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Dynamics of nonlocal graphene mindlin plate subjected to moving nanoparticles on viscoelastic support

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Abstract. This paper presents a detailed investigation into the forced vibration behavior of a nano-rectangular plate under the influence of moving nanoparticles, incorporating Coulomb friction effects. By combining Eringen's nonlocal continuum theory with the first-order shear deformation theory, the study examines the impact of the nonlocal parameter on the nanoplate's forced vibration response. The nanoplate, supported by a viscoelastic foundation modeled with a damper and Winkler modulus, is subjected to moving nanoparticles, considering their weight, inertia, and friction. The governing equations of motion and boundary conditions are derived using Mindlin's displacement field relations and Hamilton's principle. The analytical Galerkin method and Eigenfunction expansion are employed to transform the dimensionless partial differential equations into dimensionless ordinary differential equations under simply supported boundary conditions. The model's validity is confirmed through comparison with existing research. Additionally, this research explores the effects of key parameters, including the nonlocal parameter, foundation stiffness and damping coefficients, nanoparticle inertia, and velocity, on the dynamic amplitude factors of in-plane and out-of-plane displacements in detail.
Keywords: dynamic analysis; mindlin plate; moving nanoparticle; viscoelastic

1. Introduction

The future of Nanoengineering is bright and exciting. Nanotechnology and nanoscience are new branches of scientific research. Nanostructured elements have attracted the scientific community's attention due to their exceptional mechanical, thermal and electrical properties. One interesting new subgroup of nanostructure with a two-dimensional shape that has attracted a lot of attention is the nanoplate.

They could be utilized in:

• Medicine, referred to as nanomedicine and exciting possibilities in healthcare in two important aspects of nanomedicine: drug delivery and complex tissue engineering (Cells seeded into biocompatible and nanostructured scaffolds under the stimulation of growth factors spatio-temporally delivered by nanoparticles), (Shi *et al*. 2010)

• Observation of chemical reactions at the single molecule level using nano structures patterned with molecules, proteins and gold nanoparticles, (Li *et al*. 2011)

• Therapeutic applications using engineered nanostructured to encapsulate and release bioactive agents in a stimulus-responsive manner, (Li *et al*. 2011)

• Transporting of nano cars, (Shirai *et al*. 2005), (García-López *et al*. 2015), chemical and biological sensors, (Park *et al*. 2009), Yguerabide and Yguerabide (2002)

Conducting experiments with nanoscale-sized samples is proving difficult and expensive. Developing a proper mathematical model for nanostructures is also important. As a result of nanoplate application, understanding the vibration characteristics of nanoplates is crucial, making their vibration analysis a topic of significant interest in recent years.

To this end, previous research has focused on analyzing free and forced vibration of nanoplates subjected to a moving load or nanoparticle.

Jafarinezhad *et al*. (2024) presented a comprehensive analysis of functionally graded annular nanoplates using a stress-driven nonlocal elasticity model. Their findings emphasized the critical role of small-scale effects in accurately predicting transverse displacements and natural frequencies under various boundary conditions. This approach provides a more realistic representation of nanoplate behavior, that is vital for designing nanoscale devices.

Another notable contribution is the work of (Khien *et al*. 2024), who combined the Mindlin plate theory with flexoelectric effects and viscoelastic foundations. Their analytical solutions demonstrated that these additional factors significantly impact the mechanical response of nanoplates, particularly in the presence of strain gradient components.

Moreover, the vibration behavior of orthotropic nanoplates embedded in elastic media has been extensively studied. For instance, Anjomshoa and Tahani (2023) developed a continuum model based on the nonlocal Mindlin plate theory to investigate the free vibration of circular and elliptical nanoplates using the Galerkin method. Their results revealed the importance of considering orthotropic material properties in accurately predicting the natural frequencies of nanoplates.

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Furthermore, the role of nanoparticles interacting with nanoplates has been explored in several recent studies. (Mofidi *et al*. 2024) analyzed the performance of mass nanosensors based on the vibration characteristics of magneto-electro-elastic nanoplates with attached nanoparticles. Their work demonstrated how nonlocal effects and external electromagnetic potentials influence the frequency shift and sensitivity of nanosensors.

(Mashhour *et al*. 2022) investigated the effect of thickness stretching on free vibration, bending and buckling behavior of carbon nanotubes reinforced composite (CNTRC) laminated nanoplates rested on a new variable elastic foundation.

(Bouafia *et al*. 2021) examined the small-scale impact on the vibrational properties of “functionally graded” (FG) nanoplate embedded in an elastic medium.

(Malekzadeh *et al*. 2009) determined the dynamic response of cross-ply laminated thick plates subjected to moving load, they presented a solution procedure based on the three-dimensional (3D) elasticity theory. Taheri and Ting (1989) developed an algorithm based on a structural impedance approach to study the transient response of plates with arbitrary boundary conditions and subjected to moving loads. Taheri and Ting (1990) developed an algorithm based on a finite element approach to study the transient response of plates with arbitrary boundary conditions and subjected to moving loads. The effect of electrostatic and Casimir forces on the pull-in instability and vibration of single nanoplate (SNP) carrying a moving nanoparticle has been carried out by Ghorbanpour Arani and Shokravi (2012) in another study forced-vibration analysis of a coupled system of single layered graphene sheets (SLGSs) subjected to the moving nanoparticle has been carried out based on nonlocal elasticity theory of orthotropic plate by (Ghorbanpour Arani *et al*. 2013). Moving nanoparticle weight effects on the dynamic analysis and vibration response of an embedded poly-vinyli-dene fluoride (PVDF) nanoplate has been studied by (Ghorbanpour Arani *et al*. 2015) employing the classical plate and Eringen’s non-local theories, energy method and Hamilton’s principle. In another work, vibrations of an embedded nanoplate subjected to bi-axially applied loads and a moving nanoparticle has been investigated considering the mass weight of the moving nanoparticle, and its friction with the upper surface of the nanoplate. the nanoplate modeled via nonlocal Kirchhoff plate theory and a rigid body and The roles of the moving nanoparticle velocity, small-scale parameter and bi-axially tension forces on the displacements of the nanoplate have been discussed by Kiani (2013). He also investigated The vibration of elastic thin nanoplates traversed by a moving nanoparticle involving Coulomb friction using the nonlocal continuum theory of Eringen, Kiani (2013) and in another research dynamic response of a nanoplate subjected to a moving nanoparticle has been examined within the context of nonlocal continuum theory of Eringen. The fully simply supported nanoplate is modeled based on the nonlocal Kirchhoff, Mindlin, and higher-order plate theories Kiani (2013, 2011a).

It should be noted that a large part of previous studies has been focused on the analysis of forced vibration of nanoplates subjected to a moving nanoparticle weight force without considering the inertia effect of moving nanoparticles. In some proposed applications of the nanoplates such as nano cars and drug delivery systems, we should scrutinize all parameters that affect the vibration analysis. Thus, so more attention should be paid for the moving nanoparticle inertia effect on vibration analysis of micro/nanoplate.

(Ai *et al*. 2018) presented analytical research on the dynamic interaction problem between an ore-stressed plate and a transversely isotropic multilayered half-plane subjected to a moving load.

(Fayaz *et al*. 2017) investigated the dynamic response of geometrically nonlinear rectangular elastic plates subjected to moving mass loading, numerically.

(Anague Tabejieu *et al*. 2019), devoted a particular attention to analyze the effect of the foundation having fractional order viscoelastic material on a response of a beam structure.

(Pirmoradian *et al*. 2018) analysed the rectangular plates subjected to moving inertial loads via IHB method.

(Torkan *et al*. 2019) considered the instability inspection of parametric vibrating rectangular of moving masses (Pirmoradian *et al*. 2020) used an energy-based method to analyze dynamic stability and parametric resonance of Mindlin plates lying on Winkler foundations under periodic loading single-layered graphene sheets (SLGSs) embedded in thermal environment and elastic medium while carrying a nanoparticle moving along an elliptical path. (Abdelrahman *et al*. 2021) developed a nonclassical dynamic finite element model based on the nonlocal strain gradient theory to study and analyze the dynamic behavior of perforated nanobeam structures under moving mass/load.

Recent advancements in the modeling of nanostructures have emphasized the significance of incorporating size-dependent effects, viscoelastic foundations, and multi-functional material properties to capture accurate mechanical and dynamic behaviors. (Pham *et al*. 2024) introduced a novel integration of nonlocal Mindlin plate theory and flexoelectricity to analyze the bending and vibration of nanoplates supported by viscoelastic foundations, highlighting the influence of nonlocal and electromechanical coupling parameters. In a complementary study, (Jalaei *et al*. 2022) investigated the transient response of magnetically affected functionally graded nanobeams using a quasi-3D nonlocal strain gradient theory coupled with a Kelvin-Voigt viscoelastic model, emphasizing the role of porosity and magnetic fields on dynamic damping characteristics. To resolve inconsistencies in Eringen’s nonlocal elasticity theory under cantilever boundary conditions, Demir and Civalek (2017) proposed an enhanced differential model, providing analytical solutions for bending responses of nano/micro beams under various loading and support configurations. (Numanoğlu *et al*. 2018) addressed the longitudinal vibration behavior of nanorods with diverse boundary and attachment conditions, demonstrating that attached masses significantly reduce the natural frequencies, particularly in higher modes. (Civalek *et al*. 2021) analyzed forced vibration in carbon nanotube-reinforced composite beams using first-order shear deformation theory and the Ritz method, showcasing the impact of nanotube distribution on dynamic responses. From a thermoelastic perspective, (Abouelregal *et al*. 2022) developed a model based on Caputo–Fabrizio fractional derivatives to study nonlocal thermoelastic wave propagation under pulsed heat flux, underlining the effects of fractional and nonlocal parameters on thermal conduction. He and She (2024) explored nonlinear forced vibration of functionally graded beams with initial geometric imperfections in hygrothermal environments, revealing soft, hard, and mixed spring-type resonance behaviors. (Arefi *et al*. 2021) conducted an in-depth analysis of graphene nanoplatelet-reinforced cylindrical shells under thermo-mechanical loads, finding that reinforcement distribution patterns significantly influence stress and displacement outcomes. Furthermore, (Sobhani *et al*. 2022) investigated the vibration behavior of coupled hemispherical-conical shell structures embedded with CNT and GNP nanofillers, emphasizing the role of agglomeration and distribution profiles. Finally, (Alazwari *et al*. 2022) presented a comprehensive dynamic analysis of functionally graded nanobeams under thermo-magnetic fields and moving loads using nonlocal strain gradient theory, where magnetic fields were shown to increase stiffness and suppress transverse deflections. These studies collectively underscore the necessity of advanced theoretical frameworks to accurately model and predict nanoscale structural behavior under complex physical conditions.

This study investigates the vibration behavior of a nonlocal Mindlin plate with simply supported boundary conditions under the influence of moving nanoparticles. The analysis considers the effects of nanoparticle weight, inertia, friction, and the viscoelastic foundation. The selection of the nonlocal Mindlin plate theory in the present research is motivated by its unique ability to account for both small-scale (nonlocal) effects and transverse shear deformation, which are critically important for realistic modeling of nanoplates subjected to moving nanoparticles. While classical plate theories such as the Kirchhoff plate model neglect both size-dependent phenomena and shear deformation, the nonlocal Mindlin theory incorporates Eringen's nonlocal elasticity to effectively capture long-range interatomic interactions and the resulting scale effects, leading to more accurate predictions at the nanoscale. Additionally, Mindlin's first-order shear deformation theory addresses the limitations of classical thin plate theories by including transverse shear, which becomes significant in moderately thick nanoplates and when viscoelastic foundations are present. This combined approach is particularly advantageous for analyzing nanoplates under complex dynamic scenarios—such as moving nanoparticles with inertia and friction—because it provides a comprehensive and physically sound framework that local or purely classical theories cannot offer. Therefore, the nonlocal Mindlin plate theory enables a more realistic and robust dynamic analysis of nanoplates, which is essential for reliable predictions in advanced nano-technology applications. The Mindlin theory, introduced by Mindlin (1951), is an extension of classical plate theory and includes the effects of transverse shear deformation. This theory is particularly relevant for plates with moderate thickness-to-length ratios, where classical Kirchhoff plate theory fails to account for shear deformation. For further exploration of the local Mindlin plate theory, readers can refer to the works of Mindlin (1951) and Reissner (1945)**,** who provided foundational formulations for plate bending under various loading conditions.

Given the dynamic nature of the problem—where moving nanoparticles introduce inertia and frictional forces—this theory provides a more comprehensive and physically accurate representation of the system's behavior.

The vibration analysis of plates has been addressed through various analytical and numerical methods, each offering distinct advantages depending on the problem's complexity. The Finite Element Method (FEM) stands out as a powerful numerical technique capable of handling complex geometries, arbitrary boundary conditions, and diverse material properties by discretizing the plate into finite elements and approximating solutions through shape functions, Marzavan and Nastasescu (2023). For time-domain analysis of dynamic systems, the Newmark-Beta Method provides a stable numerical scheme for solving the second-order differential equations governing plate vibrations, (Wen *et al*. 2022). Alternatively, the Rayleigh-Ritz Method employs a variational approach to approximate natural frequencies and mode shapes by using admissible functions that satisfy boundary conditions while minimizing the system's potential energy, (Njim *et al*. 2021). For nonlinear vibration problems, particularly those involving axially moving plates and fluid-structure interactions, the Chebyshev Collocation Method proves highly effective by leveraging Chebyshev polynomials for solution approximation, Powmya and Narasimhan (2015).

In the present study, the Galerkin method combined with Eigenfunction expansion was utilized to derive the governing equations of motion and obtain an analytical solution under simply supported boundary conditions; however, future research could enhance the analysis by integrating numerical techniques like FEM or the Newmark-Beta Method to address more complex geometries and boundary conditions. Finally, validation of the proposed approach was achieved by comparing the results for natural frequencies and dynamic amplitude factors with existing studies, confirming the model's accuracy and reliability.

2. Definition of the problem

The coordinate system and a schematic diagram of the graphene nanoplate with a total thickness h in the z direction, length a in the $x-$direction, width b in the $y-$direction subjected to a moving nanoparticle on a viscoelastic foundation are illustrated in Fig. 1. The nano plate is of density ρ, Young's modulus E_p and Poisson ratio ν. The viscoelastic foundation is modeled as the Kelvin-Voigt foundation, consisting of an infinite set of springs and dampers. The damping coefficient and spring stiffness are denoted by C_d and K_w, respectively.

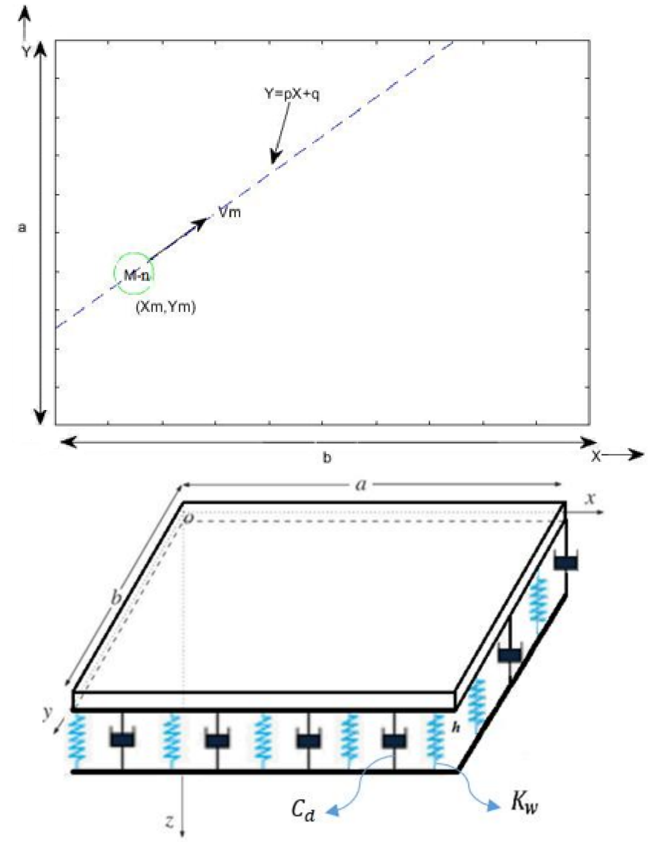


Fig. 1 A graphene nano plate subjected to moving nano particles resting on viscoelastic foundation

The moving nanoparticle remains in contact with the nanoplate during excitation, and the inertia effects of the moving nanoparticles are incorporated into the formulation of the proposed model.

The transverse vibration of the nanoplate is induced by loads resulting from the weight of moving nanoparticles on its upper surface, foundation forces, and in-plane vibration caused by friction between the moving nanoparticles and the nanoplate surface. The Coulomb friction model is employed for simply investigation of the contact effect of moving nanoparticles on the in-plane vibration of a nano plate, Kiani (2011b). The friction force is expressed as $q_s = \mu_k M_i g$ where μ_k denotes the kinetic friction coefficient and M_i as the mass of $i-th$ nanoparticle.

3. Nonlocal continuum theory

It has been shown that the small-scale effect must play a significant role in nanostructures. The classical continuum theories are not size-dependent theories and for using the present classical theory with nanoplates, the non-local effects due to the small sizes are to be incorporated into the formulation. To consider small scale effects, the nonlocal elasticity theory was proposed by Eringen which is widely used because of its efficiency and simplicity.

According to nonlocal theory of Eringen, Eringen (1983), Eringen (2002), strain at a point depends on both the stress and the spatial derivatives of the stress at that point. According to Eringen the nonlocal constitutive behavior is defined by the following differential constitutive relation:

$$(1 - l_1^2 \nabla^2)\sigma_{\alpha\beta}^{nl} = \sigma_{\alpha\beta}^{l} \tag{1}$$

where $\sigma_{\alpha\beta}^{nl}$ is the nonlocal stress tensor, $\sigma_{\alpha\beta}^{l}$ is the local stress tensor and l_1 is the first small scale parameter or nonlocal parameter.

3.1 Basic formulation and assumption (Governing Equation)

The displacement of an arbitrary point of the nanoplate can be expressed in terms of the middle surface displacement components. The local Mindlin plate theory is based on the following displacement field:

$$\begin{aligned} U_1^M(x,y,z,t) &= u_0^M(x,y,t) + z\psi_x^M(x,y,t) \\ U_2^M(x,y,z,t) &= v_0^M(x,y,t) + z\psi_y^M(x,y,t) \\ U_3^M(x,y,z,t) &= w_0^M(x,y,t) \end{aligned} \tag{2}$$

where $(u_0^M,\ v_0^M,\ w_0^M)$ denote the displacements of a point on the mid-plane $(z=0)$ of the Mindlin plate theory along $x,\ y$ and z axes, also ψ_x^M and ψ_y^M the rotation of a transverse normal about the x and y axes, respectively and t is time. Within the context of linear elasticity, the strain components of the Mindlin plate are expressed as:

$$\begin{aligned} \varepsilon_{xx}^M &= u_{0,x}^M + z\psi_{x,x}^M \\ \varepsilon_{yy}^M &= v_{0,y}^M + z\psi_{y,y}^M \\ \gamma_{xy}^M &= u_{0,y}^M + v_{0,x}^M + z\left(\psi_{x,y}^M + \psi_{y,x}^M\right) \\ \gamma_{zy}^M &= \psi_y^M + w_{0,y}^M \\ \gamma_{xz}^M &= \psi_x^M + w_{0,x}^M \end{aligned} \tag{3}$$

Moreover, the local components of the local in-plane forces $({N_{\alpha\beta}}^l)$, local in-plane moments $({M_{\alpha\beta}}^l)$ and local transverse shear forces $({Q_{\alpha\beta}}^l)$ are defined as: $(\alpha, \beta = x, y, z)$

$$\begin{aligned} ({N_x}^l, {N_y}^l, {N_{xy}}^l) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} ({\sigma_x}^l, {\sigma_y}^l, {\sigma_{xy}}^l)\, dz \\ ({M_x}^l, {M_y}^l, {M_{xy}}^l) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} ({\sigma_x}^l, {\sigma_y}^l, {\sigma_{xy}}^l) z\, dz \\ ({Q_x}^l, {Q_y}^l) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} ({\sigma_{xz}}^l, {\sigma_{yz}}^l)\, dz \end{aligned} \tag{4}$$

Based on the Mindlin plate theory and the constitutive relations between local stress and strain components within the frame work of local continuum theory for an isotropic homogeneous nanoplate and using Eq. (4), the components of $({N_{\alpha\beta}}^l, {M_{\alpha\beta}}^l, {Q_{\alpha\beta}}^l)$ are expressed in terms of midplane displacements as:

$$\begin{bmatrix} (N_{xx}^M)^l \\ (N_{yy}^M)^l \\ (N_{xy}^M)^l \end{bmatrix} = A \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial u_0^M}{\partial x} \\ \dfrac{\partial v_0^M}{\partial y} \\ \dfrac{\partial u_0^M}{\partial y} + \dfrac{\partial v_0^M}{\partial x} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix}(M_{xx}^M)^l\\(M_{yy}^M)^l\\(M_{xy}^M)^l\end{bmatrix}=D\begin{bmatrix}1&\nu&0\\\nu&1&0\\0&0&\frac{1-\nu}{2}\end{bmatrix}\begin{bmatrix}\frac{\partial\psi_x^M}{\partial x}\\\frac{\partial\psi_y^M}{\partial y}\\\frac{\partial\psi_x^M}{\partial y}+\frac{\partial\psi_y^M}{\partial x}\end{bmatrix} \tag{6}$$

$$\begin{bmatrix}(Q_x^M)^l\\(Q_y^M)^l\end{bmatrix}=K_sA\frac{1-\nu}{2}\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\psi_x^M+\frac{\partial w^M}{\partial x}\\\psi_y^M+\frac{\partial w^M}{\partial y}\end{bmatrix} \tag{7}$$

where, the in-plane and bending rigidities of the nano plate are respectively denoted by $A=\frac{Eh}{1-\nu^2}$ and $D=\frac{Eh^3}{12(1-\nu^2)}$. In Eq. (7), K_s denotes the shear correction factor, which is introduced to account for the non-uniform distribution of transverse shear strains through the thickness of the plate. For rectangular cross-sections, a typical value of K_s is 5/6, although it may vary depending on the geometry and assumptions of the model.

The equations of motion of the Mindlin plate model under moving nanoparticle are derived using Hamilton's principle. The principle can be stated in analytical form as:

$$0=\int_0^t(\delta U-\delta T-\delta W)dt \tag{8}$$

where δU, δT and δW are the variation of strain energy, kinetic energy and work done energy, respectively. The variation of strain energy is calculated by

$$\delta U=\int_V\left(\sigma_{ij}\delta\varepsilon_{ij}\right)dV \tag{9}$$

$$\delta U^M=\int_A\int_{-\frac{h}{2}}^{+\frac{h}{2}}\left(\sigma_x^M\delta\varepsilon_x^M+\sigma_y^M\delta\varepsilon_y^M+\sigma_{xy}^M\delta\varepsilon_{xy}^M+\sigma_{xz}^M\delta\varepsilon_{xz}^M+\sigma_{yz}^M\delta\varepsilon_{yz}^M\right)dzdA \tag{10}$$

By substituting stress resultants from Eq. (3) into Eq. (7), the variation of strain energy for Mindlin plate theory is:

$$\delta U^M=\int_A\begin{bmatrix}(N_x^M)^{nl}\frac{\partial\delta u_0^M}{\partial x}+(M_x^M)^{nl}\delta\psi_{x,x}^M\\+\left(N_y^M\right)^{nl}\frac{\partial\delta v_0^M}{\partial y}+\left(M_y^M\right)^{nl}\delta\psi_{y,y}^M\\+\left(N_{xy}^M\right)^{nl}\frac{\partial\delta u_0^M}{\partial y}+\left(N_{xy}^M\right)^{nl}\frac{\partial\delta v_0^M}{\partial x}\\+\left(M_{xy}^M\right)^{nl}\left(\frac{\partial\psi_x^M}{\partial y}+\frac{\partial\delta\psi_y^M}{\partial x}\right)+(Q_x^M)^{nl}\delta\psi_x^M\\+(Q_x^M)^{nl}\frac{\partial\delta w_o^M}{\partial x}+\left(Q_y^M\right)^{nl}\delta\psi_y^M\\+(Q_x^M)^{nl}\frac{\partial\delta w_o^M}{\partial y}\end{bmatrix}dA \tag{11}$$

Kinetic energy and variation of kinetic energy can be written as:

$$K=\frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_A\left(\dot{U}_1+\dot{U}_2+\dot{U}_3\right)\rho(z)dAdz+\frac{1}{2}M_n\left(\frac{\partial U_3}{\partial t}\right)^2 \tag{12}$$

$$\delta K=\int_V\left(\dot{U}_1\delta\dot{U}_1+\dot{U}_2\delta\dot{U}_2+\dot{U}_3\delta\dot{U}_3\right)\rho(z)dAdz+M_n\frac{\partial^2\delta w_0}{\partial t^2} \tag{13}$$

The variation of kinetic energy for Mindlin plate theory is:

$$\delta K^M=\int_A I_0\left(\dot{u}_0\delta\dot{u}_0+\dot{v}_0\delta\dot{v}_0+\dot{w}_0\delta\dot{w}_0\right)+I_1\left(\dot{u}_0\delta\dot{\psi}_x+\dot{v}_0\delta\dot{\psi}_y+\dot{\psi}_x\delta\dot{u}_0+\dot{\psi}_y\delta\dot{v}_0\right)+I_2\left(\dot{\psi}_x\delta\dot{\psi}_x+\dot{\psi}_y\delta\dot{\psi}_y\right)dA+M_i\frac{\partial^2\delta w_0}{\partial t^2} \tag{14}$$

In which (I_0,I_1,I_2) are the mass moments of inertia which can be calculated from Eq. (15) and dot-superscript convention indicates the differentiation with respect to the time variable t.

$$I_n=\int\rho(z)z^ndz\Rightarrow(I_0+I_1+I_2)=\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,z,z^2)\rho(z)dz \tag{15}$$

The variation of work done by external forces consists of three parts:

(1) Virtual work done by the body forces in $V=\Omega\times\left(-\frac{h}{2},\frac{h}{2}\right)$

(2) Virtual work done by surface tractions acting on the top and bottom surfaces Ω

(3) Virtual work done by surface tractions acting on the lateral surface $S=\Gamma\times\left(-\frac{h}{2},\frac{h}{2}\right)$

where, Ω denotes the middle surface of the plate and Γ is the boundry of the middle surface.

Let (f_x,f_y,f_z) be the body forces, (c_x,c_y,c_z) be the body couples, (q_x,q_y,q_z) be the tractions acting on Ω, and (t_x,t_y,t_z) and (s_x,s_y,s_z) be the Cauchy traction and surface couple, respectively, acting on S. Then, the variation of work done by external forces can be expressed as

$$\delta V=-\int_\Omega\left(f_x\delta u_0+f_y\delta v_0+f_z\delta w_0+q_x\delta u_0+q_y\delta v_0+q_z\delta w_0+C_x\delta\theta_x+C_y\delta\theta_y+C_z\delta\theta_z\right)dxdy+\int_\Gamma\left(t_x\delta u_0+t_y\delta v_0+t_z\delta w_0+S_x\delta\theta_x+S_y\delta\theta_y+S_z\delta\theta_z\right) \tag{16}$$

In which $(\theta_1,\theta_2,\theta_3)$ are the components of the rotation vector defined by:

$$\theta_1 = \frac{1}{2}\left(\frac{\partial U_3}{\partial x_2} - \frac{\partial U_2}{\partial x_3}\right)$$
$$\theta_2 = \frac{1}{2}\left(\frac{\partial U_1}{\partial x_3} - \frac{\partial U_3}{\partial x_1}\right) \quad (17)$$
$$\theta_3 = \frac{1}{2}\left(\frac{\partial U_2}{\partial x_1} - \frac{\partial U_1}{\partial x_2}\right)$$

For this case without body force f, body couple c and small rotations, the variation of work done by external forces reduce to:

$$\delta V = -\int_{\Omega} \left(q_x \delta u_0 + q_y \delta v_0 + q_z \delta w_0\right) dxdy \quad (18)$$

Here, the transverse loading on nano plate is applied by three items:

1- By the foundation in z direction (viscoelastic foundation modeled by Winkler and damper module)

$$q_{z\_w}(x,y) = K_w w_0(x,y) \quad (19)$$

$$q_{z\_c}(x,y) = C_p \frac{\partial w_0(x,y)}{\partial t} \quad (20)$$

2- By the moving nanoparticles weight

The transverse loading by the moving nanoparticles weight is obtained by:

$$q_{z\_m}(x,y) = \sum_{i=1}^{n} M_i g \Delta(x-x_m)\Delta(y-y_m)H(a - x_m)H(b-y_m) \quad (21)$$

where, Δ is the Dirac delta function, H is Heaviside step function and q_{z_m} is the transverse loading of moving nanoparticles.

3- By the friction force between nanoparticles and nano plate

$$q_s = \mu_k . M_i g \quad (22)$$

$$q_{sx}(x,y) = \mu_x M_i g \Delta(x-x_m)\Delta(y-y_m)H(a - x_m)H(b-y_m)\delta U_1 \quad (23)$$

$$q_{sy}(x,y) = \mu_y M_i g \Delta(x-x_m)\Delta(y-y_m)H(a - x_m)H(b-y_m)\delta U_2 \quad (24)$$

where q_{sx} and q_{sy} are in turn the components of the kinetic friction force in the x and y direction and (x_m, y_m) is the coordinates of the moving nanoparticle.

$$\delta V = -Kw_0\delta w_0 - C\frac{\partial w_0}{\partial t}\delta w_0 + q_{z_m}(x,y,t)\delta w_0 + \mu_x M_i g \Delta(x-x_m)\Delta(y-y_m)H(a-x_m)H(b - y_m)\delta U_1 + \mu_y M_i g \Delta(x-x_m)\Delta(y-y_m)H(a-x_m)H(b - y_m)\delta U_2 \quad (25)$$

Finally, the variation of work done by external forces for Mindlin plate theory can be expressed as:

$$\delta V^M = -Kw_0\delta w_0 - C\frac{\partial w_0}{\partial t}\delta w_0 + M_i g \Delta(x-x_m)\Delta(y-y_m)H(a-x_m)H(b-y_m)\delta w_0 + \mu_x M_i g \Delta(x-x_m)\Delta(y-y_m)H(a-x_m)H(b - y_m)\left(\delta u_0 + \frac{h}{2}\delta\psi_x\right) + \mu_y M_i g \Delta(x-x_m)\Delta(y-y_m)H(a-x_m)H(b - y_m)\left(\delta v_0 + \frac{h}{2}\delta\psi_y\right) \quad (26)$$

Substituting the expressions for δU^M, δK^M, and δV^M from Eq. (11), Eq. (14) and Eq. (26) into Eq. (8) and integrating by parts, and collecting the coefficients of $\left(\delta U_0, \delta V_0, \delta W_0, \delta\psi_x, \delta\psi_y\right)$, the following equations of motion of Mindlin plate theory are obtained

$$A\left(u_{0,xx}^M + \tfrac{1-\nu}{2}u_{0,yy}^M + \tfrac{1+\nu}{2}v_{0,xy}^M\right) = (1 - l_1^2\nabla^2)\left(I_0 u_{0,tt}^M - \mu_x M_i g\Delta(x-x_m)\Delta(y-y_m)H(a - x_m)H(b-y_m)\right) \quad (27)$$

$$A\left(v_{0,yy}^M + \tfrac{1-\nu}{2}v_{0,xx}^M + \tfrac{1+\nu}{2}u_{0,xy}^M\right) = (1 - l_1^2\nabla^2)\left(I_0 v_{0,tt}^M - \mu_y M_i g\Delta(x-x_m)\Delta(y-y_m)H(a - x_m)H(b-y_m)\right) \quad (28)$$

$$KsF\left(\psi_{x,x}^M + \psi_{y,y}^M + \nabla^2 W_0^M\right) = (1 - l_1^2\nabla^2)\left(I_0 w_{0,tt}^M - M_i g\Delta(x-x_m)\Delta(y-y_m)H(a - x_m)H(b-y_m) + kw_0^M + C\frac{\partial w_0^M}{\partial t} + M_i\Delta(x-x_m)\Delta(y-y_m)H(a - x_m)H(b-y_m)\left(\frac{\partial^2 w_0^M}{\partial t^2}\right)\right) \quad (29)$$

$$D\left(\psi_{x,xx}^M + \frac{1-\nu}{2}\psi_{x,yy}^M + \frac{1+\nu}{2}\psi_{y,xy}^M\right) - KsF\left(\psi_x^M + w_{0,x}^M\right) = (1 - l_1^2\nabla^2)\left(I_2\psi_{x,tt}^M - \mu_x M_i g\Delta(x-x_m)\Delta(y-y_m)H(a-x_m)H(b - y_m)\frac{h}{2}\right) \quad (30)$$

$$D\left(\psi_{y,yy}^M + \frac{1-\nu}{2}\psi_{y,xx}^M + \frac{1+\nu}{2}\psi_{x,xy}^M\right) - KsF\left(\psi_y^M + w_{0,y}^M\right) = (1 - l_1^2\nabla^2)\left(I_2\psi_{y,tt}^M - \mu_y M_i g\Delta(x-x_m)\Delta(y-y_m)H(a-x_m)H(b - y_m)\frac{h}{2}\right) \quad (31)$$

For more appropriate in analysis of the problem, the following dimensionless parameters are defined

$$\psi_{\bar{y}} = \frac{\psi_y}{a} \quad \psi_{\bar{x}} = \frac{\psi_x}{a} \quad \overline{W} = \frac{w_0}{a} \quad \overline{V} = \frac{v_0}{a} \quad \overline{U} = \frac{u_0}{a}$$
$$\mu_1 = \frac{l_1}{a} \quad \acute{k} = \frac{h}{a} \quad k = \frac{a}{b} \quad \bar{y} = \frac{y}{b} \quad \bar{x} = \frac{x}{a} \quad (32)$$

where, $\tau = \frac{t}{a}\sqrt{\frac{F}{I_0}}$ is the time dimensionless parameter, and $(\bar{x}, \bar{y})$ denotes the dimensionless coordinates. By introducing Eq. (32) into Eqs. (27)-(31) the dimensionless governing equation of motion of the nonlocal Mindlin plate theory under excitation of moving nanoparticles are readily obtained as:

$$\bar{A}\left(\frac{\partial^2\bar{U}_o^M}{\partial\bar{x}^2}+\frac{1-\nu}{2}k^2\frac{\partial^2\bar{U}_o^M}{\partial\bar{y}^2}+\frac{1+\nu}{2}k\frac{\partial^2\bar{V}_o^M}{\partial\bar{x}\partial\bar{y}}\right) = \left(1-\mu_1^2\left(\frac{\partial^2}{\partial\bar{x}^2}+k^2\frac{\partial^2}{\partial\bar{y}^2}\right)\right) \left\{\frac{\partial^2\bar{U}_o^M}{\partial\tau^2}-\mu_x\bar{f}\Delta(x-\bar{x})\Delta(y-\bar{y})H(1-\bar{x})H(1-\bar{y})\right\} \tag{33}$$

$$\bar{A}\left(k^2\frac{\partial^2\bar{V}_o^M}{\partial\bar{y}^2}+\frac{1-\nu}{2}\frac{\partial^2\bar{V}_o^M}{\partial\bar{x}^2}+\frac{1+\nu}{2}k\frac{\partial^2\bar{U}_o^M}{\partial\bar{x}\partial\bar{y}}\right) = \left(1-\mu_1^2\left(\frac{\partial^2}{\partial\bar{x}^2}+k^2\frac{\partial^2}{\partial\bar{y}^2}\right)\right) \left\{\frac{\partial^2\bar{V}_o^M}{\partial\tau^2}-\mu_y\bar{f}\Delta(x-\bar{x})\Delta(y-\bar{y})H(1-\bar{x})H(1-\bar{y})\right\} \tag{34}$$

$$K_s\left(\frac{\partial\psi_{\bar{x}}^M}{\partial\bar{x}}+k\frac{\partial\psi_{\bar{y}}^M}{\partial\bar{y}}+\frac{\partial^2\bar{W}_o^M}{\partial\bar{x}^2}+k^2\frac{\partial^2\bar{W}_o^M}{\partial\bar{y}^2}\right) = \left(1-\mu_1^2\left(\frac{\partial^2}{\partial\bar{x}^2}+k^2\frac{\partial^2}{\partial\bar{y}^2}\right)\right) \left\{\begin{array}{c}\frac{\partial^2\bar{W}_o^M}{\partial\tau^2}+\frac{K_w a^2}{F}\bar{W}_o^M+\frac{C_p a}{F\sqrt{\frac{I_0}{F}}}\frac{\partial\bar{W}_o^M}{\partial\tau}\\ +\frac{M_i}{I_0}\frac{1}{ab}\frac{\partial^2\bar{W}_o^M}{\partial\tau^2}\Delta(x-\bar{x})\Delta(y-\bar{y})H(1-\bar{x})H(1-\bar{y})\\ -\bar{f}\Delta(x-\bar{x})\Delta(y-\bar{y})H(1-\bar{x})H(1-\bar{y})\end{array}\right\} \tag{35}$$

$$\bar{D}\left(\frac{\partial^2\psi_{\bar{x}}^M}{\partial\bar{x}^2}+\frac{1-\nu}{2}k\frac{\partial^2\psi_{\bar{x}}^M}{\partial\bar{y}^2}+\frac{1+\nu}{2}k^2\frac{\partial^2\psi_{\bar{y}}^M}{\partial\bar{x}\partial\bar{y}}\right) -K_s\left(\psi_{\bar{x}}^M+\frac{\partial\bar{W}_o^M}{\partial\bar{x}}\right) = \left(1-\mu_1^2\left(\frac{\partial^2}{\partial\bar{x}^2}+k^2\frac{\partial^2}{\partial\bar{y}^2}\right)\right) \left\{\bar{I}_2\frac{\partial^2\psi_{\bar{x}}^M}{\partial\tau^2}-\frac{1}{2}\acute{k}\mu_x\bar{f}\Delta(x-\bar{x})\Delta(y-\bar{y})H(1-\bar{x})H(1-\bar{y})\right\} \tag{36}$$

$$\bar{D}\left(k^2\frac{\partial^2\psi_{\bar{y}}^M}{\partial\bar{y}^2}+\frac{1-\nu}{2}\frac{\partial^2\psi_{\bar{y}}^M}{\partial\bar{y}\bar{x}^2}+\frac{1+\nu}{2}k\frac{\partial^2\psi_{\bar{x}}^M}{\partial\bar{x}\partial\bar{y}}\right) -K_s\left(\psi_{\bar{y}}^M+\frac{\partial\bar{W}_o^M}{\partial\bar{y}}\right) = \left(1-\mu_1^2\left(\frac{\partial^2}{\partial\bar{x}^2}+k^2\frac{\partial^2}{\partial\bar{y}^2}\right)\right) \left\{\bar{I}_2\frac{\partial^2\psi_{\bar{y}}^M}{\partial\tau^2}-\frac{1}{2}\acute{k}\mu_y\bar{f}\Delta(x-\bar{x})\Delta(y-\bar{y})H(1-\bar{x})H(1-\bar{y})\right\} \tag{37}$$

where the dimensionless parameters in Eqs. (33)-(37) are as follows:

$$G=\frac{E}{2*(1+\nu)}, F=G*h$$
$$\bar{A}=\frac{A}{F}, \bar{B}=\frac{B}{F}, \bar{D}=\frac{D}{a^2F}\bar{f}=\frac{M_n g}{Fb} \tag{38}$$

The above system of second-order PDEs in Eqs. (33)-(37) should be solved. In this study, nanoplate is considered with simply supported boundary condition, hence at each end of the nanoplate, the displacements and moments are identical to zero.

3.2 Boundary conditions

While the present analytical solution is developed for simply supported boundary conditions, the proposed model and governing equations can be adapted to other classical boundary conditions—such as clamped, free, or mixed boundaries—by utilizing established solution methods including the Levy method, Rayleigh–Ritz method, and especially finite element analysis for complex geometries. These approaches provide flexibility in addressing a wide range of practical scenarios in the vibration analysis of Mindlin nanoplates.

Nonetheless, we acknowledge that other types of boundary conditions—such as clamped, free, or elastically restrained edges—are commonly encountered in real-world applications. When dealing with plates that have complex boundary conditions, engineers often turn to numerical methods like finite element analysis (FEA) [Eftekhari & Jafari, 2012], differential quadrature [Liu & Liew, 1999], or semi-analytical techniques such as the Rayleigh–Ritz method [Eftekhari & Jafari, 2013]. These approaches prove particularly valuable when traditional analytical solutions become too difficult to derive mathematically. Incorporating such general boundary conditions is beyond the scope of the current work. Future research can build upon this foundation by integrating alternative boundary scenarios to improve the applicability of the model to a wider range of engineering problems.

In this study, the analytical solution is explicitly presented for nanoplates with simply supported boundary conditions, which allows for the use of the Navier solution and eigenfunction expansions. It also simplifies the implementation of solution procedures such as the Galerkin or Fourier series methods. The mathematical expression of the simply supported boundary condition for nonlocal Mindlin plate theory is:

$$\begin{gathered}\bar{U}_0^M(x,0,t)=\bar{U}_0^M(x,b,t)=0\\ \bar{V}_0^M(0,y,t)=\bar{V}_0^M(a,y,t)=0\\ \bar{W}_0^M(0,y,t)=\bar{W}_0^M(a,y,t)=\bar{W}_0^M(x,0,t)=\bar{W}_0^M(x,b,t)=0\\ (N_{xx}^M)^{nl}(0,y,t)=(N_{xx}^M)^{nl}(a,y,t)=\left(N_{yy}^M\right)^{nl}(x,0,t)=\left(N_{yy}^M\right)^{nl}(x,b,t)=0\\ (M_{xx}^M)^{nl}(0,y,t)=(M_{xx}^M)^{nl}(a,y,t)=\left(M_{yy}^M\right)^{nl}(x,0,t)=\left(M_{yy}^M\right)^{nl}(x,b,t)=0\end{gathered} \tag{39}$$

4. Analytical solution

An analytical Method based on Galerkin approach and Eigen function expansion has been used to transform the

Table 1 Material properties of (a)graphene sheet (b)nano plate

(a)	
E_p	$1Tpa$
ν	0.16
ρ	$2250\,{}^{kg}/_{m^3}$
h	$0.34\,nm$
E_p	$1Tpa$
(b)	
$a = 40\ [nm]$	$E_P = 1e12\ [pa]$
$h = 1\ [nm]$	$\nu = 0.2$
$\mu_k = 0.1$	$\rho = 2500\ [kg/m^3]$

dimensionless partial differential equations in Eqs. (33)-(37) to dimensionless ordinary differential equations for simply supported boundary condition. To this aim, the following admissible displacements for Mindlin plate theory based on simply supported boundary condition are considered as below:

$$\bar{U}_0^M(\bar{x},\bar{y},\tau) = \sum_{j=1}^{M}\sum_{i=1}^{N} \bar{U}_{ij}^M(\tau)\cos(i.\pi.\bar{x})\sin(j.\pi.\bar{y}) \quad (40)$$

$$\bar{V}_0^M(\bar{x},\bar{y},\tau) = \sum_{j=1}^{M}\sum_{i=1}^{N} \bar{V}_{ij}^M(\tau)\sin(i.\pi.\bar{x})\cos(j.\pi.\bar{y}) \quad (41)$$

$$\bar{W}_0^M(\bar{x},\bar{y},\tau) = \sum_{j=1}^{N}\sum_{i=1}^{N} \bar{W}_{ij}^M \sin(\tau)(i.\pi.\bar{x})\sin(j.\pi.\bar{y}) \quad (42)$$

$$\bar{\psi}_{\bar{x}}^M(\bar{x},\bar{y},\tau) = \sum_{j=1}^{M}\sum_{i=1}^{N} \bar{\psi}_{\bar{x}ij}^M(\tau)\cos(i.\pi.\bar{x})\sin(j.\pi.\bar{y}) \quad (43)$$

$$\bar{\psi}_{\bar{y}}^M(\bar{x},\bar{y},\tau) = \sum_{j=1}^{M}\sum_{i=1}^{N} \bar{\psi}_{\bar{y}ij}^M(\tau)\sin(i.\pi.\bar{x})\cos(j.\pi.\bar{y}) \quad (44)$$

Substituting Eqs. (40)-(44) into Eqs. (33)-(37) leads to the following set of dimensionless ordinary differential equations:

$$\begin{bmatrix} \bar{m}_1 & 0 & 0 & 0 & 0 \\ 0 & \bar{m}_2 & 0 & 0 & 0 \\ 0 & 0 & \bar{m}_3 & 0 & 0 \\ 0 & 0 & 0 & \bar{m}_4 & 0 \\ 0 & 0 & 0 & 0 & \bar{m}_5 \end{bmatrix} \begin{bmatrix} \ddot{\bar{U}}_{ij}^M(\tau) \\ \ddot{\bar{V}}_{ij}^M(\tau) \\ \ddot{\bar{W}}_{ij}^M(\tau) \\ \ddot{\bar{\psi}}_{xij}^M(\tau) \\ \ddot{\bar{\psi}}_{yij}^M(\tau) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{C}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\bar{U}}_{ij}^M(\tau) \\ \dot{\bar{V}}_{ij}^M(\tau) \\ \dot{\bar{W}}_{ij}^M(\tau) \\ \dot{\bar{\psi}}_{xij}^M(\tau) \\ \dot{\bar{\psi}}_{yij}^M(\tau) \end{bmatrix} + \begin{bmatrix} \bar{k}_1 & \bar{k}_2 & 0 & 0 & 0 \\ \bar{k}_3 & \bar{k}_4 & 0 & 0 & 0 \\ 0 & 0 & \bar{k}_5 & \bar{k}_6 & \bar{k}_7 \\ 0 & 0 & \bar{k}_8 & \bar{k}_9 & \bar{k}_{10} \\ 0 & 0 & \bar{k}_{11} & \bar{k}_{12} & \bar{k}_{13} \end{bmatrix} = \begin{bmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \\ \bar{f}_4 \\ \bar{f}_5 \end{bmatrix} \quad (45)$$

Where, the coefficients are:

$$\bar{k}_1 = \bar{A}\left(i^2\pi^2 + \frac{1-\nu}{2}k^2j^2\pi^2\right), \bar{k}_2 = \bar{A}\left(\frac{1+\nu}{2}kij\pi^2\right)$$
$$\bar{k}_3 = \bar{A}\left(\frac{1+\nu}{2}kij\pi^2\right), \bar{k}_4 = \bar{A}\left(k^2j^2\pi^2 + \frac{1-\nu}{2}i^2\pi^2\right)$$
$$\bar{k}_5 = K_s(i^2\pi^2) + K_s(k^2j^2\pi^2) + \acute{\Delta}\frac{k_w a^2}{F}, \bar{k}_6 = K_s(i\pi)$$
$$\bar{k}_7 = K_s(kj\pi), \bar{k}_8 = K_s(i\pi)$$
$$\bar{k}_9 = K_s + \bar{D}\left(i^2\pi^2 + \frac{1-\nu}{2}k^2j^2\pi^2\right)$$
$$\bar{k}_{10} = \bar{D}\left(\frac{1+\nu}{2}kij\pi^2\right), \bar{k}_{11} = K_s(j\pi) \quad (46)$$
$$\bar{k}_{12} = \bar{D}\left(\frac{1+\nu}{2}kij\pi^2\right)$$
$$\bar{k}_{13} = K_s + \bar{D}\left(k^2j^2\pi^2 + \frac{1-\nu}{2}i^2\pi^2\right)$$
$$\bar{m}_1 = \acute{\Delta}, \bar{m}_2 = \acute{\Delta}$$
$$\bar{m}_3 = 4\acute{\Delta}\frac{M_n}{I_0}\frac{1}{ab}\sin(i\pi\bar{x}_m)^2\sin(j\pi\bar{y}_m)^2$$
$$\bar{m}_4 = \bar{I}_2\acute{\Delta}, \bar{m}_5 = \bar{I}_2\acute{\Delta}$$
$$\bar{C}_1 = \acute{\Delta}\frac{C_p a}{F\sqrt{\frac{I_0}{F}}}$$

and

$$\bar{f}_1 = 4\acute{\Delta}\mu_x\bar{f}\cos(i\pi\bar{x}_m)\sin(j\pi\bar{y}_m)$$
$$\bar{f}_2 = 4\acute{\Delta}\mu_y\bar{f}\sin(i\pi\bar{x}_m)\cos(j\pi\bar{y}_m)$$
$$\bar{f}_3 = 4\acute{\Delta}\bar{f}\sin(i\pi\bar{x}_m)\sin(j\pi\bar{y}_m) \quad (47)$$
$$\bar{f}_4 = \frac{1}{2}\acute{\Delta}\acute{k}\mu_x\bar{f}\cos(i\pi\bar{x}_m)\sin(j\pi\bar{y}_m)$$
$$\bar{f}_5 = \frac{1}{2}\acute{\Delta}\acute{k}\mu_y\bar{f}\sin(i\pi\bar{x}_m)\cos(j\pi\bar{y}_m)$$

5. Verifying results

In this section, to verify the accuracy of the obtained results from the present study, a comparative analysis is performed in two cases: (i) verification of natural frequencies at the nanoscale and (ii) dynamic amplitude factor in a macro-scale plate under a moving load.

(i) verifying the Natural frequencies in nanoscale

Consider a square single-layer graphene nano plate with the following data in Table 1. The predicted values of natural frequencies by the proposed model by (Ansari *et al*. 2010) have been provided in Table 2. As shown in Table 3,

Table 2 Geometry and mechanical properties of the plate

$E_p = 30e6\ [psi]$	$a = 4\ [in]$
$\rho = 10e-3\ {lb\ s^2}/{in^4}$	$b = 4\ [in]$
$\nu = 0.3$	$h = 0.1\ [in]$

Table 3 Natural frequencies of simply-supported single layered graphene sheet

Side length of square SLGS (nm)	Natural frequencies [THz], (Ansari *et al.* 2010) $(l_1 = 1.41[nm])$	Natural frequencies [THz] (Present study)
10	0.0584221	0.0581
15	0.0282888	0.0276
20	0.0164593	0.0159
25	0.0107085	0.0103
30	0.0075049	0.0072
35	0.0055447	0.0053
40	0.0042608	0.0041
45	0.0033751	0.0032
50	0.0027388	0.0026

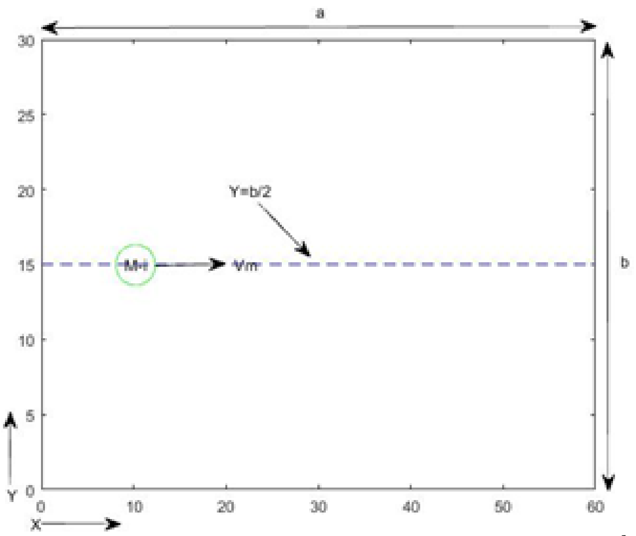


Fig. 2 The view of the load path moving on the line $y = \frac{b}{2}$

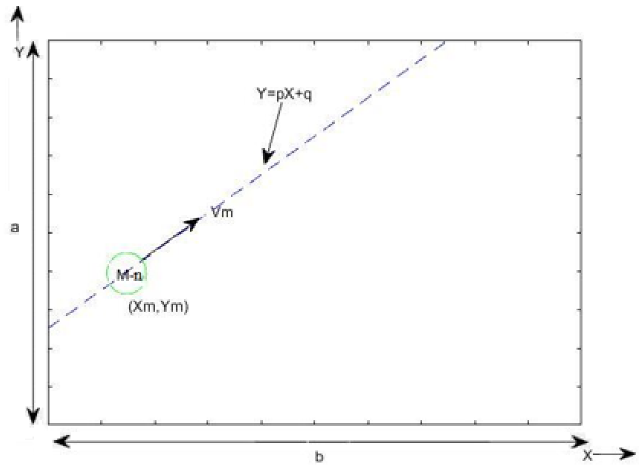
this model is justified by a good agreement between the result given by the present study and available data in the literature.

(ii) Dynamic amplitude factor in a macro-scale plate under a moving load $(l_1 = 0)$

Consider a square plate under a moving load subjected the plate on the line $y = \frac{b}{2}$ (see Fig. 2) with a constant velocity, with the following geometry and mechanical properties in Table 2.

As it is seen in Table 4, the obtained values of the proposed model in this study with the other researches have been provided. As it is shown in Table 4, there is a good agreement between the obtained results and those of other researchers for different levels of the moving load velocity.

It is worth noting that the discrepancy between the present results and the reference values becomes more



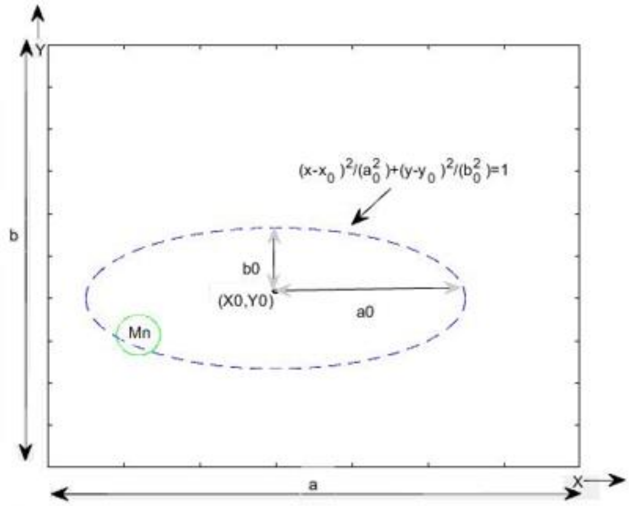


Fig. 3 The view of load path moving on (a) a straight line (b) an elliptical path

noticeable in the last two rows of Table 4. This can be attributed to the increased influence of nonlocal effects and small-scale parameters in these specific cases, which typically correspond to higher vibration modes or more sensitive structural configurations. In such conditions, small differences in modeling assumptions, numerical techniques, or boundary condition implementations can lead to larger deviations.

6. Numerical results and discussion

In this section, the dimensionless ordinary differential equations presented in Eq. (46) are solved in MATLAB and analyzed to investigate the effects of various parameters, including the nonlocal parameter, the spring stiffness and damping coefficient of the foundation, the inertia effect of moving nanoparticles, and the velocity of the moving nanoparticle.

In all presented numerical results, the effects of friction, inertia, and the viscoelastic foundation have been considered.

The result was obtained in two different cases:

Nanoparticle moves along a straight line $y = px + q$, with constant velocity V_m (Fig. 3a):

Table 4 Comparison between the obtained values for DAF_w in this study and the other research for a thin isotropic square plate under a moving load with different levels of velocities

τ_f / T^*	$V[in/s]$	DAF_w Taheri and Ting (1990)	DAF_w (Malekzadeh *et al*. 2009)	DAF_w Taheri and Ting (1989)	DAF_w (Present study-Mindlin plate theory)
$1/8$	515	1.042	1.050	1.022	1.0212
$1/4$	1030	1.088	1.096	1.032	1.0367
$1/2$	2060	1.200	1.200	1.234	1.2741
1	4120	1.568	1.572	1.525	1.7216
2	8240	1.390	1.392	1.359	1.5732

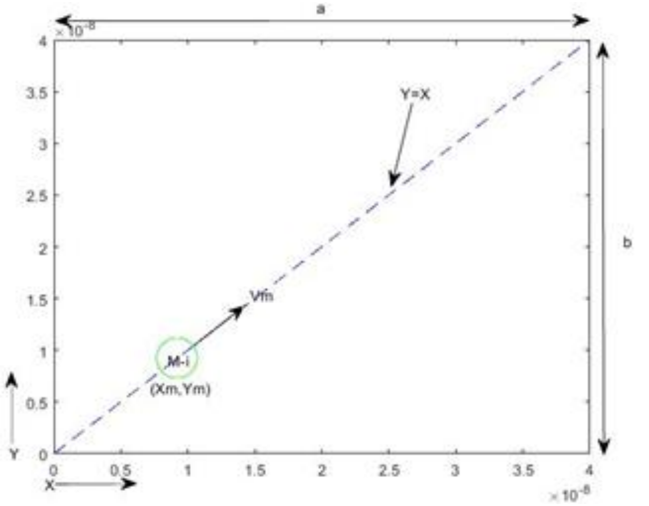


Fig. 4 The view of the nanoparticle path moving on the line $x = y$

Dimensionless location of moving nanoparticle is:

$$\bar{x}_m = \frac{V_m\sqrt{\rho}}{\sqrt{G(1+p^2)}}\tau$$
$$\bar{y}_m = \frac{pV_m k\sqrt{\rho}}{\sqrt{G(1+p^2)}}\tau + \frac{q}{b} \tag{48}$$

Nanoparticle moves along an elliptical path $\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$, with constant angular velocity ω (Fig. 3b):

Which (x_0, y_o) is the center coordinate and $(2a, 2b)$ are the minor and major diameters of the elliptical path.

Dimensionless location of moving nanoparticle is:

$$\bar{x}_m = \bar{x}_0 - \bar{a}\cos(\varpi\tau)$$
$$\bar{y}_m = \bar{y}_0 + \bar{a}\sin(\varpi\tau) \tag{49}$$

where,

$$\bar{a} = \frac{a_0}{a}, \qquad \bar{b} = \frac{b_0}{b},$$
$$\bar{x}_0 = \frac{x_0}{a}, \qquad \bar{y}_0 = \frac{y_0}{b}, \qquad \varpi = a\omega\sqrt{\frac{I_0}{F}} \tag{50}$$

Mechanical properties of nano plate are presented in Table 1.

For convenience, the following dimensionless parameters are used:

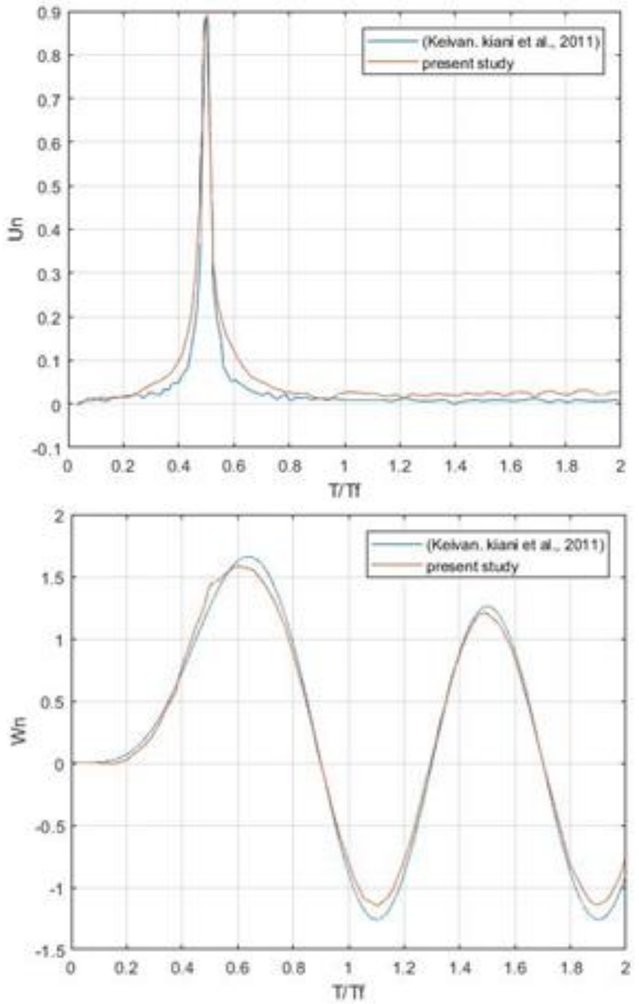


Fig. 5 Comparison between the obtained result of time history normalized displacement of the nano plate center in this study and research that has been done by Kiani (2011b)

$$U_N = \bar{U}_0/\bar{U}_{st}$$
$$W_N = \bar{W}_0/\bar{W}_{st} \tag{51}$$

where, $\bar{U}_{st}$ and $\bar{W}_{st}$ are the dimensionless components of the static displacement that analytic expressions of them have been presented in Kiani (2011b).

Case (i): Nanoparticle moves along a straight line $= px + q$, with constant velocity V_m

The normalized Velocity and dynamic amplitude factors are expressed as:

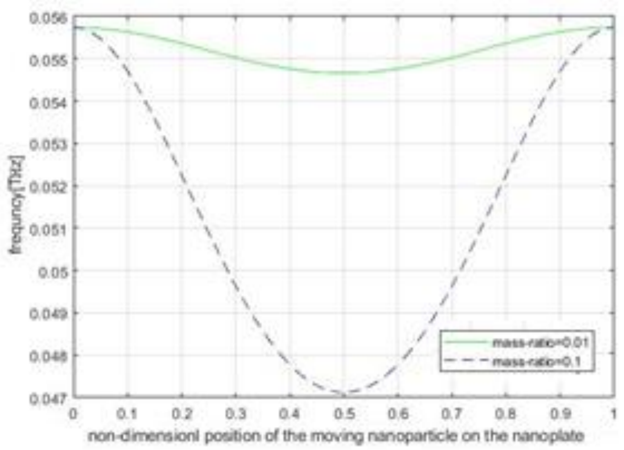


Fig. 6 Resonant frequency of nano plate vs non-dimensional position of the moving nanoparticle for different mass ratio ε

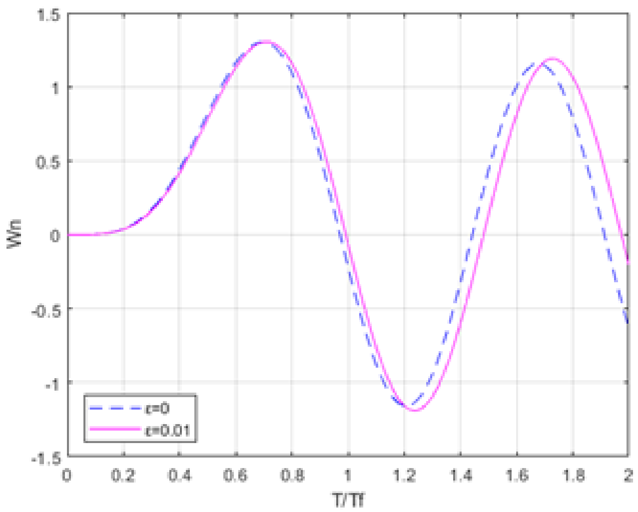


Fig. 7 Analysis inertia effect of moving nanoparticle ($\boldsymbol{\varepsilon = 0}$) and the corresponding moving load ($\boldsymbol{\varepsilon = 0.01}$) on the Normalized time history out-of-plane displacement of the nano plate center $\boldsymbol{W_N}$

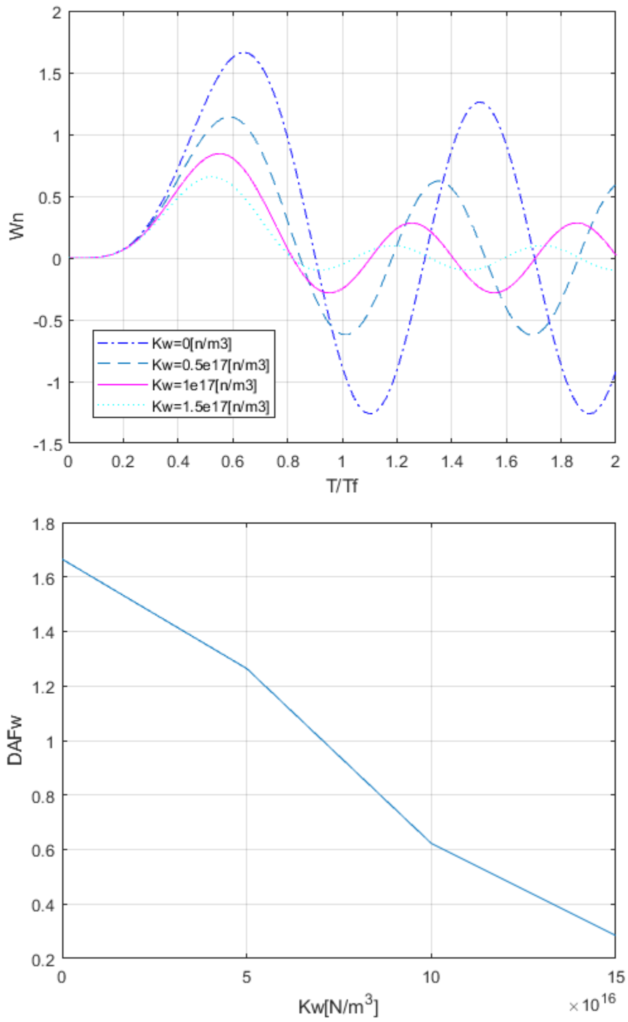


Fig. 8 Effect of the spring stiffness of viscoelastic foundation on the Normalized time history out-of-plane displacement of the nano plate center $\boldsymbol{W_N}$ and dynamic amplitude factor of the $\boldsymbol{W_N (DAF_W)}$ for different values of the spring stiffness coefficient ($\boldsymbol{K_w = 0}$, $\boldsymbol{K_w = 0.5e17\ [n/m^3]}$, $\boldsymbol{K_w = 1e17 [n/m^3]}$, $\boldsymbol{K_w = 1.5e17 [n/m^3]}$)

$$V_N = v/\acute{V}$$
$$DAF_W = \bar{W}_{0,max}/\bar{W}_{st}(0.5,0.5) \tag{52}$$

where $\acute{V}$ denotes the critical velocity of the moving nanoparticle is obtained as follows:

$$\acute{V} = \sqrt{\frac{2\pi^2 \acute{k}^2 E}{3(1-\nu^2)\rho(1+2(\pi\mu_1)^2)}} \tag{53}$$

In-plane and out-of-plane displacements of a nanoplate subjected to a moving nanoparticle are illustrated and compared with Kiani (2011a) in Fig. 5

Nanoparticle moves on a straight line with the equation $x = y$ and starts from the point (0,0).

It should be noted that the in-plane displacement in directions x and y (U_n, V_n) are the same because of the symmetry of the nanoplate geometry and the path of moving nanoparticle.

Fig. 6 illustrates the effect of moving nanoparticle inertia on the resonant frequency of the nanoplate for different mass ratios$\left(\varepsilon = \frac{M_n}{\rho hab}\right)$. It is evident that the frequency decreases, and the effect of nanoparticle inertia becomes more significant when the resonant frequency reaches its peak value at the center of the nanoplate. The out-of-plane deflection profiles for both the moving load (ε=0) and the moving nanoparticle (ε=0.01) are shown in Fig. 7. Although the out-of-plane deflection profiles for both cases are similar but the deflection profiles show a higher peak in response for the case of moving mass also the peak for the moving load is seen to be slightly shifted away from that for the moving mass.

Through Figs. 8-9, normalized time history of out-of-plane displacement of the nanoplate center W_N and dynamic amplitude factor of $W_N, (DAF_W)$ are plotted for different values of damping coefficient and spring stiffness. For a

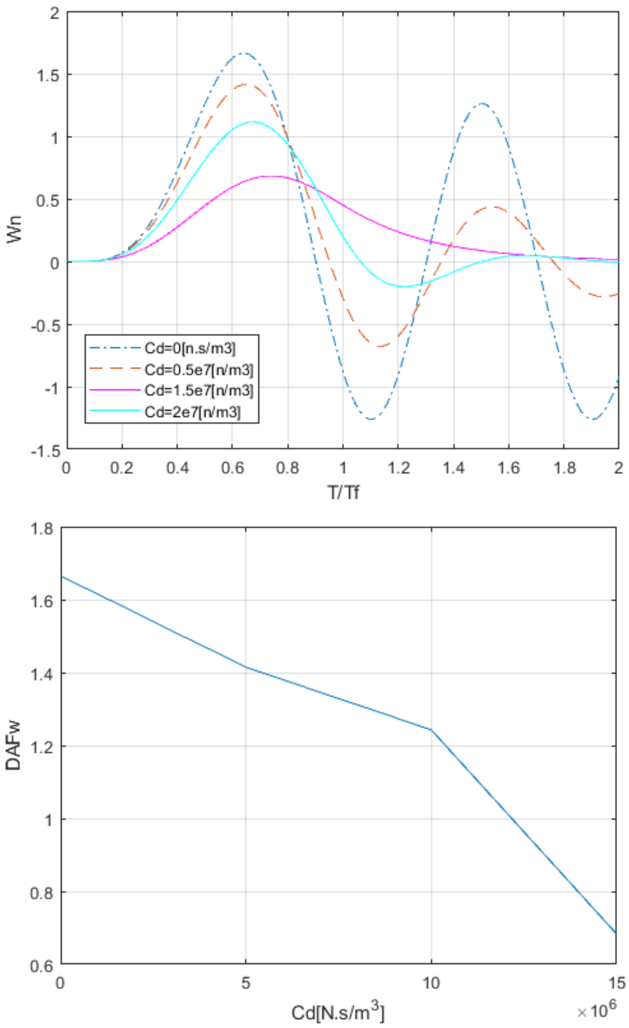


Fig. 9 Effect of the damping coefficient of viscoelastic foundation on the Normalized time history out-of-plane displacement of the nano plate center $\boldsymbol{W_N}$ and dynamic amplitude factor of the $\boldsymbol{W_N(DAF_W)}$ for different values of the damping coefficient ($\boldsymbol{C_d = 0}$, $\boldsymbol{C_d = 0.5e7[n.s/m^3]}$, $\boldsymbol{C_d = 1.5e7[n.s/m^3]}$, $\boldsymbol{C_d = 2e17[n.s/m^3]}$)

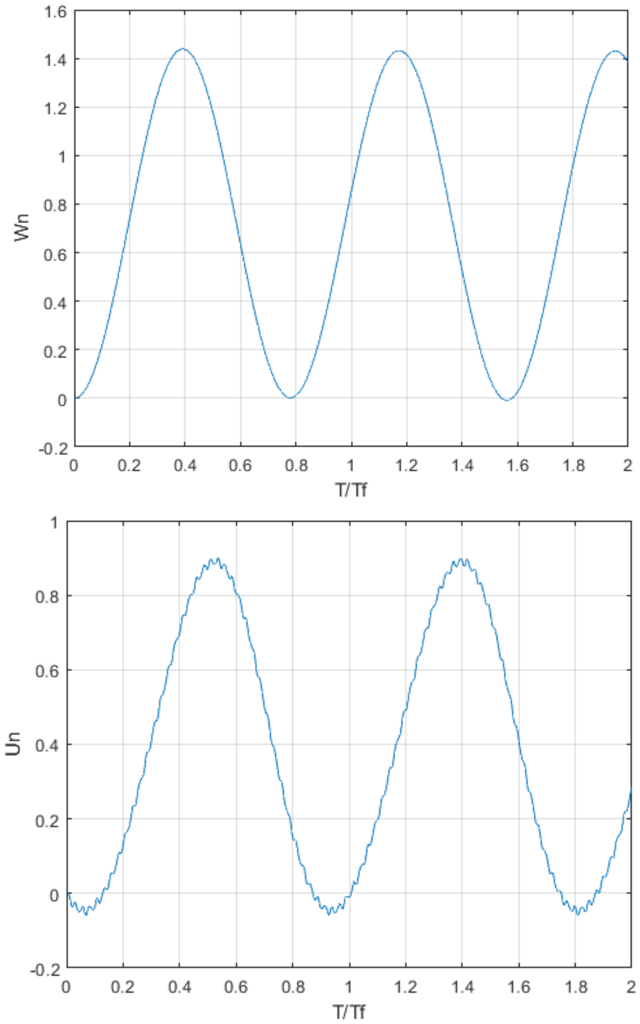


Fig. 10 Normalized time history in-plane and out-of-plane displacements of the nano plate center ($\boldsymbol{U_N, W_N}$) when the moving nanoparticle moves on an ellipse path

constant damping coefficient as the parameter of spring stiffness increases, the system stiffness increases, DAF_W decrease and frequencies increase. However, for a constant spring stiffness DAF_W and frequencies decrease as the damping coefficient decreases and it is more significant in larger values of spring stiffness.

Case (ii): Moving nanoparticle moves along an elliptical path $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$, with constant angular velocity ω:

In-plane and out of plane displacements of nanoplate subjected to moving nanoparticle that moves on an elliptical path with the Eq. $\frac{(x-x_0)^2}{a_0^2}+\frac{(y-y_0)^2}{b_0^2}=1$ and starts from the point $\left(x=\frac{a}{4}, y=\frac{b}{4}\right)$ with constant angular velocity $\omega_n = 0.1$ are plotted in Fig. 10.

In which

$$(x_0, y_0) = \left(\frac{a}{2}, \frac{b}{2}\right), (a_0, b_0) = \left(\frac{a}{4}, \frac{b}{4}\right) \tag{54}$$

And,

$$\omega_n = \frac{\omega}{\omega_{cr}^k}$$

$$\omega_{cr}^k = 2\pi^2 \acute{k}^2 (1+k^2) h\sqrt{3\rho_p(1-\nu^2)E/\left(1+\pi^2\mu_1^2(1+k^2)\right)} \tag{55}$$

Finally, the effect of moving two nanoparticles has been studied in Fig. 11. As seen in the deflection profile of U_N and W_N plotted for two masses with different normalized velocities separately on the side of the plot of a nanoplate traversed by two moving nanoparticles.

7. Conclusions

This study presents a comprehensive analytical and numerical investigation of the dynamic behavior of micro/ nano rectangular plates subjected to moving nanoparticles, incorporating three critical factors: nanoparticle inertia, Coulomb friction, and viscoelastic foundation effects. Using Eringen's nonlocal elasticity theory combined with Mindlin's

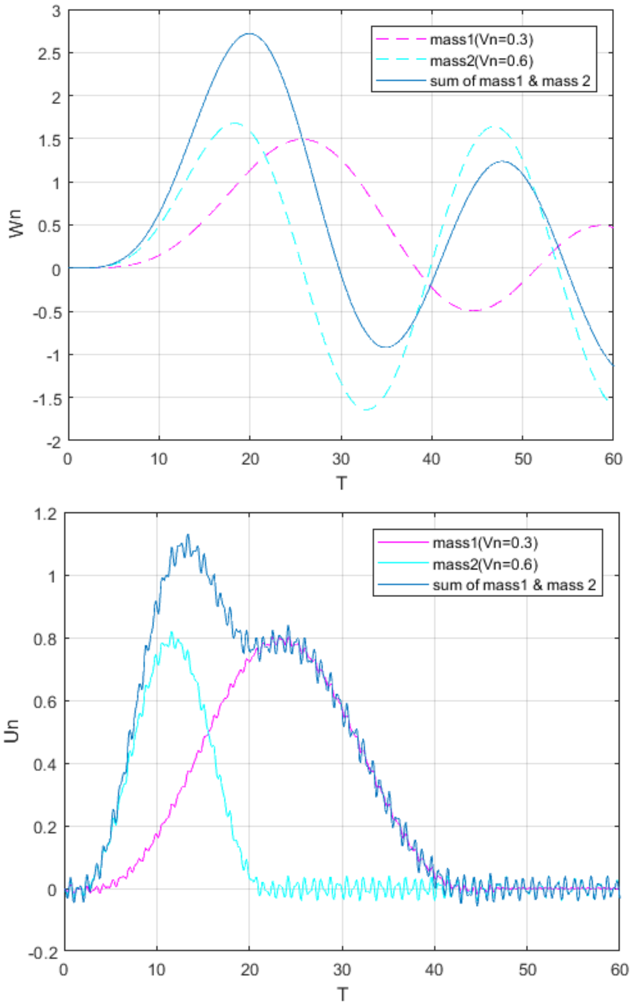


Fig. 11 Normalized time history in-plane and out-of-plane displacements of the nanoplate center ($\boldsymbol{U_N}, \boldsymbol{W_N}$), considering two masses with different velocities: (1) $\boldsymbol{V_n = 0.3}$, (2) $\boldsymbol{V_n = 0.6}$

first-order shear deformation model and Hamilton's principle, a robust framework was developed to capture small-scale effects while remaining applicable to microscale systems. The governing equations were solved analytically via the Galerkin method and numerically in MATLAB under simply supported boundary conditions, considering both linear and elliptical nanoparticle trajectories. Results show that increasing the nonlocal parameter leads to a significant reduction in displacement amplitudes—up to 12% in transverse motion for a 20% rise in the scale parameter—while nanoparticle inertia notably amplifies responses near resonance, especially at higher velocities (e.g., above 0.3), causing more than 15% deviation from static conditions. The viscoelastic foundation enhances dynamic stability by reducing deflection through its stiffness component and suppressing vibration via damping. Furthermore, Coulomb friction produces appreciable in-plane oscillations, particularly under low stiffness conditions, emphasizing the role of particle-surface interactions. Comparing straight and elliptical paths revealed that curved trajectories induce more complex mode shapes, relevant to targeted nanoscale transport systems. This work offers novel contributions, including: (1) the first detailed study of multiple interacting nanoparticles on nanoplates, (2) path-dependent dynamic characterization, and (3) quantitative evaluation of how foundation parameters affect vibration mitigation. These findings provide valuable insight for optimizing NEMS, flexible nano-structures, and advanced functional materials. Future work may include nonlinear extensions, thermal-coupled effects, multi-layered graphene systems, and experimental validation to bridge theory and practical implementation in nanoengineering.

This study provides several key innovations, including the development of an analytical framework that, for the first time, simultaneously incorporates both inertia and frictional effects of moving nanoparticles on nonlocal Mindlin nanoplates, as well as the integration of nonlocal elasticity theory and viscoelastic foundation modeling to more accurately capture the coupled dynamic behavior. In addition, a thorough quantitative parametric analysis is presented, clarifying the individual and interactive effects of nonlocal parameters, foundation properties, and nanoparticle dynamics, thereby extending the theoretical basis for vibration control and design in advanced nano-engineered devices.

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