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Wave responses in seismic FGM concrete nanobeam using deep neural network

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Abstract. In the current study, we investigate the vibration of a nano-scale beam structure composed of bi-directionally functionally graded concrete. We employ a dual approach, combining mathematical structural modeling with deep neural network analysis, to determine the natural frequency of the nanobeam. The concrete is assumed to be graded along the beam's axis and transverse direction, following a power-law model. We utilize Timoshenko beam theory (TBT) and nonlocal stress-strain gradient relations to describe the nanobeam's displacement field. Hamilton's principle is used to account for external forces and boundary conditions. A deep neural network is trained to predict the natural frequency with varying error margins. The governing equations are solved using the differential quadrature numerical method, and the results are validated against existing literature. This work introduces novelties in three key areas: 1) a model for bi-FG concrete nanobeams under in-plane loading, 2).

Keywords: bi-directional FG concrete nanobeam; differential quadrature method; nonlocal strain gradient theory; physics-informed neural networks; vibrational analysis

1. Introduction

The relentless pursuit of materials with superior thermo-mechanical properties has consistently positioned carbon and its derivatives as prime candidates for reinforcing engineering structures. The selection of reinforcement scale, whether macro or nano, remains a critical design parameter, dictated by the specific engineering objectives (Jin *et al*. 2024, Daichang *et al*. 2025, Fang *et al*. 2025, Jining *et al*. 2025, Li *et al*. 2025b, Lin *et al*. 2025, Liu *et al*. 2025b, Xia *et al*. 2025, Xiao *et al*. 2025, Yang *et al*. 2025a, b, Ying *et al*. 2025, Yuanchao *et al*. 2025). Traditionally, composites reinforced with macroscale carbon fibers (CF) have been employed to enhance structural mechanical performance. For instance, (Ma *et al*. 2023, Li *et al*. 2024d, Wang *et al*. 2024b, Chang *et al*. 2025, Liu *et al*. 2025a, Liu *et al*. 2025c, Wang *et al*. 2025, Zhou *et al*. 2025) explored the buckling behavior of fiber-reinforced Euler beams under hygrothermal conditions, utilizing Reddy's theory. However, the paradigm is shifting towards nanoscale reinforcement, where carbon nanotubes (CNTs) offer a significant leap in mechanical performance. The escalating demand for energy-efficient devices across diverse engineering applications has catalyzed a surge in research focused on lightweight micro/nano systems, where size-dependent effects play a pivotal role in stability analysis (Li *et al*. 2024c, Wang *et al*. 2024c, g, Xue *et al*. 2024, Zhang *et al*. 2024c, Zhu *et al*. 2024a, b). This drive has particularly intensified the investigation into carbon nanotube (CNT)-reinforced structures, recognizing their potential to significantly enhance mechanical performance at the nanoscale (Huang *et al*. 2024, Man *et al*. 2024, Wang *et al*. 2024f, Zhang *et al*. 2024d, Zhiqiang *et al*. 2024,

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Zisong and Habibi 2024). Building upon this foundation, researchers have extensively explored the mechanical behavior of functionally graded (FG) materials at the micro and nanoscale. (Guo *et al*. 2024, Song *et al*. 2024, Wang *et al*. 2024h, Xiao *et al*. 2024, Yin *et al*. 2024, Yu *et al*. 2024, Zhang *et al*. 2024b) initiated the development of models for elastoplastic deformations in FG rotating disks, paving the way for further investigations into the thermo-elasto-plastic analysis of thick-walled spherical pressure vessels (Ge *et al*. 2023, He *et al*. 2024, Liang *et al*. 2024, Liu *et al*. 2024, Wang *et al*. 2024a, d, Zhao *et al*. 2024) and the elastic analysis of rotating cylindrical pressure vessels with exponentially varying properties (Gu *et al*. 2023, Li *et al*. 2023, Lu *et al*. 2023a, b, Shariati *et al*. 2023, Tang *et al*. 2023, Zhang *et al*. 2023a). Subsequent work by (Dai *et al*. 2023a, Dai *et al*. 2023b, Peng *et al*. 2023, Wang *et al*. 2023, Yang *et al*. 2023, Yu *et al*. 2023, Zhang *et al*. 2023b) provided exact solutions for the deformation and stress states of FG cylindrical pressure vessels under thermal loads.

Furthermore, studies have delved into the intricacies of nano-beam and nano-plate behavior, utilizing nonlocal elasticity theory and consistent couple stress theory to account for size effects. (Habibi *et al*. 2018a, b, 2019a, b, 2022, Zhang *et al*. 2023c, Zhao *et al*. 2023, Zheng *et al*. 2023). A significant contribution to the field was made by Hadi *et al*. (2018), who undertook a comprehensive investigation into the free vibration characteristics of three-directional functionally graded (FG) nano-beams. Their work provided valuable insights into the dynamic behavior of these complex structures, particularly concerning the influence of material gradation across multiple axes. Complementing this line of inquiry, other researchers have directed their efforts towards analyzing stress distributions in rotating nano-disks featuring nonlinearly varying thickness. These studies have sought to elucidate the intricate interplay between centrifugal forces, material properties, and geometric variations, thereby expanding our understanding of the mechanical response of nano-scale rotating components (Fan *et al*. 2022, Luo *et al*. 2022, Wang *et al*. 2022, Xia *et al*. 2022). These investigations have significantly advanced our understanding of the fundamental mechanics governing these nanoscale structures. However, a persistent challenge remains: the precise fabrication of materials with controlled grading properties at such minute scales, a hurdle that necessitates further innovation in manufacturing techniques.

Recent advancements in functionally graded (FG) concrete materials have opened new avenues for creating devices with tailored properties across their material geometry (Li *et al*. 2024b, Wang *et al*. 2024e, Yang *et al*. 2025c). (Li *et al*. 2024a, Li *et al*. 2025a, Zhang *et al*. 2025) demonstrated the efficacy of artificial neural networks in predicting the bonding strength of fiber-reinforced polymers to concrete. (Bendenia *et al*. 2020, Menasria *et al*. 2020, Bakoura *et al*. 2021, Hachemi *et al*. 2021, Merazka *et al*. 2021, Zerrouki *et al*. 2021) explored the use of carbon nanotubes to enhance concrete strength, modeling various distribution patterns and analyzing the vibration of concrete beams on elastic substrates. Shariati *et al* (2019) investigated the impact of tea waste on concrete strength using a neural fuzzy system, highlighting the potential of natural reinforcement fibers for environmental sustainability. The vibrational behavior of nanoscale beam structures, often approximated by simple models, has been extensively studied. Some researchers (Niu *et al*. 2024, Wu *et al*. 2024, Rong *et al*. 2025) incorporated small-size effects and porosity into the vibrational analysis of functionally graded nanobeams using nonlocal elasticity theory. Ebrahimi Mamaghani (Mamaghani *et al*. 2016, Zheng *et al*. 2020) explored the size dependency of piezoelectric nanobeams on elastic substrates, while Ke *et al*. (2012) investigated the temperature effects on nanobeam natural frequencies. Although numerical methods offer powerful tools for analyzing complex structural behaviors, they can be computationally expensive, particularly in scenarios like crystal plasticity simulation (Amelirad and Assempour 2022). Consequently, artificial intelligence emerges as a compelling alternative, capable of directly correlating influencing parameters with material properties (Moradi *et al*. 2022). In this vein, numerous studies have leveraged artificial intelligence for the vibration and buckling analysis of small-scale structures, circumventing the complexities of mathematical modeling and numerical solutions (Huang *et al*. 2022, Yao *et al*. 2022, Zhang *et al*. 2024a).

In essence, previous work has laid the groundwork for understanding the intricate behavior of nanoscale structures. These studies have meticulously explored the effects of various parameters, from material composition and loading conditions to size effects and environmental factors. However, the complexity of these structures, particularly when dealing with functionally graded materials and nanoscale dimensions, necessitates a multifaceted approach that combines traditional analytical methods with advanced computational techniques. The current work aims to build upon this foundation by introducing a novel model for bi-directionally functionally graded concrete nanobeams, leveraging both mathematical modeling and deep neural networks to provide a comprehensive understanding of their vibrational characteristics.

Furthermore, the integration of deep neural networks into the analysis of nanoscale structures represents a significant advancement. By harnessing the power of machine learning, we can overcome the limitations of traditional analytical and numerical methods, particularly when dealing with complex material behaviors and intricate geometries. This approach not only enhances the accuracy and efficiency of our analyses but also opens up new avenues for exploring the design space of nanoscale structures, enabling us to tailor their properties for specific applications. The synergy between traditional modeling and deep learning offers a powerful tool for advancing our understanding and utilization of nanoscale materials.

In the current study, we investigate the vibration of a nano-scale beam structure composed of bi-directionally functionally graded concrete. We employ a dual approach, combining mathematical structural modeling with deep neural network analysis, to determine the natural frequency of the nanobeam. The concrete is assumed to be graded along the beam's axis and transverse direction, following a power-law model. We utilize Timoshenko beam theory

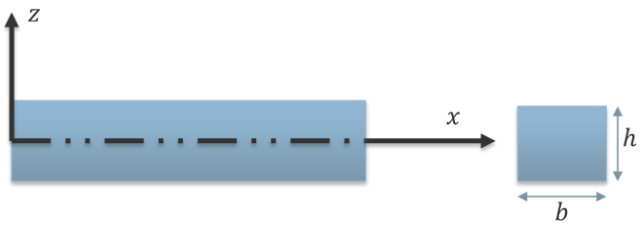


Fig. 1 Schematic of a concrete nanobeam

(TBT) and nonlocal stress-strain gradient relations to describe the nanobeam's displacement field. Hamilton's principle is used to account for external forces and boundary conditions. A deep neural network is trained to predict the natural frequency with varying error margins. The governing equations are solved using the differential quadrature numerical method, and the results are validated against existing literature. This work introduces novelties in three key areas: 1) a model for bi-FG concrete nanobeams under in-plane loading, 2).

2. Materials and methods

2.1 Concrete material graded in two-directions

The material properties of functionally graded (FG) are varied in the spatial directions. The development of the grading properties is deeply dependent on the application of the FG materials. In some cases, only mechanical properties are varied to have desirable properties in different sections of the parts. In other applications, the thermal properties of FG material are designed to be varied. In the present study, we assume that the properties of the concrete beam structure, P, is graded in two directions following the power law rule:

$$P(x,z) = P_L e^{\frac{n_x x}{L} + n_z\left(0.5+\frac{z}{h}\right)} \quad (1)$$

Parameter P_L is the reference properties and coefficients n_x and n_z determines the variation of the properties P in the x and z directions as shown in Figure 1. The constant h is the height of the beam structure. It is obvious that homogenous material could be obtained by setting $n_x = n_z = 0$.

2.2 Mathematical modeling

2.2.1 Modeling of two-dimensional functionally graded refined TBT

Fig. 1 presents a detailed schematic of the beam geometry under consideration. The beam exhibits a uniform cross-section along its longitudinal axis, ensuring consistent geometric properties throughout its length. However, the defining characteristic of this beam lies in its material composition: the material properties are functionally graded, varying continuously along both the x-axis (lengthwise) and the z-axis (thickness). This bi-directional gradation, as illustrated in the figure, allows for precise tailoring of the material's mechanical behavior at different locations within the beam. Furthermore, the beam is situated within a Winkler-Pasternak elastic foundation, a model that represents the surrounding environment's support. This environment is characterized by two parameters: The Winkler modulus, representing the stiffness of the foundation, and the Pasternak shear modulus, accounting for the shear interaction between the foundation elements. This foundation model is crucial for accurately simulating the beam's behavior in realistic applications where it interacts with surrounding media. It is essential to note that the beam's dimensions are in the nanometer range, placing it within the realm of nanoscale mechanics. This scale is significant because it introduces size-dependent effects, requiring the consideration of nonlocal theories to accurately capture the beam's behavior. The figure clearly indicates the coordinate system used for analysis, providing a clear reference for understanding the material property variations and the beam's orientation within the elastic foundation. This detailed representation of the beam's geometry, material properties, and surrounding environment is crucial for the subsequent mathematical modeling and analysis of its mechanical behavior.

2.3 Displacement field

To accurately model the deformation behavior of the nanobeam, the classical Timoshenko beam theory is employed to define the displacement field in both the longitudinal (x) and transverse (z) directions. This theory, renowned for its ability to account for shear deformation and rotational inertia, is particularly suitable for analyzing beams with relatively thick cross-sections, where shear effects cannot be neglected. Unlike the simpler Euler-Bernoulli beam theory, which assumes plane sections remain perpendicular to the neutral axis during deformation, Timoshenko theory relaxes this assumption, allowing for a more realistic representation of the beam's behavior. Specifically, the displacement field is expressed in terms of three primary variables: the axial displacement along the x-axis, the transverse displacement along the z-axis, and the rotation of the beam's cross-section. These variables are formulated as functions of the longitudinal coordinate (x) and time (t), capturing both static and dynamic deformations. By incorporating shear deformation, Timoshenko theory provides a more accurate prediction of the beam's deflection and stress distribution, especially when subjected to transverse loads or high-frequency vibrations. This displacement field forms the foundation for the subsequent derivation of the governing equations using Hamilton's principle, enabling a comprehensive analysis of the nanobeam's mechanical response.

$$\begin{aligned} u(x,z,t) &= u_0(x,t) + z u_1(x,t), \\ v(x,z,t) &= 0, \\ w(x,z,t) &= w_0(x,t), \end{aligned} \quad (2)$$

where (u,w) refer to the axial displacements of point (x,z) in the plane of the beam movement. The variables (u_0, w_0) are the axial and transverse displacement on the mid-plane of the structure. The function u_1 represents the angle of initial normal line to the axis of beam in the

deformed conditions with respect to current normal line at the deformed beam. Small deformation and rotation assumption results in the following definition of strains.

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{zz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_{xx}^{0}\\ \varepsilon_{zz}^{0}\\ \gamma_{xz}^{0}\end{Bmatrix}+z\begin{Bmatrix}k_{xx}^{0}\\ k_{zz}^{0}\\ k_{xz}^{0}\end{Bmatrix} \tag{3}$$

In addition:

$$\begin{Bmatrix}\varepsilon_{xx}^{0}\\ \varepsilon_{zz}^{0}\\ \gamma_{xz}^{0}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ 0\\ u_1+\dfrac{\partial w_0}{\partial x}\end{Bmatrix},\begin{Bmatrix}k_{xx}^{0}\\ k_{zz}^{0}\\ k_{xz}^{0}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_1}{\partial x}\\ 0\\ 0\end{Bmatrix} \tag{4}$$

The linear elastic bahvior of the concrete material is assumed to follow the below constitutive equation:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{zz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{13} & 0\\ Q_{31} & Q_{33} & 0\\ 0 & 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{zz}\\ \gamma_{xz}\end{Bmatrix} \tag{5}$$

In which:

$$\begin{aligned}Q_{11}=Q_{33}&=\frac{E(x,z)}{1-\nu^2(x,z)},\\ Q_{13}=Q_{31}&=\frac{\nu E(x,z)}{1-\nu^2(x,z)},\\ Q_{55}&=\frac{E(x,z)}{2\big(1+\nu(x,z)\big)}\end{aligned} \tag{6}$$

2.4 Hamilton's variational principle and the development of governing equations

With the strain components and constitutive equations meticulously established, the next critical step involves addressing the force equilibrium and dynamic motion of the nanobeam. This necessitates a framework that seamlessly integrates the internal stress states with external influences and boundary conditions. Hamilton's principle emerges as an exceptionally potent variational method for achieving this integration. Its strength lies in its ability to simultaneously derive the governing equations of motion and the natural boundary conditions, all while inherently incorporating the effects of external forces on the body's stress state. Unlike traditional equilibrium-based approaches, Hamilton's principle operates on the principle of least action, considering the integral of the Lagrangian over time. This Lagrangian, defined as the difference between the kinetic and potential energies of the system, provides a holistic representation of the system's energy dynamics. By minimizing the time integral of the Lagrangian, Hamilton's principle yields the equations of motion that accurately describe the system's temporal evolution. Furthermore, the principle automatically accounts for the natural boundary conditions, which are constraints that arise from the variational process itself, ensuring that the derived equations are physically consistent. The core formulation of Hamilton's principle, which elegantly encapsulates these concepts, is expressed as follows:

$$\int_{t_1}^{t_2}\big(\delta\Pi_k-\delta\Pi_e+\delta\Pi_{w_1}+\delta\Pi_{w_2}\big)dt=0 \tag{7}$$

Here, variables Π_e and Π_k represents energies associated with elastic and kinetic, respectively, the external work Π_w is split in Π_{w_1}, and Π_{w_2} whose definition depends on the elastic Winkler-Pasternak and the mechanical loading, respectively. The variation of energy expressions is defined in the following relations:

$$\delta\Pi_k=\int\rho\left[\sum_{i=1}^{3}\frac{\partial U_i}{\partial t}\frac{\partial\delta U_i}{\partial t}\right]dV \tag{8}$$

$$\delta\Pi_e=\int\{\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{zz}\delta\varepsilon_{zz}+\tau_{xz}\delta\gamma_{xz}\}dV \tag{9}$$

here $U_i=U,V,W$ for $i=1,2,3$ as the displacement components in general continuum material. In addition, work conducted by the elastic foundation on the nanobeam structure can be obtained as:

$$\delta\Pi_{w_1}=\int\{-K_w w_0+K_p\nabla^2 w_0\}\delta w_0 dA \tag{10}$$

Within the context of the elastic foundation supporting the nanobeam, $K_w\left(\frac{N}{m^3}\right)$ and $K_p\left(\frac{N}{m}\right)$ represent the Winkler and Pasternak constants, respectively. These constants are crucial for accurately modeling the interaction between the beam and its surrounding environment. The Winkler constant, *k_w*, quantifies the stiffness of the foundation, representing the resistance to vertical displacement at any given point. In essence, it describes the force per unit area required to produce a unit displacement. The Pasternak constant, K_p, on the other hand, accounts for the shear interaction between adjacent points in the foundation. It represents the resistance to shear deformation within the foundation material, effectively capturing the interconnectedness of the support medium. These constants are essential in simulating the real-world behavior of the nanobeam, particularly in applications where it rests on or interacts with a deformable substrate. The values of $K_w\left(\frac{N}{m^3}\right)$ and $K_p\left(\frac{N}{m}\right)$ are determined by the specific material properties and geometry of the foundation.

Following the establishment of these constants, the variation of work performed by internal forces within the nanobeam can be expressed. This variation, denoted as $\delta\Pi_{w_2}$, represents the change in internal work due to infinitesimal variations in the displacement field. It is a critical component in the application of Hamilton's principle, as it allows us to relate the internal stress states to the overall energy balance of the system. The mathematical expression for $\delta\Pi_{w_2}$, which incorporates the constitutive equations and strain components, is as follows:

$$\delta\Pi_{w_2}=-\int P\frac{\partial^2 w_0}{\partial x^2}\delta w_0 dA \tag{11}$$

P represents the mechanical loading. Also, by substitution of Eqs. (8-11) into Eq. (7), after a mathematical manipulation we get the following equations

$$\begin{aligned}\delta u_0 \quad &: \quad \frac{\partial N_{xx}}{\partial x}=I_0\frac{\partial^2 u_0}{\partial t^2}+I_1\frac{\partial^2 u_1}{\partial t^2},\\ \delta w_0 \quad &: \quad \frac{\partial N_{xz}}{\partial x}-K_w w_0+K_P\frac{\partial^2 w_0}{\partial x^2}-P\frac{\partial^2 w_0}{\partial x^2}=I_0\frac{\partial^2 w_0}{\partial t^2},\end{aligned} \tag{12}$$

$$\delta u_1 \quad : \quad \frac{\partial M_{xx}}{\partial x} - N_{xz} = I_1 \frac{\partial^2 u_0}{\partial t^2} + I_2 \frac{\partial^2 u_1}{\partial t^2},$$

where $\{I_0, I_1, I_2\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^L \{1, z, z^2\} \rho(x,z) dx dz$. The resultant boundary conditions are obtained as follows:

$$\begin{aligned} &\delta u_0 = 0 \quad or \quad \hat{n}_x N_{xx} = 0\,, \\ &\delta w_0 = 0 \quad or \quad \hat{n}_x N_{xz} = 0\,, \\ &\delta u_1 = 0 \quad or \quad \hat{n}_x M_{xx} = 0\,. \end{aligned} \tag{13}$$

In the above boundary conditions:

$$\begin{aligned} \left\{N_{xx}, M_{xx}\right\} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^L \{1, z\} \sigma_{xx} dx\, dz, \\ \{N_{xz}, M_{xz}\} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_0^L \{1, z\} \tau_{xz} dx dz. \end{aligned} \tag{14}$$

2.5 The nonlocal stress-strain gradient formulation

The nonlocal elasticity theory and strain gradient theory were developed separately. They were found to be useful and provide satisfactory results in certain small-scale analyses. Specifically, the nonlocal theory is worthy in the nano-scale problems and strain gradient theory in micro-scale systems. For the structure containing both nano and micro effects, a combination of these two theories is developed which is called nonlocal stress-strain gradient theory. The main formulation of this theory is to add the extra terms that arise for stress components to the stress components of the classical elasticity:

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla \sigma_{ij}^{(1)} \tag{15}$$

In the above equation, the final stress σ_{ij} is a combination of nonlocal stress $\sigma_{ij}^{(0)}$ and gradient of stress $\sigma_{ij}^{(1)}$ arises from the gradient of the strain tensor $\nabla \varepsilon_{ij}$. These two terms are defined as:

$$\sigma_{ij}^{(0)} = \int_0^L \alpha_0(\boldsymbol{x}, \boldsymbol{x}', e_0 a/l) \sigma_{ij}^c(\boldsymbol{x}')\, d\boldsymbol{x}' \tag{16a}$$

$$\sigma_{ij}^{(1)} = l^2 \int_0^L \alpha_1(\boldsymbol{x}, \boldsymbol{x}', e_1 a/l) \nabla \sigma_{ij}^c(\boldsymbol{x}')\, d\boldsymbol{x}' \tag{16b}$$

In the above definitions, e_i is a constant to be calibrated for each material. Parameter a represents a characteristic length associated with the crystal structure of the material and l is the length of external micro-size features of the body like the crack length. Kernels of the integral define the dependency of the corresponding stress component on the classical stress and gradient of classical stress for nonlocal and strain gradient theories, respectively. The length of the concrete beam in this study is denoted by L. With the appropriate selection of the kernels α_i, the integral form converts to differential form with the following operator:

$$\mathbb{L}_i = 1 - (e_i a)^2 \nabla^2 \quad \text{for } i = 0,1 \tag{17}$$

Finally, using the above differential operator the final form of the nonlocal-stress-strain gradient becomes:

$$\begin{aligned} &[1 - (e_1 a)^2 \nabla^2][1 - (e_0 a)^2 \nabla^2] \sigma_{ij} \\ &= [1 - (e_1 a)^2 \nabla^2] \sigma_{ij}^c - [1 - (e_0 a)^2 \nabla^2] l^2 \nabla^2 \sigma_{ij}^c \end{aligned} \tag{18}$$

here ∇^2 is the Laplacian operator. Since we dealing with small deformations, the higher-order differential could be omitted to obtain the following form of the equation which will be utilized in the next calculations:

$$[1 - (ea)^2 \nabla^2] \sigma_{ij} = (1 - l^2 \nabla^2) \sigma_{ij}^c \tag{19}$$

The final form of separated differential equations based on the displacement components are:

$$\begin{aligned} &\delta u_0 \quad : \quad \left(1 - l^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial N_{xx}}{\partial x} \\ &= \left(1 - \mu^2 \frac{\partial^2}{\partial x^2}\right) \left(I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 u_1}{\partial t^2}\right) \\ &\delta w_0 \quad : \quad \left(1 - l^2 \frac{\partial^2}{\partial x^2}\right) \frac{\partial N_{xz}}{\partial x} + \left(1 - \mu^2 \frac{\partial^2}{\partial x^2}\right) \\ &\left(-K_w w_0 + K_P \frac{\partial^2 w_0}{\partial x^2} - P \frac{\partial^2 w_0}{\partial x^2}\right) \\ &= \left(1 - \mu^2 \frac{\partial^2}{\partial x^2}\right) \left(I_0 \frac{\partial^2 w_0}{\partial t^2}\right), \\ &\delta u_1 \quad : \quad \left(1 - l^2 \frac{\partial^2}{\partial x^2}\right) \left(\frac{\partial M_{xx}}{\partial x} - N_{xz}\right) \\ &= \left(1 - \mu^2 \frac{\partial^2}{\partial x^2}\right) \left(I_1 \frac{\partial^2 u_0}{\partial t^2} + I_2 \frac{\partial^2 u_1}{\partial t^2}\right) \end{aligned} \tag{20}$$

where $\mu = ea$.

3. Solution procedure

To precisely illustrate the numerical approximations central to the Differential Quadrature Approximation (DQA), we consider a one-dimensional function *f(x)*. The DQA method elegantly converts the continuous operation of differentiation into a discrete algebraic form. This is achieved by expressing the *p*th derivative of *f(x)* at a specific node, x_i, as a linear combination of the function values at all other nodes within the domain. The coefficients of this combination, known as weighting coefficients, are meticulously determined to ensure accuracy. The subsequent formula encapsulates this fundamental transformation, revealing how the derivative is approximated through a weighted summation of the function's values at discrete points:

$$\begin{aligned} &\frac{\partial^p f(x)}{\partial x^p} = \sum_{j=1}^{N} A_{ij}^{(p)} f(x) \\ &for\ i = 1,2,\ldots,N\ and\ p = 1,2,\ldots,N-1, \end{aligned} \tag{21}$$

To implement the Differential Quadrature Approximation (DQA), the solution domain is partitioned into N discrete grid points. These points are the foundation for the numerical approximation of derivatives. The weighting

coefficients, represented by $A_{ij}^{(p)}$, are the mathematical constructs that enable the DQA to approximate the *p*th derivative at the *i*th grid point using the function's values at the *j*th grid point. The accurate computation of these coefficients is paramount to the validity of the DQA method. The following procedure outlines the detailed steps involved in calculating these weighting coefficients, which are essential for transforming continuous derivatives into discrete algebraic expressions.

3.1 Differential quadrature approach (DQA)

The Differential Quadrature Approximation (DQA) employs a recursive relation to derive the weight coefficients, $A_{ij}^{(p)}$, for derivatives of second order and higher (i.e., p $\geq$ 2). This recursive formulation is mathematically derived to maintain consistency and accuracy in the approximation of higher-order derivatives. The following expression provides the mathematical basis for this recursive calculation:

$$A_{ij}^{(p)} = n\left(A_{ii}^{(p-1)}A_{ij}^{(1)} - \frac{A_{ij}^{(p-1)}}{x_i - x_j}\right), \quad p = 2,3,\dots,N-1 \; and \; i,j = 1,2,\dots,N, \tag{22}$$

here $A_{ij}^{(1)}$ would be determined as

$$A_{ij}^{(1)} = \frac{M^{(1)}(x_i)}{M^{(1)}(x_j)} \times \frac{1}{x_i - x_j}, i,j = 1,2,\dots,N. \tag{23}$$

The weight coefficients for the case of $i = j$ would be determined as

$$A_{ii}^{(p)} = -\sum_{j=1, j\neq i}^{N} A_{ij}^{(p)}, \quad i = 2,3,\dots,N \; and \; p = 1,2,\dots,N-1, \tag{24}$$

In Eq. (21), $M^{(1)}$ would be obtained as follow

$$M^{(1)}(x_k) = -\sum_{j=1, j\neq k}^{N} (x_k - x_j), for k = 1,2,3,\dots,N. \tag{25}$$

The matric form of eigenvalue problems is encapsulated as follows:

$$\{[M]\omega^2 + [K]\}\begin{Bmatrix} u \\ w \end{Bmatrix} = 0 \tag{26}$$

Eq. (26) is used to extract the natural frequency of the process The below dimensionless factors are defined to be used in presenting results:

$$\bar{\omega} = \omega L^2\sqrt{\frac{\rho A}{EI}}, \quad \bar{K}_w = K_w\frac{L^5}{EI}, \quad \bar{K}_p = K_p\frac{L^3}{EI}, \quad \bar{P} = \frac{P}{E} \tag{27}$$

4. Deep learning-driven comparative evaluation

The deep neural network uses mathematical modeling to directly relate the affecting parameters to the desired outputs without engaging the fundamental physical modeling of the system. In the current study, there are several factors affecting the vibrational behavior of concrete nanobeam structure including small-scale effects, substrate properties, functionally grading properties, and the method of formulation and solution. The inclusion of all these parameters in mathematical modeling, besides the approximations carried out in the modeling, results in a cumbersome solution procedure. In the DNN method, all the procedures are bypassed and the inputs are directly connected to outputs through simple special regression methods. To this aim, the input parameters influencing natural frequency are selected to be a combination of material and geometrical properties as well as parameters of physical modeling $X = \left\{\frac{L}{h}, \mu, l, n_x, n_z, K_w, K_p, P\right\}^T$. The output of the network is the natural frequency of the structure which is the main parameter in determining the vibrational behavior of the concrete nanobeam $Y = \omega$. The training process of the DNN is carried out by comparing the differences between predicted natural frequency by DNN and the calculated values obtained by nonlocal stress-strain gradient $\hat{Y}$. In this regard, the mean squared error (MSE) is used to compare these two values:

4.1 Adaptive parameter optimization of DNNs using ADADELTA

Training a DNN involves a series of processes of adjusting the internal parameters of the network. This adjusting process is carried out using optimization processes. There are several optimization approaches developed for this purpose. The descent gradient is of the famous approaches. However, the newly developed method of ADADELTA is proven to be more effective and requires less manual intervention to adjust the parameter. This method is insensitive to hyperparameters while setting the learning rate automatically. There are several studies comparing the performance of this method to other methods. The superior efficiency of this optimization method is the reason that we utilized it in the training process of DNN.

5. Result

Having completed the formulation and solution procedure description, the results of this methodology are now compared with two other references mentioned in Table 1. In these data, the slenderness of the concrete nanobeam and nonlocal parameters are varied to find the dimensionless natural frequency of the structure. As could be seen, the results of the Timoshenko beam theory with nonlocal stress-strain gradient theory are in close correlation with the results of the other two references. However, the results of TBT are slightly lower than the results achieved by other theories.

The second approach to obtain the dimensionless frequency of concrete nanobeam structure is using DNN method. In the training process of the DNN, the error could be achieved in very low values maintaining enough epochs in the process. The error margin for different slenderness of the concrete nano-scale beam is seen in Table 2.

Table 1 Validation of dimensionless frequency results using the current methodology by comparison to two other references

L/h	μ	Ref. (Nejad *et al*. 2016)	Ref. (Eltaher *et al*. 2013)	TBT
10	0	22.2594	22.4926	22.2112
	1	20.9742	21.1688	20.9251
	2	19.8843	20.0484	19.8138
	3	18.9454	19.085	18.9361
	4	18.1260	18.2458	18.1271
	5	17.4031	17.5056	17.3891
20	0	22.3446	22.4022	22.3361
	1	21.0751	21.1236	21.0692
	2	19.9953	20.0368	19.9882
	3	19.0631	19.099	19.0561
	4	18.2481	18.2795	18.2382
	5	17.5279	17.5556	17.5195
100	0	22.3721	22.3744	22.3642
	1	21.1076	21.1096	21.0991
	2	20.0313	20.033	20.0232
	3	19.1013	19.1028	19.0998
	4	18.2877	18.289	18.2731
	5	17.5685	17.5696	17.5582

Table 2 Values of dimensionless natural frequency as obtained by the DNN method for different MSE margins

		Predicted			
		MSE_{Train} $= 0.15 \times 10^{-6}$	MSE_{Train} $= 0.5 \times 10^{-6}$	MSE_{Train} $= 0.55 \times 10^{-6}$	MSE_{Train} $= 0.6 \times 10^{-6}$
$l = 0$	16.6146	16.8261	16.6154	16.6151	16.6151
$l = h$	16.7857	16.9172	16.7867	16.7860	16.7860
$l = 2h$	17.2012	17.4091	17.2021	17.2015	17.2015
$l = 3h$	17.8356	17.9901	17.8370	17.8364	17.8364
$l = 4h$	18.6339	18.8032	18.6349	18.6344	18.6344

Table 3 Convergence study of the dimensionless first fundamental frequency for various boundary conditions with L/h=50, $\mu = h,\ l = h,\ n_x = 0.5,\ n_z = 0.5,\ \bar{P} = 0.01,\ \bar{K}_w = 1000$, and $\bar{K}_p = 1000$

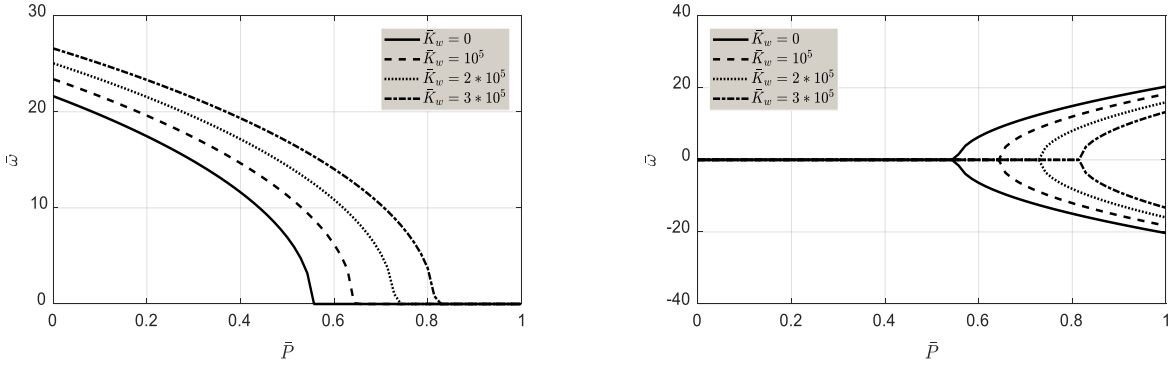
	(N)			
	10	15	20	25
SS	8.0720	8.0718	8.0718	8.0718
CS	14.0463	14.0593	14.0593	14.0593
CC	21.3113	21.3071	21.3071	21.3071

The numerical solution approach in the current study is highly dependent on the discretization of the domain. The number of grids N leading to convergence in frequency results is presented in Table 3 for different types of boundary conditions. As seen, at least 15 grids are required to reach converge results for first mode natural frequency.

Effects of several parameters that appeared in the nonlocal stress-strain gradient formulation of the concrete nanobeam are presented in the followings. In Fig. 2, the condition of vibrational stability of concrete nanobeam is depicted as a function of variation in the K_w parameter of Winkler-Pasternak substrate. As seen, an increase in the parameter K_w results in more stability of the concrete nanobeam by shifting the critical point of instability to the higher values of $\bar{P}$.

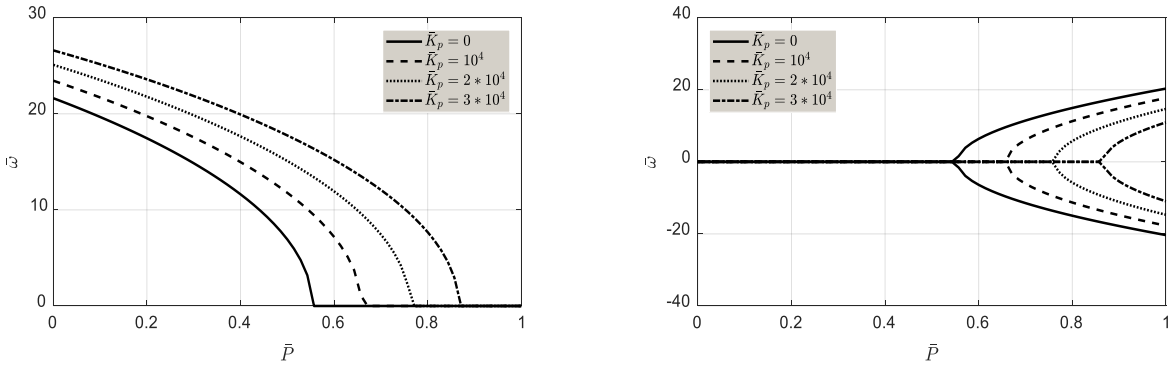
The influence of parameter $\bar{K}_p$ is similar to the effect of parameter $\bar{K}_w$ as observed in Fig. 3. Increase in the parameter $\bar{K}_p$ results in more stability of the concrete nanobeam by shifting the crtitical point of instability to the higher values of $\bar{P}$.

The dimensional aspect ratio of the concrete nanobeam could alter the state of the stability of vibrational motions. It



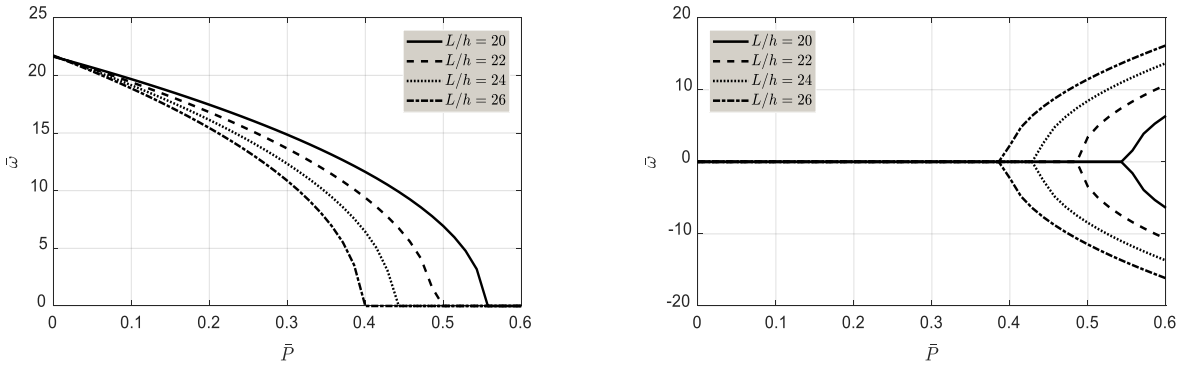
(a) Real part (b) Imaginary part

Fig. 2 Frequency coefficient vs. mechanical variation, for different elastic foundation coefficients



(a) Real part (b) Imaginary part

Fig. 3 Frequency coefficient vs. mechanical variation, for different Pasternak foundation coefficients



(a) Real part (b) Imaginary part

Fig. 4 Frequency coefficient vs. mechanical variation, for different L/h

is clear with the higher slenderness of the nanobeam the condition of the stability worsened as deduced also from curves in Fig. 3. In this figure, shorter nanobeam manifest higher stability as the critical stability point moved to the higher $\bar{P}$.

Fig. 5 elucidates the impact of size-dependent parameters, namely nonlocality and length scale, on the natural frequency of the concrete nanobeam, specifically under varying levels of the dimensionless axial load parameter, denoted as $\bar{P}$. This analysis reveals a distinct relationship between nonlocality and vibrational behavior. As the nonlocality parameter increases, signifying a greater influence of long-range interatomic interactions, the natural frequency of the nanobeam systematically diminishes. This observation implies that enhanced nonlocality reduces the effective stiffness of the nanobeam, leading to a decrease in its inherent vibrational frequencies. Conversely, an increase in the length scale parameter, which accounts for the influence of strain gradients and the material's internal microstructure, results in a notable improvement in the nanobeam's stability. This improvement is manifested as an increase in the natural frequency, indicating that a larger length scale contributes to increased structural rigidity and resistance to deformation. Furthermore, the dimensionless

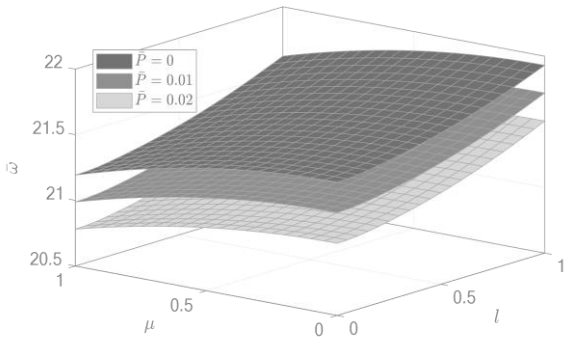


Fig. 5 Frequency coefficient vs. μ, and l, for different mechanical loading

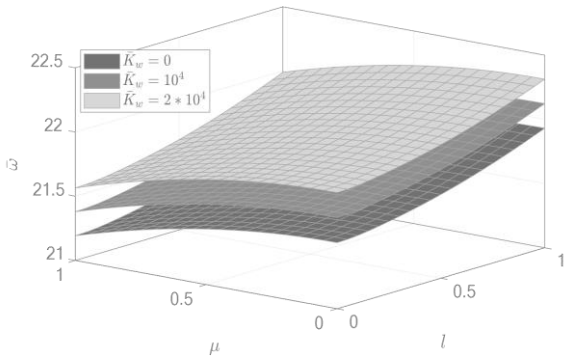


Fig. 6 Frequency coefficient vs. μ, and l, for different elastic foundation coefficients

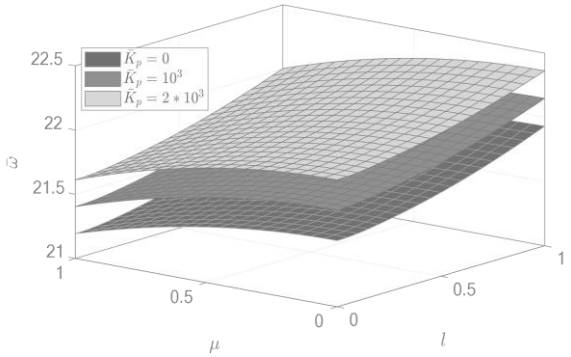
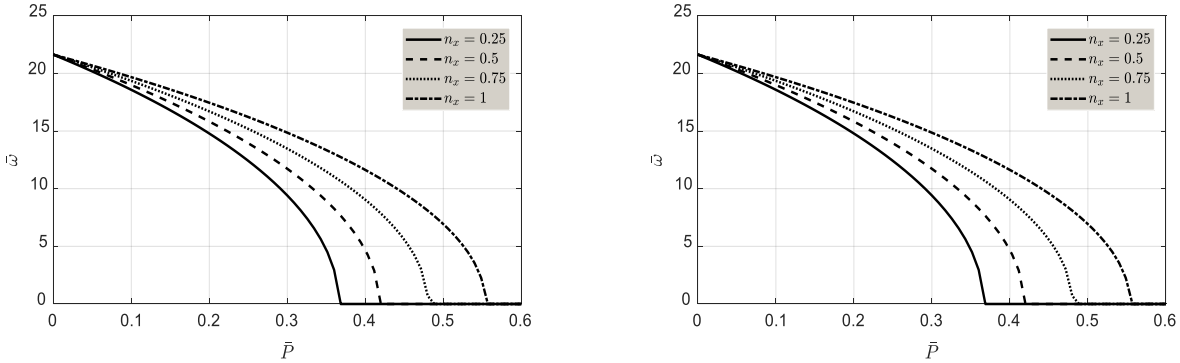


Fig. 7 Frequency coefficient vs. μ, and l, for different Pasternak foundation coefficients

axial load parameter, $\bar{\mathrm{P}}$, consistently exhibits a destabilizing effect on the nanobeam, as observed in previous figures. An increase in $\bar{\mathrm{P}}$, which represents the magnitude of the axial load applied to the nanobeam, results in a decrease in the natural frequency. This trend suggests that higher axial loads weaken the nanobeam's resistance to vibrations, making it more susceptible to instability. The combined effects of nonlocality, length scale, and axial load, as depicted in Fig. 5, underscore the complex interplay of these parameters in determining the dynamic behavior of nanoscale structures. These insights are crucial for the accurate design and analysis of nanobeams in various engineering applications.

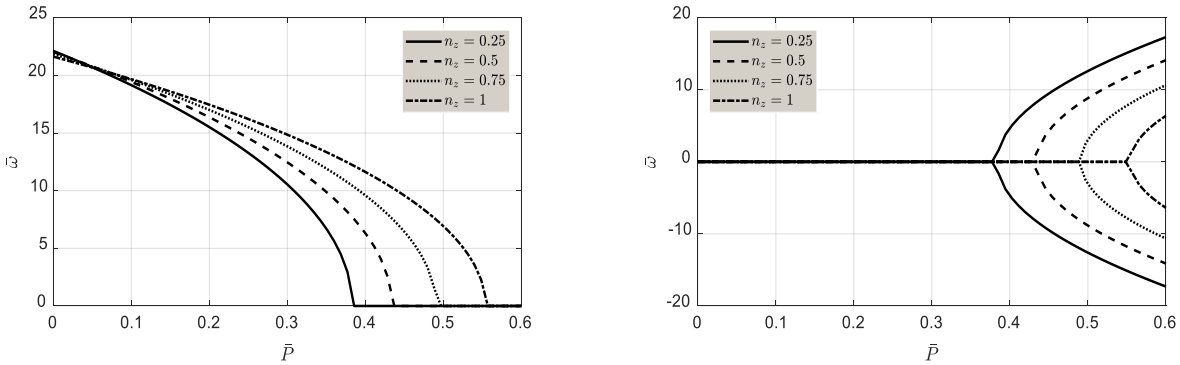
Expanding upon the analysis of material properties, Fig. 6 delves into the influence of size-dependent parameters—specifically, nonlocality and the length scale—on the natural frequency of the concrete nanobeam, while considering varying values of the dimensionless Winkler parameter, denoted as $\bar{K}_w$. This figure reveals a clear trend regarding the impact of nonlocality: as the nonlocality parameter increases, indicating a stronger influence of long-range interatomic forces, the natural frequency of the nanobeam consistently decreases. This suggests that heightened nonlocality reduces the effective stiffness of the nanobeam, thereby lowering its vibrational frequencies. Conversely, an increase in the length scale parameter, which reflects the material's internal microstructure and captures the effects of strain gradients, leads to a noticeable enhancement in the nanobeam's stability. This manifests as an increase in the natural frequency, signifying that a larger length scale contributes to increased stiffness and resistance to deformation. This reinforces the importance of considering microstructural effects when analyzing nanoscale structures. Furthermore, the dimensionless Winkler parameter, $\bar{K}_w$, which represents the stiffness of the elastic foundation supporting the nanobeam, consistently demonstrates its stabilizing effect, as observed in previous figures. An increase in $\bar{K}_w$ leads to an improvement in the natural frequency, indicating that a stiffer foundation enhances the overall rigidity of the nanobeam. This observation underscores the synergistic interplay between the foundation stiffness, size-dependent parameters, and the material's internal microstructure in determining the vibrational characteristics of the nanobeam. The complex interactions highlighted in Figure 6 emphasize the necessity for a comprehensive understanding of these parameters in the design and optimization of nanoscale structures for specific applications.

Building upon the observed trends, Fig. 7 delves into the intricate interplay between size-dependent parameters—specifically, nonlocality and the length scale—and the natural frequency of the functionally graded concrete nanobeam. This investigation is conducted across a spectrum of parameter values, allowing for a comprehensive understanding of their influence. The graph reveals a distinct inverse relationship between nonlocality and natural frequency. As the nonlocality parameter increases, indicating a stronger influence of long-range interatomic forces, the natural frequency of the nanobeam systematically decreases. This suggests that heightened nonlocality diminishes the stiffness of the nanobeam, thereby lowering its vibrational frequencies. Conversely, an increase in the length scale parameter, which reflects the material's internal microstructure, leads to a pronounced enhancement in the nanobeam's stability. This manifests as an elevation in the natural frequency, implying that a larger length scale contributes to increased stiffness and resistance to deformation. Furthermore, the parameter, consistently observed to augment the natural frequency in preceding analyses, continues to exhibit its stabilizing effect in this context. Its influence reinforces the nanobeam's rigidity, further contributing to the observed frequency trends. This synergistic interplay of size-dependent parameters and the



(a) Real part (b) Imaginary part

Fig. 8 Frequency coefficient vs. mechanical variation, for different $\boldsymbol{n_x}$



(a) Real part (b) Imaginary part

Fig. 9 Frequency coefficient vs. mechanical variation, for different $\boldsymbol{n_z}$

parameter underscores the complex mechanics governing the vibrational behavior of these nanoscale structures, highlighting the necessity for precise control of these parameters in engineering applications.

The distribution of constituent materials within the concrete nanobeam, characterized by its grading properties, exerts a decisive influence upon the dimensionless fundamental frequency of the structure. Fig. 8 presents a graphical representation of the correlation between variations in the grading index along the x-axis, n_x, and the stability and dimensionless natural frequency of the nanobeam. The empirical evidence indicates a direct proportionality between the magnitude of the grading index, n_x, and the structural stability of the concrete nanobeam. Specifically, an incremental increase in n_x results in a discernible displacement of instability points towards higher parameter values. This phenomenon suggests that a higher grading index confers enhanced resistance to structural instability, thereby augmenting the overall dynamic integrity of the nanobeam. This observation underscores the critical role of material gradation in the design and optimization of nanoscale structures for specific operational environments.

The impact of variations in the grading index along the z-axis, represented by n_z, on the stability and dimensionless natural frequency of the concrete nanobeam is graphically presented in Fig. 9. The data presented therein demonstrates a direct correlation between the magnitude of n_z and the structural stability of the nanobeam. An increase in n_z results in a significant displacement of instability points towards higher values of the parameter $\bar{P}$. This observable phenomenon indicates that a higher grading index in the z-direction confers enhanced resistance to structural instability, thereby augmenting the overall dynamic integrity of the nanobeam. The findings underscore the critical role of material gradation in the design and optimization of nanoscale structures for demanding operational environments.

Effects of variation in slenderness on the dimensionless natural frequency of the nanobeam structure are examined in more detail in the following. Fig. 10 shows the decrease in the dimensionless natural frequency of the current structure as a result of an increase in the ratio of length to the height of the nanobeam. The influence of the increase in L/h is more prominent in the high values of $\bar{P}$ parameter. In low values of $\bar{P}$, the dimensionless natural frequency seems to increase slightly at low slenderness and become constant subsequently.

Fig. 11 demonstrates the increase in the dimensionless natural frequency of the nanobeam structure as a result of an increase in the ratio of length to height L/h of the structure. The influence of an increase in L/h is more prominent in the low values of $\bar{K}_w$ parameter. In high values of $\bar{K}_w$, the dimensionless natural frequency seems to increase slightly at low slenderness and become constant subsequently.

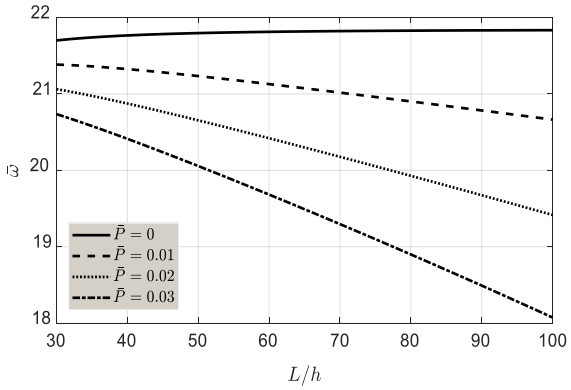


Fig. 10 Frequency coefficient vs. L/h variation, for different mechanical loading

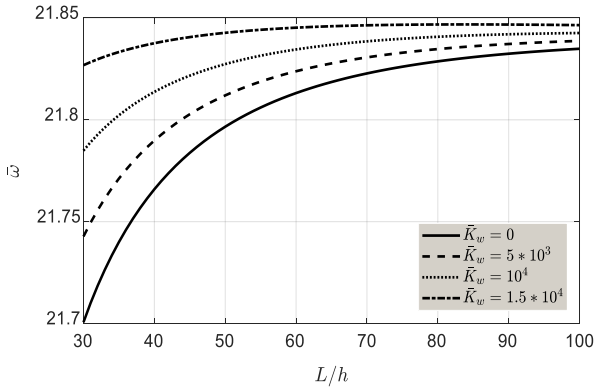


Fig. 11 Frequency coefficient vs. L/h variation, for different elastic foundation coefficient

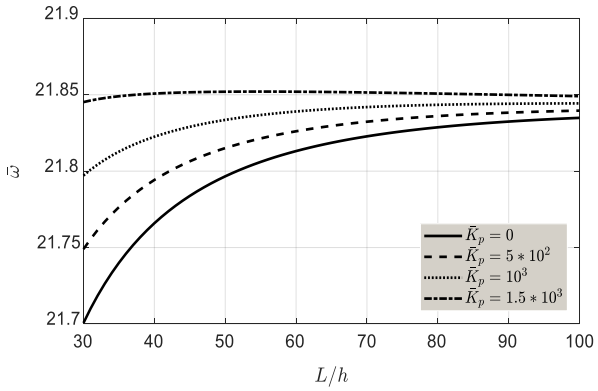


Fig. 12 Frequency coefficient vs. L/h variation, for different Pasternak foundation coefficient

Figs. 11 and 12 demonstrate the increase in the dimensionless natural frequency of the nanobeam structure as a result of an increase in the ratio of length to height L/h of the structure. The influence of an increase in L/h is more prominent in the low values of $\bar{K}_p$ parameter. In high values of $\bar{K}_p$, the dimensionless natural frequency seems to increase slightly at low slenderness and become constant subsequently. However, the effect of $\bar{K}_p$ parameter is not important as $\bar{K}_w$ parameter.

6. Conclusions

In the present study, the vibration of a nanoscale beam structure made of functionally graded concrete material was examined. In doing so, two different approaches of mathematical structural modeling and deep neural network were utilized to obtain the natural frequency of the nanobeam structure. The concrete was assumed to be graded in two directions namely the axis of the constant cross-section beam in the transverse direction. The grading properties followed the power-law model. The displacement field was assumed to be TBT and nonlocal stress-strain gradient related to the state of stress and strain in the nanobeam. On the other hand, the influence of external forces and boundary conditions were taken into account using Hamilton's principle. The deep neural network was also trained to gain different error margins in predicting natural frequency. The extracted governing equations were solved using DQM. The results of the current approach were validated by comparing results with other similar problems in the literature. An extensive parametric study was also conducted to observe various material, geometrical and Winkler_Paternal substrate parameters on the stability and natural frequency of the concrete nanobeam structure. It was revealed that:

- The slenderness of concrete nanobeam does not necessarily decrease the dimensionless natural frequency and substrate constant could diversely change its effect.
- An increase in the grading index significantly improves the stability of concrete nanobeam structure as the instability points shifted to larger values of $\bar{P}$.
- An increase in the nonlocality decreases the dimensionless natural frequency while an increase in the length scale parameter improves the stability by increasing the dimensionless natural frequency.

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