Advances in Nano Research, *Vol. 18, No. 6 (2025) 529-535*
https://doi.org/10.12989/anr.2025.18.6.529

Multidirectional understanding of chiral structure on the honeycomb sandwich composite sheet: Control sensing of density

Mohamed Amine Khadimallah*[1], Djamel Ouis[2,3], Rahul Saini[4] and Elimam Ali[1]

[1]Department of Civil Engineering, College of Engineering in Al-Kharj, Prince Sattam Bin Abdulaziz University, Al-Kharj, 11942, Saudi Arabia
[2]Architectural Engineering and Construction Management Department, King Fahd University of Petroleum and Minerals,
Dhahran 31261, Saudi Arabia
[3]Interdisciplinary Research Center for Construction and Building Materials, King Fahd University of Petroleum, Dhahran 31261, Saudi Arabia
[4]Department of Mathematics (Applied Sciences), School of Engineering and Technology, Hemvati Nandan,
Bahuguna Garhwal University (A Central University), Srinagar 246174, Uttarakhand, India

(*Received December 31, 2024, Revised March 15, 2025, Accepted March 17, 2025*)

Abstract. This study used a modified orthotropic elastic shell model to study the vibration of chiral single-walled carbon nanotubes. Budiansky and Sanders (1963) are the source of the stress and strain equations. Both the impact of height-to-diameter ratios and boundary conditions are included in this model. The governing equations are expressed in eigenform using the complex approach. The fundamental frequencies of SWCNTs are obtained by solving this eigenform using MATLAB software. The impact of density and boundary conditions on frequency behavior is examined. For chiral nanotubes, the frequency pattern with two boundary conditions seems to be parallel. As density increased, the frequencies dropped. Frequencies will be higher with a higher index. Compared to the equivalent C-F situation, the C-C frequencies are higher. There is a significant frequency shift between the C-C and C-F boundary condition curves for shorter tubes and shorter chiral indices. This frequency study is anticipated by the author for high frequencies in intriguing electromagnetic devices.

Keywords: boundary conditions; chiral SWCNTs; computer software; density variation; nano-sized structures; orthotropic model

1. Introduction

CNTs are effective for moving fluids and gases due to their cylindrical mechanism and exceptional mechanical properties. The main tool for continuum mechanics (Qi *et al*. 2024, Zhang *et al*. 2023) is its computational capacity to generate results of large range systems. Various models provide significant computing efficiency and accuracy, making them a viable choice for further research in vibration field (Yang *et al*. 2025a, Wang *et al*. 2023). Kroto *et al*. (1985) and associates discovered Buckminster Fullerene, or C60, a novel kind of carbon molecule. Ru (2000) investigated the bending of carbon nanotubes using a multilinear model. The mechanical behavior of different materials is explored by Xu *et al*. (2023) and Yang *et al*. 2025b. Wang *et al*. (2005) investigated the limitations and uses of the multiwalled carbon nanotubes.

Wang (2005) investigated the vibration of carbon nanotubes using the shell model. The bending of multi-walled carbon nanotubes is described using a multi-elastic shell model. Experimental and numerical studies are observed by Zhang *et al*. (2024) and Yu *et al*. (2022). Reddy (2007) updated the nonlocal relation beam ideas found in the body of current literature. A simpler Flugge shell for the vibration of double-walled carbon nanotubes was presented by Natsuki *et al*. (2008). These tubes are completely submerged in liquid. The interaction of van der Waals forces as two exclusive beams was clearly demonstrated by the free vibrations of double-walled carbon nanotubes (Xu *et al*. 2008). The nested CNT tubes with double walls were modeled by Xu *et al*. (2008) as individual elastic beams. According to their research, double-walled CNT remained unchanged under specific edge conditions at a given invariable frequency. Ke *et al*. (2009) examined the free nonlinear vibrations of double-walled carbon nanotubes (CNTs) using nonlocal Timoshenko beam theory, and they used differential quadrature approach to derive frequency equations. For multi-walled carbon nanotubes with nonlinear geometric and physical consequences for nonlinear vibrations of tubes, Adali (2009) suggested the principle of variation. Shima (2011) and Wang *et al*. (2006) demonstrated carbon nanotube buckling. The investigation's primary focus was on small-scale diameters and aspect ratios, which demonstrated the applicability of Donnell shell theory for vibration analysis. The buckling analysis of carbon nanotubes was studied by Chandrasekhar *et al*. (2009) and Hollerer and Celigoj (2013) using an atomistic approach and the Cosserat rod model. The continuum elastic energy is parameterized and produced. Liu *et al*. (2025) and Ni *et al*. (2024) investigated shear effect of composite plates and interlayer ofCu nanoparticles. Hu *et al*. (2012) and Budarapu *et al*. (2014) taking into account the continuum bars for carbon nanotube vibrations. The frequency vibrations of chiral double-walled CNT were

---

*Corresponding author, Ph.D., Assistant Professor,
E-mail: am.khadimallah@gmail.com;
mohamedamine.khadimallah@fsgf.rnu.tn

http://www.techno-press.org/?journal=journal=anr&subpage=5
ISSN: 2287-237X (Print), 2287-2388 (Online)

demonstrated by Chemi *et al*. (2015). Nonlocal Euler Bernoulli beam theory was utilized to model the set of governing equations. Peng *et al*. (2025) and Gao *et al*. (2023) utitllized the sequence table for composite laminated plate and inspiration for bionic bimanual robot teleoperation. Bouadi *et al*. (2018) developed a new model displacement field for the nonlocal buckling properties of a single graphene sheet. The Eringen relation was used for the theoretical formation with the length scale parameter. Li *et al*. (2024) and Cong *et al*. (2024) investigated shear wave ultrasonography and composite cylindrical shells. Fatahi-Vajari *et al*. (2019) investigated the torsional vibration of SWCNTs using second order PDEs. These equations are reduced by GT, and the torsional frequency equation is then generated using HPM. Malikan and Eremeyev (2020) predicted the buckling analysis of CNTs using the Winkler matrix with different boundary conditions. The Hamilton's notion was used to measure the carbon nanotubes' hardness and softness-stiffness. Alves *et al*. (2023) and Qiu *et al*. (2024) investigated the control of soft robotic hands and diesel engine damper design.

Pourasghar *et al*. (2021) demonstrated the transient heat conduction and vibration of SWCNTs based on Eringen's and heat conduction theories. The energy scale is found using the nonlocal term with heat conduction theory. Zhang *et al*. (2023) and Su *et al*. (2023) investigated heterogeneous random porous materials and concealable grippers. Wang *et al*. (2023, 2025) investigated the plastic deformation of calcareous sand and the wrinkling monitoring of metal tubes using nonlocal elasticity theory. Simsek (2010) used nonlocal EBM with supported edge situations to determine the forced vibration of SWCNTs. For effects, aspect ratio and velocity are carefully calculated. Moreover, the excitation frequency and load velocity have a significant impact on the nanostructures.

Madihav *et al*. (2011) developed the model that was utilized in conjunction with the Timoshenko and Euler Bernoulli beam theories using a harmonic balanced approach. They introduced the two-way vibrational behavior and the Timoshenko beam model of single-walled carbon nanotubes. Additionally, they investigated how van der Waal forces affected the frequency spectra of medium and nearby double-walled carbon nanotubes.

Both Zhang *et al*. (2025) and Zou *et al*. (2024) investigate the dispersion of Lamb waves and dual-loop control of QUAV trajectory. Lei *et al*. (2012) took into consideration surface effects in conducting vibration of double-walled carbon nanotubes using the Timoshenko beam model and nonlocal theory. The significance of surface effects and nonlocality, with the largest modes of vibration in shorter tubes is uttilized. In 2022, Ghasemi and Gouran assessed the vibration of SWCNTs lying on the Pasternak foundation and filled with fluid. For pipe flow, He *et al*. (2024) and Cai *et al*. (2025) employed particle-based absorber-roller systems and Lagrangian particle models.

Different researcher used different techniques for material investigations (Samaniego *et al*. 2020, Zhuang *et al*. 2021, Penna 2023, Civalek *et al*. 2023, Barretta *et al*. 2022, Civalek and Baltacıoğlu 2018, Benvenuti and Simone 2013, Safaei *et al*. 2019, Benmansour *et al*. 2019, Akbaş 2020, Forsat *et al*. 2021, Luo *et al*. 2022, Moradi*et al*. 2023, Hussain 2024, Hussain *et al*. 2017, Hussain 2020, 2022, Alghaffari *et al*. 2023, Qazaq *et al*. 2022, Ghandourah *et al*. 2023, 2024, Taj *et al*. 2023, Hussain *et al*. 2023, Khedher *et al*. 2023, 2024, Hussain *et al*. 2024).

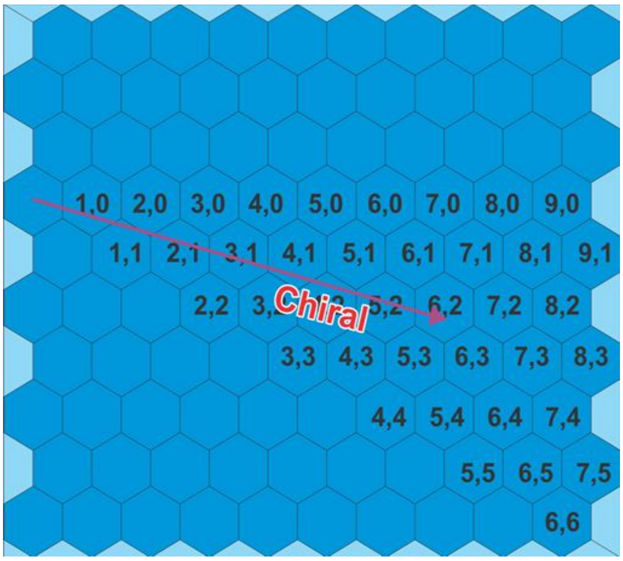

Fig 1 The graphene sheet showing chiral SWCNTs schematically

In order to determine the natural frequencies against density under clamped-clamped (C-C) and clamped-free boundary conditions, the current study is used for vibration of chiral SWCNTs. MD simulations demonstrated that the parameters is appropriately selected, and the findings of this model are in good agreement with those of the Timoshenko beam model and beam element. MATLAB coding has been used to obtain the frequency equation's eigen solutions. The current thorough findings might also be useful in designing the basic frequencies of nanostructures.

2. Mathematical formulation

Because of different radii in different structure, two types of carbon nanotube configurations are found: single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs). A graphene sheet is considered SWCNTs once it has been rolled once to form a cylinder with an end cape. Fig. 1 shows the graphene sheet with a schematic representation of chiral SWCNTs. In this case, the axial coordinate is x, and the angular coordinate is θ. Axial displacement is denoted by u, circumferential displacement by v, and inward deflection by w. The density is denoted by ρ. In the longitudinal direction, ν_x is the Poisson ratio, and in the circumferential direction, ν_θ. R is the radius of the carbon nanotubes, and h is the tube's thickness.

The longitudinal modulus of elasticity is denoted by ε_x, while the circumferential modulus is represented by ε_θ. Similarly ν_x is poison's ratio in longitudinal direction and ν_θ s in circumferential direction.

2.1 Strain-displacement relationships

The strain-displacement relations from Budiansky and Sanders (1963) theory are furnished as

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\frac{\partial v}{R\partial\theta} - \varepsilon_\theta = \frac{w}{R} \tag{1}$$

$$\gamma_{x\theta} - \frac{\partial u}{R\partial\theta} = \frac{\partial v}{\partial x}$$

The stress and strain relationship is given as

$$\frac{(1 - v_x v_\theta)}{(\varepsilon_x + v_x\varepsilon_\theta)} = \frac{E_x}{\sigma_x}$$

$$\frac{(1 - v_x v_\theta)}{(\varepsilon_\theta + v_\theta\varepsilon_x)} = \frac{E_\theta}{\sigma_\theta} \tag{2}$$

$$\tau_{x\theta} = \gamma_{x\theta} G_{x\theta}$$

The axial and circumferential prestresses are

$$N_\theta = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\theta dz$$

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz \tag{3}$$

$$N_{x\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{x\theta} dz.$$

Furthermore longitudinal and circumferential Poisson ratios and Young's moduli are denoted by v_x , v_θ and E_x , E_θ respectively. The four parameters E_x, v_x, α, and β can be used to characterize the orthotropic elastic shell model. Additionally, D_x, D_θ, and $D_{x\theta}$, respectively, reflect effective bending stiffness (Pablo *et al.* 2003, Sirenko *et al.* 1996). The mathematical formulation can be seen (Taj and Zhang 2011). The three-equation orthotropic elastic shell model that was created (Eslami and Javaheri 1999) describes the forces that cause the vibration of chiral SWCNTs.

$$\begin{bmatrix} (K_x + N_x)R^2 \frac{\partial^2}{\partial x^2} + 2RN_{x\theta}\frac{\partial^2}{\partial x\partial\theta} \\ +(\frac{K_{x\theta}R^2 + D_{x\theta}}{R^2} + N_\theta)\frac{\partial^2}{\partial\theta^2} \end{bmatrix} u + \left[R(v_x K_\theta + K_{x\theta})\frac{\partial^2}{\partial x\partial\theta} \right] v + \begin{bmatrix} -R(v_\theta K_x - N_\theta)\frac{\partial}{\partial x} + \\ RD_x\frac{\partial^3}{\partial x^3} - \frac{D_{x\theta}}{R}\frac{\partial^3}{\partial x\partial\theta^2} \end{bmatrix} w = 0 \tag{4}$$

$$\left[R(v_\theta K_x + K_{x\theta})\frac{\partial^2}{\partial x\partial\theta} \right] u + \begin{bmatrix} (K_\theta + N_\theta)\frac{\partial^2}{\partial\theta^2} + 2RN_{x\theta}\frac{\partial^2}{\partial x\partial\theta} \\ +(\frac{K_{x\theta}R^2 + 3D_{x\theta}}{R^2} + N_x)R^2\frac{\partial^2}{\partial x^2} \end{bmatrix} v + \begin{bmatrix} -(K_\theta + N_\theta)\frac{\partial}{\partial\theta} - 2RN_{x\theta}\frac{\partial}{\partial x} + \\ (v_\theta D_x + 3D_{x\theta})\frac{\partial^3}{\partial x^2\partial\theta} \end{bmatrix} w = 0 \tag{5}$$

$$\left[R(v_\theta K_x - N_\theta)\frac{\partial}{\partial x} - RD_x\frac{\partial^3}{\partial x^3} + \frac{D_{x\theta}}{R}\frac{\partial^3}{\partial x\partial\theta^2} \right] u + \begin{bmatrix} (K_\theta + N_\theta)\frac{\partial}{\partial\theta} + 2RN_{x\theta}\frac{\partial}{\partial x} \\ -(v_\theta D_x + 3D_{x\theta})\frac{\partial^3}{\partial x^2\partial\theta} \end{bmatrix} v + \begin{bmatrix} -R^2 D_x\frac{\partial^4}{\partial x^4} + 2RN_{x\theta}\frac{\partial^2}{\partial x\partial\theta} \\ -(2v_\theta D_x + 4D_{x\theta})\frac{\partial^4}{\partial x^2\partial\theta^2} \\ -\frac{D_\theta}{R^2}(\frac{\partial^2}{\partial\theta^2} + 1)^2 + N_\theta\frac{\partial^2}{\partial\theta^2} \\ +N_x R^2\frac{\partial^2}{\partial x^2} - K_\theta \end{bmatrix} w = 0 \tag{6}$$

Here CNTs are supported as simply supported, the vibration analysis of free CNTs is considered by the solution of the form,

$$u(x,\theta,t) = \alpha_m U(x)\, cos\, n\,\theta e^{i\omega t} \tag{7}$$

$$v(x,\theta,t) = \beta_m V(x)\, sin\, n\,\theta e^{i\omega t} \tag{8}$$

$$w(x,\theta,t) = \gamma_m W(x)\, cos\, n\,\theta e^{i\omega t} \tag{9}$$

where amplitudes in longitudinal, circumferential, and radial directions are represented by the constants *U*, *V*, and *W*, respectively. In the axial direction, *m* is the half wave number and *L* is the tube's length.

k_m is the wave vector in longitudinal direction. ω is the angular frequency and n is the circumferential wave number.

$$U(x) = cos\, k_m\, x$$

$$V(x) = W(x) = sin\, k_m\, x$$

where (10)

$$k_m = \frac{m\pi}{L} \quad \text{(C-C)}$$

$$k_m = (2\frac{m-1)\pi}{2L} \quad \text{(C-F)}$$

Putting (10) into (2), the eigen value form is attained.

3. Results and discussion

Here, the deep analysis of carbon nanotubes is studied using the orthotropic elastic shell model. With the current model, the impacts of density against fundamental frequencies are found.

The first step in determining the vibration characteristics

Table 1 Analyzing frequency levels with Duan *et al.* (2007)

		f (THz)					
	Aspect ratio	5.26	5.62	5.99	6.35	6.71	7.07
Clamped-clamped	Duan *et al.* (2007)	0.975	0.887	0.809	0.741	0.681	0.628
	Present	0.965	0.881	0.799	0.733	0.671	0.618
Clamped -free	Duan *et al.* (2007)	0.212	0.188	0.167	0.150	0.136	0.123
	Present	0.201	0.179	0.159	0.139	0.126	0.111

Table 2 Analyzing frequency levels with Arghavan and Singh (2011)

		f (THz)				
	Mode No.	1	2	3	4	5
Clamped-clamped	Arghavan and Singh (2011)	0.4323	0.4323	1.0582	1.0582	1.2661
	Present	0.4311	0.4311	1.0569	1.0573	1.2647
Clamped -free	Arghavan and Singh (2011)	0.0746	0.0746	0.4316	0.4316	0.6296
	Present	0.0732	0.0732	0.4301	0.4301	0.6281

Table 3 Analyzing frequency levels with Zhang *et al.* (2009)

	f (THz)				
Aspect Ratio	21.06	24.66	28.85	31.85	35.53
MD simulation (Zhang *et al.* 2009)	0.07629	0.05798	0.04578	0.03662	0.03052
Present	0.07618	0.05786	0.04560	0.03654	0.03039

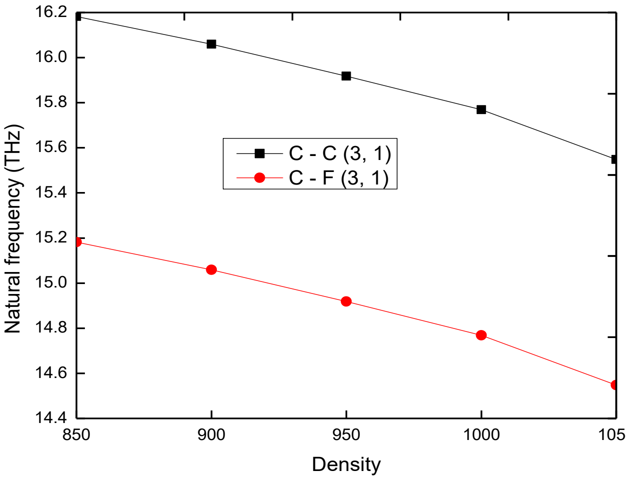

Fig. 2 Chiral (3, 1) SWCNTs' density effects on (C-C, C-F) frequencies

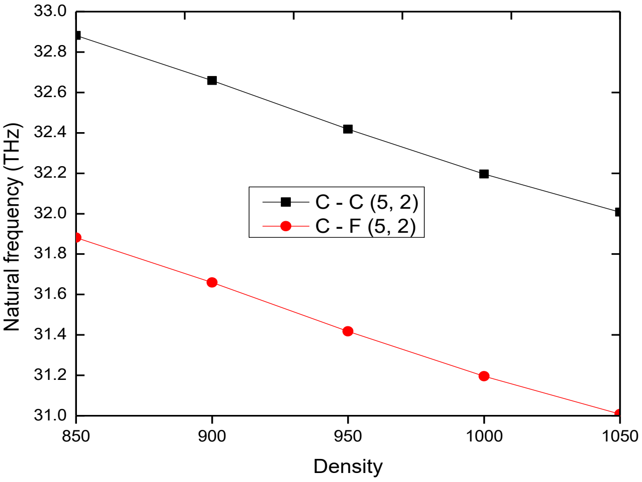

Fig. 3 Chiral (5, 2) SWCNTs' density effects on (C-C, C-F) frequencies

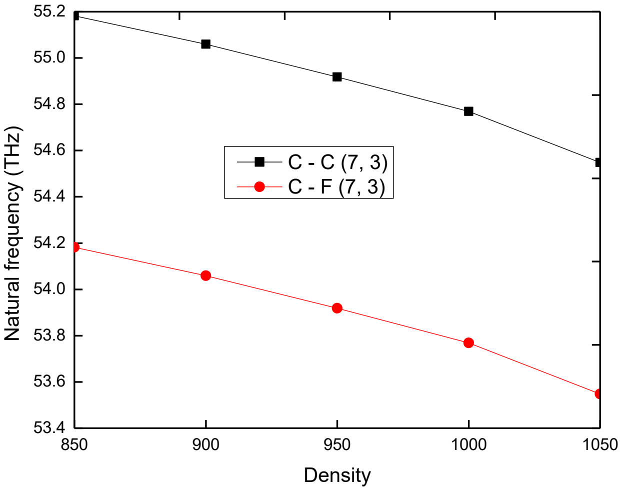

Fig. 4 Chiral (7, 3) SWCNTs' density effects on (C-C, C-F) frequencies

of chiral (3, 1), (5, 2), and (7, 3) SWCNTs using a continuum model is to validate the results with Duan *et al.* (2007), Arghavan and Singh (2011), and Zhang *et al.* (2009) to ensure accuracy as indicated in Tables 1-3. The frequencies for two distinct boundary conditions-one with a fixed diameter and length and the other parameters with a changing mode number. The frequencies is found to be in closed agreement. Following validation, it seems to have outstanding accuracy. The current model's results offer a potent method for examining the behavior of SWCNTs. Figs. 2-4 illustrate how vibrating chiral (3, 1), (5, 2), and (7, 3) single-walled carbon nanotubes affect density variation. The calculated results from present model for chiral index C - C (= 3, 1) at ρ (=850, 900, 950, 1000, 1050), the first nine frequencies are 16.182, 16.059, 15.918, 15.769, 15.548 and for chiral index C - C (= 5, 2), the frequencies are 32.882, 32.659, 32.418, 32.196, 32.008 and frequencies for chiral index C - C (= 7, 3), are 55.182¸ 55.059, 54.918, 54.769, 54.548. The frequency at which the system resonates is the nanotube's inherent frequency. Two elements contribute to the natural frequency in this case: the mass and the stiffness of the nanotube, which serves as a spring. The mass of the chiral single-walled carbon nanotubes is inversely correlated with the vibration frequencies. So due to increase of density results the decrease in resonance frequency. As a result, natural frequencies likewise fall. We turn next to examine in more detail chiral index C - F (= 3, 1) at ρ (= 850 ~ 1050) the first none frequencies are 15.182, 15.059, 14.918, 14.769, 14.548, and for chiral index C - F (= 5, 2), the frequencies are

31.882, 31.659, 31.418¸ 31.196, and for chiral index C - F (= 7, 3), are 54.182, 54.059, 53.918, 53.769, 53.548. The C–F boundary condition shows a similar pattern of frequencies decreasing as density increases. It is evident that the frequencies for the C-C, C-F (3, 1), (5, 2), and (7, 3) chiral indices drop off quite quickly. The chiral indices (3, 1), (5, 2), and (7, 3) all exhibit the same fall in frequency pattern. However, as the index rises from (3, 1) to (5, 2) and (7, 3), the frequencies rise as well.

For two boundary conditions, the chiral index (5, 2) lies between the chiral indexes (3, 1) and (7, 3). Furthermore, was concluded that the chiral index (3, 1) has a smaller frequency gap than the other two indexes. Compared to the C-C condition, the C-F border condition occurs less frequently. It is quite evident that the density of nanotubes significantly affects the shell's inherent frequencies. It has been shown that when density grows, so do natural frequencies. The frequencies drop with increasing density. This is primarily due to the system's mass having the opposite effect on the natural frequency.

4. Conclusions

For the vibration properties of chiral single-walled carbon nanotubes, theoretical formulations based on the extended orthotropic shell model are investigated. In order to extract the frequencies of CNTs, the governing equation of motion and boundary conditions are expressed in the form of an eigenvalue using a complicated approach. Compared to the previous approaches, this improved model is less complicated. The effect of density, boundary conditions for chiral (3, 1), (5, 2) and (7, 3) on the frequencies are calculated. It is observed that the increment in the density, the frequencies decreases for chiral tubes. Both clamped-clamped and clamped-free end situations exhibit these frequency fluctuations. For general density values, the frequency pattern appears to be parallel with the two specified boundary constraints. The frequency curves with the lowest results are also observed when the density of the C-F boundary condition is changed. However, it has a significant impact on the tubes' distortion. The current frequencies are compared with several models using carefully selected parameters. It is evident from the comparisons that the current model can produce results that are suitable for frequency prediction.

Acknowledgement

The authors extend their appreciation to Prince Sattam bin Abdulaziz University for funding this research work through the project number (PSAU/2024/01/31613)

References

Adali, S. (2009), "Variational principles for multi-walled carbon nanotubes undergoing non-linear vibrations by semi-inverse method", *Micro Nano Lett.*, **4**(4), 198-203. https://doi.org/10.1049/mnl.2009.0084

Akbaş, Ş.D. (2020), "Modal analysis of viscoelastic nanorods under an axially harmonic load", *Adv. Nano Res.*, **8**(4), 277. https://doi.org/10.12989/anr.2020.8.4.277

Alghaffari, S., Hussain, M., Khadimallah, M.A., Al Thobiani, F. and Sulaimani, H.T. (2023), "Simply supported boundary condition for bifurcation analysis of functionally graded material: Thickness control by exponential fraction law", *Adv. Nano Res.*, **14**(4), 303-312. https://doi.org/10.12989/anr.2023.14.4.303

Alves, S., Babcinschi, M., Silva, A., Neto, D., Fonseca, D. and Neto, P. (2023), "Integrated design fabrication and control of a bioinspired multimaterial soft robotic hand", *Cyborg Bionic Syst.*, **4**, 0051. https://doi.org/10.34133/cbsystems.0051

Arghavan, S. and Singh, A.V. (2011), "On the vibrations of single-walled carbon nanotubes", *J. Sound Vib.*, **330**(13), 3102-3122. https://doi.org/10.1016/j.jsv.2011.01.032

Barretta, R., Marotti de Sciarra, F., Pinnola, F.P. and Vaccaro, M. S. (2022), "On the nonlocal bending problem with fractional hereditariness", *Meccanica*, **57**(4), 807-820. https://doi.org/10.1007/s11012-021-01366-8

Benmansour, D.L., Kaci, A., Bousahla, A.A., Heireche, H., Tounsi, A., Alwabli, A.S., Alhebshi, A.M., Al-ghmady, K. and Mahmoud, S.R. (2019), "The nano scale bending and dynamic properties of isolated protein microtubules based on modified strain gradient theory", *Adv. Nano Res.*, **7**(6), 443. https://doi.org/10.12989/anr.2019.7.6.443

Benvenuti, E. and Simone, A. (2013), "One-dimensional nonlocal and gradient elasticity: Closed-form solution and size effect", *Mech. Res. Commun.*, **48**, 46-51. https://doi.org/10.1016/j.mechrescom.2012.12.001

Budarapu, P.R., Yb, S.S., Javvaji, B. and Mahapatra, D.R. (2014), "Vibration analysis of multi-walled carbon nanotubes embedded in elastic medium," *Front. Struct. Civil Eng.*, **8**, 151-159. https://doi.org/10.1007/s11709-014-0247-9

Budiansky, B. and Sanders, J.L. (1963), "On the best first-order linear shell theory", *Prog. Appl. Mech.*, The Prager Anniversary, **1**. https://cir.nii.ac.jp/crid/1570572699128144512

Cai, Y., Wei, J., Hou, Q., Fan, H. and Tijsseling, A.S. (2025), "A Lagrangian particle model for one-dimensional transient pipe flow with moving boundary", *Eng. Appl. Comput. Fl. Mech.*, **19**(1), 2452360. https://doi.org/10.1080/19942060.2025.2452360

Chandraseker, K., Mukherjee, S., Paci, J.T. and Schatz, G.C. (2009), "An atomistic-continuum Cosserat rod model of carbon nanotubes", *J. Mech. Phys. Solid.*, **57**(6), 932-958. https://doi.org/10.1016/j.jmps.2009.02.005

Chemi, A., Heireche, H., Zidour, M., Rakrak, K. and Bousahla, A.A. (2015), "Critical buckling load chiral double-walled carbon nanotubes using non-local elasticity theory", *Adv. Neno Res.*, **3**(4), 193-206. https://doi.org/10.12989/anr.2015.3.4.193

Civalek, Ö. and Baltacıoğlu, A.K. (2018), "Vibration of carbon nanotube reinforced composite (CNTRC) annular sector plates by discrete singular convolution method", *Compos. Struct.*, **203**, 458-465.https://doi.org/10.1016/j.compstruct.2018.07.037

Civalek, Ö., Uzun, B. and Yaylı, M.Ö. (2023), "On nonlinear stability analysis of saturated embedded porous nanobeams", *Int. J. Eng. Sci.*, **190**, 103898. https://doi.org/10.1016/j.ijengsci.2023.103898

Cong, F., Zhang, R., Li, W., Jin, Y., Yu, G. and Wu, L. (2024), "Buckling analysis of moderately thick carbon fiber composite cylindrical shells under hydrostatic pressure", *Appl. Ocean Res.*, **153**, 104272. https://doi.org/10.1016/j.apor.2024.104272

Duan, W.H., Wang, C.M. and Zhang, Y.Y. (2007), "Calibration of nonlocal scaling effect parameter for free vibration of carbon nanotubes by molecular dynamics", *J. Appl. Phys.*, **101**(2), 024305. https://doi.org/10.1063/1.2423140

Fatahi-Vajari, A., Azimzadeh, Z. and Hussain, M. (2019), "Nonlinear coupled axial–torsional vibration of single-walled carbon nanotubes using homotopy perturbation method", *Micro Nano Lett*., **14**(14), 1366-1371. https://doi.org/10.1049/mnl.2019.020

Forsat, M., Musharavati, F., Eltai, E., Zain, A.M., Mobayen, S. and Mohamed, A.M. (2021), "Vibration characteristics of microplates with GNPs-reinforced epoxy core bonded to piezoelectric-reinforced CNTs patches", *Adv. Nano Res*., **11**(2), 115. https://doi.org/10.12989/anr.2021.11.2.115

Gao, Q., Deng, Z., Ju, Z. and Zhang, T. (2023), "Dual-hand motion capture by using biological inspiration for bionic bimanual robot teleoperation", *Cyborg Bionic Syst*., **4**, 52. https://doi.org/10.34133/cbsystems.0052

Ghandourah, E., Hussain, M., Khadimallah, A., Alhawsawi, A., Banoqitah, E.M. and Ali, M.R. (2023), "Vibration analysis of double-walled carbon nanotubes based on Timoshenko beam theory and wave propagation approach", *Adv. Nano Res*., **14**(6), 521-525. https://doi.org/10.12989/anr.2023.14.6.521

Ghandourah, E., Hussain, M., Khadimallah, M. A., Ayed, H., Ahmad, M., Azrar, L. and Mouldi, A. (2024), "Dynamic response with fraction laws: Eigen solution of clamped-simply supported rotating shell", *Adv. Nano Res*., **17**(5), 435-444. https://doi.org/10.12989/anr.2024.17.5.435

Ghasemi, S.E. and Gouran, S. (2022), "Nonlinear analysis on flow-induced vibration of single-walled carbon nanotubes employing analytical methods", *Int. J. Struct. Stabil. Dyn*., **22**(11), 2250115. https://doi.org/10.1142/S0219455422501152

He, D., Xu, H., Wang, M. and Wang, T. (2024), "Transmission and dissipation of vibration in a dynamic vibration absorber-roller system based on particle damping technology", *Chinese J. Mech. Eng*., **37**(1), 108. https://doi.org/10.1186/s10033-024-01107-

Hollerer, S. and Celigoj, C.C. (2013), "Buckling analysis of carbon nanotubes by a mixed atomistic and continuum model", *Comput. Mech*., **51**, 765-789. https://doi.org/10.1007/s00466-012-0757-z

Hu, Y.G., Liew, K.M. and Wang, Q. (2012), "Modeling of vibrations of carbon nanotubes", *Procedia Eng*., **31**, 343-347. https://doi.org/10.1016/j.proeng.2012.01.1034

Hussain, M. (2020), "Application of Kelvin's approach for material structure of CNT: polynomial volume fraction law", *Struct. Eng. Mech*., **76**(1), 129-139. https://doi.org/10.12989/sem.2020.76.1.129

Hussain, M. (2022), "FG-based computational fracture of frequency up-conversion for bistablity of rotating shell: An effective numerical scheme", *Adv. Concr. Constr*., **13**(5), 367-376. https://doi.org/10.12989/acc.2022.13.5.367

Hussain, M. (2024), *Small-scale Computational Vibration of Carbon Nanotubes: Composite Structure*, CRC Press.

Hussain, M., Ali, M.R., Benslimane, A., Sharif, H., Khadimallah, M.A., Naeem, M.N., Harbaoui, I., Helaili, S., Majeed, A. and Tounsi, A. (2023), "Service ability design of vibrating chiral SWCNTs: Validation and parametric study", *Comput. Concr*., **32**(4), 393. https://doi.org/10.12989/cac.2023.32.4.393

Hussain, M., Khadimallah, M.A., Ayed, H., Ghandourah, E., Mouldi, A. and Tounsi, A. (2024), "Closed form interaction for safety assessment of DWCNTs: Mechanical vibration", *Adv. Nano Res*., **17**(4), 315-321. https://doi.org/10.12989/anr.2024.17.4.315

Hussain, M., Naeem, M. N., Shahzad, A. and He, M. (2017), "Vibrational behavior of single-walled carbon nanotubes based on cylindrical shell model using wave propagation approach", *AIP Adv*., **7**(4). https://doi.org/10.1063/1.4979112

Ke, L.L., Xiang, Y., Yang, J. and Kitipornchai, S. (2009), "Nonlinear free vibration of embedded double-walled carbon nanotubes based on nonlocal Timoshenko beam theory", *Comput. Mater. Sci*., **47**(2), 409-417. https://doi.org/10.1016/j.commatsci.2009.09.002

Khedher, K.M., Chattah, S.A., Khadimallah, M.A., Ahmad, I., Hussain, M., Muntazir, R.M.A., ... and Tounsi, A. (2024), "Fluid bounding effect on FG cylindrical shell using Hankel's functions of second kind", *Adv. Nano Res*., **16**(6), 565-577. https://doi.org/10.12989/anr.2024.16.6.565

Khedher, K.M., Hussain, M., Munir, R., Alsulamy, S. and Alluqmani, A.E. (2023), "Application of Hamilton variational principle for vibration of fluid filled structure", *Adv. Nano Res*., **15**(5), 401. https://doi.org/10.12989/anr.2023.15.5.401

Kroto, H.W., Heath, J.R., OBrien, S.C., Curl, R.F. and Smalley, R.E. (1985), "C60 buckyminsterfullerence", *Nature*, **318**, 162. https://doi.org/10.1038/318162a0

Lei, X.W., Natsuki, T., Shi, J.X. and Ni, Q.Q. (2012), "Surface effects on the vibrational frequency of double-walled carbon nanotubes using the nonlocal Timoshenko beam model", *Compos. Part B Eng*., **43**(1), 64-69. https://doi.org/10.1016/j.compositesb

Li, X., Wang, S., Li, Z., Yang, R. and Li, Z. (2024), "Measurement of bolt axial stress using a combination of trailing wave and shear wave ultrasound", *NDT E Int*., **143**, 103056. https://doi.org/10.1016/j.ndteint.2024.103056

Liu, Y., Su, J., He, D., Hao, P., Liu, Y., Wang, Z., ... Wang, T. (2025), "Analytical model for corrugated rolling of composite plates considering the shear effect", *J. Manuf. Proc*., **134**, 1069-1081. https://doi.org/10.1016/j.jmapro.2025.01.02

Luo, J., Lv, M., Hou, S., Nasihatgozar, M. and Behshad, A. (2022), "Dynamic analysis of viscoelastic concrete plates containing nanoparticle subjected to low velocity impact load", *Adv. Nano Res*., **13**(4), 369-378. https://doi.org/10.12989/anr.2022.13.4.369

Malikan, M. and Eremeyev, V.A. (2020), "Post-critical buckling of truncated conical carbon nanotubes considering surface effects embedding in a nonlinear Winkler substrate using the Rayleigh-Ritz method", *Mater. Res. Express*, **7**(2), 025005. https://doi.org/10.1088/2053-1591/ab691c10.1088/2053-1591/ab691c

Moradi, Z., Ebrahimi, F. and Davoudi, M. (2023), "Vibration control, energy harvesting and forced vibration of the piezoelectric NEMS via paradox-free local/nonlocal theory", *Adv. Nano Res*., **14**(4), 335-353. https://doi.org/10.12989/anr.2023.14.4.335

Natsuki, T., Ni, Q.Q. and Endo, M. (2008), "Vibrational analysis of fluid-filled carbon nanotubes using the wave propagation approach", *Appl. Phys. A*, **90**, 441-445. https://doi.org/10.1007/s00339-007-4297-x

Ni, Z. L., Li, B. H., Nazarov, A. A., Ma, J. S., Yuan, Z. P., Wang, X. X. and Li, H. (2024), "Simulation of ultrasonic welding of Cu/Cu joints with an interlayer of Cu nanoparticles", *Mater. Today Commun*., **39**, 109330. https://doi.org/10.1016/j.mtcomm.2024.109330

Pablo, P.J., Schaap, I.A.T., Mackintosh, F.C. and Schmidt, C.F. (2003), "Deformation and collapse of microtubules on the nanometer scale", *Phys. Rev. Lett*., **91**(9), 098101-098114. https://doi.org/10.1103/PhysRevLett.91.098101

Peng, X., Chen, K., Jia, W., Li, K., Huang, C. and Liu, X. (2025), "Stacking sequence optimization of variable thickness composite laminated plate based on multi-peak stacking sequence table", *Composite Structures*, **356**, 118886. https://doi.org/10.1016/j.compstruct.2025.118886

Penna, R. (2023), "Bending analysis of functionally graded nanobeams based on stress-driven nonlocal model incorporating surface energy effects", *Int. J. Eng. Sci*., **189**, 103887. https://doi.org/10.1016/j.ijengsci.2023.103887

Pourasghar, A., Yang, W., Brigham, J. and Chen, Z. (2021), "Nonlocal thermoelasticity: Transient heat conduction effects on the linear and nonlinear vibration of single-walled carbon nanotubes", *Mech. Based Des. Struct*., 1-17. https://doi.org/10.1080/15397734.2021.1985516

Qazaq, A., Hussain, M., Mujalli, M. and Tounsi, A. (2022), “Fundamental computer assessment of ring support with exponent of trigonometric function: Safety geometrical perfection”, *Adv. Concr. Constr.*, **14**(6), 381. https://doi.org/10.12989/acc.2022.14.6.381

Qi, H., Ding, L., Zheng, M., Huang, L., Gao, H., Liu, G. and Deng, Z. (2024), “Variable wheelbase control of wheeled mobile robots with worm-inspired creeping gait strategy”, *IEEE T Robot*, **40**, 3271-3289. https://doi.org/10.1109/TRO.2024.3400947

Qiu, Z., Chen, R., Gan, X. and Wu, C. (2024), “Torsional damper design for diesel engine: theory and application”, *Physica Scripta*, **99**(12), 125214. https://doi.org/10.1088/1402-4896/ad8af8

Reddy, J. (2007), “Nonlocal theories for bending, buckling and vibration of beams”, *Int. J. Eng. Sci.*, **45**(2-8), 288-307. https://doi.org/10.1016/j.ijengsci.2007.04.004

Ru, C.Q. (2000), “Column buckling of multiwalled carbon nanotubes with interlayer radial displacements”, *Phys. Rev. B*, **62**(24), 16962. https://doi.org/10.1103/PhysRevB.62.16962

Safaei, B., Khoda, F.H. and Fattahi, A.M. (2019), “Non-classical plate model for single-layered graphene sheet for axial buckling”, *Adv Nano Res*, **7**(4), 265-275. https://doi.org/10.12989/anr.2019.7.4.265

Samaniego, E., Anitescu, C., Goswami, S., Nguyen-Thanh, V.M., Guo, H., Hamdia, K., Zhuang, X. and Rabczuk, T. (2020), “An energy approach to the solution of partial differential equations in computational mechanics via machine learning: Concepts, implementation and applications”, *Comput. Meth. Appl. Mech. Eng.*, **362**, 112790. https://doi.org/10.1016/j.cma.2019.112790

Shima, H. (2011), “Buckling of carbon nanotubes: Astate of the art review”, *Materials*, **5**(1), 47-84. https://doi.org/10.3390/ma5010047

Simsek, M. (2010), “Vibration analysis of a single-walled carbon nanotube under action of a moving harmonic load based on nonlocal elasticity theory”, *Physica E*, **43**,182-191. https://doi.org/10.1016/j.physe.2010.07.003

Sirenko, M., Stroscio, M. and Kim, K.W. (1996), “Elastic vibrations of microtubules in a fluid”, *Phys. Rev. E*, **53**(1), 1003-1010. https://doi.org/10.1103/PhysRevE.53.1003

Su, Y., Zhu, J., Long, X., Zhao, L., Chen, C. and Liu, C. (2023), “Statistical effects of pore features on mechanical properties and fracture behaviors of heterogeneous random porous materials by phase-field modeling”, *Int. J. Solids Struct.*, **264**, 112098. https://doi.org/10.1016/j.ijsolstr.2022.112098

Taj, M., Ahmad, M., Khadimallah, M.A., Akram, S., Hussain, M., Tahir, M., Butt, F.M. and Tounsi, A. (2023), “Lubrication phenomenon in the stagnation point flow of Walters-B nanofluid”, *Adv. Concr. Constr.*, **15**(5), 303-312. https://doi.org/10.12989/acc.2023.15.5.303

Taj, M. and Zhang, J.Q. (2011), “Buckling of embedded micro-tubules in elastic medium”, *Appl. Math. Mech.*, **32**(3), 293-300. https://doi.org/10.1007/s10483-011-1415-x

Wang, C., Ru, C. and Mioduchowski, A. (2005), “Buckling and free vibration of multiwall carbon nanotubes”, Doctoral Thesis, Department of Mechanical Engineering, *AIP Publishing*, Edmonton, Alberta, Canada.

Wang, K., Chen, Z., Wang, Z., Chen, Q. and Ma, D. (2023), “Critical dynamic stress and cumulative plastic deformation of calcareous sand filler based on shakedown theory”, *J. Marine Sci. Eng.*, **11**(1), 195. https://doi.org/10.3390/jmse11010195

Wang, Q. Varadan, V.K. and Quek, S.T. (2006), “Small scale effect on elastic buckling of carbon nanotubes with nonlocal continuum models”, *Phys. Lett. A.*, **357**(2), 130-135. https://doi.org/10.1016/j.physleta.2006.04.026.

Wang, S., He, J., Fan, J., Sun, P. and Wang, D. (2023), “A time-domain method for free vibration responses of an equivalent viscous damped system based on a complex damping model”, *J. Low Freq. Noise Vib. Active Control*, **42**(3), 1531-1540. https://doi.org/10.1177/14613484231157514

Wang, Z., Li, J., Yuan, Y., Zhang, S., Hu, W., Ma, J. and Tan, J. (2025), “Digital-twin-enabled online wrinkling monitoring of metal tube bending manufacturing: A multi-fidelity approach using forward-convolution-GAN”, *Appl. Soft Comput.*, **171**, 112684. https://doi.org/10.1016/j.asoc.2024.112684

Xu, J., Chang, L., Chen, T., Ren, T., Zhang, Y. and Cai, Z. (2023), “Study of the bending properties of variable stiffness chain mail fabrics”, *Compos. Struct.*, **322**, 117369. https://doi.org/10.1016/j.compstruct.2023.117369

Xu, K.Y., Guo, X.N. and Ru, C.Q. (2006), “Vibration of a double-walled carbon nanotube aroused by nonlinear intertube van der Waals forces”, *J. Appl. Phys.*, **99**(6), 64303. https://doi.org/10.1063/1.2179970

Yang, Q., Li, H., Zhang, L., Guo, K. and Li, K. (2025a), “Nonlinear flutter in a wind-excited double-deck truss girder bridge: experimental investigation and modeling approach”, *Nonlinear Dyn.*, **113**(7), 6427-6445. https://doi.org/10.1007/s11071-024-10496-z

Yang, Q., Zeng, X., Guo, K., Cao, S., Wei, K., Shan, W. and Tamura, Y. (2025b), “Analysis of vortex-induced vibration in flexible risers using a physically-meaningful wake-oscillator model”, *Eng. Struct.*, **325**, 119415. https://doi.org/10.1016/j.engstruct.2024.119415

Yu, H., Wang, H. and Lian, Z. (2022), “An assessment of seal ability of tubing threaded connections: A hybrid empirical-numerical method”, *J. Energy Resour. Technol.*, **145**(5). https://doi.org/10.1115/1.4056332

Zhang, S., Li, F., Fu, R., Li, H., Zou, S., Ma, N., Qu, S. and Li, J. (2023), “A versatile continuum gripping robot with a concealable gripper”, *Cyborg Bionic Syst.*, **4**, 3. https://doi.org/10.34133/cbsystems.0003

Zhang, W., Huang, J., Lin, J., Lin, B., Yang, X. and Huan, Y. (2024), “Experimental and numerical investigation of mechanical behavior of segmental joint of shield tunneling strengthened by prestressed CFRP plates”, *Structures*, **70**, 107634. https://doi.org/10.1016/j.istruc.2024.107634

Zhang, W., Kang, S., Liu, X., Lin, B. and Huang, Y. (2023), “Experimental study of a composite beam externally bonded with a carbon fiber-reinforced plastic plate”, *J. Build. Eng.*, **71**, 106522. https://doi.org/10.1016/j.jobe.2023.106522

Zhang, X., Shen, S., Luo, Y., Elmaimouni, L. and Yu, J. (2025), “Dispersion and attenuation characteristics of Lamb waves in multilayered piezoelectric semiconductor plates with imperfect interfaces”, *Mech. Adv. Mater. Struct.*, 1-14. https://doi.org/10.1080/15376494.2025.2456681

Zhang, Y.Y., Wang, C.M. and Tan, V.B.C. (2009), “Assessment of Timoshenko beam models for vibrational behavior of single-walled carbon nanotubes using molecular dynamics”, *Adv. Appl. Math. Mech.*, **1**(1), 89-106.

Zhuang, X., Guo, H., Alajlan, N., Zhu, H. and Rabczuk, T. (2021), “Deep autoencoder based energy method for the bending, vibration, and buckling analysis of Kirchhoff plates with transfer learning”, *Eur. J. Mech. A Solids*, **87**, 104225. https://doi.org/10.1016/j.euromechsol.2021.104225

Zou, Z., Yang, S. and Zhao, L. (2024), “Dual-loop control and state prediction analysis of QUAV trajectory tracking based on biological swarm intelligent optimization algorithm”, *Sci. Rep.*, **14**(1), 19091. https://doi.org/10.1038/s41598-024-69911-5

CC