Advances in Nano Research, *Vol. 19, No. 4 (2025) 347-365*
https://doi.org/10.12989/anr.2025.19.4.347

Future of metamaterial-computer interaction: The combination of graphene origami and piezoelectric intelligent for application in sport pole vault

Weiwei Wang[1], Defang Chen*[1], Mostafa Habibi[2,3], Monire Norouzi[4] and Y. Duang

[1]*Nanchang Institute of Science & Technology, Nanchang 330108, Jiangxi, China*
[2]*Department of Biomaterials, Saveetha Dental College and Hospital, Saveetha Institute of Medical and Technical Sciences, Chennai, 600077, India*
[3]*Department of Mechanical Engineering, Faculty of Engineering, Haliç University, Istanbul, Turkey*
[4]*Department of Computer Engineering, Faculty of Engineering, Halic University, 34060, Istanbul, Turkey*
[5]*School of Computer Science, Production Engineering Group, department of Construction, Kuala Lumpur, Malaysia*

(*Received March 19, 2025, Revised September 7, 2025, Accepted September 9, 2025*)

Abstract. This study presents a comprehensive multi-physics analysis of wave propagation characteristics in a shear-deformable sandwich beam, employing a novel higher-order thickness-stretched model. The physical system comprises a graphene origami-reinforced copper matrix core, integrated with piezoelectric and piezomagnetic face-sheets, operating under combined thermal, electrical, and magnetic excitations. The formulation rigorously incorporates two-dimensional constitutive relations for the shear-deformable structure, coupled with the governing electric potential and magnetic induction equations. Hamilton's principle is applied to derive the system's governing equations, explicitly accounting for the interdependent effects of thermal gradients, applied electric potentials, and magnetic inductions. The results of this analysis and proposed structure can be used for application in sport equipment such as pole vault. The constitutive behavior of the innovative GOri-copper composite core is critically modeled using temperature-dependent modifier functions within the Halpin-Tsai micromechanical framework. This captures the influence of key parameters—including graphene volume fraction, origami folding degree, and thermal load—on the effective modulus of elasticity, Poisson's ratio, thermal expansion coefficient, and density of the core material. An analytical methodology is developed for the multi-field (thermal-electro-magneto-mechanical) and multi-material analysis of the composite beam structure. This approach enables systematic investigation of wave propagation sensitivity to variations in core morphology (folding degree, volume fraction), environmental conditions (temperature), and excitation parameters (electric potential, magnetic induction). A detailed verification study establishes the validity and accuracy of the proposed higher-order model and analytical solution by benchmarking against established theories and available results. The derived numerical results demonstrate significant potential for the application of this smart sandwich structure in both sensor and actuator systems. The integrated face-sheets, coupled with the tailored GOri core, provide a robust platform for real-time measurement of mechanical deformation, strain, or stress states within composite structures via the embedded electromagnetic response. Furthermore, the model offers critical insights for the design and optimization of advanced multifunctional sandwich composites operating in complex thermal and electromagnetic environments, particularly for structural health monitoring and adaptive structural control applications.
Keywords: auxetic metamaterial; folding; electro-magneto-elastic results; graphene origami; initial electric/magnetic potentials; shear and normal deformation theory

1. Introduction

The advancement of modern technology is fundamentally intertwined with the development and application of novel materials and sophisticated control systems. This multi-disciplinary endeavor spans from large-scale environmental remediation and civil engineering to the precise micro-scale design of advanced functional materials and the complex dynamics of mechanical and electrical systems. A persistent challenge across these fields is the management of material behavior under stress, corrosion, and dynamic operational conditions, while simultaneously striving for enhanced efficiency, sustainability, and intelligence.

In the realm of environmental and construction materials, significant research is focused on repurposing industrial waste to promote a circular economy. For instance, another work demonstrates the potential of industrial by-products like red mud and fly ash. These materials can be effectively used for the remediation of heavy metal pollutants in water and engineered into high-strength geopolymers and cementitious materials. However, the long-term durability of such materials is contingent on their resistance to environmental stressors, particularly chemical corrosion from acids and alkalis, which can compromise their structural integrity and containment capabilities.

Parallel to material science innovations, the field of mechanical and manufacturing engineering continuously seeks to understand and control complex processes. Studies on keyhole stability in deep penetration welding and the

*Corresponding author, Ph.D.,
E-mail: 550352730@qq.com

http://www.techno-press.org/?journal=journal=anr&subpage=5 ISSN: 2287-237X (Print), 2287-2388 (Online)

solidification microstructure of alloy joints are critical for achieving high-integrity manufacturing. Furthermore, the prediction and management of friction and wear, as seen in the modeling of elastohydrodynamic lubrication (EHL) in bearings, are essential for improving the longevity and reliability of mechanical systems. The accurate measurement and monitoring of mechanical components, such as through advanced piezoelectric ultrasonic methods for bolt pretension stress, are equally vital for ensuring structural safety. Recent advancements in materials science and engineering have been profoundly shaped by the integration of nanotechnology and computational modeling, leading to the development of next-generation composites and smart structures with tailored mechanical properties. This interdisciplinary field explores the behavior of materials and systems across multiple scales, from the nano and micro to the macro level. Key areas of focus include the enhancement of structural stability, dynamic response, and functional performance in diverse applications. Research by Cao *et al*. (2025) employs machine learning and a three-dimensional layerwise theory to investigate delamination and bending in cracked, graphene-reinforced curved panels, while Huang *et al*. (2025) and Li *et al*. (2025) utilize deep neural networks and hybrid intelligent models to analyze wave responses in seismic nanobeams and complex piezo-electric shells, respectively. Concurrently, studies explore the mechanical fundamentals of advanced materials, from the performance of auxetic metamaterials (Hu *et al*. 2025) and the strengthening of 3D-printed polymers (Fang *et al*. 2025) to the evaluation of textile microfibers (Liu *et al*. 2025) and the wear-reducing properties of nano-lubricants (Tan *et al*. 2025). The evolution towards smarter systems is marked by the integration of sensing, control, and advanced materials. This is evident in the development of multi-functional self-sensing components for structural health monitoring and the creation of composite fibers with enhanced piezoelectricity for energy applications. In power electronics, robust stability design for inverters ensures reliable energy conversion. Similarly, in automotive engineering, precise control strategies from clutch friction estimation and gearshift management to real-time combustion phase optimization in engines are paramount for improving performance and efficiency. Underpinning many of these technological advancements is the critical need to model and understand complex dynamic behaviors. This includes the development of novel thermodynamic constitutive models for granular materials like soils, accounting for phenomena such as particle breakage (Bai *et al*. 2025), as well as the application of advanced computational methods like the differential quadrature and point interpolation mesh-free methods to analyze the vibrations and frequency characteristics of sophisticated nanostructures and smart panels. Even the design of meta-materials for omnidirectional sound wave absorption (Bai *et al*. 2025) relies on a deep understanding of wave propagation and material interaction.

This introduction sets the stage for a broader discussion on these interconnected research fronts. The following sections will explore in greater detail the latest breakthroughs in mention your specific focus area, e.g., sustainable material design, intelligent system control, advanced manufacturing processes, or multi-functional material applications], highlighting how the insights from the referenced studies contribute to overcoming existing challenges and driving innovation forward. The relentless pursuit of enhanced performance, durability, and intelligence in mechanical, materials, and energy systems drives innovation across engineering disciplines. A central challenge in this endeavor is the accurate prediction and control of complex physical phenomena—from dynamic vibrations and material failure to energy conversion and fluid dynamics—often under extreme or variable operating conditions. Modern research addresses these challenges through a powerful synergy of advanced materials, sophisticated computational modeling, and data-driven methodologies, moving beyond traditional approaches to create smarter, more adaptive, and more efficient solutions. Furthermore, these principles are applied to revolutionize fields like biomedical engineering, with work on hemodynamic responses (Chang *et al*. 2025), protein tissue stability (Chen *et al*. 2025), and nanodevice stability for drug delivery (Chen *et al*. 2025), as well as sports science, through the analysis of equipment geometry (Dai *et al*. 2022), ball dynamics (Daichang *et al*. 2025), and athletic technique (Liu *et al*. 2025). Together, these works from 2025 demonstrate a powerful synergy between experimental validation and advanced computational intelligence to solve complex problems in modern mechanics. The integration of advanced materials science and cutting-edge engineering analysis is revolutionizing the design and functionality of sports equipment and athletic infrastructure. Contemporary research is increasingly focused on enhancing performance, safety, and innovation through the application of nanotechnology, smart materials, and sophisticated computational modeling. A significant body of work in 2025 demonstrates this trend, exploring the stability and dynamic response of sports structures. A critical frontier lies in the domain of dynamic response and vibration control. The mitigation of detrimental vibrations, such as the vertical chatter in industrial rolling mills, is being revolutionized by smart materials like magnetorheological fluids, which enable real-time, adaptive damping systems. Concurrently, accurately predicting the dynamic behavior of complex structures, from the torsional responses of metal-foam beams to the bending of advanced composite plates and foldable shells, requires increasingly sophisticated analytical and numerical models. The stability of intricate structures, such as cable domes, further underscores the need for precise mechanical analysis to ensure structural integrity.

The pursuit of enhanced mechanical performance in advanced structures has led to a significant paradigm shift towards the integration of innovative materials and sophisticated modeling techniques. A prominent research thrust, as evidenced by recent studies, focuses on improving the static and dynamic stability of shell structures under extreme thermomechanical loads. For instance, Ebrahimi and collaborators have extensively explored the use of bio-inspired architectures, demonstrating that sandwich composite toroidal shells with an auxetic core can significantly enhance nonlinear static stability under axial compression (Ebrahimi *et al*. 2025a) and thermomechanical loading (Ebrahimi *et al*. 2025b). Further expanding the application of these smart materials, their work on star-shaped auxetic metabeams showcases their viability for advanced engineering applications like piezoelectric vibration energy harvesting (Ebrahimi *et al*. 2025c). The complexity of dynamic responses in novel composites is also a critical area of investigation, with studies revealing the chaotic and nonlinear behavior of hybrid nanocomposite cylindrical shells enabled by graphene origami configurations (Ebrahimi *et al*. 2025d). Concurrently, the vibration and flutter instability of fluid-conveying conical shells have been addressed through the use of functionally graded graphene platelet (FG-GPL) and carbon nanotube (FG-CNT) reinforced composites, offering pathways to more stable and resilient designs (Ghasemi *et al*. 2025). This body of work builds upon foundational research into the vibrational characteristics of nanoscale materials, such as circular graphene sheets in thermal environments (Ebrahimi *et al*. 2022). Complementing these material-centric approaches, the field is increasingly adopting data-driven methods, as seen in the application of machine learning for the optimal design of fluid-conveying pipes to mitigate weak vibrations (Deng *et al*. 2025). Collectively, these studies underscore a multidisciplinary movement towards creating next-generation structures that are not only lighter and stronger but also intelligently designed to withstand complex operational demands through the synergistic use of auxetic materials, nanocomposites, and computational intelligence.

Underpinning the reliability of these systems is the fundamental issue of fatigue and structural failure. Traditional life prediction models are being transcended by novel approaches that integrate machine learning with the underlying physics of damage accumulation, enabling more accurate multi-mode fatigue life predictions essential for aerospace, automotive, and infrastructure applications (Hao *et al*. 2025). Furthermore, the integrity of welded and joined components remains paramount, with research focusing on how factors like axial misalignment affect the microstructure and subsequent mechanical and corrosion properties of joints. The push for greater efficiency also extends to fluid dynamics and energy systems. Research into cavitation-induced noise and its abatement through biomimetic designs like leading-edge protuberances highlights the interdisciplinary effort to improve the performance and environmental footprint of marine and hydraulic systems. In energy conversion, precision is key; deep learning is now being applied to optimize fuel injection strategies in high-pressure common rail systems to maximize energy efficiency, while novel materials like cellulose ionogels are opening new pathways for harvesting low-grade thermal energy. Beyond equipment, the field is also advancing athlete care, with innovative approaches utilizing stem cells in reconstructive treatments for sports injuries (Yu *et al*. 2023). This collective effort, encompassing everything from foldable metamaterials for pressure vessels (Ying *et al*. 2025) to wave propagation in poroelastic biocomposite beams (Yuanchao *et al*. 2025b), underscores a multi-disciplinary movement towards creating a new generation of high-performance, intelligent, and reliable solutions for the world of sports. Finally, these advancements are enabled by breakthroughs in materials science and measurement technology. The development of novel catalysts for biofuel conversion (Li *et al*. 2022), the design of wavelength-selective organic detectors for advanced sensing and the creation of techniques for extreme environment measurement—such as strain gauging at temperatures up to 3000°C collectively expand the toolbox available to engineers. The integration of artificial intelligence, exemplified by neural networks for predicting the stability of microscale structures (Liang *et al*. 2024), signifies a paradigm shift towards intelligent design and analysis. This introduction frames the current interdisciplinary effort to solve complex engineering problems. The following discussion will delve deeper into how these interconnected themes—spanning dynamic control, failure prediction, energy optimization, and advanced materials—are collectively shaping the future of engineering systems, with a specific focus on [mention your specific focus area, e.g., intelligent vibration suppression, data-driven life prediction, or sustainable energy harvesting. The frontier of modern engineering is being reshaped by the convergence of advanced materials, intelligent systems, and multi-physics modeling. This interdisciplinary push aims to overcome longstanding challenges in performance, sustainability, and adaptability across a vast spectrum of applications, from micro-scale sensors to large-scale infrastructure. Central to this evolution is the development of novel materials with tailored properties—such as piezoelectric composites, graphene reinforcements, and smart meta-materials—and the sophisticated computational frameworks required to predict their complex behavior under mechanical, electrical, and environmental stimuli.

A significant thrust of contemporary research focuses on the design, analysis, and optimization of advanced composite structures. The integration of nano-reinforcements like graphene nanoplatelets and origami-inspired architectures into polymers and sandwiches has unlocked unprecedented mechanical performance, enabling lightweight yet high-strength components for aerospace, sports equipment, and biomedical applications. Accurately predicting the static and dynamic responses of these complex structures, especially at micro-scales where size-dependent effects become critical, necessitates advanced theoretical frameworks like modified strain gradient theory and sophisticated numerical methods such as the generalized differential quadrature element solution. Furthermore, understanding the multi-load effect on deformation and wave propagation in imperfect, functionally graded materials is essential for ensuring reliability in real-world. Parallel to these advancements in structural mechanics is the rapid innovation in functional materials for sensing and energy applications. The field of piezoelectric materials is particularly vibrant, with progress ranging from transferred PMN-PT thick films for actuators to the development of high-sensitivity, self-powered flexible pressure sensors based on multi-scale structured composites. These materials are the cornerstone of a new generation of smart devices, and cellulose-based platforms for electrochemical sensing. Innovations like Janus-type nanoribbon hydrogel arrays for stimulus response sensors and graphene-based electrodes for triboelectric nanogenerators further highlight the move towards multifunctional, energy-autonomous systems.

Underpinning the functionality of these smart systems is the critical need for intelligent control and noise mitigation. This is evident in the development of improved gain-scheduling robust model predictive control (MPC) for the path following of autonomous vehicles amidst uncertainties, as well as in the sophisticated modeling and suppression of magnetic noise in nanocrystalline shielding systems. The concept of smart meta-devices that can harness stray energy for both shielding and detection represents a paradigm shift towards green, self-powered intelligent systems. For instance, Xia *et al*. (2025) highlighted the importance of Micro-Electro-Mechanical Systems (MEMS) for testing athlete performance through computational stability analysis, while Xiao *et al*. (2025) investigated lightweight functionally graded tubular structures to pave a pathway for sports innovation. Concurrently, the development of novel materials is a key focus, with studies on polyvinyl chloride-based nanocomposites for equipment enhancement (Yang *et al*. 2025) and graphene nanoplatelet reinforcements for aerobic sport plates (Zhaowei *et al*. 2025). The role of smart technology is also paramount, as researchers develop applicable sensors made from smart nanomaterials to solve volleyball sport problems (Xu *et al*. 2025) and perform electroelastic analysis of piezoelectric shells for springboard and gymnastic training (Zhu *et al*. 2025). Furthermore, this research extends to understanding complex environmental interactions, such as the wave propagation response of porous equipment under thermal loading (YaJie *et al*. 2025) and the influence of humid-thermal environments on material dispersion (Yuanchao *et al*. 2025a). Finally, these advancements are supported by progress in manufacturing and foundational material processes. Research into ultrasonic spot welding of joints informs better assembly techniques, while studies on penetration grouting and slope deformation patterns provide critical insights for geotechnical engineering and construction. The very discovery of novel physical phenomena, such as reversible negative compressibility in meta-materials, continues to expand the design space for future technologies (Zha and Zhang 2024).

This introduction synthesizes the key drivers of innovation as illustrated by the referenced literature. The following discussion will explore in greater detail how the interplay between [Choose one or two key themes, e.g., “multi-functional nanocomposites,” “intelligent sensing and control,” or “advanced modeling of micro-structures”] is addressing critical engineering challenges and enabling the next generation of technological solutions.

2. Higher-order analysis

In this section, a higher-order shear deformation theory (HOSDT) is employed to conduct a comprehensive static analysis of stress, strain, and deformation fields within a cylindrical shell structure reinforced with graphene origami-enabled auxetic metamaterial (GOAM). This advanced kinematic framework accounts for nonlinear shear strain distributions through the shell thickness, overcoming limitations of classical shell theories for moderately thick structures and auxetic composites with complex micro-architectures. The governing partial differential equations are rigorously derived by integrating the generalized Hooke’s law, which defines the stress-strain relationships for the anisotropic GOAM-reinforced laminate, with Hamilton’s principle of virtual work. The resulting constitutive relations, which explicitly couple the material anisotropy of GOAM to the shell’s deformation kinematics, are formulated as follows:

$$\begin{aligned} \sigma_{xx} &= \frac{E}{(1-\vartheta^2)}\left\{\varepsilon_{xx} - \alpha(T-T_0) + \vartheta\big(\varepsilon_{zz} - \alpha(T-T_0)\big)\right\} \\ \sigma_{zz} &= \frac{E}{(1-\vartheta^2)}\left\{\vartheta\big(\varepsilon_{xx} - \alpha(T-T_0)\big) + \varepsilon_{zz} - \alpha(T-T_0)\right\}, \\ \tau_{xz} &= \frac{E}{(1-\vartheta^2)}\frac{1-\vartheta}{2}\gamma_{xz} \end{aligned} \quad (1)$$

The effective macroscopic constitutive properties governing the cylindrical shell’s elastic response specifically its equivalent modulus of elasticity (E_{eff}), Poisson’s ratio (ν_{eff}), and density (ρ_{eff}) are defined at the structural scale based on the multiscale architecture of the graphene origami auxetic metamaterial (GOAM) reinforcement. These homogenized properties, which capture the emergent auxetic behavior (negative Poisson’s ratio) and stiffness enhancement induced by the GOAM’s nanoscale origami patterning within the host matrix, are derived through micromechanical homogenization as follows:

$$\begin{aligned} E_{eff} &= \frac{1+\xi\eta V_{Gr}}{1-\eta V_{Gr}} E_{Cu} f_E(H_{Gr}, V_{Gr}, T), \\ \nu_{eff} &= (\nu_{Gr}V_{Gr} + \nu_{Cu}V_{Cu}) f_\nu(H_{Gr}, V_{Gr}, T), \\ \rho_{eff} &= (\rho_{Gr}V_{Gr} + \rho_{Cu}V_{Cu}) f_\rho(H_{Gr}, V_{Gr}, T), \\ \alpha_{eff} &= (\alpha_{Gr}V_{Gr} + \alpha_{Cu}V_{Cu}) f_\alpha(H_{Gr}, V_{Gr}, T), \end{aligned} \quad (2)$$

in which, are used as content percentage of Cu as matrix and graphene origami as reinforcement. Furthermore, (ν_{Gr}, ν_{Cu}), (ρ_{Gr}, ρ_{Cu}) and $(\alpha_{Gr}, \alpha_{Cu})$ are Poisson’s ratio, density and heat expansion coefficients of graphene origami and Cu matrix, respectively.

Where V_{Cu}, V_{Gr} represent the volume fractions of graphene origami reinforcement and copper matrix, respectively, constrained by the relation $V_{Cu} + V_{Gr} = 1$. The constituent material properties required for micromechanical homogenization include: Poisson’s ratios (ν_{Gr}, ν_{Cu}), where graphene origami exhibits intrinsic auxetic behavior ($\rho_{Gr} < 0$) due to its reconfigurable nanoscale topology Mass densities (ρ_{Gr}, ρ_{Cu}), with graphene origami’s ultra-low density enabling lightweight design Coefficients of thermal expansion $(\alpha_{Gr}, \alpha_{Cu})$, capturing differential thermomechanical responses

To account for real-world microstructural effects, environmental conditions, and scale-dependent phenomena not captured by the idealized homogenization in Eq. (1), empirically calibrated modification functions are introduced. These functions dynamically adjust the homogenized constitutive properties based on critical experimental parameters:

$$V_{Gr} = \frac{W_{Gr}}{W_{Gr} + \frac{\rho_{Gr}}{\rho_{Cu}}(1 - W_{Gr})}, V_{Cu} = 1 - V_{Gr} \tag{3}$$

To establish universal scaling relationships independent of absolute dimensions, the non-dimensional governing parameters characterizing the effective modulus of elasticity are derived through Buckingham π-theorem. These parameters synthesize the competing influences of graphene origami's nanoscale architecture and copper matrix properties into fundamental dimensionless groups defined as:

$$\eta = \frac{\frac{E_{Gr}}{E_{Cu}} - 1}{\frac{E_{Gr}}{E_{Cu}} + \xi}, \xi = 2\frac{l_{Gr}}{t_{Gr}} \tag{4}$$

Where E_{Gr}, E_{Cu} represent the intrinsic elastic moduli of the graphene origami nanostructure and copper matrix, respectively. The geometric anisotropy of graphene origami is characterized by its unit cell length l_{Gr} and thickness t_{Gr} critical parameters governing fold-induced auxetic behavior and stress transfer efficiency at nanoscale interfaces. The empirically derived modification functions $f_E(H_{Gr}, V_{Gr}, T), f_v(H_{Gr}, V_{Gr}, T), f_\rho(H_{Gr}, V_{Gr}, T), f_\alpha(H_{Gr}, V_{Gr}, T)$ incorporate three-dimensional dependency on:

$$\begin{aligned} &f_E(H_{Gr}, V_{Gr}, T) = 1.11 - 1.22V_{Gr} - 0.134\left(\frac{T}{T_0}\right) \\ &+0.559V_{Gr}\left(\frac{T}{T_0}\right) - 5.5V_{Gr}H_{Gr} + 38V_{Gr}^2H_{Gr} \\ &-20.6V_{Gr}^2H_{Gr}^2, \\ &f_v(H_{Gr}, V_{Gr}, T) = 1.01 - 1.43V_{Gr} + 0.165\left(\frac{T}{T_0}\right) \\ &-1.1V_{Gr}H_{Gr}\left(\frac{T}{T_0}\right) - 16.8V_{Gr}H_{Gr} + 16V_{Gr}^2H_{Gr}^2 \\ &f_\alpha(H_{Gr}, V_{Gr}, T) = 0.794 - 16.8V_{Gr}^2 - 0.0279\left(\frac{T}{T_0}\right)^2 \\ &+0.182(1 + V_{Gr})\left(\frac{T}{T_0}\right), \\ &f_\rho(H_{Gr}, V_{Gr}, T) = 1.01 - 2.01V_{Gr}^2 - 0.0131\left(\frac{T}{T_0}\right) \end{aligned} \tag{5}$$

Within these functions, T denotes the local instantaneous temperature during shell operation, while T_0=298K represents the reference temperature at which constituent material properties are characterized. This temperature differential $\triangle T = T - T_0$ governs critical thermomechanical phenomena.

The two components of displacement are:

$$\begin{aligned} u_x(x, \mathrm{z}) &= u(x) - \mathrm{z}\frac{\partial w_b(x)}{\partial x} - f(\mathrm{z})\frac{\partial w_s(x)}{\partial x}, \\ u_z(x, \mathrm{z}) &= w_b(x) + w_s(x) + g(\mathrm{z})\chi(x), \end{aligned} \tag{6}$$

Using the relations, the strain components are extended:

$$\begin{aligned} &\varepsilon_x = \frac{\partial u(x)}{\partial x} - \mathrm{z}\frac{\partial^2 w_b(x)}{\partial x^2} - f(\mathrm{z})\frac{\partial^2 w_s(x)}{\partial x^2}, \\ &\varepsilon_z = \frac{dg(\mathrm{z})}{dz}\chi(x),\ \gamma_{xz} = g(\mathrm{z})\left(\frac{\partial w_s(x)}{\partial x} + \frac{\partial \chi(x)}{\partial x}\right) \end{aligned} \tag{7}$$

The constitutive relations in the presence of the mechanical, electric and magnetic field components are derived as follows:

$$\begin{aligned} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \tau_{xz} \end{Bmatrix} &= \begin{bmatrix} C_{xxxx} & C_{xxzz} & 0 \\ C_{xxzz} & C_{zzzz} & 0 \\ 0 & 0 & C_{xzxz} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} - \alpha(T - T_0) \\ \varepsilon_{zz} - \alpha(T - T_0) \\ \gamma_{xz} \end{Bmatrix} \\ &- \begin{bmatrix} 0 & e_{xxz} \\ 0 & e_{zzz} \\ e_{xzx} & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_z \end{Bmatrix} - \begin{bmatrix} 0 & q_{xxz} \\ 0 & q_{zzz} \\ q_{xzx} & 0 \end{bmatrix} \begin{Bmatrix} H_x \\ H_z \end{Bmatrix}, \end{aligned} \tag{8}$$

Using the strain components and electric and magnetic potential components, the extended stress components are derived as follows:

$$\begin{aligned} &\sigma_{xx} = C_{xxxx}\left(\frac{\partial u}{\partial x} - \mathrm{z}\frac{\partial^2 w_b}{\partial x^2} - f(\mathrm{z})\frac{\partial^2 w_s}{\partial x^2} - \alpha(T - T_0)\right) \\ &+C_{xxzz}\left(\frac{dg(\mathrm{z})}{dz}\chi - \alpha(T - T_0)\right) \\ &+e_{xxz}\left(\frac{2\Psi_0}{h} + \frac{\pi}{h}\psi(x)sin\left(\frac{\pi z}{h}\right)\right) \\ &+q_{xxz}\left(\frac{2\Phi_0}{h} + \frac{\pi}{h}\phi(x)sin\left(\frac{\pi z}{h}\right)\right) \\ &\sigma_{zz} = C_{xxzz}\left(\frac{\partial u}{\partial x} - \mathrm{z}\frac{\partial^2 w_b}{\partial x^2} - f(\mathrm{z})\frac{\partial^2 w_s}{\partial x^2} - \alpha(T - T_0)\right) \\ &+C_{zzzz}\left(\frac{dg(\mathrm{z})}{dz}\chi - \alpha(T - T_0)\right) \\ &+e_{zzz}\left(\frac{2\Psi_0}{h} + \frac{\pi}{h}\psi(x)sin\left(\frac{\pi z}{h}\right)\right) \\ &+q_{zzz}\left(\frac{2\Phi_0}{h} + \frac{\pi}{h}\phi(x)sin\left(\frac{\pi z}{h}\right)\right) \\ &\tau_{xz} = C_{xzxz}g(\mathrm{z})\left(\frac{\partial w_s}{\partial x} + \frac{\partial \chi}{\partial x}\right) - e_{xzx}\frac{\partial \psi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right) \\ &-q_{xzx}\frac{\partial \phi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right) \end{aligned} \tag{9}$$

The electric displacement and magnetic induction are introduced as follows

$$\begin{aligned} \begin{Bmatrix} D_x \\ D_z \end{Bmatrix} &= \begin{bmatrix} 0 & 0 & e_{xxz} \\ e_{zxx} & e_{zzz} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} - \alpha(T - T_0) \\ \varepsilon_{zz} - \alpha(T - T_0) \\ \gamma_{xz} \end{Bmatrix} \\ &+ \begin{bmatrix} \epsilon_{xx} & 0 \\ 0 & \epsilon_{zz} \end{bmatrix} \begin{Bmatrix} E_x \\ E_z \end{Bmatrix} + \begin{bmatrix} m_{xx} & 0 \\ 0 & m_{zz} \end{bmatrix} \begin{Bmatrix} H_x \\ H_z \end{Bmatrix}, \end{aligned} \tag{10}$$

The magnetic induction relations in the presence of the mechanical loads, electric and magnetic potentials are derived in the following format:

$$\begin{aligned} \begin{Bmatrix} B_x \\ B_z \end{Bmatrix} &= \begin{bmatrix} 0 & 0 & q_{xxz} \\ q_{zxx} & q_{zzz} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} - \alpha(T - T_0) \\ \varepsilon_{zz} - \alpha(T - T_0) \\ \gamma_{xz} \end{Bmatrix} \\ &+ \begin{bmatrix} m_{xx} & 0 \\ 0 & m_{zz} \end{bmatrix} \begin{Bmatrix} E_x \\ E_z \end{Bmatrix} + \begin{bmatrix} \mu_{xx} & 0 \\ 0 & \mu_{zz} \end{bmatrix} \begin{Bmatrix} H_x \\ H_z \end{Bmatrix}, \end{aligned} \tag{11}$$

Electric and magnetic field components are derived as:

$$\psi(x, z) = \frac{2z\Omega_0}{h} - \psi(x)cos\left(\frac{\pi z}{h}\right), \tag{12}$$

$$\phi(x,z)=\frac{2z\Theta_0}{h}-\phi(x)cos\left(\frac{\pi z}{h}\right),$$

Using the defined electric and magnetic potential, one can arrive at electric and magnetic field components:

$$\begin{aligned}E_x&=-\frac{\partial\psi(x,z)}{\partial x}=\frac{\partial\psi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right),\\E_z&=-\frac{\partial\psi(x,z)}{\partial z}=-\frac{2\Psi_0}{h}-\frac{\pi}{h}\psi(x)sin\left(\frac{\pi z}{h}\right)\\H_x&=-\frac{\partial\phi(x,z)}{\partial x}=\frac{\partial\phi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right),\\H_z&=-\frac{\partial\phi(x,z)}{\partial z}=-\frac{2\Phi_0}{h}-\frac{\pi}{h}\phi(x)sin\left(\frac{\pi z}{h}\right)\end{aligned}\quad(13)$$

Substitution yields to:

$$\begin{aligned}D_x&=e_{xxz}g(\mathrm{z})\left(\frac{\partial w_s}{\partial x}+\frac{\partial\chi}{\partial x}\right)\\&-\epsilon_{xx}\left(\frac{\partial\psi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right)\right)+m_{xx}\left(\frac{\partial\phi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right)\right)\\D_z&=e_{zxx}\begin{pmatrix}\frac{\partial u(x)}{\partial x}-\mathrm{z}\frac{\partial^2w_b(x)}{\partial x^2}\\-f(\mathrm{z})\frac{\partial^2w_s(x)}{\partial x^2}-\alpha(T-T_0)\end{pmatrix}\\&+e_{zzz}\left(\frac{dg(\mathrm{z})}{dz}\chi(x)-\alpha(T-T_0)\right)\\&+\epsilon_{zz}\left(-\frac{2\Psi_0}{h}-\frac{\pi}{h}\psi(x)sin\left(\frac{\pi z}{h}\right)\right)\\&+m_{zz}\left(-\frac{2\Phi_0}{h}-\frac{\pi}{h}\phi(x)sin\left(\frac{\pi z}{h}\right)\right)\end{aligned}\quad(14)$$

and

$$\begin{aligned}B_x&=q_{xxz}g(\mathrm{z})\left(\frac{\partial w_s}{\partial x}+\frac{\partial\chi}{\partial x}\right)\\&-m_{xx}\left(\frac{\partial\psi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right)\right)+\mu_{xx}\left(\frac{\partial\phi(x)}{\partial x}cos\left(\frac{\pi z}{h}\right)\right)\\B_z&=q_{zxx}\begin{pmatrix}\frac{\partial u(x)}{\partial x}-\mathrm{z}\frac{\partial^2w_b(x)}{\partial x^2}\\-f(\mathrm{z})\frac{\partial^2w_s(x)}{\partial x^2}-\alpha(T-T_0)\end{pmatrix}\\&+q_{zzz}\left(\frac{dg(\mathrm{z})}{dz}\chi(x)-\alpha(T-T_0)\right)\\&+m_{zz}\left(-\frac{2\Psi_0}{h}-\frac{\pi}{h}\psi(x)sin\left(\frac{\pi z}{h}\right)\right)\\&+\mu_{zz}\left(-\frac{2\Phi_0}{h}-\frac{\pi}{h}\phi(x)sin\left(\frac{\pi z}{h}\right)\right)\end{aligned}\quad(15)$$

The governing equations that describe the mechanical behavior of the system are rigorously derived using the Principle of Virtual Work, a fundamental concept in analytical mechanics and the finite element method. The PVW states that for a system in equilibrium, the total virtual work done by all internal and external forces under any arbitrary, kinematically admissible virtual displacement is zero:

$$\delta U=\iiint_V\begin{pmatrix}\sigma_x\delta\varepsilon_x+\sigma_z\delta\varepsilon_z+\tau_{xz}\delta\gamma_{xz}\\-D_x\delta E_x-D_z\delta E_z-B_x\delta H_x-B_z\delta H_z\end{pmatrix}\mathrm{d}V\quad(16)$$

An update version of the strain energy variation in the electromagnetic field, one can arrive at the more detailed one as follows:

$$\delta U=\iiint_V\begin{pmatrix}\sigma_x\begin{bmatrix}\frac{\partial\delta u(x)}{\partial x}-\mathrm{z}\frac{\partial^2\delta w_b(x)}{\partial x^2}\\-f(\mathrm{z})\frac{\partial^2\delta w_s(x)}{\partial x^2}\end{bmatrix}\\+\sigma_z\left[\frac{dg(\mathrm{z})}{dz}\delta\chi(x)\right]\\+\tau_{xz}\left[g(\mathrm{z})\left(\frac{\partial\delta w_s(x)}{\partial x}+\frac{\partial\delta\chi(x)}{\partial x}\right)\right]\\-D_xcos\left(\frac{\pi z}{h}\right)\frac{\partial\delta\psi(x)}{\partial x}+D_z\frac{\pi}{h}\delta\psi(x)sin\left(\frac{\pi z}{h}\right)\\-B_xcos\left(\frac{\pi z}{h}\right)\frac{\partial\delta\phi(x)}{\partial x}+B_z\frac{\pi}{h}\delta\phi(x)sin\left(\frac{\pi z}{h}\right)\end{pmatrix}\mathrm{d}V\quad(17)$$

To facilitate the formulation of the governing equations for a structural element (e.g., a plate, shell, or beam), the abstract variation of strain energy, expressed in terms of the three-dimensional stress and strain fields, must be translated into a practical form utilizing stress resultants. These resultants are mechanically equivalent forces and moments that represent the integrated effect of the stresses through the thickness of the structure. This dimensional reduction is critical for developing efficient, two-dimensional models from three-dimensional continuum mechanics.

$$\delta U=\iiint_V\begin{pmatrix}\left[N_x\frac{\partial\delta u(x)}{\partial x}-M_x\frac{\partial^2\delta w_b(x)}{\partial x^2}-S_x\frac{\partial^2\delta w_s(x)}{\partial x^2}\right]\\+[N_z\delta\chi(x)]+\left[N_{xz}\left(\frac{\partial\delta w_s(x)}{\partial x}+\frac{\partial\delta\chi(x)}{\partial x}\right)\right]\\-\mathcal{D}_x\frac{\partial\delta\psi(x)}{\partial x}+\mathcal{D}_z\delta\psi(x)-\mathcal{B}_x\frac{\partial\delta\phi(x)}{\partial x}\\+\mathcal{B}_z\delta\phi(x)\end{pmatrix}\mathrm{d}x\quad(18)$$

By definition of resultant components as:

$$\begin{aligned}\{N_x,M_x,S_x\}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_x\{1,z,f(\mathrm{z})\}\mathrm{dz},\\\{N_z,M_{xy},S_{xy}\}&=\int_{-h/2}^{h/2}\frac{dg(\mathrm{z})}{dz}\sigma_z\mathrm{d}\mathbb{Z},\\\{N_{xz}\}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\tau_{xz}g(\mathrm{z})\mathrm{d}\mathbb{Z}.\\\{\mathcal{D}_x,\mathcal{B}_x\}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}cos\left(\frac{\pi z}{h}\right)\{D_x,B_x\}\mathrm{dz},\\\{\mathcal{D}_z,\mathcal{B}_z\}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}D_z\frac{\pi}{h}sin\left(\frac{\pi z}{h}\right)\{D_z,B_z\}\mathrm{dz}\end{aligned}\quad(19)$$

We will have variation of strain energy as follows:

$$\delta U=\iiint_V\begin{pmatrix}-\frac{\partial N_x}{\partial x}\delta u(x)-\frac{\partial^2M_x}{\partial x^2}\delta w_b(x)-\frac{\partial^2S_x}{\partial x^2}\delta w_s(x)\\-\frac{\partial N_{xz}}{\partial x}\delta w_s(x)+N_z\delta\chi(x)-\frac{\partial N_{xz}}{\partial x}\delta\chi(x)\\+\frac{\partial\mathcal{D}_x}{\partial x}\delta\psi(x)+\mathcal{D}_z\delta\psi(x)+\frac{\partial\mathcal{B}_x}{\partial x}\delta\phi(x)\\+\mathcal{B}_z\delta\phi(x)\end{pmatrix}\mathrm{d}x\quad(20)$$

In addition, the external works is defined as follows:

$$\delta W=\int_0^L\begin{bmatrix} q\delta u_z\big|_{z=+\frac{h}{2}}-F_f\delta u_z\big|_{z=-\frac{h}{2}} \\ -(N_x^T+N_x^E+N_x^M)\dfrac{d^2w}{dx^2}\end{bmatrix}\mathrm{d}x \qquad (21)$$

Substitution of transverse deflection into external work yields:

$$\begin{aligned}\delta W &= \int_0^L\Big[q\big(\delta w_b(x)+\delta w_s(x)+g(z=+h/2)\delta\chi(x)\big)\big|_{z=+h/2} \\ &- F_f\big(\delta w_b(x)+\delta w_s(x)+g(z=-h/2)\delta\chi(x)\big)\big|_{z=-h/2} \\ &- (N_x^T+N_x^E \\ &+ N_x^M)\frac{\partial^2\big(w_b(x)+w_s(x)+g(0)\chi(x)\big)}{\partial x^2}\big(\delta w_b(x)+\delta w_s(x) \\ &+ \delta\chi(x)\big)\Big]\mathrm{d}x\end{aligned} \qquad (22)$$

Variation in kinetic energy per unit width yields to:

$$\begin{aligned}\delta T &= \iiint_V (u_x\delta\dot{u}_x+u_z\delta\dot{u}_z)\mathrm{d}z\mathrm{d}x \\ &= \int_0^L\int_{-h/2}^{+h/2}\left\{\begin{matrix}(\dot{u}-\mathrm{z}\dot{w}_{b,x}-f(\mathrm{z})\dot{w}_{s,x}) \\ (\delta\dot{u}-\mathrm{z}\delta\dot{w}_{b,x}-f(\mathrm{z})\delta\dot{w}_{s,x}) \\ +(\dot{w}_b+\dot{w}_s+g(\mathrm{z})\dot{\chi}) \\ (\delta\dot{w}_b+\delta\dot{w}_s+g(\mathrm{z})\delta\dot{\chi})\end{matrix}\right\}\mathrm{bdzd}x\end{aligned} \qquad (23)$$

Using definition of some integration constants, one can arrive at kinetic energy as more compatible form. The integration constants are defined as follows:

$$\begin{aligned}&\ni_1=\int_{-h/2}^{+h/2}\rho dz\,,\ni_2=\int_{-h/2}^{+h/2}\rho zdz, \\ &\ni_3=\int_{-h/2}^{+h/2}\rho f(\mathrm{z})dz\,,\ni_4=\int_{-h/2}^{+h/2}\rho\mathrm{z}^2dz\,, \\ &\ni_5=\int_{-h/2}^{+h/2}\rho zf(\mathrm{z})dz,\ \ni_6=\int_{-h/2}^{+h/2}\rho f^2(\mathrm{z})dz\,, \\ &\ni_7=\int_{-h/2}^{+h/2}\rho g(\mathrm{z})dz\,,\ni_8=\int_{-h/2}^{+h/2}\rho g^2(\mathrm{z})dz\end{aligned}$$

Using the above definitions, one can arrive at variation in kinetic energy as follows:

$$\delta T=\int_0^L\left\{\begin{matrix}-\ni_1\ddot{u}(x)\delta u+\ni_2\ddot{w}_{b,x}\delta u+\ni_3\ddot{w}_{s,x}\delta u \\ -\ni_2\ddot{u}_{,x}\delta w_b+\ni_4\ddot{w}_{b,x,x}\delta w_b+\ni_5\ddot{w}_{s,x,x}\delta w_b \\ -\ni_3\ddot{u}_{,x}\delta w_s+\ni_5\ddot{w}_{b,x,x}\delta w_s+\ni_6\ddot{w}_{s,x,x}\delta w_s \\ -\ni_1\ddot{w}_b\delta w_b-\ni_1\ddot{w}_s\delta w_b-\ni_7\ddot{\chi}\delta w_b \\ -\ni_1\ddot{w}_b\delta w_s-\ni_1\ddot{w}_s\delta w_s-\ni_7\ddot{\chi}\delta w_s \\ -\ni_7\ddot{w}_b\delta\chi-\ni_7\ddot{w}_s\delta\chi-\ni_8\ddot{\chi}\delta\chi\end{matrix}\right\}\mathrm{d}x. \qquad (24)$$

Rearranging the variables leads to:

$$\begin{aligned}\delta T &= \int_0^L\left\{\begin{matrix}\big[-\ni_1\ddot{u}(x)+\ni_2\ddot{w}_{b,x}+\ni_3\ddot{w}_{s,x}\big]\delta u(x) \\ +\begin{bmatrix}-\ni_2\ddot{u}_{,x}+\ni_4\ddot{w}_{b,x,x}+\ni_5\ddot{w}_{s,x,x} \\ -\ni_1\ddot{w}_b(x)-\ni_1\ddot{w}_s(x)-\ni_7\ddot{\chi}(x)\end{bmatrix}\delta w_b(x) \\ +\begin{bmatrix}-\ni_3\ddot{u}_{,x}+\ni_5\ddot{w}_{b,x,x}+\ni_6\ddot{w}_{s,x,x} \\ -\ni_1\ddot{w}_b(x)-\ni_1\ddot{w}_s(x)-\ni_7\ddot{\chi}(x)\end{bmatrix}\delta w_s(x) \\ +[-\ni_7\ddot{w}_b(x)-\ni_7\ddot{w}_s(x)-\ni_8\ddot{\chi}(x)]\delta\chi(x)\end{matrix}\right\}\mathrm{d}x\end{aligned} \qquad (25)$$

The final governing equations of motion for the system are rigorously derived through the application of Hamilton's Principle, a variational fundamental that provides a powerful and systematic framework for mechanics. Hamilton's Principle states that the true path of a dynamical system between two specified states at times t_1 and t_2 is the one for which the Hamiltonian action integral, Π, is stationary (i.e., its first variation is zero).

$$\begin{aligned}&\delta u:-\frac{\partial\mathcal{N}_x}{\partial x}=-\ni_1\ddot{u}+\ni_2\ddot{w}_{b,x}+\ni_3\ddot{w}_{s,x}, \\ &\delta w_b:-\frac{\partial^2M_x}{\partial x^2}=q-F_f-(\mathcal{N}_x^T+\mathcal{N}_x^E+\mathcal{N}_x^M)\frac{\partial^2w_b}{\partial x^2} \\ &\quad-\ni_2\ddot{u}_{,x}+\ni_4\ddot{w}_{b,x,x}+\ni_5\ddot{w}_{s,x,x}-\ni_1\ddot{w}_b-\ni_1\ddot{w}_s-\ni_7\ddot{\chi}, \\ &\delta w_s:-\frac{\partial^2S_x}{\partial x^2}-\frac{\partial N_{xz}}{\partial x}=q-F_f-(\mathcal{N}_x^T+\mathcal{N}_x^E+\mathcal{N}_x^M)\frac{\partial^2w_b}{\partial x^2} \\ &\quad-\ni_3\ddot{u}_{,x}+\ni_5\ddot{w}_{b,x,x}+\ni_6\ddot{w}_{s,x,x}-\ni_1\ddot{w}_b-\ni_1\ddot{w}_s-\ni_7\ddot{\chi}, \\ &\delta\chi:\mathcal{N}_z-\frac{\partial\mathcal{N}_{xz}}{\partial x}=-(\mathcal{N}_x^T+\mathcal{N}_x^E+\mathcal{N}_x^M)\frac{\partial^2w_b}{\partial x^2} \\ &\quad-\ni_7\ddot{w}_b-\ni_7\ddot{w}_s-\ni_8\ddot{\chi}, \\ &\delta\psi:\frac{\partial\mathcal{D}_x}{\partial x}+\mathcal{D}_z=0,\delta\phi:\frac{\partial\mathcal{B}_x}{\partial x}+\mathcal{B}_z=0\end{aligned} \qquad (26)$$

Substitution of strain components into stress components and then into resultant components leads to:

$$\begin{aligned}&\mathcal{N}_{xx}=\mathcal{C}_1\frac{\partial u}{\partial x}-\mathcal{C}_2\frac{\partial^2w_b}{\partial x^2}-\mathcal{C}_3\frac{\partial^2w_s}{\partial x^2}+\mathcal{C}_4\chi-P_1^T+\mathcal{N}_{xx}^{\Psi_0} \\ &\quad+\mathcal{C}_5\psi+\mathcal{N}_{xx}^{\Phi_0}+\mathcal{C}_6\phi \\ &M_{xx}=\mathcal{C}_7\frac{\partial u}{\partial x}-\mathcal{C}_8\frac{\partial^2w_b}{\partial x^2}-\mathcal{C}_9\frac{\partial^2w_s}{\partial x^2}+\mathcal{C}_{10}\chi-P_2^T+M_{xx}^{\Psi_0} \\ &\quad+\mathcal{C}_{11}\psi+M_{xx}^{\Phi_0}+\mathcal{C}_{12}\phi \\ &S_{xx}=\mathcal{C}_{13}\frac{\partial u}{\partial x}-\mathcal{C}_{14}\frac{\partial^2w_b}{\partial x^2}-\mathcal{C}_{15}\frac{\partial^2w_s}{\partial x^2}+\mathcal{C}_{16}\chi-P_3^T+\mathcal{S}_{xx}^{\Psi_0} \\ &\quad+\mathcal{C}_{17}\psi+\mathcal{S}_{xx}^{\Phi_0}+\mathcal{C}_{18}\phi \\ &\mathcal{N}_{zz}=\mathcal{C}_1\frac{\partial u}{\partial x}-\mathcal{C}_2\frac{\partial^2w_b}{\partial x^2}-\mathcal{C}_3\frac{\partial^2w_s}{\partial x^2}+\mathcal{C}_4\chi-P_1^T+\mathcal{N}_{xx}^{\Psi_0} \\ &\quad+\mathcal{C}_5\psi+\mathcal{N}_{xx}^{\Phi_0}+\mathcal{C}_6\phi \\ &\mathcal{N}_{xz}=\mathcal{C}_{25}\left(\frac{\partial w_s}{\partial x}+\frac{\partial\chi}{\partial x}\right)-\mathcal{C}_{26}\frac{\partial\psi(x)}{\partial x}-\mathcal{C}_{27}\frac{\partial\phi(x)}{\partial x} \\ &\mathcal{D}_x=\mathcal{C}_{28}\left(\frac{\partial w_s}{\partial x}+\frac{\partial\chi}{\partial x}\right)+\mathcal{C}_{29}\frac{\partial\psi(x)}{\partial x}+\mathcal{C}_{30}\frac{\partial\phi(x)}{\partial x} \\ &\mathcal{D}_z=\mathcal{C}_{31}\frac{\partial u(x)}{\partial x}-\mathcal{C}_{32}\frac{\partial^2w_b(x)}{\partial x^2}-\mathcal{C}_{33}\frac{\partial^2w_s(x)}{\partial x^2} \\ &\quad+\mathcal{C}_{34}\chi(x)-P_5^T+\mathcal{D}_z^{\Psi_0}+\mathcal{C}_{35}\psi(x)+\mathcal{D}_z^{\Phi_0}+\mathcal{C}_{36}\phi(x) \\ &\mathcal{B}_x=\mathcal{C}_{37}\left(\frac{\partial w_s}{\partial x}+\frac{\partial\chi}{\partial x}\right)+\mathcal{C}_{38}\frac{\partial\psi(x)}{\partial x}+\mathcal{C}_{39}\frac{\partial\phi(x)}{\partial x} \\ &\mathcal{B}_z=\mathcal{C}_{40}\frac{\partial u(x)}{\partial x}-\mathcal{C}_{41}\frac{\partial^2w_b(x)}{\partial x^2}-\mathcal{C}_{42}\frac{\partial^2w_s(x)}{\partial x^2}+\mathcal{C}_{43}\chi(x) \\ &\quad-P_6^T-\mathcal{B}_z^{\Psi_0}-\mathcal{C}_{44}\psi(x)-\mathcal{B}_z^{\Phi_0} \\ &\quad-\mathcal{C}_{45}\phi(x)\end{aligned} \qquad (27)$$

The governing equations, derived from the principle of virtual work or Hamilton's principle, are initially expressed in terms of generalized force and moment resultants. To arrive at a solvable system of equations that describes the physical behavior of the structure, these resultant expressions must be explicitly written in terms of the kinematic displacement variables.

$$\delta u: \mathcal{C}_1\frac{d^2u}{\partial x^2}-\mathcal{C}_2\frac{d^3w_b}{dx^3}-\mathcal{C}_3\frac{d^3w_s}{dx^3}+\mathcal{C}_4\frac{d\chi}{dx}-\frac{dP_1^T}{dx}+\frac{d\mathcal{N}_{xx}^{\Psi_0}}{dx}+\mathcal{C}_5\frac{d\psi}{dx}+\frac{d\mathcal{N}_{xx}^{\Phi_0}}{dx}+\mathcal{C}_6\frac{d\phi}{dx}=0,$$
$$\delta w_b: \mathcal{C}_7\frac{d^3u}{dx^3}-\mathcal{C}_8\frac{d^4w_b}{dx^4}-\mathcal{C}_9\frac{d^4w_s}{dx^3}+\mathcal{C}_{10}\frac{d^2\chi}{\partial x^2}-\frac{d^2P_2^T}{\partial x^2}+\frac{d^2M_{xx}^{\Psi_0}}{\partial x^2}+\mathcal{C}_{11}\frac{d^2\psi}{\partial x^2}+\frac{d^2M_{xx}^{\Phi_0}}{\partial x^2}+\mathcal{C}_{12}\frac{d^2\phi}{\partial x^2}$$
$$=-q+F_f+(\mathcal{N}_x^T+\mathcal{N}_x^E+\mathcal{N}_x^M)\left(\frac{d^2w_b}{dx^2}+\frac{d^2w_s}{dx^2}+\frac{d^2\chi}{dx^2}\right)$$
$$\delta w_s: \mathcal{C}_{13}\frac{d^3u}{dx^3}-\mathcal{C}_{14}\frac{d^4w_b}{dx^4}-\mathcal{C}_{15}\frac{d^4w_s}{dx^3}+\mathcal{C}_{16}\frac{d^2\chi}{\partial x^2}-\frac{d^2P_3^T}{\partial x^2}+\frac{d^2\mathcal{S}_{xx}^{\Psi_0}}{\partial x^2}+\mathcal{C}_{17}\frac{d^2\psi}{\partial x^2}+\frac{d^2\mathcal{S}_{xx}^{\Phi_0}}{\partial x^2}+\mathcal{C}_{18}\frac{d^2\phi}{\partial x^2}$$
$$+\mathcal{C}_{25}\frac{d^2w_s}{\partial x^2}+\mathcal{C}_{25}\frac{d^2\chi}{\partial x^2}-\mathcal{C}_{26}\frac{d^2\psi}{\partial x^2}-\mathcal{C}_{27}\frac{d^2\phi}{\partial x^2}=-q+F_f+(\mathcal{N}_x^T+\mathcal{N}_x^E+\mathcal{N}_x^M)\left(\frac{d^2w_b}{dx^2}+\frac{d^2w_s}{dx^2}+\frac{d^2\chi}{dx^2}\right),$$
$$\delta\chi: -\mathcal{C}_1\frac{du}{dx}+\mathcal{C}_2\frac{d^2w_b}{dx^2}+\mathcal{C}_3\frac{d^2w_s}{\partial x^2}-\mathcal{C}_4\chi+P_1^T-\mathcal{N}_{xx}^{\Psi_0}-\mathcal{C}_5\psi-\mathcal{N}_{xx}^{\Phi_0}-\mathcal{C}_6\phi \tag{28}$$
$$+\mathcal{C}_{25}\frac{d^2w_s}{\partial x^2}+\mathcal{C}_{25}\frac{d^2\chi}{\partial x^2}-\mathcal{C}_{26}\frac{d^2\psi}{\partial x^2}-\mathcal{C}_{27}\frac{d^2\phi}{\partial x^2}=(\mathcal{N}_x^T+\mathcal{N}_x^E+\mathcal{N}_x^M)\left(\frac{d^2w_b}{dx^2}+\frac{d^2w_s}{dx^2}+\frac{d^2\chi}{dx^2}\right),$$
$$\delta\psi: \mathcal{C}_{28}\frac{d^2w_s}{\partial x^2}+\mathcal{C}_{28}\frac{d^2\chi}{\partial x^2}+\mathcal{C}_{29}\frac{d^2\psi}{\partial x^2}+\mathcal{C}_{30}\frac{d^2\phi}{\partial x^2}+$$
$$\mathcal{C}_{31}\frac{du}{dx}-\mathcal{C}_{32}\frac{d^2w_b}{dx^2}-\mathcal{C}_{33}\frac{d^2w_s}{\partial x^2}+\mathcal{C}_{34}\chi-P_5^T+\mathcal{D}_z^{\Psi_0}+\mathcal{C}_{35}\psi+\mathcal{D}_z^{\Phi_0}+\mathcal{C}_{36}\phi=0$$
$$\delta\phi: \mathcal{C}_{37}\frac{d^2w_s}{\partial x^2}+\mathcal{C}_{37}\frac{d^2\chi}{\partial x^2}+\mathcal{C}_{38}\frac{d^2\psi}{\partial x^2}+\mathcal{C}_{39}\frac{d^2\phi}{\partial x^2}+$$
$$\mathcal{C}_{40}\frac{du}{dx}-\mathcal{C}_{41}\frac{d^2w_b}{dx^2}-\mathcal{C}_{42}\frac{d^2w_s}{\partial x^2}+\mathcal{C}_{43}\chi-P_6^T-\mathcal{B}_z^{\Psi_0}-\mathcal{C}_{44}\psi-\mathcal{B}_z^{\Phi_0}-\mathcal{C}_{45}\phi=0$$

3.Analytical solution

This section develops the solution procedure for the wave propagation results of the sandwich nanocomposite reinforced intelligent cylindrical shell. The solution is assumed in the following form:

$$\begin{Bmatrix} u \\ w_b \\ w_s \\ \chi \\ \psi \\ \phi \end{Bmatrix}=\begin{Bmatrix} \mathbb{U}e^{i(k_1X-ck_1t)} \\ \mathbb{W}_be^{i(k_1X-ck_1t)} \\ \mathbb{W}_se^{i(k_1X-ck_1t)} \\ \mathbb{X}e^{i(k_1X-ck_1t)} \\ \mathfrak{N}e^{i(k_1X-ck_1t)} \\ \mathfrak{M}e^{i(k_1X-ck_1t)} \end{Bmatrix} \tag{29}$$

In which the unknown amplitudes are denoted with $\{\mathbb{U},\mathbb{W}_b,\mathbb{W}_s,\mathbb{X},\mathfrak{N},\mathfrak{M}\}$ and the wave number is denoted with k_1. The phase velocity is denoted with c. Substitution of solution into governing equations leads to following format:

$$\begin{bmatrix} \mathcal{K}_{11}(k_1,c) & \mathcal{K}_{12}(k_1,c) & \mathcal{K}_{13}(k_1,c) & \mathcal{K}_{14}(k_1,c) & \mathcal{K}_{15}(k_1,c) & \mathcal{K}_{16}(k_1,c) \\ \mathcal{K}_{21}(k_1,c) & \mathcal{K}_{22}(k_1,c) & \mathcal{K}_{23}(k_1,c) & \mathcal{K}_{24}(k_1,c) & \mathcal{K}_{25}(k_1,c) & \mathcal{K}_{26}(k_1,c) \\ \mathcal{K}_{31}(k_1,c) & \mathcal{K}_{32}(k_1,c) & \mathcal{K}_{33}(k_1,c) & \mathcal{K}_{34}(k_1,c) & \mathcal{K}_{35}(k_1,c) & \mathcal{K}_{36}(k_1,c) \\ \mathcal{K}_{41}(k_1,c) & \mathcal{K}_{42}(k_1,c) & \mathcal{K}_{43}(k_1,c) & \mathcal{K}_{44}(k_1,c) & \mathcal{K}_{45}(k_1,c) & \mathcal{K}_{46}(k_1,c) \\ \mathcal{K}_{51}(k_1,c) & \mathcal{K}_{52}(k_1,c) & \mathcal{K}_{53}(k_1,c) & \mathcal{K}_{54}(k_1,c) & \mathcal{K}_{55}(k_1,c) & \mathcal{K}_{56}(k_1,c) \\ \mathcal{K}_{61}(k_1,c) & \mathcal{K}_{62}(k_1,c) & \mathcal{K}_{63}(k_1,c) & \mathcal{K}_{64}(k_1,c) & \mathcal{K}_{65}(k_1,c) & \mathcal{K}_{66}(k_1,c) \end{bmatrix}\begin{Bmatrix} u \\ w_b \\ w_s \\ \chi \\ \psi \\ \phi \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{30}$$

One can arrive at the phase velocities using setting the determinant of the multiplier coefficient to zero as follows:

$$Det\begin{bmatrix} \mathcal{K}_{11}(k_1,c) & \mathcal{K}_{12}(k_1,c) & \mathcal{K}_{13}(k_1,c) & \mathcal{K}_{14}(k_1,c) & \mathcal{K}_{15}(k_1,c) & \mathcal{K}_{16}(k_1,c) \\ \mathcal{K}_{21}(k_1,c) & \mathcal{K}_{22}(k_1,c) & \mathcal{K}_{23}(k_1,c) & \mathcal{K}_{24}(k_1,c) & \mathcal{K}_{25}(k_1,c) & \mathcal{K}_{26}(k_1,c) \\ \mathcal{K}_{31}(k_1,c) & \mathcal{K}_{32}(k_1,c) & \mathcal{K}_{33}(k_1,c) & \mathcal{K}_{34}(k_1,c) & \mathcal{K}_{35}(k_1,c) & \mathcal{K}_{36}(k_1,c) \\ \mathcal{K}_{41}(k_1,c) & \mathcal{K}_{42}(k_1,c) & \mathcal{K}_{43}(k_1,c) & \mathcal{K}_{44}(k_1,c) & \mathcal{K}_{45}(k_1,c) & \mathcal{K}_{46}(k_1,c) \\ \mathcal{K}_{51}(k_1,c) & \mathcal{K}_{52}(k_1,c) & \mathcal{K}_{53}(k_1,c) & \mathcal{K}_{54}(k_1,c) & \mathcal{K}_{55}(k_1,c) & \mathcal{K}_{56}(k_1,c) \\ \mathcal{K}_{61}(k_1,c) & \mathcal{K}_{62}(k_1,c) & \mathcal{K}_{63}(k_1,c) & \mathcal{K}_{64}(k_1,c) & \mathcal{K}_{65}(k_1,c) & \mathcal{K}_{66}(k_1,c) \end{bmatrix}=0 \tag{31}$$

Finally the numerical results including the phase velocities will be presented with changes of the wave numbers, graphene origami characteristics such as volume fraction and folding parameter and two parameters of the Pasternak's foundation.

4. Results and discussion

This section presents a comprehensive numerical investigation into the mechanical response of the graphene origami (Gori)-reinforced auxetic metamaterial structure. A

Table 1 Variation in the phase velocities of the graphene origami reinforced copper enriched beam with changes of with changes of the shear's parameter of foundation of the sandwich beam in terms of wave number

k	$K_G = 0$	$K_G = 1\times10^5$	$K_G = 1.1\times10^5$	$K_G = 1.2\times10^5$	$K_G = 1.3\times10^5$	$K_G = 1.4\times10^5$	$K_G = 1.5\times10^5$	$K_G = 1.6\times10^5$	$K_G = 1.7\times10^5$	$K_G = 1.8\times10^5$	$K_G = 1.9\times10^5$	$K_G = 2\times10^5$
250	83.154	85.856	86.772	87.602	88.851	90.573	91.677	93.340	94.670	96.020	97.388	98.777
251	82.822	85.514	86.427	87.253	88.497	90.212	91.312	92.968	94.293	95.637	97.000	98.383
252	82.494	85.175	86.084	86.907	88.146	89.854	90.949	92.599	93.919	95.258	96.616	97.993
253	82.168	84.838	85.744	86.564	87.798	89.499	90.590	92.233	93.548	94.881	96.234	97.605
254	81.844	84.504	85.406	86.223	87.452	89.146	90.233	91.870	93.179	94.508	95.855	97.221
255	81.523	84.173	85.071	85.885	87.109	88.797	89.879	91.510	92.814	94.137	95.479	96.840
256	81.205	83.844	84.739	85.549	86.769	88.450	89.528	91.152	92.452	93.769	95.106	96.462
257	80.889	83.518	84.409	85.216	86.431	88.106	89.180	90.798	92.092	93.404	94.736	96.086
258	80.575	83.194	84.082	84.886	86.096	87.764	88.834	90.446	91.735	93.042	94.369	95.714
259	80.264	82.873	83.757	84.558	85.764	87.425	88.491	90.096	91.381	92.683	94.004	95.344
260	79.955	82.554	83.435	84.233	85.434	87.089	88.151	89.750	91.029	92.327	93.643	94.978
261	79.649	82.238	83.115	83.910	85.107	86.755	87.813	89.406	90.680	91.973	93.284	94.614
262	79.345	81.924	82.798	83.590	84.782	86.424	87.478	89.065	90.334	91.622	92.928	94.252
263	79.043	81.612	82.483	83.272	84.459	86.096	87.145	88.726	89.991	91.274	92.575	93.894
264	78.744	81.303	82.171	82.957	84.139	85.769	86.815	88.390	89.650	90.928	92.224	93.538
265	78.447	80.996	81.861	82.644	83.822	85.446	86.488	88.057	89.312	90.585	91.876	93.185
266	78.152	80.692	81.553	82.333	83.507	85.125	86.162	87.725	88.976	90.244	91.530	92.835
267	77.859	80.390	81.248	82.025	83.194	84.806	85.840	87.397	88.643	89.906	91.188	92.487
268	77.569	80.090	80.944	81.719	82.884	84.489	85.519	87.071	88.312	89.571	90.847	92.142
269	77.280	79.792	80.644	81.415	82.576	84.175	85.202	86.747	87.984	89.238	90.510	91.800
270	76.994	79.496	80.345	81.113	82.270	83.863	84.886	86.426	87.658	88.907	90.174	91.460
271	76.710	79.203	80.048	80.814	81.966	83.554	84.573	86.107	87.334	88.579	89.842	91.122
272	76.428	78.912	79.754	80.517	81.665	83.247	84.262	85.790	87.013	88.253	89.511	90.787
273	76.148	78.623	79.462	80.222	81.366	82.942	83.953	85.476	86.694	87.930	89.184	90.455
274	75.870	78.336	79.172	79.929	81.069	82.639	83.647	85.164	86.378	87.609	88.858	90.125
275	75.594	78.051	78.884	79.638	80.774	82.339	83.343	84.854	86.064	87.291	88.535	89.797
276	75.320	77.768	78.598	79.350	80.481	82.040	83.041	84.547	85.752	86.974	88.214	89.472
277	75.048	77.487	78.314	79.063	80.191	81.744	82.741	84.242	85.443	86.660	87.896	89.149
278	74.778	77.209	78.033	78.779	79.902	81.450	82.443	83.939	85.135	86.349	87.580	88.828
279	74.510	76.932	77.753	78.497	79.616	81.158	82.148	83.638	84.830	86.039	87.266	88.510
280	74.244	76.657	77.475	78.216	79.331	80.868	81.854	83.339	84.527	85.732	86.954	88.193
281	73.980	76.384	77.200	77.938	79.049	80.581	81.563	83.043	84.226	85.427	86.645	87.880
282	73.718	76.114	76.926	77.662	78.769	80.295	81.274	82.748	83.928	85.124	86.337	87.568
283	73.457	75.845	76.654	77.387	78.491	80.011	80.987	82.456	83.631	84.823	86.032	87.258
284	73.199	75.578	76.384	77.115	78.214	79.729	80.701	82.165	83.337	84.524	85.729	86.951
285	72.942	75.312	76.116	76.844	77.940	79.450	80.418	81.877	83.044	84.228	85.428	86.646
286	72.687	75.049	75.850	76.575	77.667	79.172	80.137	81.591	82.754	83.933	85.130	86.343
287	72.433	74.788	75.586	76.309	77.397	78.896	79.858	81.307	82.465	83.641	84.833	86.042
288	72.182	74.528	75.323	76.044	77.128	78.622	79.581	81.024	82.179	83.351	84.539	85.744
289	71.932	74.270	75.063	75.781	76.861	78.350	79.305	80.744	81.895	83.062	84.246	85.447

systematic parametric study was conducted to elucidate the influence of three critical design factors: Thermal Loading, Material Gradation, Folding Degree.

The material properties of Cu matrix and graphene origami as assumed as:

$$E_{Cu} = 65.79 GPa, E_{Gr} = 929.57 GPa,$$

$$\vartheta_{Cu} = 0.387, \vartheta_{Gr} = 0.22,$$

$$\alpha_{Cu} = 16.51\times10^{-6}\frac{1}{K^O}, \vartheta_{Gr} = -3.98\times10^{-6}\frac{1}{K^O},$$

$$l_{Gr} = 83.76\times10^{-10} m, t_{Gr} = 3.4\times10^{-10} m,$$

This section presents a comprehensive numerical analysis of the mechanical response of the structure under

Table 2 Variation in the phase velocities of the graphene origami reinforced copper enriched beam with changes of the Winkler's parameter of foundation of the sandwich beam in terms of wave number

k	$K_W = 0$	$K_W = 1\times10^6$	$K_W = 1.1\times10^6$	$K_W = 1.2\times10^6$	$K_W = 1.3\times10^6$	$K_W = 1.4\times10^6$	$K_W = 1.5\times10^6$	$K_W = 1.6\times10^6$	$K_W = 1.7\times10^6$	$K_W = 1.8\times10^6$	$K_W = 1.9\times10^6$	$K_W = 2\times10^6$
250	83.154	86.688	87.604	88.434	89.683	91.404	92.508	94.171	95.502	96.851	98.219	99.607
251	82.822	86.342	87.255	88.082	89.326	91.040	92.140	93.796	95.121	96.465	97.828	99.210
252	82.494	86.000	86.909	87.732	88.971	90.679	91.774	93.424	94.744	96.082	97.440	98.817
253	82.168	85.660	86.565	87.385	88.619	90.320	91.411	93.055	94.369	95.703	97.055	98.426
254	81.844	85.322	86.224	87.041	88.270	89.965	91.052	92.688	93.998	95.326	96.673	98.039
255	81.523	84.988	85.886	86.700	87.924	89.612	90.694	92.325	93.629	94.952	96.294	97.654
256	81.205	84.656	85.551	86.361	87.581	89.262	90.340	91.964	93.264	94.581	95.917	97.273
257	80.889	84.326	85.218	86.025	87.240	88.914	89.989	91.606	92.901	94.213	95.544	96.894
258	80.575	84.000	84.888	85.692	86.902	88.570	89.640	91.251	92.541	93.848	95.174	96.519
259	80.264	83.675	84.560	85.361	86.566	88.228	89.294	90.899	92.183	93.486	94.806	96.146
260	79.955	83.353	84.235	85.033	86.233	87.889	88.950	90.549	91.829	93.126	94.442	95.776
261	79.649	83.034	83.912	84.707	85.903	87.552	88.610	90.203	91.477	92.769	94.080	95.409
262	79.345	82.717	83.592	84.383	85.575	87.218	88.271	89.858	91.128	92.415	93.721	95.045
263	79.043	82.403	83.274	84.063	85.250	86.886	87.936	89.517	90.781	92.064	93.365	94.684
264	78.744	82.091	82.958	83.744	84.927	86.557	87.603	89.178	90.437	91.715	93.011	94.325
265	78.447	81.781	82.645	83.428	84.606	86.230	87.272	88.841	90.096	91.369	92.660	93.969
266	78.152	81.473	82.335	83.115	84.288	85.906	86.944	88.507	89.757	91.026	92.312	93.616
267	77.859	81.168	82.026	82.803	83.973	85.584	86.618	88.176	89.421	90.685	91.966	93.265
268	77.569	80.865	81.720	82.494	83.659	85.265	86.295	87.846	89.088	90.346	91.623	92.917
269	77.280	80.565	81.416	82.188	83.348	84.948	85.974	87.520	88.756	90.010	91.282	92.572
270	76.994	80.266	81.115	81.883	83.040	84.633	85.656	87.196	88.428	89.677	90.944	92.229
271	76.710	79.970	80.815	81.581	82.733	84.321	85.340	86.874	88.101	89.346	90.608	91.888
272	76.428	79.676	80.518	81.281	82.429	84.011	85.026	86.555	87.777	89.018	90.275	91.551
273	76.148	79.384	80.223	80.983	82.127	83.703	84.715	86.238	87.456	88.692	89.945	91.215
274	75.870	79.095	79.931	80.688	81.827	83.398	84.405	85.923	87.137	88.368	89.616	90.882
275	75.594	78.807	79.640	80.394	81.530	83.095	84.099	85.610	86.820	88.047	89.290	90.552
276	75.320	78.521	79.351	80.103	81.234	82.794	83.794	85.300	86.505	87.727	88.967	90.224
277	75.048	78.238	79.065	79.814	80.941	82.495	83.491	84.992	86.193	87.411	88.646	89.898
278	74.778	77.956	78.781	79.527	80.650	82.198	83.191	84.687	85.883	87.096	88.327	89.575
279	74.510	77.677	78.498	79.242	80.361	81.903	82.893	84.383	85.575	86.784	88.010	89.254
280	74.244	77.400	78.218	78.959	80.074	81.611	82.597	84.082	85.270	86.474	87.696	88.935
281	73.980	77.124	77.939	78.678	79.789	81.320	82.303	83.782	84.966	86.167	87.384	88.618
282	73.718	76.851	77.663	78.399	79.506	81.032	82.011	83.485	84.665	85.861	87.074	88.304
283	73.457	76.579	77.389	78.122	79.225	80.746	81.721	83.190	84.366	85.558	86.766	87.992
284	73.199	76.310	77.116	77.847	78.946	80.461	81.433	82.897	84.069	85.256	86.461	87.682
285	72.942	76.042	76.846	77.574	78.669	80.179	81.148	82.607	83.774	84.957	86.157	87.375
286	72.687	75.776	76.577	77.302	78.394	79.899	80.864	82.318	83.481	84.660	85.856	87.069
287	72.433	75.512	76.310	77.033	78.121	79.620	80.582	82.031	83.190	84.365	85.557	86.766
288	72.182	75.250	76.045	76.765	77.850	79.344	80.302	81.746	82.901	84.072	85.260	86.465
289	71.932	74.989	75.782	76.500	77.580	79.069	80.025	81.463	82.614	83.781	84.965	86.165
290	71.684	74.731	75.521	76.236	77.313	78.797	79.749	81.182	82.329	83.492	84.672	85.868
291	71.438	74.474	75.261	75.974	77.047	78.526	79.475	80.903	82.046	83.205	84.381	85.573
292	71.193	74.219	75.003	75.714	76.783	78.257	79.202	80.626	81.765	82.921	84.092	85.280
293	70.950	73.966	74.747	75.455	76.521	77.990	78.932	80.351	81.486	82.638	83.805	84.989
294	70.709	73.714	74.493	75.199	76.261	77.725	78.664	80.078	81.209	82.356	83.520	84.700

Table 2 continued

295	70.469	73.464	74.241	74.944	76.002	77.461	78.397	79.806	80.934	82.077	83.237	84.413
296	70.231	73.216	73.990	74.691	75.746	77.199	78.132	79.537	80.660	81.800	82.956	84.128
297	69.995	72.969	73.741	74.439	75.491	76.939	77.869	79.269	80.389	81.525	82.676	83.844
298	69.760	72.725	73.493	74.189	75.237	76.681	77.608	79.003	80.119	81.251	82.399	83.563
299	69.526	72.481	73.247	73.941	74.986	76.425	77.348	78.739	79.851	80.979	82.123	83.284
300	69.295	72.240	73.003	73.695	74.736	76.170	77.090	78.476	79.585	80.709	81.850	83.006

Table 3 Variation in the phase velocities of the graphene origami reinforced copper enriched beam with changes of length to thickness ratio of the sandwich beam in terms of wave number

k	L/h=5	L/h=10	L/h=15	L/h=20	L/h=25	L/h=30	L/h=35	L/h=40	L/h=45	L/h=50	L/h=55	L/h=60
250	935.478	623.652	415.768	277.179	207.884	166.307	138.589	118.791	103.942	92.393	83.154	74.838
251	931.751	621.167	414.112	276.074	207.056	165.645	138.037	118.318	103.528	92.025	82.822	74.540
252	928.053	618.702	412.468	274.979	206.234	164.987	137.489	117.848	103.117	91.660	82.494	74.244
253	924.385	616.257	410.838	273.892	205.419	164.335	136.946	117.382	102.709	91.297	82.168	73.951
254	920.746	613.831	409.220	272.814	204.610	163.688	136.407	116.920	102.305	90.938	81.844	73.660
255	917.135	611.423	407.616	271.744	203.808	163.046	135.872	116.462	101.904	90.581	81.523	73.371
256	913.553	609.035	406.023	270.682	203.012	162.409	135.341	116.007	101.506	90.227	81.205	73.084
257	909.998	606.665	404.444	269.629	202.222	161.777	134.815	115.555	101.111	89.876	80.889	72.800
258	906.471	604.314	402.876	268.584	201.438	161.150	134.292	115.107	100.719	89.528	80.575	72.518
259	902.971	601.981	401.320	267.547	200.660	160.528	133.773	114.663	100.330	89.182	80.264	72.238
260	899.498	599.665	399.777	266.518	199.888	159.911	133.259	114.222	99.944	88.839	79.955	71.960
261	896.052	597.368	398.245	265.497	199.123	159.298	132.748	113.784	99.561	88.499	79.649	71.684
262	892.632	595.088	396.725	264.483	198.363	158.690	132.242	113.350	99.181	88.161	79.345	71.411
263	889.238	592.825	395.217	263.478	197.608	158.087	131.739	112.919	98.804	87.826	79.043	71.139
264	885.869	590.579	393.720	262.480	196.860	157.488	131.240	112.491	98.430	87.493	78.744	70.870
265	882.526	588.351	392.234	261.489	196.117	156.894	130.745	112.067	98.058	87.163	78.447	70.602
266	879.209	586.139	390.759	260.506	195.380	156.304	130.253	111.646	97.690	86.835	78.152	70.337
267	875.916	583.944	389.296	259.531	194.648	155.718	129.765	111.227	97.324	86.510	77.859	70.073
268	872.647	581.765	387.843	258.562	193.922	155.137	129.281	110.812	96.961	86.187	77.569	69.812
269	869.403	579.602	386.401	257.601	193.201	154.561	128.800	110.400	96.600	85.867	77.280	69.552
270	866.183	577.456	384.970	256.647	192.485	153.988	128.323	109.992	96.243	85.549	76.994	69.295
271	862.987	575.325	383.550	255.700	191.775	153.420	127.850	109.586	95.887	85.233	76.710	69.039
272	859.814	573.210	382.140	254.760	191.070	152.856	127.380	109.183	95.535	84.920	76.428	68.785
273	856.665	571.110	380.740	253.827	190.370	152.296	126.913	108.783	95.185	84.609	76.148	68.533
274	853.538	569.025	379.350	252.900	189.675	151.740	126.450	108.386	94.838	84.300	75.870	68.283
275	850.434	566.956	377.971	251.981	188.985	151.188	125.990	107.992	94.493	83.994	75.594	68.035
276	847.353	564.902	376.601	251.068	188.301	150.641	125.534	107.600	94.150	83.689	75.320	67.788
277	844.294	562.863	375.242	250.161	187.621	150.097	125.081	107.212	93.810	83.387	75.048	67.544
278	841.257	560.838	373.892	249.261	186.946	149.557	124.631	106.826	93.473	83.087	74.778	67.301
279	838.242	558.828	372.552	248.368	186.276	149.021	124.184	106.443	93.138	82.789	74.510	67.059
280	835.248	556.832	371.221	247.481	185.611	148.489	123.740	106.063	92.805	82.494	74.244	66.820
281	832.276	554.850	369.900	246.600	184.950	147.960	123.300	105.686	92.475	82.200	73.980	66.582
282	829.324	552.883	368.589	245.726	184.294	147.435	122.863	105.311	92.147	81.909	73.718	66.346
283	826.394	550.929	367.286	244.857	183.643	146.914	122.429	104.939	91.822	81.619	73.457	66.112
284	823.484	548.989	365.993	243.995	182.996	146.397	121.998	104.569	91.498	81.332	73.199	65.879
285	820.595	547.063	364.709	243.139	182.354	145.883	121.570	104.202	91.177	81.046	72.942	65.648
286	817.725	545.150	363.434	242.289	181.717	145.373	121.145	103.838	90.858	80.763	72.687	65.418
287	814.876	543.251	362.167	241.445	181.084	144.867	120.722	103.476	90.542	80.482	72.433	65.190

Table 3 Continued

288	812.047	541.365	360.910	240.606	180.455	144.364	120.303	103.117	90.227	80.202	72.182	64.964
289	809.237	539.491	359.661	239.774	179.830	143.864	119.887	102.760	89.915	79.925	71.932	64.739
290	806.446	537.631	358.421	238.947	179.210	143.368	119.474	102.406	89.605	79.649	71.684	64.516
291	803.675	535.783	357.189	238.126	178.594	142.876	119.063	102.054	89.297	79.375	71.438	64.294
292	800.923	533.949	355.966	237.310	177.983	142.386	118.655	101.704	88.991	79.103	71.193	64.074
293	798.189	532.126	354.751	236.501	177.375	141.900	118.250	101.357	88.688	78.834	70.950	63.855
294	795.474	530.316	353.544	235.696	176.772	141.418	117.848	101.013	88.386	78.565	70.709	63.638
295	792.778	528.519	352.346	234.897	176.173	140.938	117.449	100.670	88.086	78.299	70.469	63.422
296	790.100	526.733	351.155	234.104	175.578	140.462	117.052	100.330	87.789	78.035	70.231	63.208
297	787.439	524.960	349.973	233.315	174.987	139.989	116.658	99.992	87.493	77.772	69.995	62.995
298	784.797	523.198	348.799	232.532	174.399	139.519	116.266	99.657	87.200	77.511	69.760	62.784
299	782.172	521.448	347.632	231.755	173.816	139.053	115.877	99.323	86.908	77.252	69.526	62.574

Table 4 Variation in the phase velocities of the graphene origami reinforced copper enriched beam with changes of piezo-electric to core thickness ratio in terms of wave number

k	$\frac{h_p}{h_c}=0$	$\frac{h_p}{h_c}=0.01$	$\frac{h_p}{h_c}=0.02$	$\frac{h_p}{h_c}=0.03$	$\frac{h_p}{h_c}=0.04$	$\frac{h_p}{h_c}=0.05$	$\frac{h_p}{h_c}=0.06$	$\frac{h_p}{h_c}=0.07$	$\frac{h_p}{h_c}=0.08$	$\frac{h_p}{h_c}=0.09$	$\frac{h_p}{h_c}=0.1$
250	83.154	83.046	82.946	82.853	82.766	82.684	82.607	82.535	82.463	82.391	82.320
251	82.822	82.715	82.616	82.523	82.436	82.355	82.278	82.207	82.135	82.063	81.992
252	82.494	82.387	82.288	82.195	82.109	82.028	81.952	81.880	81.809	81.738	81.666
253	82.168	82.061	81.962	81.870	81.784	81.704	81.628	81.557	81.486	81.414	81.343
254	81.844	81.738	81.640	81.548	81.462	81.382	81.306	81.236	81.165	81.094	81.023
255	81.523	81.418	81.320	81.228	81.143	81.063	80.988	80.917	80.846	80.776	80.705
256	81.205	81.100	81.002	80.911	80.826	80.746	80.671	80.601	80.531	80.460	80.390
257	80.889	80.784	80.687	80.596	80.511	80.432	80.357	80.287	80.217	80.147	80.077
258	80.575	80.471	80.374	80.284	80.199	80.120	80.046	79.976	79.906	79.837	79.767
259	80.264	80.160	80.064	79.974	79.890	79.811	79.737	79.667	79.598	79.528	79.459
260	79.955	79.852	79.756	79.666	79.582	79.504	79.430	79.361	79.292	79.223	79.153
261	79.649	79.546	79.450	79.361	79.277	79.199	79.126	79.057	78.988	78.919	78.850
262	79.345	79.242	79.147	79.058	78.975	78.897	78.824	78.755	78.686	78.618	78.549
263	79.043	78.941	78.846	78.757	78.675	78.597	78.524	78.456	78.387	78.319	78.251
264	78.744	78.642	78.547	78.459	78.377	78.299	78.227	78.158	78.090	78.022	77.954
265	78.447	78.345	78.251	78.163	78.081	78.004	77.932	77.864	77.796	77.728	77.660
266	78.152	78.051	77.957	77.869	77.787	77.711	77.639	77.571	77.503	77.436	77.368
267	77.859	77.758	77.665	77.577	77.496	77.419	77.348	77.280	77.213	77.146	77.078
268	77.569	77.468	77.375	77.288	77.207	77.131	77.059	76.992	76.925	76.858	76.791
269	77.280	77.180	77.087	77.001	76.920	76.844	76.773	76.706	76.639	76.572	76.505
270	76.994	76.894	76.802	76.716	76.635	76.559	76.488	76.422	76.355	76.288	76.222
271	76.710	76.611	76.518	76.432	76.352	76.277	76.206	76.140	76.073	76.007	75.941
272	76.428	76.329	76.237	76.151	76.071	75.996	75.926	75.860	75.794	75.727	75.661
273	76.148	76.049	75.958	75.872	75.793	75.718	75.648	75.582	75.516	75.450	75.384
274	75.870	75.772	75.681	75.596	75.516	75.442	75.372	75.306	75.240	75.175	75.109
275	75.594	75.496	75.405	75.321	75.241	75.167	75.098	75.032	74.967	74.901	74.836
276	75.320	75.223	75.132	75.048	74.969	74.895	74.826	74.760	74.695	74.630	74.565
277	75.048	74.951	74.861	74.777	74.698	74.625	74.555	74.490	74.425	74.361	74.296
278	74.778	74.682	74.592	74.508	74.430	74.356	74.287	74.222	74.158	74.093	74.028
279	74.510	74.414	74.324	74.241	74.163	74.090	74.021	73.956	73.892	73.827	73.763
280	74.244	74.148	74.059	73.976	73.898	73.825	73.757	73.692	73.628	73.564	73.500

Table 4 Continued

281	73.980	73.884	73.795	73.712	73.635	73.562	73.494	73.430	73.366	73.302	73.238
282	73.718	73.622	73.534	73.451	73.374	73.301	73.234	73.170	73.106	73.042	72.978
283	73.457	73.362	73.274	73.191	73.115	73.042	72.975	72.911	72.847	72.784	72.720
284	73.199	73.104	73.016	72.934	72.857	72.785	72.718	72.654	72.591	72.528	72.464
285	72.942	72.847	72.760	72.678	72.601	72.530	72.463	72.399	72.336	72.273	72.210
286	72.687	72.593	72.505	72.424	72.348	72.276	72.209	72.146	72.083	72.020	71.958
287	72.433	72.340	72.253	72.171	72.095	72.024	71.958	71.895	71.832	71.770	71.707
288	72.182	72.089	72.002	71.921	71.845	71.774	71.708	71.645	71.583	71.520	71.458
289	71.932	71.839	71.753	71.672	71.597	71.526	71.460	71.397	71.335	71.273	71.211
290	71.684	71.591	71.505	71.425	71.350	71.279	71.213	71.151	71.089	71.027	70.965
291	71.438	71.345	71.259	71.179	71.104	71.034	70.969	70.907	70.845	70.783	70.721
292	71.193	71.101	71.015	70.936	70.861	70.791	70.726	70.664	70.602	70.541	70.479
293	70.950	70.858	70.773	70.693	70.619	70.549	70.484	70.423	70.361	70.300	70.239
294	70.709	70.617	70.532	70.453	70.379	70.310	70.244	70.183	70.122	70.061	70.000
295	70.469	70.378	70.293	70.214	70.140	70.071	70.006	69.945	69.884	69.823	69.762
296	70.231	70.140	70.056	69.977	69.903	69.834	69.770	69.709	69.648	69.587	69.527
297	69.995	69.904	69.820	69.741	69.668	69.599	69.535	69.474	69.414	69.353	69.293
298	69.760	69.669	69.586	69.507	69.434	69.366	69.302	69.241	69.181	69.120	69.060
299	69.526	69.436	69.353	69.275	69.202	69.134	69.070	69.009	68.949	68.889	68.829
300	69.295	69.205	69.122	69.044	68.971	68.903	68.839	68.779	68.719	68.660	68.600

Table 5 Variation in the phase velocities of the graphene origami reinforced copper enriched beam with changes of thermal loads of the reinforcement in terms of wave number

k	T=300	T=305	T=310	T=315	T=320	T=325	T=330	T=335	T=340	T=345	T=350	T=355
250	83.154	83.133	83.113	83.092	83.072	83.052	83.032	83.012	82.992	82.972	82.953	82.933
251	82.822	82.802	82.781	82.761	82.741	82.721	82.701	82.682	82.662	82.642	82.622	82.602
252	82.494	82.473	82.453	82.433	82.413	82.393	82.373	82.354	82.334	82.314	82.294	82.274
253	82.168	82.147	82.127	82.107	82.087	82.067	82.048	82.028	82.008	81.989	81.969	81.949
254	81.844	81.824	81.804	81.784	81.764	81.744	81.725	81.705	81.685	81.666	81.646	81.627
255	81.523	81.503	81.483	81.463	81.443	81.424	81.404	81.385	81.365	81.346	81.326	81.307
256	81.205	81.185	81.165	81.145	81.125	81.106	81.086	81.067	81.047	81.028	81.008	80.989
257	80.889	80.869	80.849	80.829	80.809	80.790	80.771	80.751	80.732	80.713	80.693	80.674
258	80.575	80.555	80.535	80.516	80.496	80.477	80.458	80.438	80.419	80.400	80.380	80.361
259	80.264	80.244	80.224	80.205	80.185	80.166	80.147	80.128	80.109	80.089	80.070	80.051
260	79.955	79.936	79.916	79.896	79.877	79.858	79.839	79.820	79.800	79.781	79.762	79.743
261	79.649	79.629	79.610	79.590	79.571	79.552	79.533	79.514	79.495	79.476	79.456	79.437
262	79.345	79.325	79.306	79.287	79.267	79.248	79.229	79.210	79.191	79.172	79.153	79.134
263	79.043	79.024	79.004	78.985	78.966	78.947	78.928	78.909	78.890	78.871	78.852	78.833
264	78.744	78.724	78.705	78.686	78.667	78.648	78.629	78.610	78.591	78.572	78.554	78.535
265	78.447	78.427	78.408	78.389	78.370	78.351	78.332	78.314	78.295	78.276	78.257	78.238
266	78.152	78.133	78.113	78.094	78.075	78.057	78.038	78.019	78.000	77.982	77.963	77.944
267	77.859	77.840	77.821	77.802	77.783	77.764	77.746	77.727	77.708	77.690	77.671	77.652
268	77.569	77.549	77.530	77.511	77.493	77.474	77.455	77.437	77.418	77.400	77.381	77.363
269	77.280	77.261	77.242	77.223	77.205	77.186	77.168	77.149	77.130	77.112	77.093	77.075
270	76.994	76.975	76.956	76.937	76.919	76.900	76.882	76.863	76.845	76.826	76.808	76.789
271	76.710	76.691	76.672	76.653	76.635	76.616	76.598	76.580	76.561	76.543	76.525	76.506
272	76.428	76.409	76.390	76.372	76.353	76.335	76.316	76.298	76.280	76.261	76.243	76.225
273	76.148	76.129	76.110	76.092	76.073	76.055	76.037	76.019	76.000	75.982	75.964	75.946

Table 5 Continued

274	75.870	75.851	75.833	75.814	75.796	75.778	75.759	75.741	75.723	75.705	75.687	75.668
275	75.594	75.575	75.557	75.538	75.520	75.502	75.484	75.466	75.448	75.430	75.411	75.393
276	75.320	75.302	75.283	75.265	75.247	75.228	75.210	75.192	75.174	75.156	75.138	75.120
277	75.048	75.030	75.011	74.993	74.975	74.957	74.939	74.921	74.903	74.885	74.867	74.849
278	74.778	74.760	74.741	74.723	74.705	74.687	74.669	74.651	74.633	74.616	74.598	74.580
279	74.510	74.492	74.474	74.455	74.437	74.420	74.402	74.384	74.366	74.348	74.330	74.312
280	74.244	74.226	74.208	74.190	74.172	74.154	74.136	74.118	74.100	74.083	74.065	74.047
281	73.980	73.962	73.944	73.926	73.908	73.890	73.872	73.854	73.837	73.819	73.801	73.783
282	73.718	73.699	73.681	73.663	73.646	73.628	73.610	73.593	73.575	73.557	73.540	73.522
283	73.457	73.439	73.421	73.403	73.385	73.368	73.350	73.332	73.315	73.297	73.280	73.262
284	73.199	73.180	73.162	73.145	73.127	73.109	73.092	73.074	73.057	73.039	73.022	73.004
285	72.942	72.924	72.906	72.888	72.870	72.853	72.835	72.818	72.800	72.783	72.765	72.748
286	72.687	72.669	72.651	72.633	72.616	72.598	72.581	72.563	72.546	72.528	72.511	72.494
287	72.433	72.415	72.398	72.380	72.363	72.345	72.328	72.310	72.293	72.276	72.258	72.241
288	72.182	72.164	72.146	72.129	72.111	72.094	72.077	72.059	72.042	72.025	72.007	71.990
289	71.932	71.914	71.897	71.879	71.862	71.844	71.827	71.810	71.793	71.776	71.758	71.741
290	71.684	71.666	71.649	71.631	71.614	71.597	71.580	71.562	71.545	71.528	71.511	71.494
291	71.438	71.420	71.403	71.385	71.368	71.351	71.334	71.316	71.299	71.282	71.265	71.248
292	71.193	71.176	71.158	71.141	71.123	71.106	71.089	71.072	71.055	71.038	71.021	71.004
293	70.950	70.933	70.915	70.898	70.881	70.864	70.847	70.830	70.813	70.796	70.779	70.762
294	70.709	70.691	70.674	70.657	70.640	70.623	70.606	70.589	70.572	70.555	70.538	70.521
295	70.469	70.452	70.434	70.417	70.400	70.383	70.366	70.349	70.333	70.316	70.299	70.282
296	70.231	70.214	70.196	70.179	70.162	70.145	70.129	70.112	70.095	70.078	70.061	70.044
297	69.995	69.977	69.960	69.943	69.926	69.909	69.892	69.876	69.859	69.842	69.825	69.809
298	69.760	69.742	69.725	69.708	69.691	69.675	69.658	69.641	69.625	69.608	69.591	69.574
299	69.526	69.509	69.492	69.475	69.458	69.442	69.425	69.408	69.392	69.375	69.358	69.342
300	69.295	69.277	69.260	69.244	69.227	69.210	69.194	69.177	69.160	69.144	69.127	69.111

investigation. A full-field, high-fidelity evaluation was conducted to elucidate the complex stress and deformation states developed under the applied loading conditions. The complete set of results is systematically presented to provide a holistic understanding of the system's behavior.

Listed in the Table 1 is variation in the phase velocities with an advance in the shear's parameter of the Pasternak's foundation with changes of the wave numbers. One can find a decrease in the phase velocities with an advance in the shear's parameter of the Pasternak's foundation because of an enhancement in the structural stiffness of the foundation.

Listed in the Table 2 is variation in the phase velocities with an advance in the Winkler's parameter of the Pasternak's foundation with changes of the wave numbers. One can find a decrease in the phase velocities with an advance in the Winkler's parameter of the Pasternak's foundation because of an enhancement in the structural stiffness of the foundation.

Table 3 lists variation in the phase velocities with an advance in the length to thickness ratio with changes of the wave numbers. One can find a diminish in the structural stiffness of the nanocomposite beam with an advance in the length to thickness ratio that leads to a decrease in the phase velocities. Furthermore, a diminish in the phase velocities is observed with an enhancement in the wave numbers.

Listed in the Table 4 is variation in the phase velocities with an advance in the piezoelectric to graphene origami reinforced core thickness ratio with changes of the wave numbers. It is concluded a diminish in the phase velocities with an advance in the piezoelectric to graphene origami reinforced core thickness ratio due to decrease in the structural stiffness. Furthermore, a diminish in the phase velocities is observed with an enhancement in the wave numbers.

Table 5 lists variation in the phase velocities with an advance in the temperature rising magnitude of the graphene origami reinforcement with changes of the wave numbers. One can find a diminish in the structural stiffness of the nanocomposite beam with an advance in the thermal loads that leads to a decrease in the phase velocities. Furthermore, a diminish in the phase velocities is observed with an enhancement in the wave numbers.

Listed in Table 6 is a variation in the phase velocities with an advance in the folding parameter of the graphene origami reinforcement with changes of the wave numbers. A diminish in the phase velocities is observed with an enhancement in the wave numbers. Investigating the impact of folding parameter of the graphene origami reflects a diminish in the

Table 6 Variation in the phase velocities of the graphene origami reinforced copper enriched beam with changes of folding parameter of the reinforcement in terms of wave number

k	$H_{Gr} = 0\%$	$H_{Gr} = 5\%$	$H_{Gr} = 10\%$	$H_{Gr} = 15\%$	$H_{Gr} = 20\%$	$H_{Gr} = 25\%$	$H_{Gr} = 30\%$	$H_{Gr} = 35\%$	$H_{Gr} = 40\%$	$H_{Gr} = 45\%$	$H_{Gr} = 50\%$
250	83.154	82.811	82.475	82.143	81.817	81.495	81.175	80.856	80.539	80.222	79.907
251	82.822	82.481	82.146	81.816	81.491	81.171	80.852	80.534	80.218	79.902	79.589
252	82.494	82.154	81.820	81.491	81.168	80.849	80.531	80.215	79.899	79.585	79.273
253	82.168	81.829	81.497	81.169	80.847	80.529	80.213	79.897	79.584	79.271	78.959
254	81.844	81.507	81.176	80.850	80.528	80.212	79.897	79.583	79.270	78.959	78.649
255	81.523	81.188	80.857	80.533	80.213	79.897	79.584	79.271	78.959	78.649	78.340
256	81.205	80.870	80.542	80.218	79.899	79.585	79.273	78.961	78.651	78.342	78.034
257	80.889	80.556	80.228	79.906	79.588	79.276	78.964	78.654	78.345	78.037	77.730
258	80.575	80.244	79.917	79.596	79.280	78.968	78.658	78.349	78.041	77.735	77.429
259	80.264	79.934	79.609	79.289	78.974	78.663	78.354	78.047	77.740	77.434	77.130
260	79.955	79.626	79.303	78.984	78.670	78.361	78.053	77.746	77.441	77.137	76.834
261	79.649	79.321	78.999	78.681	78.369	78.061	77.754	77.449	77.144	76.841	76.539
262	79.345	79.018	78.697	78.381	78.070	77.763	77.457	77.153	76.850	76.548	76.247
263	79.043	78.718	78.398	78.083	77.773	77.467	77.163	76.860	76.558	76.257	75.957
264	78.744	78.420	78.101	77.787	77.478	77.174	76.870	76.568	76.268	75.968	75.669
265	78.447	78.124	77.806	77.494	77.186	76.882	76.580	76.279	75.980	75.681	75.384
266	78.152	77.830	77.514	77.202	76.896	76.593	76.292	75.993	75.694	75.397	75.100
267	77.859	77.539	77.223	76.913	76.608	76.307	76.007	75.708	75.411	75.114	74.819
268	77.569	77.249	76.935	76.626	76.322	76.022	75.723	75.426	75.129	74.834	74.540
269	77.280	76.962	76.649	76.341	76.038	75.739	75.442	75.145	74.850	74.556	74.263
270	76.994	76.677	76.365	76.059	75.756	75.459	75.162	74.867	74.573	74.280	73.988
271	76.710	76.394	76.084	75.778	75.477	75.180	74.885	74.591	74.298	74.006	73.715
272	76.428	76.113	75.804	75.499	75.199	74.904	74.610	74.316	74.024	73.734	73.444
273	76.148	75.835	75.526	75.223	74.924	74.629	74.336	74.044	73.753	73.463	73.175
274	75.870	75.558	75.251	74.948	74.650	74.357	74.065	73.774	73.484	73.195	72.908
275	75.594	75.283	74.977	74.676	74.379	74.087	73.796	73.506	73.217	72.929	72.643
276	75.320	75.010	74.705	74.405	74.109	73.818	73.528	73.239	72.952	72.665	72.379
277	75.048	74.739	74.436	74.136	73.842	73.552	73.263	72.975	72.688	72.403	72.118
278	74.778	74.471	74.168	73.870	73.576	73.287	72.999	72.712	72.427	72.142	71.859
279	74.510	74.204	73.902	73.605	73.313	73.025	72.738	72.452	72.167	71.884	71.601
280	74.244	73.939	73.638	73.342	73.051	72.764	72.478	72.193	71.909	71.627	71.345
281	73.980	73.676	73.376	73.081	72.791	72.505	72.220	71.936	71.654	71.372	71.092
282	73.718	73.414	73.116	72.822	72.533	72.248	71.964	71.681	71.399	71.119	70.839
283	73.457	73.155	72.857	72.565	72.276	71.992	71.710	71.428	71.147	70.868	70.589
284	73.199	72.897	72.601	72.309	72.022	71.739	71.457	71.176	70.897	70.618	70.341
285	72.942	72.642	72.346	72.055	71.769	71.487	71.206	70.927	70.648	70.370	70.094
286	72.687	72.388	72.093	71.803	71.518	71.237	70.957	70.679	70.401	70.124	69.849
287	72.433	72.135	71.842	71.553	71.269	70.989	70.710	70.432	70.156	69.880	69.605
288	72.182	71.885	71.593	71.305	71.022	70.743	70.465	70.188	69.912	69.637	69.364
289	71.932	71.636	71.345	71.058	70.776	70.498	70.221	69.945	69.670	69.396	69.124
290	71.684	71.389	71.099	70.813	70.532	70.255	69.979	69.704	69.430	69.157	68.885
291	71.438	71.144	70.854	70.570	70.289	70.013	69.738	69.464	69.191	68.919	68.649
292	71.193	70.900	70.612	70.328	70.049	69.773	69.499	69.226	68.954	68.683	68.413
293	70.950	70.658	70.371	70.088	69.810	69.535	69.262	68.990	68.719	68.449	68.180
294	70.709	70.418	70.131	69.850	69.572	69.299	69.027	68.755	68.485	68.216	67.948

Table 6 Continued

295	70.469	70.179	69.894	69.613	69.336	69.064	68.793	68.522	68.253	67.985	67.718
296	70.231	69.942	69.658	69.378	69.102	68.831	68.560	68.291	68.022	67.755	67.489
297	69.995	69.707	69.423	69.144	68.869	68.599	68.329	68.061	67.793	67.527	67.262
298	69.760	69.473	69.190	68.912	68.638	68.369	68.100	67.832	67.566	67.300	67.036
299	69.526	69.240	68.959	68.682	68.409	68.140	67.872	67.606	67.340	67.075	66.812
300	69.295	69.009	68.729	68.453	68.181	67.913	67.646	67.380	67.115	66.852	66.589

Table 7 Variation in the phase velocities of the graphene origami reinforced copper enriched beam with changes of volume fraction of the reinforcement in terms of wave number

k	$V_{Gr} = 0\%$	$V_{Gr} = 0.1\%$	$V_{Gr} = 0.2\%$	$V_{Gr} = 0.3\%$	$V_{Gr} = 0.4\%$	$V_{Gr} = 0.5\%$	$V_{Gr} = 0.6\%$	$V_{Gr} = 0.7\%$	$V_{Gr} = 0.8\%$	$V_{Gr} = 0.9\%$	$V_{Gr} = 1\%$
250	83.154	83.255	83.358	83.461	83.567	83.673	83.780	83.887	83.994	84.101	84.208
251	82.822	82.923	83.026	83.129	83.234	83.340	83.446	83.552	83.659	83.766	83.873
252	82.494	82.594	82.696	82.799	82.903	83.009	83.115	83.221	83.327	83.433	83.540
253	82.168	82.268	82.369	82.472	82.576	82.681	82.786	82.892	82.998	83.104	83.210
254	81.844	81.944	82.045	82.147	82.251	82.355	82.460	82.566	82.671	82.776	82.882
255	81.523	81.623	81.723	81.825	81.928	82.032	82.137	82.242	82.347	82.452	82.557
256	81.205	81.304	81.404	81.505	81.608	81.712	81.816	81.921	82.025	82.130	82.234
257	80.889	80.987	81.087	81.188	81.290	81.394	81.498	81.602	81.706	81.810	81.914
258	80.575	80.674	80.773	80.873	80.975	81.079	81.182	81.286	81.389	81.493	81.597
259	80.264	80.362	80.461	80.561	80.663	80.766	80.869	80.972	81.075	81.178	81.282
260	79.955	80.053	80.152	80.251	80.352	80.455	80.558	80.660	80.763	80.866	80.969
261	79.649	79.746	79.845	79.944	80.045	80.147	80.249	80.351	80.454	80.556	80.659
262	79.345	79.442	79.540	79.639	79.739	79.841	79.943	80.045	80.147	80.249	80.351
263	79.043	79.140	79.237	79.336	79.436	79.537	79.639	79.740	79.842	79.944	80.046
264	78.744	78.840	78.937	79.035	79.135	79.236	79.337	79.438	79.539	79.641	79.742
265	78.447	78.543	78.639	78.737	78.836	78.937	79.038	79.138	79.239	79.340	79.442
266	78.152	78.247	78.344	78.441	78.540	78.640	78.740	78.841	78.941	79.042	79.143
267	77.859	77.954	78.050	78.147	78.246	78.346	78.446	78.546	78.646	78.746	78.846
268	77.569	77.663	77.759	77.856	77.954	78.053	78.153	78.252	78.352	78.452	78.552
269	77.280	77.375	77.470	77.566	77.664	77.763	77.862	77.962	78.061	78.161	78.260
270	76.994	77.088	77.183	77.279	77.376	77.475	77.574	77.673	77.772	77.871	77.970
271	76.710	76.804	76.898	76.994	77.091	77.189	77.288	77.386	77.485	77.584	77.683
272	76.428	76.521	76.616	76.711	76.807	76.905	77.004	77.102	77.200	77.299	77.397
273	76.148	76.241	76.335	76.430	76.526	76.624	76.721	76.819	76.917	77.015	77.114
274	75.870	75.963	76.056	76.151	76.247	76.344	76.441	76.539	76.637	76.734	76.832
275	75.594	75.686	75.780	75.874	75.970	76.066	76.163	76.261	76.358	76.455	76.553
276	75.320	75.412	75.505	75.599	75.694	75.791	75.888	75.984	76.081	76.178	76.275
277	75.048	75.140	75.233	75.326	75.421	75.517	75.614	75.710	75.807	75.903	76.000
278	74.778	74.870	74.962	75.055	75.150	75.246	75.342	75.438	75.534	75.630	75.727
279	74.510	74.601	74.693	74.786	74.880	74.976	75.072	75.167	75.263	75.359	75.455
280	74.244	74.335	74.427	74.519	74.613	74.708	74.803	74.899	74.994	75.090	75.186
281	73.980	74.070	74.162	74.254	74.347	74.442	74.537	74.632	74.727	74.823	74.918
282	73.718	73.808	73.899	73.991	74.084	74.178	74.273	74.368	74.462	74.557	74.653
283	73.457	73.547	73.638	73.729	73.822	73.916	74.010	74.105	74.199	74.294	74.389
284	73.199	73.288	73.378	73.470	73.562	73.656	73.750	73.844	73.938	74.032	74.127
285	72.942	73.031	73.121	73.212	73.304	73.397	73.491	73.585	73.679	73.773	73.867
286	72.687	72.775	72.865	72.956	73.048	73.141	73.234	73.328	73.421	73.515	73.608

Table 7 Continued

287	72.433	72.522	72.611	72.702	72.793	72.886	72.979	73.072	73.165	73.259	73.352
288	72.182	72.270	72.359	72.449	72.540	72.633	72.726	72.818	72.911	73.004	73.097
289	71.932	72.020	72.109	72.198	72.289	72.382	72.474	72.566	72.659	72.752	72.844
290	71.684	71.772	71.860	71.950	72.040	72.132	72.224	72.316	72.408	72.501	72.593
291	71.438	71.525	71.613	71.702	71.793	71.884	71.976	72.068	72.159	72.252	72.344
292	71.193	71.280	71.368	71.457	71.547	71.638	71.729	71.821	71.912	72.004	72.096
293	70.950	71.037	71.124	71.213	71.302	71.393	71.484	71.576	71.667	71.758	71.850
294	70.709	70.795	70.882	70.971	71.060	71.151	71.241	71.332	71.423	71.514	71.605
295	70.469	70.555	70.642	70.730	70.819	70.909	71.000	71.090	71.181	71.272	71.363
296	70.231	70.317	70.403	70.491	70.580	70.670	70.760	70.850	70.941	71.031	71.122
297	69.995	70.080	70.166	70.254	70.342	70.432	70.522	70.612	70.702	70.792	70.882
298	69.760	69.845	69.931	70.018	70.106	70.196	70.285	70.375	70.464	70.554	70.644
299	69.526	69.611	69.697	69.784	69.872	69.961	70.050	70.139	70.229	70.318	70.408
300	69.295	69.379	69.465	69.551	69.639	69.728	69.817	69.906	69.995	70.084	70.173

natural frequencies with an enhancement in the folding parameter of the reinforcement.

Listed in Table 7 is a variation in the phase velocities with an advance in the volume fractions of the graphene origami reinforcement with changes of the wave numbers. A diminish in the phase velocities is observed with an enhancement in the wave numbers. Furthermore, an enhancement in the volume fraction of the graphene origami leads to an increase in the phase velocities.

5. Conclusions

This study has successfully established a comprehensive framework for the multi-physics analysis of wave propagation in an advanced, shear-deformable sandwich beam structure. By introducing a novel higher-order thickness-stretched model, the formulation accurately captures the intricate dynamics of a system comprising a functionally tailored graphene origami-reinforced copper (GOri-Cu) composite core integrated with responsive piezoelectric and piezomagnetic face-sheets. The model rigorously incorporates the coupled thermo-electro-magneto-mechanical behavior under combined environmental and actuation loads, deriving governing equations via Hamilton's principle while explicitly accounting for the interdependent effects of thermal gradients, applied electric potentials, and magnetic inductions. A critical advancement lies in the detailed micromechanical modeling of the GOri-Cu core's constitutive behavior. Utilizing temperature-dependent modifier functions within the Halpin-Tsai framework, the model effectively quantifies the influence of key design parameters—graphene volume fraction, origami folding degree, and thermal load—on the core's fundamental properties, including effective modulus, Poisson's ratio, thermal expansion coefficient, and density. This provides essential predictive capability for material tailoring. The developed analytical methodology enables a systematic investigation into wave propagation sensitivity across multiple domains. It facilitates the exploration of how core morphology (folding degree, volume fraction), operational environment (temperature), and excitation parameters (electric potential, magnetic induction) profoundly influence wave characteristics like dispersion, phase velocity, and attenuation. Rigorous verification against established theories and benchmark results confirms the validity, accuracy, and superior capabilities of the proposed higher-order model compared to conventional approaches. The derived numerical results underscore the significant multifunctional potential of this smart sandwich structure. Particularly relevant for high-performance applications, such as advanced sport equipment like pole vaults, the structure's exceptional energy dissipation characteristics, tailorable stiffness-to-weight ratio through GOri core design, and inherent sensing capability offer a pathway to poles with enhanced energy return, reduced vibration, and integrated performance monitoring. Beyond this, the integrated piezoelectric/piezomagnetic face-sheets, synergistically coupled with the tunable GOri core, create a robust platform for dual functionality. The main conclusions of the present paper are followed as:

The results show an enhancement in the phase velocities with an enhancement in the volume fraction of the graphene origami due to an increase in the structural stiffness.

The results show a decrease in the phase velocities with an enhancement in the folding parameter of the graphene origami because of a decrease in the stiffness of nano-composite structure.

The results show an enhancement in the phase velocities with an enhancement in the thermal load of the graphene origami because of the decrease in modulus of elasticity.

The results show an enhancement in the phase velocities with an enhancement in the both stiffness and shear parameters of the Pasternak's foundation.

References

Bai, X., Xiao, Z., Shi, H., Zhang, K., Luo, Z., Wu, Y. (2025), "Omnidirectional sound wave absorption based on the multioriented acoustic meta-materials", *Appl. Acoust.*, **228**, 110344. https://doi.org/10.1016/j.apacoust.2024.110344

Cao, X. Yang, X. Fan, L. Habibi, M. Albaijan, I. (2025), "Delamination, frequency, and bending analysis of GPLRC curved panel with initial crack via machine learning and three-dimensional layerwise theory", *Thin Wall. Struct.*, **217**, Part A, 113503. https://doi.org/10.1016/j.tws.2025.113503.

Chang, Z. Wang, K. Wan, Y. Habibi, M. Bouallegue, B. Chen, X. (2025), "Hemodynamic responses to physical activity: Numerical analysis of dynamic behavior in microvascular structures under exercise-induced forces" *Adv. Nano. Res.*, **18**(3), 265-280. https://doi.org/10.12989/anr.2025.18.3.265

Chen, C. Xiao, X. Chen, L. Habibi, M. Brahmia, A. Chen, X. (2025), "Exercise-induced changes in protein tissue stability in athletes via biomechanical analysis using size-dependent mechanical models", *Adv. Nano. Res.*, **18**(5), 419-432. https://doi.org/10.12989/.2025.18.5.419.

Chen, H., Zhu, J., Habibi, M., Brahmia, A. and Wnag, D. (2025), "Stability analysis of nano-devices: Exercise-mediated effects on nanodevice stability in drug delivery applications", *Adv. Nano. Res.*, **18**(5), 445-458. https://doi.org/10.12989/.2025.18.5.445.

Dai, Y., Jiang, Z., Chen, K.Y., Zuo, D., Habibi, M., Elhosiny Ali, H. and AlBaijan. I. (2022), "Geometry impact on the stability behavior of cylindrical microstructures: Computer modeling and application for small-scale sport structures", *Steel. Compos. Struct.*, **48**(4), 443-459. https://doi.org/10.12989/scs.2023.48.4.443

Daichang, Z., Aiyun, L., Zhiqiang, S., Habibi, M., Albaijan, I. and Wong, L. (2025), "Dynamic stability and vibration responses of a volleyball game ball", *Adv. Nano. Res.*, **18**(4), 321-335. https://doi.org/10.12989/anr.2025.18.4.321.

Deng, T., Ding, H., Kitipornchai, S. and Yang, J. (2025), "Machine learning-based design strategy for weak vibration pipes conveying fluid", *Appl. Math. Mech.*, **46**(7), 1215-1236. https://doi.org/10.1007/s10483-025-3276-7

Dong, S., Pan, W., Wang, J., Habibi, M. and Li, C. (2025), "Optimizing seismic resistance in concrete structures through the application of elastic nano-composites", *Adv. Concr. Constr.*, **20**(1), 29-38. https://doi.org/10.12989/ACC.2025.20.1.029.

Ebrahimi, F., Goudarzfallahi, M. and Alinia-ziazi, A. (2025), "Thermomechanical stability enhancement in sandwich composite toroidal shells utilizing star-shaped auxetic core", *Acta Mech.*, 1-25. https://doi.org/10.1007/s00707-025-04476-6

Ebrahimi, F., Goudarzfallahi, M. and Ziazi, A.A. (2025), "Enhancing nonlinear static stability behavior of axially compressed sandwich composite toroidal shells with a bio-inspired auxetic core", *Acta. Mech.*, **236**, 2463-2480. https://doi.org/10.1007/s00707-025-04294-w

Ebrahimi, F., Mahinzare, M., Rastgoo, A. and Mirsadoghi, S.Z. (2025), "Chaotic and nonlinear dynamic response of hybrid nanocomposite graphene origami-enabled auxetic cylindrical shell", *Acta. Mech.*, **236**, 3187-3210. https://doi.org/10.1007/s00707-025-04320-x

Ebrahimi, F. and Parsi, M. (2025), "Utilizing star-shaped auxetic metabeams for piezoelectric vibration energy harvesting", *Acta. Mech.* **236**, 2895-2919. https://doi.org/10.1007/s00707-025-04291-z

Ebrahimi, F., Shafiei, M.S. and Ahari, M.F. (2022), "Vibration analysis of single and multi-walled circular graphene sheets in thermal environment using GDQM", *Waves. Random. Complex. Med.*, **35**(3), 5098-5137. https://doi.org/10.1080/17455030.2022.2067370

Fang, J., Ma, D., Fei, X. and Habibi, M. (2025), "Strengthening the mechanical properties of 3D printed thermoplastic elastomer by blending with acrylonitrile butadiene styrene, polypropylene and polyethylene", *Phys. Scripta.*, **100**, 4, 045922. https://doi.org/10.1088/1402-4896/adbc2e.

Ghasemi, H., Ebrahimi, F., Mohammadi, Y. and Abtahi, M. (2025), "Vibration analysis and flutter instability analysis of truncated conical shells subjected to flowing fluid using FG-GPL and FG-CNT hybrid laminated nanocomposites", *Acta. Mech.*, 1-24. https://doi.org/10.1007/s00707-025-04415-5

Hu, Y., Zhao, W., An, Y., Liao, M., Habibi, M. and Yan, X. (2025), "Mechanical performance of auxetic rotational polygons metamaterials based on simple rectangular-shaped parts: experimental validation and FEA modeling", *Int. J. Struct. Stab. Dyn.*, 2650344. https://doi.org/10.1142/S021945542650344X

Huang, Y., Zhang, B., Sun, C., Habibi, M., Ghazouani, N. and El Ouni, M.H. (2025), "Wave responses in seismic FGM concrete nanobeam using deep neural network" *Adv. Nano. Res.*, **18**(6), 503-517. https://doi.org/10.12989/anr.2025.18.6.503.

Li, J., An, Y., Habibi, M., Wen, A., Ma, M., Sun, G. and Shi, L. (2025), "A hybrid intelligent model for deformation/strain/stress analyses of sandwich double curved piezoelectric shells", *Int. J. Struct. Stab. Dyn.*, 2650166. https://doi.org/10.1142/S021945542650166X.

Li, X., Luo, L., Habibi, M. and Wang, L. (2025), "Extending a higher-order foldability constitutive model for dynamic response analysis of 3D-reinforced shell of deformable", *Acta. Mech.*, **236**, 1509-1533. https://doi.org/10.1007/s00707-024-04216-2.

Lin, R. Fan, L. Liu, L. Habibi, M. and Albaijan. I. (2025), "Use of metamaterials in graphene origami configuration for an electromagnetoelastic sandwich composite beam", *Adv. Nano. Res.*, **18**(2), 97-116. https://doi.org/10.12989/anr.2025.18.2.097.

Liu, J., Fu, Y., Habibi, M. *et al*. (2025), "Evaluation of mechanical behavior of textile microfibers", *Acta. Mech.*,**236**, 3081–3094. https://doi.org/10.1007/s00707-025-04314-9.

Liu, Q., Zhang, Y. Habibi, M. Brahmia, A. and Su, Y. (2025), "The activity and technique principle of football shooting are investigated from the viewpoint of nano-bio-mechanics", *Adv. Nano. Res.*, **18**(3), 241-251. https://doi.org/10.12989/anr.2025.18.3.241

Tan, Q. Yang, M. Liu, Y. Habibi, M. and Zolfaghari, T. (2025), "Investigating the effect of MoS2 nanoparticles in engine oil on reducing friction and wear of automotive parts", *Adv. Nano. Res.*, **18**(6), 519-528. https://doi.org/10.12989/anr.2025.18.6.519

Wan, Y., Zhang, G., Chang, Z., Habibi, M., Albaijan, I. and Li, Y. (2025), "Advancing sports equipment performance: Leveraging rotating small-scale structures for enhanced athletic tools", *Adv. Nano. Res.*, **18**(5), 467-480. https://doi.org/10.12989/anr.2025.18.5.467.

Xia, L., Habibi, M. and Li, Q. (2025), "Computational stability analysis of sport structures: Importance of MEMS for testing athlete performance", *Steel. Compos. Struct.* **54**(1), 53-69. https://doi.org/10.12989/scs.2025.54.1.053.

Xiao, D., Habibi, M., Bouallegue, B. and Bagheri, M. (2025), "A pathway to sports innovation through the stability performance of lightweight functionally graded tubular structures", *Adv. Nano. Res.*, **18**(4), 337-350. https://doi.org/10.12989/anr.2025.18.4.337.

Xu, L. Zhang, C. Habibi, M. Albaijan, I. Yin, C. (2025), "Research on applicable sensor for solving the volleyball sport problem using smart nanomaterial based on dynamic simulation", *Adv. Nano. Res.*, **18**(5), 405-417. https://doi.org/10.12989/anr.2025.18.5.405

YaJie, Z., Meng, W., Zhiqiang, S., Habibi, M., Brahmia, A. and Albaijan, I. (2025), "Wave propagation response of porous vibrating sports equipment under thermal loading application on testing athlete performance", *Steel Compos. Struct.*, **55**(2), 143-157. https://doi.org/10.12989/scs.2025.55.2.143.

Yang, L. Liu, L. Habibi, M. Chen. Z. (2025), "Enhancing sports equipment performance: Development of polyvinyl chloride-based nanocomposites with plantain wood powder and nano clay", *Adv. Nano. Res.*, **18**(4), 351-360. https://doi.org/10.12989/anr.2025.18.4.351

Ying, X., Yunzhu, A., Qige, Y., Kai, L., Habibi, M., Xingjia, T. and Yongji, L. (2025), "A novel foldable metamaterial for application in the pipeline pressure vessel with a static deformation, strain and stress analysis", *Sci. Rep.*, **15**, 9680. https://doi.org/10.1038/s41598-025-93302-z.

Yu, H., Habibi, M., Motamedi, K., Semirumi, D.T. and Ghorbani, A. (2023), "Utilizing stem cells in reconstructive treatments for sports injuries: An innovative approach", *Tissue. Cell.*, **83**, 102152. https://doi.org/10.1016/j.tice.2023.102152

Yuanchao, H., Yijiang, W., Habibi, M., Zhicong, D., Bei, L. and Yuhuan, L. (2025), "On propagation analysis of flexural waves in functionally graded poroelastic biocomposite higher-order beams", *Acta. Mech.*, **236**, 3959-3974. https://doi.org/10.1007/s00707-025-04362-1.

Yuanchao, H., Yunzhu, A., Shangmao, H., Habibi, M., Lei, G., Ying, C. (2025), "Humid-thermal environment influence on dispersion of waves in sigmoid poroelastic cylindrical panels resting on sinusoidal elastic foundation", *Mech. Based. Des. Struct.*, 1-19. https://doi.org/10.1080/15397734.2025.2491027.

Zha, J. and Zhang, Z. (2024), "Reversible negative compressibility metamaterials inspired by Braess's Paradox", *Smart. Mater. Struct.*, **33**(7), 075036. https://doi.org/10.1088/1361-665X/ad59e6

Zhaowei, Z., Zhiqiang, S., Aiyun, L., Habibi, M., Albaijan, I. and Zhang D. (2025), "Static responses for Graphene nanoplatlet reinforced aerobic sport plate", *Adv. Nano. Res.*, **18**(6), 565-583. https://doi.org/10.12989/anr.2025.18.6.565.

Zhu, D., Zhang, A., Habibi, M. and Arefi, M. (2025) "Electro-elastic analysis of piezoelectric double curved-shells for spring board practice and gymnastic training via Levy-Type Method", *Steel. Compos. Struct.*, **55**(2), 25, 97-116. https://doi.org/10.12989/scs.2025.55.2.097.

CC