\right)u_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial x} + k_1 A' f(z)\frac{\partial \theta(x,y,t)}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial y} + k_2 B' f(z)\frac{\partial \theta(x,y,t)}{\partial y} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{3}$$

in which the parameters A', B', k_1 and k_2 can be expressed by

$$A' = -1/\alpha^2\,,\quad B' = -1/\beta^2\,,\quad k_1 = \alpha^2,\quad k_2 = \beta^2 \tag{4}$$

where

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b} \tag{5}$$

In this study, the shape function is taken according to Zenkour (2015) as a combination of hyperbolic and polynomial functions to achieve the zero shear stress conditions on the upper and lower surfaces of the cylindrical shell. This function is assumed by defining

$$f(z) = h\,sinh\left(\frac{z}{h}\right) - \frac{3z^3}{2h^2} \tag{6}$$

According to the displacement fields given in Eq. (3), the strain-displacement relations for a circular cylindrical shell of any material system, including the effects of transverse shear deformation, are given below

$$\begin{gathered}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u}{\partial x} + \dfrac{w}{R}\\ \dfrac{\partial v}{\partial y}\\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}\varepsilon_x^1\\ \varepsilon_y^1\\ \gamma_{xy}^1\end{Bmatrix} + f(z)\begin{Bmatrix}\varepsilon_x^2\\ \varepsilon_y^2\\ \gamma_{xy}^2\end{Bmatrix},\\ \begin{Bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} - \dfrac{u_0}{R}\\ \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{xz}^0\\ \gamma_{yz}^0\end{Bmatrix}\end{gathered} \tag{7}$$

where

$$\begin{gathered}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x} + \dfrac{w_0}{R}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}\varepsilon_x^1\\ \varepsilon_y^1\\ \gamma_{xy}^1\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\,\partial y}\end{Bmatrix},\\ \begin{Bmatrix}\varepsilon_x^2\\ \varepsilon_y^2\\ \gamma_{xy}^2\end{Bmatrix} = \begin{Bmatrix}k_1 A' \dfrac{\partial^2\theta}{\partial x^2}\\ k_2 B' \dfrac{\partial^2\theta}{\partial y^2}\\ (k_1 A' + k_2 B')\dfrac{\partial^2\theta}{\partial x\,\partial y}\end{Bmatrix},\end{gathered} \tag{8a}$$

$$\begin{Bmatrix}\gamma_{xz}^0\\ \gamma_{yz}^0\end{Bmatrix} = \begin{Bmatrix}k_1 A' \dfrac{\partial\theta}{\partial x}\\ k_2 B' \dfrac{\partial\theta}{\partial y}\end{Bmatrix} \tag{8b}$$

and

$$g(z) = \frac{df(z)}{dz} = cosh\left(\frac{z}{h}\right) - \frac{9z^2}{2h^2} \tag{9}$$

In this paper, each layer of the laminated composite shell is considered to be orthotropic in a two-dimensional stress state. For this purpose, the constitutive equations employed to obtain the stress-strain transformation method between the k^{th} orthotropic layer and the global coordinate system of the cylindrical shell are expressed as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(k)} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)} \tag{10}$$

where $\{\sigma\}^k$ and $\{\varepsilon\}^k$ are the stress and strain vectors, respectively. While $\overline{Q}_{ij}^{k}$ are the transformed elastic coefficients, and are yielded by Reddy (1984)

$$\begin{Bmatrix} \bar{Q}_{11}^k \\ \bar{Q}_{12}^k \\ \bar{Q}_{22}^k \\ \bar{Q}_{66}^k \end{Bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 4c^2s^2 \\ c^2s^2 & c^4+s^4 & c^2s^2 & -4c^2s^2 \\ s^4 & 2c^2s^2 & c^4 & 4c^2s^2 \\ c^2s^2 & -2c^2s^2 & c^2s^2 & (c^2-s^2)^2 \end{bmatrix} \begin{Bmatrix} Q_{11} \\ Q_{12} \\ Q_{22} \\ Q_{66} \end{Bmatrix}, \quad \begin{Bmatrix} \bar{Q}_{44}^k \\ \bar{Q}_{55}^k \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 \\ s^2 & c^2 \end{bmatrix} \begin{Bmatrix} Q_{44} \\ Q_{55} \end{Bmatrix} \tag{11}$$

where $c = cos\,\theta^k$, $s = sin\,\theta^k$ and θ^k denotes to the orientation of the k^{th} layer with respect to the global coordinate system of the cylindrical shell. Therefore Q_{ij} are the reduced elastic coefficients which can be determined using the engineering constants as follows

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}},\ Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}},\ Q_{12} = \frac{\nu_{12}E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{66} = G_{12},\ Q_{44} = G_{23},\ Q_{55} = G_{13} \tag{12}$$

2.3 Governing equations

The governing differential equations associated with the proposed four-variable displacement field would then be obtained using the principle of virtual work. This principle can be expressed in the following analytical form (Karami *et al.* 2018, Remil *et al.* 2019, Sayyad and Ghugal 2022)

$$\begin{aligned} &\int_{-h/2}^{h/2}\int_A \left(\sigma_x^{(k)}\delta\varepsilon_x + \sigma_y^{(k)}\delta\varepsilon_y + \tau_{xy}^{(k)}\delta\gamma_{xy} + \tau_{yz}^{(k)}\delta\gamma_{yz} + \tau_{xz}^{(k)}\delta\gamma_{xz}\right) dAdz \\ &- \int_A q(x,y)\delta w dA - \int_{-h/2}^{h/2}\int_A \rho^{(k)}(\dot{u}\delta\,\dot{u} + \dot{v}\delta\,\dot{v} + \dot{w}\delta\,\dot{w})\,dAdz = 0 \end{aligned} \tag{13}$$

where δ denotes the variational operator, $q(x,y)$ is the transverse load. Whereas, A and ρ are the top surface and the mass density of the cylindrical shell, respectively. By substituting the strains given in Eq. (7) into Eq. (13) and integrating with respect to z-direction yields the following expression

$$\int_0^a \int_0^b \left(\begin{array}{c} N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta\varepsilon_x^1 \\ +M_y^b \delta\varepsilon_y^1 + M_{xy}^b \delta\gamma_{xy}^1 + M_x^s \delta\varepsilon_x^2 + M_y^s \delta\varepsilon_y^2 \\ +M_{xy}^s \delta\gamma_{xy}^2 + S_{yz}^s \delta\gamma_{yz}^0 + S_{xz}^s \delta\gamma_{xz}^0 - q(x,y)\delta w_0 \\ -I_0(\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0) \\ +I_1\left(\dot{u}_0 \frac{\partial\delta\dot{w}_0}{\partial x} + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y}\right) \\ +I_1\left(\left(\frac{\partial\dot{w}_0}{\partial x} - \frac{2}{R}\dot{u}_0\right)\delta\dot{u}_0 + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right) \\ +I_2\left(\left(\frac{1}{R}\frac{\partial\dot{w}_0}{\partial x} - \frac{1}{R^2}\dot{u}_0\right)\delta\dot{u}_0 - \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) \\ +I_2\left(\frac{1}{R}\dot{u}_0 - \frac{\partial\dot{w}_0}{\partial x}\right)\frac{\partial\delta\dot{w}_0}{\partial x} - I_3 k_1 A' \frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0 \\ -I_3\left(k_1 A' \dot{u}_0 \frac{\partial\delta\dot{\theta}}{\partial x} + k_2 B'\left(\frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0 + \dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y}\right)\right) \\ -I_4 k_1 A'\left(\left(\frac{1}{R}\delta\dot{u}_0 - \frac{\partial\delta\dot{w}_0}{\partial x}\right)\frac{\partial\dot{\theta}}{\partial x}\right) \\ -I_4 k_1 A'\left(\frac{1}{R}\dot{u}_0 - \frac{\partial\dot{w}_0}{\partial x}\right)\frac{\partial\delta\dot{\theta}}{\partial x} \\ +I_4 k_2 B'\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) \\ -I_5\left((k_1 A')^2\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + (k_2 B')^2\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right) \end{array} \right) dydx = 0 \tag{14}$$

where point-superscript convention indicates time-related differentiation. The subscripts "*b*" and "*s*" are the moment resultants according to classical shell theory and the moment resultants deriving from shear deformation, respectively. While N, M^b, M^s and S^s are the force and moment resultants arising from the present theory and I_i $(i = 0, 1, 2, 3, 4, 5)$ are the inertia coefficients that can be defined by the following integrations over the thickness coordinates

$$\begin{gathered} \begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} \left(\sigma_x^{(k)}, \sigma_y^{(k)}, \tau_{xy}^{(k)}\right) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \\ \left(S_{xz}^s, S_{yz}^s\right) = \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} \left(\tau_{xz}^{(k)}, \tau_{yz}^{(k)}\right) g(z)\, dz \\ (I_0, I_1, I_2, I_3, I_4, I_5) = \sum_{k=1}^{N}\int_{h_k}^{h_{k+1}} \rho^{(k)}(1, z, z^2, f(z), z\, f(z), f^2(z))d\, z \end{gathered} \tag{15}$$

By substituting the relations from Eqs. (7) and (10) into the Eq. (15), the stress resultants of the present approach can be obtained in terms of strains by the following expressions

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & E_{11} & E_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & E_{12} & E_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & E_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & F_{11} & F_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & F_{12} & F_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & F_{66} \\ E_{11} & E_{12} & 0 & F_{11} & F_{12} & 0 & H_{11} & H_{12} & 0 \\ E_{12} & E_{22} & 0 & F_{12} & F_{22} & 0 & H_{12} & H_{22} & 0 \\ 0 & 0 & E_{66} & 0 & 0 & F_{66} & 0 & 0 & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \\ \varepsilon_x^2 \\ \varepsilon_y^2 \\ \gamma_{xy}^2 \end{Bmatrix} \tag{16a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{16b}$$

where $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}$ and A_{ij}^s are the shell stiffness coefficients given by

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right) = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \bar{Q}_{ij}^{(k)} \left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) d\,z, \quad i,j = 1,2,6 \tag{17a}$$

$$A_{ij}^s = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \overline{Q}_{ij}^{(k)} g^2(z) dz, \quad i,j = 4,5 \tag{17b}$$

By substituting the strains and stresses components shown in Eqs. (8) and (10) into Eq. (14) and after integration by parts, we can obtain the governing differential equations for the laminated cylindrical shell as follows

$$\begin{aligned}
&\delta u_0\!: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \left(I_0 + 2\frac{I_1}{R} + \frac{I_2}{R^2}\right)\ddot{u}_0 - \left(I_1 + \frac{I_2}{R}\right)\frac{\partial \ddot{w}_0}{\partial x} + k_1 A' \left(I_3 + \frac{I_4}{R}\right)\frac{\partial \ddot{\theta}}{\partial x} \\
&\delta v_0\!: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + I_3 k_2 B' \frac{\partial \ddot{\theta}}{\partial y} \\
&\delta w_0\!: \quad \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\,\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - \frac{N_x}{R} + q = I_0 \ddot{w}_0 \\
&+\left(I_1 + \frac{I_2}{R}\right)\frac{\partial \ddot{u}_0}{\partial x} + I_1 \frac{\partial \ddot{v}_0}{\partial y} - I_2 \frac{\partial^2 \ddot{w}_0}{\partial x^2} - I_2 \frac{\partial^2 \ddot{w}_0}{\partial y^2} + I_4 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + I_4 k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} \\
&\delta \theta\!: \quad -k_1 A' \frac{\partial^2 M_x^s}{\partial x^2} - k_2 B' \frac{\partial^2 M_y^s}{\partial y^2} - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x\,\partial y} \\
&+k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = -k_1 A' \left(I_3 + \frac{I_4}{R}\right)\frac{\partial \ddot{u}_0}{\partial x} \\
&-k_2 B' I_3 \frac{\partial \ddot{v}_0}{\partial y} + I_4 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + I_4 k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} - I_5 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} - I_5 (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2}
\end{aligned} \tag{18}$$

The simply supported boundary conditions on all four edges of the laminated composite cylindrical shells can be expressed as

$$\begin{aligned}
&v_0(0,y) = v_0(a,y) = u_0(x,0) = u_0(x,b) = 0,\\
&w_0(0,y) = w_0(a,y) = w_0(x,0) = w_0(x,b) = 0,\\
&N_x(0,y) = N_x(a,y) = N_y(x,0) = N_y(x,b) = 0,\\
&M_x^b(0,y) = M_x^b(a,y) = M_y^b(x,0) = M_y^b(x,b) = 0,\\
&M_x^s(0,y) = M_x^s(a,y) = M_y^s(x,0) = M_y^s(x,b) = 0,\\
&\theta(0,y) = \theta(a,y) = \theta(x,0) = \theta(x,b) = 0
\end{aligned} \tag{19}$$

3. Analytical solutions

In this work, the analytical solutions of Eq. (18) for the bending and free vibration responses of a simply supported laminated composite cylindrical shell are presented using Navier's method which fulfills the following boundary conditions described in Eq. (19). In the case of the bending analysis of laminated fiber-reinforced composite cylindrical shells, we note that the displacement variables can be formulated into a double-Fourier series as follows

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \begin{Bmatrix} U_{mn} \cos(\alpha x) \sin(\beta y) \\ V_{mn} \sin(\alpha x) \cos(\beta y) \\ W_{mn} \sin(\alpha x) \sin(\beta y) \\ \Theta_{mn} \sin(\alpha x) \sin(\beta y) \end{Bmatrix} \tag{20}$$

where U_{mn}, V_{mn}, W_{mn} and Θ_{mn} are unknown coefficients, so the parameters α and β are already defined in Eq. (5). It is considered that the transverse mechanical load $q(x,y)$ acting on the top surface of the shell can also be stated by the double-Fourier sine series as

$$q(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin(\alpha x) \sin(\beta y) \tag{21}$$

where q_{mn} is the Fourier coefficient, which can be represented with the maximum intensity for typical loads, in this case by a sinusoidal distributed load ($q_{mn} = q_0$) and (m, n) are odd integers. Substitution of this solution of Eqs. (20) and (21) into the governing equations Eq. (18), the analytical solutions for the bending problem can be derived from the following an algebraic equation

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Theta_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_0 \\ 0 \end{Bmatrix} \tag{22}$$

where the components of the symmetric stiffness matrix $[K_{ij}]$ can be defined as follows

$$\begin{aligned}
K_{11} &= \alpha^2 A_{11} + \beta^2 A_{66},\\
K_{12} &= \alpha\beta(A_{12} + A_{66}),\\
K_{13} &= -\alpha \frac{A_{11}}{R} - \alpha^3 B_{11} - \alpha\beta^2(B_{12} + 2B_{66}),\\
K_{14} &= k_1 A' \alpha(\beta^2 E_{66} + \alpha^2 E_{11}) + k_2 B' \alpha\beta^2 (E_{12} + E_{66}),
\end{aligned}$$

$$
\begin{aligned}
K_{22} &= \beta^2 A_{22} + \alpha^2 A_{66},\\
K_{23} &= -\beta \frac{A_{12}}{R} - \alpha^2\beta(B_{12} + 2B_{66}) - \beta^3 B_{22},\\
K_{24} &= k_1 A'\alpha^2\beta(E_{12} + E_{66}) + k_2 B'\beta(\alpha^2 E_{66} + \beta^2 E_{22}),\\
S_{33} &= \frac{A_{11}}{R^2} + 2\alpha^2\frac{B_{11}}{R} + 2\beta^2\frac{B_{12}}{R} + \alpha^4 D_{11} + \beta^4 D_{22} + 2\alpha^2\beta^2(D_{12} + 2D_{66}),\\
S_{34} &= -k_1 A'\alpha^4 F_{11} - k_1 A'\alpha^2\beta^2(F_{12} + 2F_{66}) - k_2 B'\beta^4 F_{22}\\
&\quad - k_2 B'\alpha^2\beta^2(F_{12} + 2F_{66}) - k_1 A'\alpha^2\frac{E_{11}}{R} - k_2 B'\beta^2\frac{E_{12}}{R},\\
S_{44} &= (k_1A')^2\alpha^4 H_{11} + 2k_1k_2A'B'\alpha^2\beta^2(H_{12} + H_{66}) + (k_2B')^2\beta^4 H_{22}\\
&+(k_2B')^2\beta^2 A^s_{44} + (k_1A')^2\alpha^2 A^s_{55} + (k_1A')^2\alpha^2\beta^2 H_{66} + (k_2B')^2\alpha^2\beta^2 H_{66}
\end{aligned} \tag{23}
$$

Consequently, for the case of the free vibration analysis of laminated composite cylindrical shells, the four unknowns u_0, v_0, w_0, θ are expressed as a more appropriate form of Fourier series expansion

$$
\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \begin{Bmatrix} U_{mn} e^{i\omega t} \cos(\alpha x)\sin(\beta y) \\ V_{mn} e^{i\omega t} \sin(\alpha x)\cos(\beta y) \\ W_{mn} e^{i\omega t} \sin(\alpha x)\sin(\beta y) \\ \Theta_{mn} e^{i\omega t} \sin(\alpha x)\sin(\beta y) \end{Bmatrix} \tag{24}
$$

where $i = \sqrt{-1}$ and ω is the natural frequency of free vibration of the cylindrical shell. Substituting Eq. (24) into Eq. (18), we obtain the following resolution of the eigenvalue problem

$$
\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{12} & M_{22} & M_{23} & M_{24} \\ M_{13} & M_{23} & M_{33} & M_{34} \\ M_{14} & M_{24} & M_{34} & M_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Theta_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}
$$

where the elements of the mass matrix $[M_{ij}]$ can be defined as follows

$$
\begin{aligned}
&M_{11} = I_0 + 2\frac{I_1}{R} + \frac{I_2}{R^2}, \quad M_{12} = 0,\\
&M_{13} = -\alpha\left(I_1 + \frac{I_2}{R}\right), \; M_{14} = k_1 A'\alpha\left(I_3 + \frac{I_4}{R}\right),\\
&M_{22} = I_0, \quad M_{23} = -\beta I_1, \quad M_{24} = k_2 B'\beta I_3,\\
&M_{33} = I_0 + (\alpha^2 + \beta^2) I_2, M_{34} = -(k_1 A'\alpha^2 + k_2 B'\beta^2) I_4,\\
&M_{44} = ((k_1A')^2\alpha^2 + (k_2B')^2\beta^2) I_5
\end{aligned} \tag{26}
$$

4. Numerical results and discussions

This research investigates various numerical examples to verify the accuracy of the present theoretical model for predicting the bending and free vibration responses of laminated fiber-reinforced composite cylindrical shells. The relevance of the proposed theory can be demonstrated by comparison with previously published results. In order to achieve this objective, appropriate sets of material properties will be utilized in the numerical studies as follows.

Table 1 Mechanical properties of cylindrical shells (Bhimaraddi *et al.*1992)

Materials	Material properties
Material 1	$E_1 = E_2 = E_3 = E = 210$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = \nu = 0.3$
Material 2	$E_1 = 36.0885$ GPa, $E_2 = 26.2818$ GPa, $G_{12} = 4.9033$ GPa, $G_{31} = 4.4130$ GPa, $G_{23} = 4.0208$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.105$
Material 3	$E_1/E_2 = 25, G_{23}/E_2 = 0.2, G_{12}/E_2 = G_{13}/E_2 = 0.5,$ $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$

For comparison purposes, the results obtained for displacements, stresses, and fundamental frequencies are displayed in the following non-dimensional forms

$$\begin{gathered}\bar{w}\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{E_1 w}{q_0 a}, \quad \bar{\sigma}_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right) = \frac{\sigma_x}{q_0}, \\ \bar{\tau}_{xz}\left(0,\frac{b}{2},\frac{z}{h}\right) = \frac{\tau_{xz}}{q_0}, \\ \bar{\omega} = \omega(a^2/h)\sqrt{\rho/E_2}\end{gathered} \tag{27}$$

The percentage difference between the result of a particular theory and the result of the exact elasticity solution is calculated in the following manner

$$\%\ \text{diff.} = \left(\frac{\text{value by a particular theory}}{\text{value by exact elasticity solution}} - 1\right) \times 100 \tag{28}$$

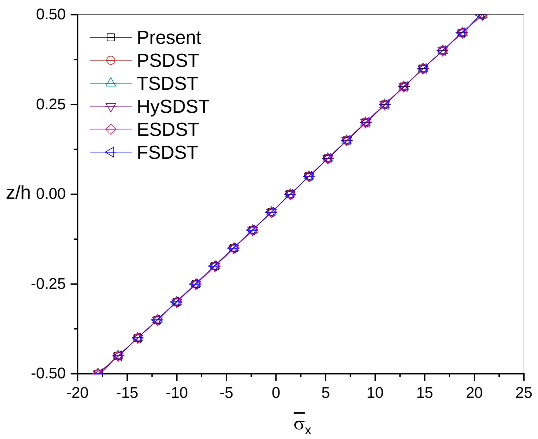
4.1 Bending analysis of isotropic cylindrical shells

To investigate the accuracy and applicability of the present theory, the first example is performed for static bending analysis of a homogenous isotropic cylindrical shell subjected to sinusoidal transverse load. The shell is made of material 1 as shown in Table 1, and this example is solved using Eq. (22) in the absence of the variables that depend on time. The variation of transverse displacements and stresses with respect to various values of curvature-radius-to-side *R/a* ratios, ranging from 5 to 20 is presented in Table 2. The present results are compared with the previous results reported by Sayyad and Ghugal (2022) using the generalized displacement field model based on various shell theories, namely, the parabolic shear deformation shell theory “PSDST” of Reddy (1984), the trigonometric shear deformation shell theory “TSDST” developed by Touratier (1991), the hyperbolic shear deformation shell theory “HySDST” given by Soldatos (1992), the exponential shear deformation shell theory “ESDST” proposed by Karama *et al.* (2009), the first-order shear deformation shell theory “FSDST” proposed by Mindlin (1951), and those computed using the classical shell theory “CST” of Kirchhoff (1850), as well as the exact elasticity solution given by Bhimaraddi *et al.* (1992). It must be emphasized that the current approach gives more accurate results in predicting the displacements and stresses when compared to the PSDST and HySDST. The percentage errors predicted by current theory for displacement, axial and transverse shear stresses when $R/a = 5$ are 0.86%, 1.57% and 1.38%, respectively, as compared to the exact elasticity solution. However, the FSDST underestimates the transverse

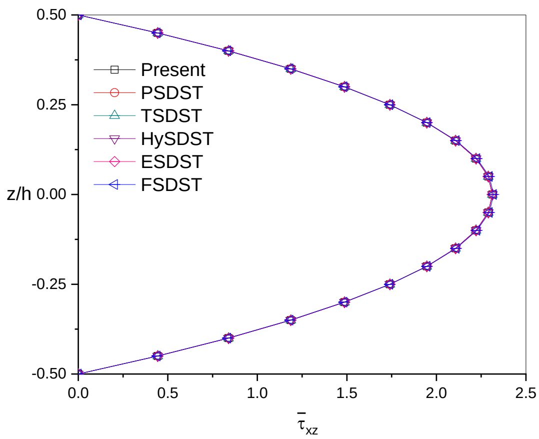
Table 2 Comparison of non-dimensional displacements and stresses in isotropic cylindrical shells with different *R/a* values (*h/a*=0.1, *a/b*=1, material 1)

R/a	Theory	Model	$\bar{w}$	% diff.	$\bar{\sigma}_x$	% diff.	$\bar{\tau}_{xz}$	% diff.
5	Present	RSDST	28.754	–0.86	20.807	–1.57	2.3132	–1.38
	Reddy (1984)	PSDST[(a)]	28.756	–0.85	20.808	–1.56	2.3132	–1.38
	Touratier (1991)	TSDST[(a)]	28.753	–0.86	20.820	–1.50	2.3125	–1.41
	Soldatos (1992)	HySDST[(a)]	28.756	–0.85	20.807	–1.57	2.3133	–1.37
	Karama *et al.* (2009)	ESDST[(a)]	28.760	–0.84	20.860	–1.32	2.3162	–1.25
	Mindlin (1951)	FSDST[(a)]	28.508	–1.71	20.620	–2.45	2.3193	–1.12
	Kirchhoff (1850)	CST	27.262	–6.0	20.582	–2.63	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	29.003	0.00	21.138	0.00	2.3455	0.00
10	Present	RSDST	29.388	0.03	20.531	–0.91	2.3642	–0.61
	Reddy (1984)	PSDST[(a)]	29.390	0.04	20.532	–0.90	2.3642	–0.61
	Touratier (1991)	TSDST[(a)]	29.386	0.02	20.544	–0.84	2.3635	–0.64
	Soldatos (1992)	HySDST[(a)]	29.390	0.04	20.532	–0.90	2.3642	–0.61
	Karama *et al.* (2009)	ESDST[(a)]	29.394	0.05	20.585	–0.65	2.3672	–0.48
	Mindlin (1951)	FSDST[(a)]	29.130	–0.85	20.342	–1.82	2.3699	–0.37
	Kirchhoff (1850)	CST	27.831	–5.27	20.316	–1.95	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	29.379	0.00	20.719	0.00	2.3787	0.00
20	Present	RSDST	29.551	0.36	20.275	–0.68	2.3773	–0.31
	Reddy (1984)	PSDST[(a)]	29.552	0.36	20.277	–0.67	2.3773	–0.31
	Touratier (1991)	TSDST[(a)]	29.549	0.35	20.288	–0.61	2.3766	–0.34
	Soldatos (1992)	HySDST[(a)]	29.552	0.36	20.277	–0.67	2.3773	–0.31
	Karama *et al.* (2009)	ESDST[(a)]	29.557	0.38	20.330	–0.41	2.3803	–0.18
	Mindlin (1951)	FSDST[(a)]	29.290	–0.53	20.088	–1.59	2.3830	–0.07
	Kirchhoff (1850)	CST	27.977	–4.99	20.073	–1.67	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	29.445	0.00	20.413	0.00	2.3847	0.00

[(a)] Results taken from reference of Sayyad and Ghugal (2022)



(a) Axial normal stress $\bar{\sigma}_x$



(b) Transverse shear stress $\bar{\tau}_{xz}$

Fig. 2 Variation of non-dimensional stresses through the thickness of an isotropic cylindrical shell subjected to a sinusoidal distributed load for thickness ratio $a/h = 10$ and curvature ratio $R/a = 5$, material 1

Table 3 Comparison of non-dimensional displacements and stresses in orthotropic cylindrical shells with different *R/a* values (*h/a*=0.1, *a/b*=1, material 2)

R/a	Theory	Model	$\bar{w}$	% diff.	$\bar{\sigma}_x$	% diff.	$\bar{\tau}_{xz}$	% diff.
5	Present	RSDST	54.414	2.09	28.015	–1.58	2.5836	–1.70
	Reddy (1984)	PSDST[a]	54.434	2.13	27.914	–1.94	2.5736	–2.08
	Touratier (1991)	TSDST[a]	54.425	2.11	27.944	–1.83	2.5721	–2.14
	Soldatos (1992)	HySDST[a]	54.480	2.22	27.831	–2.23	2.5656	–2.39
	Karama *et al.* (2009)	ESDST[a]	54.446	2.15	28.052	–1.45	2.5798	–1.85
	Mindlin (1951)	FSDST[a]	53.609	0.58	27.449	–3.57	2.5876	–1.55
	Kirchhoff (1850)	CST	49.442	–7.23	27.515	–3.34	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	53.298	0.00	28.466	0.00	2.6283	0.00
10	Present	RSDST	55.363	0.74	27.923	–0.82	2.6286	3.19
	Reddy (1984)	PSDST[a]	55.384	0.78	27.820	–1.19	2.6185	2.79
	Touratier (1991)	TSDST[a]	55.374	0.76	27.850	–1.08	2.6170	2.74
	Soldatos (1992)	HySDST[a]	55.432	0.86	27.735	–1.49	2.6104	2.48
	Karama *et al.* (2009)	ESDST[a]	55.396	0.79	27.960	–0.69	2.6249	3.05
	Mindlin (1951)	FSDST[a]	54.530	–0.78	27.348	–2.86	2.6320	3.33
	Kirchhoff (1850)	CST	50.224	–8.61	27.423	–2.59	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	54.958	0.00	28.154	0.00	2.5473	0.00
20	Present	RSDST	55.605	1.06	27.752	–0.56	2.6401	–0.58
	Reddy (1984)	PSDST[a]	55.627	1.10	27.649	–0.93	2.6300	–0.96
	Touratier (1991)	TSDST[a]	55.617	1.08	27.680	–0.82	2.6285	–1.01
	Soldatos (1992)	HySDST[a]	55.675	1.18	27.564	–1.23	2.6218	–1.27
	Karama *et al.* (2009)	ESDST[a]	55.639	1.12	27.790	–0.42	2.6364	–0.72
	Mindlin (1951)	FSDST[a]	54.765	–0.47	27.179	–2.61	2.6434	–0.45
	Kirchhoff (1850)	CST	50.424	–8.36	27.267	–2.30	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	55.023	0.00	27.908	0.00	2.6554	0.00

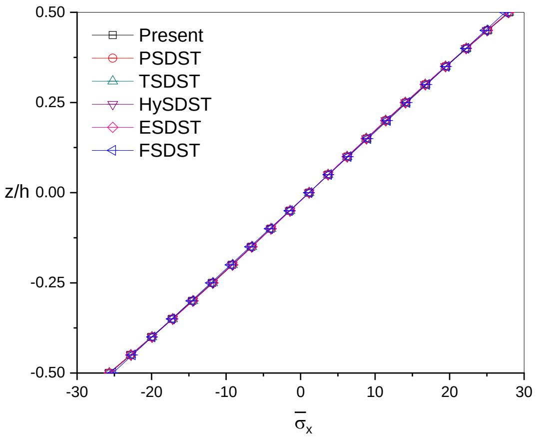
[a] Results taken from reference of Sayyad and Ghugal (2022)

displacements and stresses for all values of curvature R/a ratio. It is also noted that the displacements and transverse shear stresses tend to increase with an increase in the value of the curvature ratio.

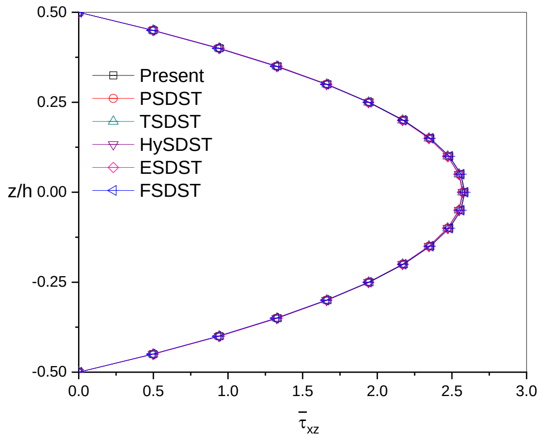
Fig. 2 shows the variation in axial normal and transverse shear stresses through the thickness of an isotropic cylindrical shell subjected to a sinusoidal distributed load for a curvature $R/a = 5$ ratio. It is apparent that the results obtained from the present refined shear deformation shell theory converge very well with those of other shear deformation shell theories.

4.2 Bending analysis of orthotropic cylindrical shells

In the next example, the accuracy and correctness of the present refined shear deformation shell theory is checked for the bending analysis of simply supported orthotropic cylindrical shells under sinusoidal loads. The cylindrical shells are composed of material 2 as mentioned in Table 1. The numerical results of non-dimensional displacements and stresses are listed in Table 3 for various values of curvature-radius-to-side ratios $R/a = 5, 10$ and 20. Examination of Table 3 also reveals



(a) Axial normal stress $\bar{\sigma}_x$



(b) Transverse shear stress $\bar{\tau}_{xz}$

Fig. 3 Variation of non-dimensional stresses through the thickness of an orthotropic cylindrical shell subjected to a sinusoidal distributed load for thickness ratio $a/h = 10$ and curvature ratio $R/a = 5$, material 2

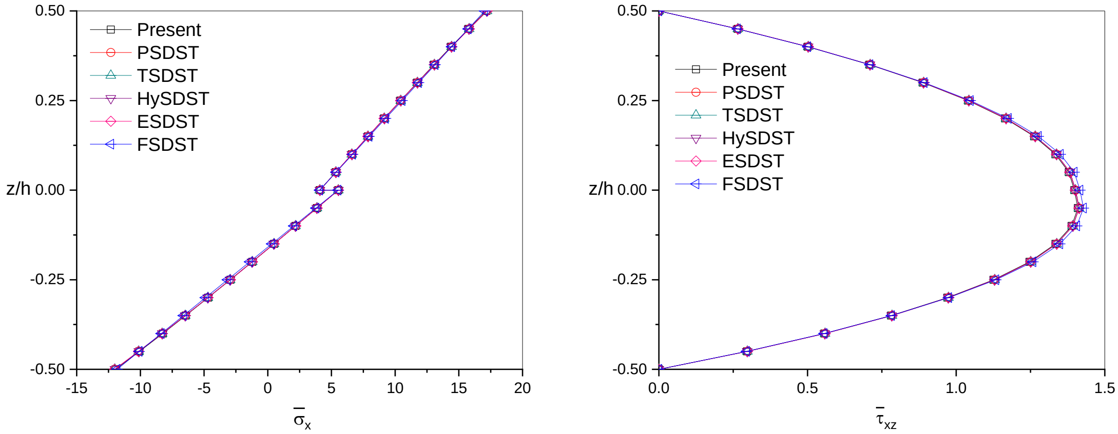
that, the transverse displacements predicted by the present analytical approach are in an excellent agreement with those reported by Sayyad and Ghugal (2022) based on the PSDST and TSDST. According to the proposed analytical model, the percentage errors predicted for displacement, axial stress and transverse shear stress are 2.09%, 1.58% and 1.70% respectively, when $R/a = 5$, as compared to the exact solution. However, ESDST predicts axial normal stresses $\bar{\sigma}_x$ in excellent agreement with the three-dimensional elasticity solutions given by Bhimaraddi *et al.* (1992) (in general, the axial stresses of orthotropic cylindrical shells are underestimated by 1.45%, 0.69% and 0.42% for the corresponding curvature ratios $R/a = 5$, 10 and 20, respectively).

Moreover, we find that the classical shell theory underestimates the transverse displacement values for all curvature-radius-to-side ratios. On the other hand, the results obtained show that increasing value of R/a leads to an increase in the non-dimensional value of transverse displacements, whereas the value of axial normal stresses decreases.

The variation of non-dimensional axial normal and transverse shear stresses through the thickness of an orthotropic cylindrical shell for curvature $R/a = 5$ ratio is shown in Fig. 3. It should be noted again that the results obtained by using the present formulations are in excellent agreement with those given by the conventional HSDST models whereas the results provided by the CST are underestimated.

4.3 Bending analysis of laminated cylindrical shells

The bending analysis of antisymmetric two-layer (0°/90°) and symmetric three-layer (0°/90°/0°) cross-ply laminated composite cylindrical shells is investigated in this example. All orthotropic layers have the same thickness and are made up of material 2. The numerical results are presented in Tables 4 and 5 for both lamination schemes, and for $R/a = 1$, $h/a = 0.1$ and $a/b = 1$. In order to ensure the accuracy of the present theory, the numerical results obtained for this example are compared with the results predicted by Sayyad and Ghugal (2022) using the five-unknown displacement model based on different higher-order shear deformation shell theories and the three-dimensional elasticity solutions given by Bhimaraddi *et al.* (1992). From the examination



(a) Axial normal stress $\bar{\sigma}_x$ (b) Transverse shear stress $\bar{\tau}_{xz}$

Fig. 4 Variation of non-dimensional stresses through the thickness of antisymmetric two-layer (0°/90°) laminated cylindrical shell subjected to a sinusoidal distributed load for thickness ratio $a/h = 10$ and curvature ratio $R/a = 1$, material 2

Table 4 Comparison of non-dimensional displacements and stresses in two-layer (0°/90°) laminated cylindrical shells with different *R/a* values (*h/a*=0.1, *a/b*=1, material 2)

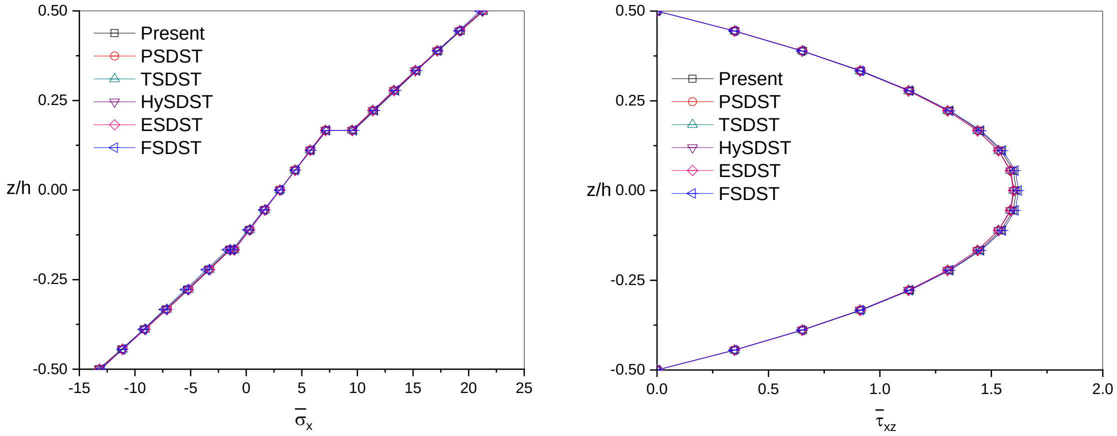
R/a	Theory	Model	$\bar{w}$	% diff.	$\bar{\sigma}_x$	% diff.	$\bar{\tau}_{xz}$	% diff.
1	Present	RSDST	35.357	–14.47	17.157	–27.61	1.3975	–21.33
	Reddy (1984)	PSDST[a]	35.358	–14.47	17.198	–27.43	1.4140	–20.40
	Touratier (1991)	TSDST[a]	35.362	–14.46	17.213	–27.37	1.4134	–20.43
	Soldatos (1992)	HySDST[a]	35.343	–14.51	17.761	–25.06	1.4139	–20.40
	Karama *et al.* (2009)	ESDST[a]	35.223	–14.80	17.229	–27.30	1.4128	–20.46
	Mindlin (1951)	FSDST[a]	35.020	–15.29	17.030	–28.14	1.4284	–19.59
	Kirchhoff (1850)	CST	33.247	–19.58	17.030	–28.14	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	41.340	00.00	23.700	00.00	1.7763	00.00

[a] Results taken from reference of Sayyad and Ghugal (2022)

Table 5 Comparison of non-dimensional displacements and stresses in three-layer (0°/90°/0°) laminated cylindrical shells with different *R/a* values (*h/a*=0.1, *a/b*=1, material 2)

R/a	Theory	Model	$\bar{w}$	% diff.	$\bar{\sigma}_x$	% diff.	$\bar{\tau}_{xz}$	% diff.
1	Present	RSDST	35.074	–14.08	21.360	–10.95	1.6124	–11.84
	Reddy (1984)	PSDST[a]	35.097	–14.03	21.246	–11.43	1.6189	–11.49
	Touratier (1991)	TSDST[a]	35.094	–14.03	21.264	–11.35	1.6179	–11.54
	Soldatos (1992)	HySDST[a]	35.079	–14.07	21.300	–11.20	1.6190	–11.48
	Karama *et al.* (2009)	ESDST[a]	35.103	–14.01	21.333	–11.06	1.6176	–11.56
	Mindlin (1951)	FSDST[a]	34.743	–14.89	21.031	–12.32	1.6382	–10.43
	Kirchhoff (1850)	CST	32.938	–19.32	21.031	–12.32	-	-
	Bhimaraddi *et al.* (1992)	Exact 3D	40.823	00.00	23.987	00.00	1.8290	00.00

[a] Results taken from reference of Sayyad and Ghugal (2022)



(a) Axial normal stress $\bar{\sigma}_x$ (b) Transverse shear stress $\bar{\tau}_{xz}$

Fig. 5 Variation of non-dimensional stresses through the thickness of symmetric three-layer (0°/90°/0°) laminated cylindrical shell subjected to a sinusoidal distributed load for thickness ratio $a/h = 10$ and curvature ratio $R/a = 1$, material 2

of Tables 4 and 5, it can be observed that the numerical results of transverse displacements and stresses obtained by using the present four-unknown model are exactly matching with the results of the conventional model based on PSDST and TSDST. However, the percentage errors predicted by the present theoretical formulation for displacement, axial stress and transverse shear stress of three-layer laminated cylindrical shells are 14.08%, 10.95% and 11.84%, respectively, as compared to the exact elasticity solution. The CST underestimates the transverse displacement and axial normal stresses for both types of laminated shells due to neglecting shear deformation effects.

The variations of non-dimensional axial normal and transverse shear stresses $\bar{\sigma}_x, \bar{\tau}_{xz}$ through the thickness of a moderately thick two-layer antisymmetric (0°/90°) and three-layer symmetric (0°/90°/0°) cross-ply laminated composite shells subjected to sinusoidal loads are shown in Figs. 4 and 5, respectively. In laminated cylindrical shell structures, we emphasize that transverse shear stresses induce a discontinuity at the interface between layers when derived from the constitutive relations. In this case, these stresses are obtained by utilizing the equilibrium equations of 3D elasticity, and they exactly fulfill the conditions of zero shear stress on the upper and lower surfaces of the cylindrical shells.

It can be seen again that the obtained results by using the present theory are in good agreement with those presented by the conventional HSDST with each other for both types of laminated shells. From this study, we can conclude that the maximum axial normal stresses are detected on the lower surface (i.e., $z = h/2$) of the shell, whereas the maximum transverse shear stresses are detected on the middle surface of the cylindrical shell.

4.4 Free vibration analysis of isotropic and laminated cylindrical shells

In the last example, the proposed shell model is also verified for the free vibration response of simply supported isotropic and laminated fiber-reinforced composite cylindrical shells, whose material properties are given in Table 1. Table 6 summarizes the non-dimensional first natural

Table 6 Comparison of non-dimensional natural frequencies in cylindrical shells with different *R*/*a* values (*h*/*a*=0.1, *a*/*b*=1, material 3)

R/a	Theory	Model	Isotropic	Orthotropic	0°/90°	0°/90°/0°
0.5	Present	RSDST	10.0164	14.2985	14.2643	15.8379
	Reddy (1984)	PSDST[(a)]	10.0200	14.0875	14.2221	15.2534
	Touratier (1991)	TSDST[(a)]	10.0199	14.0847	14.2240	15.2282
	Soldatos (1992)	HySDST[(a)]	10.0205	14.0975	14.2275	15.2557
	Karama *et al.* (2009)	ESDST[(a)]	10.0211	14.1178	14.2400	15.2417
	Mindlin (1951)	FSDST[(a)]	10.0302	14.2902	14.2538	15.6721
	Asadi *et al.* (2012)	FSDTQ	9.59610	-	13.7710	15.2500
	Kirchhoff (1850)	CST	10.0804	15.7619	14.6138	17.2289
1	Present	RSDST	7.34353	13.3176	10.9484	13.8427
	Reddy (1984)	PSDST[(a)]	7.34706	13.0226	10.9307	12.9914
	Touratier (1991)	TSDST[(a)]	7.34689	13.0188	10.9345	12.9541
	Soldatos (1992)	HySDST[(a)]	7.34782	13.0368	10.9390	12.9948
	Karama *et al.* (2009)	ESDST[(a)]	7.34898	13.0655	10.9585	12.9742
	Mindlin (1951)	FSDST[(a)]	7.36499	13.3080	10.9594	13.6064
	Asadi *et al.* (2012)	FSDTQ	7.09000	-	10.6660	13.1870
	Kirchhoff (1850)	CST	7.45550	15.3241	11.4376	15.8083
2	Present	RSDST	6.22419	12.9509	9.57485	13.0959
	Reddy (1984)	PSDST[(a)]	6.22746	12.6228	9.57321	12.1277
	Touratier (1991)	TSDST[(a)]	6.22725	12.6185	9.57804	12.0849
	Soldatos (1992)	HySDST[(a)]	6.22841	12.6386	9.58313	12.1316
	Karama *et al.* (2009)	ESDST[(a)]	6.22990	12.6707	9.60642	12.1080
	Mindlin (1951)	FSDST[(a)]	6.25008	12.9410	9.60041	12.8294
	Asadi *et al.* (2012)	FSDTQ	6.09130	-	9.45770	12.4430
	Kirchhoff (1850)	CST	6.36521	15.1641	10.1408	15.2958

[(a)] Results taken from reference of Sayyad and Ghugal (2022)

frequencies $\bar{\omega}$ of isotropic, orthotropic, and laminated cylindrical shells for different values of curvature-radius-to-side ratios $(R/a = 0.5, 1, 2)$ and for thickness-to-side $(h/a = 0.1)$ ratio. Whereas Table 7 represents the first five natural frequencies of two-layer (0°/90°) cross-ply laminated composite cylindrical shells. All layers have the same thickness and are made up of material 3. It is observed that the results from the present refined shear deformation shell theory agree extremely with the corresponding results of various shear deformation shell theories (PSDST, TSDST, HySDST, ESDST) for all values of curvature R/a ratio. This indicates that the constitutive equations of the proposed model are validated. However, the first-order shear deformation shell model predicted by Asadi *et al.* (2012) “FSDTQ” based on the equations developed by Quati (1999) underestimates the value of natural frequencies for all cylindrical shell structures. Through this investigation, we found that the 3D elasticity solution for the free vibration of laminated cylindrical shells is not available in the literature. Moreover, we found that increasing the curvature R/a ratio has an impact on reducing natural frequencies for all types of materials.

Table 7 Comparison of non-dimensional natural frequencies in two-layer (0°/90°) laminated cylindrical shells with different *R*/*a* values (*h*/*a*=0.1, *a*/*b*=1, material 3)

R/a	Theory	Model	ω_1	ω_2	ω_3	ω_4	ω_5
0.5	Present	RSDST	14.2643	22.6819	30.2797	32.6132	40.5739
	Reddy (1984)	PSDST[(a)]	14.2221	22.6425	29.8696	32.5766	40.3561
	Touratier (1991)	TSDST[(a)]	14.2240	22.6697	29.8837	32.6103	40.4125
	Soldatos (1992)	HySDST[(a)]	14.2275	22.7138	29.9156	32.6829	40.6236
	Karama *et al.* (2009)	ESDST[(a)]	14.2400	22.9038	30.0395	32.9716	41.4293
	Mindlin (1951)	FSDST[(a)]	14.2538	22.7374	29.9785	32.7823	40.4045
	Asadi *et al.* (2012)	FSDTQ	13.7710	21.0370	29.5710	31.2000	38.0730
	Kirchhoff (1850)	CST	14.6138	26.7431	32.7925	39.0631	55.1805
1	Present	RSDST	10.9484	22.4224	24.9398	31.1599	40.6170
	Reddy (1984)	PSDST[(a)]	10.9307	22.3397	24.5762	31.1716	40.3531
	Touratier (1991)	TSDST[(a)]	10.9345	22.3691	24.5999	31.2102	40.4138
	Soldatos (1992)	HySDST[(a)]	10.9390	22.4141	24.6417	31.2855	40.6210
	Karama *et al.* (2009)	ESDST[(a)]	10.9585	22.6127	24.8177	31.5948	41.2947
	Mindlin (1951)	FSDST[(a)]	10.9594	22.4171	24.6705	31.3472	40.3429
	Asadi *et al.* (2012)	FSDTQ	10.6660	21.7050	24.0900	30.3680	38.7220
	Kirchhoff (1850)	CST	11.4376	26.4513	28.3556	37.7796	54.6515
2	Present	RSDST	9.57485	22.2606	22.9228	30.6584	40.5139
	Reddy (1984)	PSDST[(a)]	9.57321	22.1518	22.6620	30.6846	40.2263
	Touratier (1991)	TSDST[(a)]	9.57804	22.1818	22.6901	30.7248	40.2888
	Soldatos (1992)	HySDST[(a)]	9.58313	22.2267	22.7350	30.8001	40.4933
	Karama *et al.* (2009)	ESDST[(a)]	9.60642	22.4266	22.9309	31.1130	41.2947
	Mindlin (1951)	FSDST[(a)]	9.60041	22.2207	22.7412	30.8446	40.1893
	Asadi *et al.* (2012)	FSDTQ	9.45770	21.6760	22.1500	29.9590	38.6080
	Kirchhoff (1850)	CST	10.1408	26.2216	26.7015	37.2424	54.2365

[(a)] Results taken from reference of Sayyad and Ghugal (2022)

5. Conclusions

In this study, a new hybrid refined shear deformation shell theory is developed for the static bending and free vibration analysis of simply supported isotropic and laminated composite cylindrical shells under sinusoidal transverse mechanical loading. The displacement field of the proposed model contains four unknown variables, and involves a hyperbolic shape function to account for an appropriate distribution of the transverse shear strains through the thickness without tangential stress on the upper and lower surfaces of the shell. Therefore, a shear correction coefficient is not required. The shell governing equations and their boundary conditions are derived by using the dynamic version of the principle of virtual work and the analytical solutions of displacements, stresses, and natural frequencies were obtained using Navier's solution method. The correctness and reliability of the present approach are checked by comparing it with various shear deformation theories, taking into account the influence of different parameters such as the curvature-radius-to-side ratio and the lamination schemes on the bending and free vibration behavior of laminated fiber-reinforced composite cylindrical shells. The numerical and graphical results show that the proposed refined shell theory is in excellent agreement with other higher-order shear deformation theories for the evaluation of stresses and free vibration response of isotropic and laminated cylindrical shells.

The principal conclusions that can be drawn from these results are as follows:

- The proposed shell theory fulfils the shear stress-free boundary conditions on the upper and lower surfaces of the cylindrical shells without the need for a shear correction factor;
- The classical shell theory "CST" underestimates the transverse displacement values for all curvature-radius-to-side ratios;
- The present study has shown that the CST overestimates the natural frequencies of all types of cylindrical shell structures, and also for all curvature-radius-to-side ratios;
- The FSDST and CST underestimate the transverse displacement and axial normal stresses for laminated composite cylindrical shells due to neglecting shear deformation effects;
- The study clearly shows that the displacements and transverse shear stresses increase in conjunction with an increase in the value of the curvature ratio;
- It was found that increasing the curvature ratio has a significant effect on reducing natural frequencies for all types of laminated cylindrical shells;
- It has been concluded that the developed refined shear deformation shell theory produces accurate results and can be applied to the solutions of engineering problems for different laminated cylindrical shells.

References

Asadi, E., Wang, W. and Qatu, M.S. (2012), "Static and vibration analyses of thick deep laminated cylindrical shells using 3D and various deformation theories", *Compos. Struct.*, **94**(2), 494-500. https://doi.org/10.1016/j.compstruct.2011.08.011.

Bhaskar, K. and Varadan, T.K. (1991), "A higher-order theory for bending analysis of laminated shells of revolution", *Comput. Struct.*, **40**(4), 815-819. https://doi.org/10.1016/0045-7949 (91)90310-I.

Bhimaraddi, A. and Chandrashekhara, K. (1992), "Three-dimensional elasticity solution for static response of simply supported orthotropic cylindrical shells", *Compos. Struct.*, **20**(4), 227-235. https://doi.org/10.1016/0263-8223(92)90028-B.

Civalek, O. (2013), "Vibration analysis of laminated composite conical shells by the method of discrete singular convolution based on the shear deformation theory", *Compos. Part B Eng.*, **45**(1), 1001-1009. https://doi.org/10.1016/j.compositesb.2012. 05.018.

Dewangan, H.C. and Panda, S.K. (2022), "Nonlinear thermoelastic numerical frequency analysis and experimental verification of cutout abided laminated shallow shell structure", *J. Press. Ves. Technol.*, **144**(6), 061903. https://doi.org/10.1115/1.4054843.

Draiche, K., Tounsi, A., Ibrahim, K.D. and Tlidji, Y. (2024), "An improved mathematical model for static and dynamic analysis of functionally graded doubly-curved shells", *Arch. Appl. Mech.*, **94**, 1589-1611. https://doi.org/10.1007/s00419-024-02595-5.

Errico, F., Franco, F., Ichchou, M., De Rosa, S. and Petrone, G. (2019), "An investigation on the vibrations of laminated shells under aeroacoustic loads using a WFE approach", *Adv. Aircraft Spacecraft Sci.*, **6**(6), 463-478. https://doi.org/10.12989/aas.2019.6.6.463.

Ghasemi, M.A., Yazdani, M. and Hosseini, S.M. (2013), "Analysis of effective parameters on the buckling of grid stiffened composite shells based on first order shear deformation theory", *Modares Mech. Eng.*, **13**(10), 51-61.

Jafari, A.A., Khalili, S.M.R. and Azarafza, R. (2005), "Transient dynamic response of composite circular cylindrical shells under radial impulse load and axial compressive loads", *Thin Wall. Struct.*, **43**(11), 1763-1786. https://doi.org/10.1016/j.tws.2005. 06.009.

Kar, V.R., Mahapatra, T.R. and Panda, S.K. (2015), "Nonlinear flexural analysis of laminated composite flat panel under hygro-thermo-mechanical loading", *Steel Compos. Struct.*, **19**(4), 1011-1033. https://doi.org/10.12989/scs.2015.19.4.1011.

Karama, M., Afaq, K.S. and Mistou, S. (2009), "A new theory for laminated composite plates", *Proc. Inst. Mech. Eng., Part L: J. Mater.: Des. Appl.*, **223**(2), 53-62. https://doi.org/10.1243/14644207JMDA189.

Kirchhoff, G.R. (1850), "Uber das Gleichgewicht und die Bewegung einer elastichen Scheibe", *J. Pure Appl. Math.*, **40**, 51-88. https://doi.org/10.1515/crll.1850.40.51.

Kumar, A., Chakrabarti, A. and Bhargava, P. (2013), "Vibration of laminated composite cylindrical shells with cutouts using higher order theory", *Int. J. Sci. Eng. Res.*, **4**(5), 199-202.

Kumar, P., Arya, R., Sharma, N., Hirwani, C.K. and Panda, S.K. (2023), "Curved fiber-reinforced laminated composite panel and variable stiffness influence on eigenfrequency responses: A higher-order FE approach", *J. Vib. Eng. Technol.*, **11**, 2349-2359. https://doi.org/10.1007/s42417-022-00706-6.

Lai, A., Jia, J., Zhou, Z., Xu, X. and Lim, C.W. (2022), "Homotopic analysis for post-buckling of cylindrical shells with local thickness defects", *Acta Astronautica*, **193**, 44-55. https://doi.org/10.1016/j.actaastro.2022. 01.005.

Lam, K.Y. and Loy, C.T. (1994), "On vibrations of thin rotating laminated composite cylindrical shells", *Compos. Eng.*, **4**(11), 1153-1167. https://doi.org/10.1016/0961-9526(95)91289-S.

Li, M., Guedes Soares, C., Liu, Z. and Zhang, P. (2024), "Free and forced vibration analysis of carbon/glass hybrid composite laminated plates under arbitrary boundary conditions", *Appl. Compos. Mater.*, **31**, 1687-1710. https://doi.org/10.1007/s10443-024-10235-y.

Li, X., Du, C.C. and Li, Y.H. (2018), "Parametric resonance of a FG cylindrical thin shell with periodic rotating angular speeds in thermal environment", *Appl. Math. Model.*, **59**, 393-409. https://doi.org/10.1016/j.apm.2018.01.048.

Liew, K.M. and Lim, C.W. (1996), "A higher-order theory for vibration of doubly curved shallow shells", *ASME J. Appl. Mech.*, **63**(3), 587-593. https://doi.org/10.1115/1.2823338.

Mahapatra, T.R. and Panda, S.K. (2010), "Thermoelastic vibration analysis of laminated doubly curved shallow panels using non-Linear FEM", *J. Therm. Stress.*, **38**(1), 39-68. https://doi.org/10.1080/01495739.2014.976125.

Mantari, J.L., Oktem, A.S. and Guedes Soares, C. (2011), "Static and dynamic analysis of laminated composite and sandwich plates and shells by using a new higher-order shear deformation theory", *Compos. Struct.*, **94**(1), 37-49. https://doi.org/10.1016/j.compstruct.2011.07.020.

Matsunaga, H. (2007), "Vibration and stability of cross-ply laminated composite shallow shells subjected to in-plane stresses", *Compos. Struct.*, **78**(3), 377-391. https://doi.org/10.1016/j.compstruct.2005.10.013.

Mindlin, R.D. (1951), "Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates", *J. Appl. Mech.*, **18**(1), 31-38. https://doi.org/10.1115/1.4010217.

Monge, J.C., Mantari, J.L., Yarasca, J. and Arciniega, R.A. (2019), "On bending response of doubly curved laminated composite shells using hybrid refined models", *J. Appl. Comput. Mech.*, **5**(5), 875-899. https://doi.org/10.22055/jacm.2019.27297.1397.

Naghdi, P.M. (1973), "The theory of shells and plates", *Linear Theories of Elasticity and Thermoelasticity: Linear and Nonlinear Theories of Rods, Plates, and Shells*, Springer Berlin Heidelberg, Berlin, Heidelberg.

Panda, S.K. and Singh, B.N. (2010), "Nonlinear free vibration analysis of thermally post-buckled composite spherical shell panel", *Int. J. Mech. Mater. Des.*, **6**, 175-188. https://doi.org/10.1007/s10999-010-9127-1.

Qatu, M.S. (1999), "Accurate equations for laminated composite deep thick shells", *Int. J. Solid. Struct.*, **36**(19), 2917-2941. https://doi.org/10.1016/S0020-7683(98)00134-6.

Ramteke, P.M., Mehar, K., Sharma, N. and Panda, S.K. (2021), "Numerical prediction of deflection and stress responses of functionally graded structure for grading patterns (power-Law, sigmoid, and exponential) and variable porosity (even/uneven)", *Scientia Iranica*, **28**(2), 811-829. https://doi.org/10.24200/sci. 2020.55581.4290.

Reddy, J.N. (1984), "A simple higher-order theory for laminated composite plates", *J. Appl. Mech.*, **51**(4), 745-752. https://doi.org/10.1115/1.3167719.

Reddy, J.N. and Liu, C.F. (1985), "A higher-order shear deformation theory of laminated elastic shells", *Int. J. Eng. Sci*., **23**(3), 319-330. https://doi.org/10.1016/0020-7225(85)90051-5.

Reissner, E. (1955), "Non-linear effects in vibrations of cylindrical shells", Ramo-Wooldridge Corporation, Guided Missile Research Division, Aeromechanics Section.

Remil, A., Benrahou, K.H., Draiche, K., Bousahla, A.A. and Tounsi, A. (2019), "A simple HSDT for bending, buckling and dynamic behavior of laminated composite plates", *Struct. Eng. Mech*., **70**(3), 325-337. https://doi.org/10.12989/sem.2019.70. 3.325.

Saad, A.L., Nuwairan, M. and Javed, S. (2021), "Free vibration of composite cylindrical shells based on third-order shear deformation theory", *J. Math.*, **2021**(1), 3792164. https://doi.org/10.1155/2021/3792164.

Sahoo, S.S., Panda, S.K. and Sen, D. (2016), "Effect of delamination on static and dynamic behavior of laminated composite plate", *AIAA J.*, **54**(8), 2530-2544. https://doi.org/10.2514/1.J054908.

Sahu, P., Sharma, N., Dewangan, H.C. and Panda, S.K. (2022), "Thermo-mechanical transient flexure of Glass-Carbon-Kevlar-Reinforced hybrid curved composite shell panels: An experimental verification", *Int. J. Appl. Mech*., **14**(01), 2150120. https://doi.org/10.1142/S1758825121501209.

Satankar, R.K., Sharma, N., Panda, S.K. and Mohapatra, S.S. (2020), "Experimental and simulation study of eigen frequency responses of Luffa cylindrica sponge fibre polymer composite", *Mater. Today: Proc.*, **33**, 5561-5565. https://doi.org/10.1016/j.matpr.2020.03.552.

Sayyad, A.S. and Ghugal, Y.M. (2022), "Assessment of refined higher order theories for the static and vibration analysis of laminated composite cylindrical shells", *J. Mech. Eng. Sci*., **16**(2), 8848-8861. https://doi.org/10.15282/jmes.16.2.2022.04. 0700.

Sharma, N., Mahapatra, T.R. and Panda, S.K. (2017), "Vibro-acoustic behaviour of shear deformable laminated composite flat panel using BEM and the higher order shear deformation theory", *Compos. Struct.*, **180**, 116-129. https://doi.org/10.1016/j.compstruct.2017.08.012.

Sharma, N., Mahapatra, T.R., Panda, S.K. and Hirwani, C.K. (2018), "Acoustic radiation and frequency response of higher-order shear deformable multilayered composite doubly curved shell panel-An experimental validation", *Appl. Acoust.*, **133**, 38-51. https://doi.org/10.1016/j.apacoust.2017.12.013.

Sheng, G.G. and Wang, X. (2018), "The dynamic stability and nonlinear vibration analysis of stiffened functionally graded cylindrical shells", *Appl. Math. Model*., **56**, 389-403. https://doi.org/10.1016/j.apm.2017.12.021.

Shinde, B.M and Sayyad, A.S. (2020), "Analysis of laminated and sandwich spherical shells using a new higher-order theory", *Adv. Aircraft Spacecraft Sci*., **7**(1), 19-40. https://doi.org/10.12989/aas.2020.7.1.019.

Simitses, G.J. and Chen, Z. (1988), "Buckling of delaminated, long, cylindrical panels under pressure", *Comput. Struct*., **28**(2), 173-184. https://doi.org/10.1016/0045-7949(88)90037-5.

Singh, V.K., Mahapatra, T.R. and Panda, S.K. (2016), "Nonlinear transient analysis of smart laminated composite plate integrated with PVDF sensor and AFC actuator", *Compos. Struct*., **157**, 121-130. https://doi.org/10.1016/j.compstruct.2016.08.020.

Sivadas, K.R. and Ganesan, N. (1991), "Vibration analysis of laminated conical shells with variable thickness", *J. Sound Vib*., **148**(3), 477-491. https://doi.org/10.1016/0022-460X(91)90479-4.

Sobhani, E. (2023), "Improvement of vibrational characteristics of multipurpose structures (plate and shells) used in aerospace components by deploying Graphene Oxide Powders (GOPs) in a matrix as a nano-reinforcement: A comprehensive study", *Eng. Anal. Bound. Elem*., **146**, 598-635. https://doi.org/10.1016/j.enganabound.2022.11.014.

Soldatos, K.P. (1992), "A transverse shear deformation theory for homogeneous monoclinic plates", *Acta Mechanica*., **94**(3) 195-220. https://doi.org/10.1007/BF01176650.

Swami, S.K. and Ghugal, Y.M. (2021), "Thermoelastic bending analysis of laminated plates subjected to linear and nonlinear thermal loads", *Adv. Aircraft Spacecraft Sci*., **8**(3), 213-237. https://doi.org/10.12989/aas.2021.8.3.213.

Tang, D., Yao, X., Wu, G. and Peng, Y. (2017), "Free and forced vibration analysis of multi-stepped circular cylindrical shells with arbitrary boundary conditions by the method of reverberation-ray matrix", *Thin Wall. Struct.*, **116**, 154-168. https://doi.org/10.1016/j.tws.2017.03.023.

Thakur, S.N., Ray, C. and Chakraborty, S. (2017), "A new efficient higher-order shear deformation theory for a doubly curved laminated composite shell", *Acta Mechanica*, **228**(1), 69-87. https://doi.org/10.1007/s00707-016-1693-3.

Topal, U. (2009), "Multiobjective optimization of laminated composite cylindrical shells for maximum frequency and buckling load", *Mater. Des.*, **30**(7), 2584-2594. https://doi.org/10.1016/j.matdes.2008.09.020.

Torkamani, Sh., Navazi, H.M., Jafari, A.A. and Bagheri, M. (2009), "Structural similitude in free vibration of orthogonally stiffened cylindrical shells", *Thin Wall. Struct.*, **47**(11), 1316-1330. https://doi.org/10.1016/j.tws.2009.03.013.

Tornabene, F., Viola, E. and Fantuzzi, N. (2013), "General higher-order equivalent single layer theory for free vibrations of doubly-curved laminated composite shells and panels", *Compos. Struct.*, **104**, 94-117. https://doi.org/10.1016/j.compstruct.2013.04.009.

Touratier, M. (1991), "An efficient standard plate theory", *Int. J. Eng. Sci.*, **29**(8), 901-916. https://doi.org/10.1016/0020-7225(91)90165-Y.

Zenkour, A.M. (2015), "Thermal bending of layered composite plates resting on elastic foundations using four-unknown shear and normal deformations theory", *Compos. Struct.*, **122**, 260-270. https://doi.org/10.1016/j.compstruct.2014.11.064.

Zhong, R., Tang, J., Wang, A., Shuai, C. and Wang, Q. (2019), "An exact solution for free vibration of cross-ply laminated composite cylindrical shells with elastic restraint ends", *Comput. Math. Appl.*, **77**(3), 641-661. https://doi.org/10.1016/j.camwa.2018.10.006.

CC