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Advancing sports equipment performance: Leveraging rotating small-scale structures for enhanced athletic tools

Yuan Wan[1], Guizhi Zhang[1], Zimin Chang*[1], Mostafa Habibi[2,3,4], Ibrahim Albaijan[5] and Yang Li[6]

[1]College of Sports and Health, Nanchang Institute of Science and Technology, Nanchang 330108, Jiangxi, China
[2]Universidad UTE, Facultad de Arquitectura y Urbanismo, Calle Rumipamba S/N y Bourgeois, Quito 170147, Ecuador
[3]Department of Biomaterials, Saveetha Dental College and Hospital, Saveetha Institute of Medical and Technical Sciences, Chennai 600 077, India
[4]Institute of Research and Development, Duy Tan University, Da Nang 550000, Viet Nam
[5]Mechanical Engineering Department, College of Engineering at Al Kharj, Prince Sattam Bin Abdulaziz University, Al Kharj, 11942, Saudi Arabia
[6]Institute Sciences and Design of AL-Kharj, Dubai, United Arab Emirates

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Abstract. The use of sophisticated materials and nanoscale structures in the design of sports equipment is recognized as a key strategy for boosting athletic performance. The study of spinning small-scale structures, such as nanobeams and nanotubes, is centered on their potential use in the creation of next-generation sporting equipment. The distinct characteristics of these constructions, such as improved stiffness, vibration damping, and longevity, play an important role in improving the efficiency, control, and responsiveness of various athletic equipment. Nanomaterials are used in tennis rackets, golf clubs, and hockey sticks to efficiently eliminate undesired vibrations while increasing energy transfer upon impact, boosting player comfort and performance. These structures' rotational dynamics closely resemble real-world circumstances encountered by sports equipment, such as the swinging motion of a bat and the bending of a ski. The nonlocal strain gradient theory provides useful insights for improving material behavior in dynamic loading situations, notably in terms of size effects at the nanoscale. Case studies and practical examples demonstrate how these innovations support athletes in improving their power, accuracy, and the longevity of their equipment. A connection exists between nanotechnology and sports engineering, facilitating the development of lighter, stronger, and more efficient technologies that enhance athletic performance capabilities. The significance of diverse methods for enhancing sports technology is emphasized, providing advantages for both elite athletes and recreational users.
Keywords: athletic performance; material optimization; nanoscale structures; nonlocal strain gradient theory; rotating dynamics; sports equipment

1. Introduction

The convergence of materials science and sports engineering has unveiled novel pathways for enhancing athletic performance via innovative design and cutting-edge technologies. In recent decades, the integration of advanced materials, including composites and nanomaterials, into sports equipment has resulted in notable enhancements in efficiency, control, and responsiveness (Gao *et al*. 2025, He *et al*. 2025, Wang *et al*. 2025c). Contemporary tennis rackets, golf clubs, and hockey sticks are crafted from advanced lightweight materials that reduce vibrations, enhance energy transfer, and increase durability (Zhang *et al*. 2022, Sakamoto *et al*. 2023, Wang *et al*. 2023b). In light of these advancements, it is imperative to enhance the mechanical performance of sports equipment when subjected to dynamic loading conditions, especially those characterized by rotational or bending movements (Man *et al*. 2024, Wang *et al*. 2024i, Zhao *et al*. 2024).

*Corresponding author, M.S.,
E-mail: chang_zimin@163.com

1.1 Background on nanomaterials in sports equipment

Nanomaterials, distinguished by their exceptional characteristics at the nanoscale, have surfaced as compelling contenders for the advancement of sports technologies in the future. These materials demonstrate exceptional stiffness, effective vibration damping properties, and remarkable fatigue resistance, rendering them highly suitable for advanced athletic equipment. Carbon nanotubes (CNTs) have undergone significant examination due to their remarkable mechanical, thermal, and electrical characteristics. Their incorporation into composite materials has already been observed in a range of sports applications, such as tennis rackets, bicycle frames, and running shoes (He and Deng 2023, Li *et al*. 2023, Li 2023, Ma *et al*. 2023, Song *et al*. 2023, Wang *et al*. 2023a, 2024c, Yan *et al*. 2024).

Recent investigations have elucidated the promise of nanomaterials in tackling significant challenges encountered by athletes. CNT-reinforced composites demonstrate a significant capacity to mitigate undesirable vibrations in tennis rackets, thereby enhancing player comfort and control throughout the game (Li *et al*. 2024c, Wei *et al*. 2024, Wu *et al*. 2024). In golf clubs, the incorporation of

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nanomaterials enhances the efficiency of energy transfer during impact, enabling players to attain greater distances with reduced exertion. Moreover, the integration of nanomaterials into protective equipment, including helmets and pads, provides superior shock absorption while minimizing weight, thus enhancing safety without sacrificing mobility (Omidi *et al*. 2013, Mousavi *et al*. 2017, Shahabinejad *et al*. 2018).

Research demonstrates the importance of accounting for size-dependent effects in the analysis of nanomaterial behavior. Dai *et al*. (2021) examined the frequency characteristics and sensitivity of size-dependent laminated nanoshells, demonstrating that material heterogeneity and surface effects substantially affect their dynamic responsiveness. Li *et al*. (2022) utilized a two-dimensional continuum mechanics framework to forecast the bending responses of graphene-reinforced higher-order annular plates, offering valuable insights into their structural behavior under diverse loading conditions. Zhang *et al*. (2024c) investigated the bending responses of sandwich cylindrical micro-panels reinforced with graphene nano-platelets and integrated piezoelectric layers, emphasizing the role of nanomaterials in improving the performance of sporting equipment. Huang *et al*. (2024) proposed a bending-based solution methodology for the analysis of foldable reinforced golf club cylinder shells through eigenvalue-eigenvector methods, emphasizing the significance of advanced modeling techniques in the optimization of sports equipment design.

1.2 Importance of rotating dynamics in sports

Even though the static properties of nanomaterials have been well documented, the way nanomaterials behave in dynamic conditions, particularly in rotational dynamics, hasn't been explored in depth. Rotational motion is crucial in many athletic activities, like swinging a bat, throwing a javelin, or bending while skiing. To create sporting equipment that works well in real-life scenarios, it's essential to understand how small-scale structures behave mechanically in those situations. The behavior of sports equipment can be modeled well using small rotating objects like nanobeams and carbon nanotubes (CNTs), which have specific characteristics that make them ideal for simulation. For instance, nanobeams could be utilized to mimic the bending and twisting of a ski effectively. At the same time, carbon nanotubes (CNTs) might be applied to represent the rotating motion of a golf club shaft (Guo *et al*. 2024, Jin *et al*. 2024, Wang *et al*. 2024h, j, Xiao *et al*. 2024, Yu *et al*. 2024, Zhiqiang *et al*. 2024, Zisong and Habibi 2024). The high level of flexibility and strength in these structures allows them to handle the stresses and strains of sports activities. lightweight means they won't add unnecessary bulk to the gear, helping athletes maintain the agility and speed they require. Chen *et al*. (2022) studied composite curving pipes' dynamic stability and aeroelastic characteristics. The study emphasized the importance of considering boundary conditions and environmental factors when analyzing rotating structures. The findings of their study emphasize the importance of creating more comprehensive models that can effectively capture the complex interactions between geometry, material properties, and external stresses. Wang *et al*. (2024d) also looked into how stable a volleyball game ball is by applying generalized differential quadrature (GDQ) and some analytical methods. This study highlighted the importance of accurate modeling for effectively predicting how sports-related systems respond dynamically. Zhu *et al*. (2024b) used revised higher-order shear deformation theory to analyze how bulk waves disperse in Poro-elastic gymnastics beams affected by hygrothermal conditions during athlete training. This study shows that it's essential to consider environmental conditions when designing sports equipment. Ma *et al*. (2024) also did some static and dynamic evaluations of sandwich micro-plates by using modified strain gradient theory. The analyses really emphasized how critical size-dependent factors are in shaping the mechanical behavior of small-scale structures.

1.3 Research gap and objectives

Even though there's a lot of excitement about using nanotechnology in sports, not much research has been done on how small-scale structures rotate in this area. Many studies tend to concentrate on static or quasi-static analyses, often overlooking the dynamic factors that are essential for real-world performance (He *et al*. 2024, Liu *et al*. 2024, 2025, Jining *et al*. 2025, Li *et al*. 2025, Zhou *et al*. 2025). Additionally, the use of nonlocal strain gradient theory (NSGT) in sports-related issues hasn't been thoroughly investigated, creating a gap in our understanding of how these theories can lead to practical enhancements for athletes. This study seeks to address this gap by exploring the mechanical behavior of rotating small-scale structures through the use of NSGT. The performance of nanobeams and carbon nanotubes (CNTs) under dynamic loading circumstances is the focus of this work, with a particular emphasis on the potential applications of these materials in sporting goods (Wang *et al*. 2022, 2024a, 2025a, Jia *et al*. 2023, Zhang *et al*. 2023a, b, c, Qi *et al*. 2024). In this matter, Li *et al*. (2024b) investigated linear and nonlinear buckling in nonuniform functionally graded micro-tubes for sports equipment using AI-driven prediction approaches, emphasizing the importance of computational methods in sports engineering. Wang *et al*. (2024e) also conducted an electro-magneto-elastic investigation on a sandwich composite beam used as a diving board in swimming and built of graphene origami metamaterials, demonstrating nanomaterials' applicability in a range of sports applications. Specifically, there are three primary goals:

• Development of a Mathematical Model: A comprehensive mathematical model is developed to predict the behavior of rotating nanoscale structures under dynamic loading conditions. Governing equations based on NSGT are derived, and appropriate boundary conditions reflecting real-world scenarios are defined (Yin *et al*. 2024).

• Validation Through Numerical Simulations: The developed model is validated through extensive numerical simulations, comparing the results with analytical solutions and experimental data from previous studies. This ensures the accuracy and reliability of the findings (Alam *et al*. 2025, Zhan *et al*. 2025).

• Demonstration of Practical Implications: The practical implications of the findings are demonstrated for improving athletic tools. By relating the results to specific sports applications, such as tennis rackets, golf clubs, and skis, the insights gained from this study inform the design of next-generation sports equipment (Wang *et al*. 2024g, Zhang *et al*. 2024b, Zhao *et al*. 2025).

1.4 Significance of the study

The significance of this research arises from its interdisciplinary approach, which incorporates scientific principles from materials science, sports technology, and mechanical engineering. The research makes a substantial contribution to the advancement of sports engineering and the creation of lighter, stronger, and more efficient sporting equipment (Cong *et al*. 2024, Wang *et al*. 2024k, Zhu *et al*. 2024c). This is accomplished by using nanoparticles' unique characteristics and the predictive power of nanoscale gas transfer (NSGT). It is possible that the results will help both elite athletes and leisure users, since they have the potential to improve their performance and overall experience. Additionally, the results of this study emphasize the need to factor in dynamic loading conditions when designing sports equipment. Conventional approaches, which typically prioritize static attributes, often overlook essential elements that influence actual performance. By integrating rotational dynamics, this research enhances our understanding of how nanomaterials can be tailored to meet specific sports requirements. For example, Xia *et al*. (2025) demonstrated the importance of microelectromechanical systems (MEMS) in evaluating athlete performance. The findings indicate that sophisticated computational methods can significantly enhance the design and assessment of sporting gear. Zhu *et al*. (2024a) investigated the use of G-Ori metamaterials as baseball bats in electro-magnetoelastic sandwich composite beams. This allowed the researchers to demonstrate how innovative materials may enhance the performance of classic sports equipment. Similarly, Zhang *et al*. (2024a) conducted a static analysis of foldable pressurized and thermally loaded cylindrical shells utilized as expanders in sports equipment and augmented with G-Ori nanofiller. This study emphasizes the significance of material innovation in the field of sports engineering.

Despite notable progress in the use of nanomaterials and sophisticated modeling techniques for the design of sports equipment, several deficiencies and gaps persist in the current literature. Many studies have concentrated on static or quasi-static analyses, overlooking the dynamic factors essential for real-world performance in rotational or bending motions (Huo *et al*. 2021, Zhang *et al*. 2021, Zhu *et al*. 2022, Li *et al*. 2024a, Xue *et al*. 2024, Wang *et al*. 2025b). Additionally, while nonlocal strain gradient theory (NSGT) has been used to explore a variety of small-scale structures, its application in assessing spinning nanoscale components for sports applications is currently under investigation (Wang *et al*. 2024f). Moreover, there has been limited research into incorporating size-dependent effects and environmental factors into the design of sports equipment, especially for high-performance items like golf clubs, tennis rackets, and skis.

This study examines the mechanical behavior of rotating small-scale structures, including nanobeams and carbon nanotubes (CNTs), under dynamic loading conditions utilizing NSGT to address existing gaps in the literature. The proposed framework integrates material heterogeneity and surface effects, providing a more precise depiction of the actual material response in athletic activities. The study clarifies the real-world implications of its results through case studies pertinent to specific sports applications, providing innovative insights into enhancing the performance and durability of advanced sports equipment. This research integrates advanced computational methodologies with empirical validation, addressing existing limitations and establishing a foundation for future interdisciplinary explorations in the fields of sports engineering and nanotechnology (Liang *et al*. 2024, Wang *et al*. 2024b).

1.5 Organization of the paper

The subsequent sections of this work are structured as follows: Section 2 delineates the mathematical formulation and simulation configuration, including the governing equations and boundary conditions. Section 3 delineates the numerical solution methods used to resolve the equations. Section 4 presents the outcomes of the simulations, emphasizing significant discoveries and their implications for sports engineering. Ultimately, Section 5 encapsulates the main results and proposes avenues for further study.

2. Mathematical formulation and simulation

The application of minuscule structures, including nanobeams and carbon nanotubes (CNTs), offers a distinctive approach to augmenting athletic gear and elevating the performance of sports equipment. The application of thorough engineering analysis, grounded in mechanical and mathematical modeling, enhances the dynamic performance of these structures in the practical contexts faced during athletic endeavors. The rotational dynamics of a golf club shaft and the bending-twisting motion of a ski can be accurately emulated through the application of nanoscale materials like carbon nanotubes and nanobeams, which demonstrate superior stiffness, vibration dampening, and resistance to fatigue. These models clarify how structural reactions to dynamic stresses provide insights into the performance impacts of size-dependent factors and material variability. Figure 1 illustrates the application of rotating small-scale structures in sports equipment design, showcasing how engineering analysis can integrate theoretical models with practical applications to produce gear that is lighter, more durable, and more efficient for athletes. This section emphasizes the transformative potential of these structures in sports technology, detailing the mathematical frameworks and simulation techniques employed to analyze their dynamics.

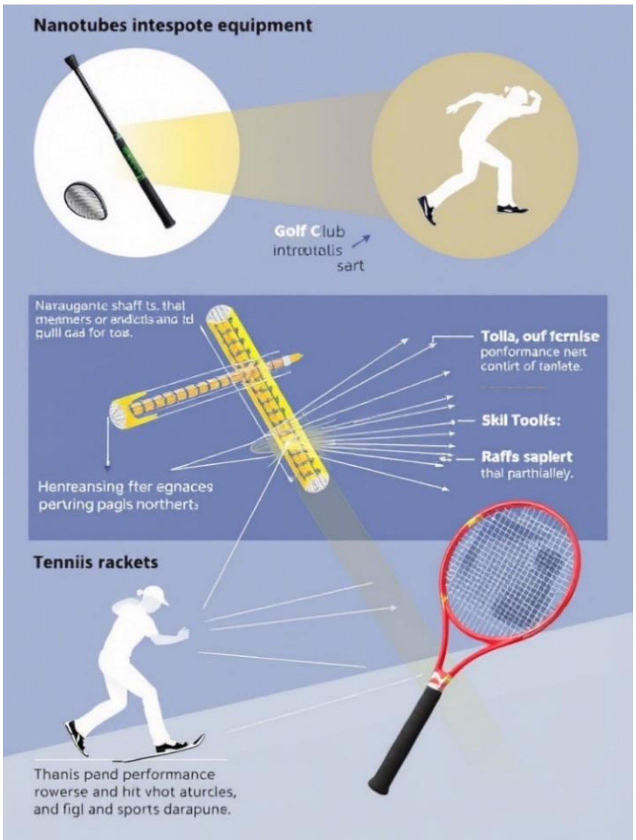

Fig. 1 Schematic representation of the application of rotating small-scale structures (e.g., nanobeams and CNTs) in sports equipment, demonstrating their role in enhancing the performance of golf clubs, skis, and tennis rackets through engineering analysis

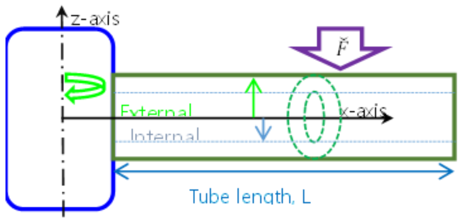

Fig. 2 A schematic of a tube under rotating and external dynamic load

To account for size-dependent effects and material heterogeneity under dynamic loading conditions, the governing equations for a spinning nanotube are derived by combining sophisticated theoretical frameworks, such as high-order tube theory and nonlocal strain gradient theory (NSGT). Hamilton's concept of formulating the kinetic and potential energy terms captures the mechanical behavior of the nanotube during rotating motion. Compared to traditional continuum mechanics, the NSGT gives a more realistic description of structure response by including additional length-scale parameters that quantify the influence of surface effects and material gradients at the nanoscale. The high-order tube theory incorporates rotational inertia effects and shear deformation to simulate the bending-twisting behavior of nanotubes in sports applications such as ski poles or golf club shafts. This section examines the critical motion assumptions and structural interactions pertaining to the spinning nanotube. Utilizing Hamilton's theory facilitates the derivation of the system's governing partial differential equations (PDEs). The model incorporates boundary constraints that more accurately reflect real-life scenarios, such as clamped or free ends. This enhances the capacity to produce a broader variety of sports equipment. The equations assess stability, energy transfer efficiency, and vibrational performance in spinning nanotubes used in sports gear. This design seeks to improve the effectiveness of spinning nanotubes in athletic applications (Afshari Behzad *et al*. 2022, Mirjavadi *et al*. 2022b, 2023, Kazemi *et al*. 2024).

Consider a tube via 'L' as its length, 'Ri' as the internal radius, and 'Re' as the external radius rotating around its left side (x = 0). This tube structure is the main suggested structure in the current problem. According to the Hamilton principle (H), which was employed to generate the governing equations, the following mathematical relation is supposed to exist (Mirjavadi *et al*. 2020b, c, d, e).

$$\int_{t_1}^{t_2} \delta H dt = \int_{t_1}^{t_2} \delta V + \delta P - \delta K dt = 0 \tag{1}$$

where 'δV' is the virtual energy of the external loading due to the rotation (C) and dynamic external load (F) displayed in Fig. 2, and it is calculated as follows:

$$\delta V = \iiint C \frac{\partial w}{\partial x}\Big|_0^L \delta(w) - \int_0^L \frac{\partial}{\partial x}\left(C\frac{\partial w}{\partial x}\right) dx\, \delta(w) + F dV \tag{2}$$

Also, 'C' and 'F' are calculated as follows:

$$F = \breve{F} \sin\left(\frac{n\pi}{L} x\right) \sin(\vartheta t) \tag{3a}$$

$$C = \int_x^L \int_A \rho \psi^2 (\chi + x) dA dx \tag{3b}$$

Here 'ϑ' refers to the external excitation frequency, 'χ' signifies the hub radius (hub radii), and 'ψ' is the angular velocity.

According to the Hamilton principle, 'P' is the potential energy, and it is calculated as follows (Mirjavadi *et al*. 2020a, 2022a, c, Kazemi *et al*. 2023):

$$P = \frac{1}{2} \iiint \varepsilon_{ij} : \sigma_{ij} dV \tag{4}$$

Based on the high-order tube beam theory which the displacement fields are defined as follows:

$$U_x = u(x,t) - z\left(\frac{\partial w(x,t)}{\partial x}\right) + \xi\left(\frac{\partial w(x,t)}{\partial x} + \varphi(x,t)\right) \tag{5a}$$

$$U_y = 0 \tag{5b}$$

$$U_z = w(x,t) \tag{5c}$$

where 'ξ' is the parameter to define the high-order tube theory and mathematically calculated as follows:

$$\xi = r \sin(\theta)\left(1 - \frac{\frac{\frac{r^2}{3} - Ri^2 Re^2}{r^2}}{Ri^2 + Re^2}\right) \tag{5d}$$

Here The strain and components are calculated as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\left(\frac{\partial^2 w}{\partial x^2}\right) + \xi\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x}\right) \tag{6a}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial \xi}{\partial y}\right)\left(\frac{\partial w}{\partial x} + \varphi\right) \tag{6b}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial \xi}{\partial z}\right)\left(\frac{\partial w}{\partial x} + \varphi\right) \tag{6c}$$

Also, the stresses are:

$$\sigma_{xx} = E\xi\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x}\right) - Ez\left(\frac{\partial^2 w}{\partial x^2}\right) + E\frac{\partial u}{\partial x} \tag{7a}$$

$$\sigma_{xy} = \frac{1}{2}G\left(\frac{\partial \xi}{\partial y}\right)\left(\frac{\partial w}{\partial x} + \varphi\right) \tag{7b}$$

$$\sigma_{xz} = \frac{1}{2}G\left(\frac{\partial \xi}{\partial z}\right)\left(\frac{\partial w}{\partial x} + \varphi\right) \tag{7c}$$

where 'E' represents the elastic modulus, and 'G' is shear modulus and calculated as '$G = E/(2 + 2\nu)$', where 'ν' is the Poisson ratio. Then, utilizing the stress and strain components, the strain energy (Eq. (4)) is reformulated as follows:

$$\begin{aligned} &P = \frac{1}{2}\iiint \delta\bar{P}\,dV, \\ &\bar{P} = E\left(\frac{\partial u}{\partial x}\right)^2 - 2Ez\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) + 2E\xi\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial \varphi}{\partial x}\right) \\ &+2E\xi\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) + Ez^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2Ez\xi\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \\ &+2E\xi^2\left(\frac{\partial \varphi}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) + G\left(\frac{\partial \xi}{\partial z}\right)^2 \varphi\left(\frac{\partial w}{\partial x}\right) + E\xi^2\left(\frac{\partial \varphi}{\partial x}\right)^2 \\ &+E\xi^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{1}{2}G\left(\frac{\partial \xi}{\partial z}\right)^2 \varphi^2 \\ &+\frac{1}{2}G\left(\frac{\partial \xi}{\partial z}\right)^2\left(\frac{\partial w}{\partial x}\right)^2 - 2Ez\xi\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial \varphi}{\partial x}\right) \end{aligned} \tag{8}$$

Finally, the Kinetic energy (K) based on the displacement fields are defined as follows:

$$K = \frac{1}{2}\iiint \rho\left(\frac{\partial U_x}{\partial t}\right)^2 + \rho\left(\frac{\partial U_z}{\partial t}\right)^2 dV \tag{9}$$

where 'ρ' is the density, then, the expand format of virtual Kinetic energy calculated as follows:

$$\begin{aligned} &K = \delta\iiint [\bar{K}]dV, \\ &\bar{K} = \frac{1}{2}\rho\left(\frac{\partial u}{\partial t}\right)^2 - \rho z\left(\frac{\partial u}{\partial t}\right)\left(\frac{\partial^2 w}{\partial x \partial t}\right) + \rho\xi^2\left(\frac{\partial \varphi}{\partial t}\right)\left(\frac{\partial^2 w}{\partial x \partial t}\right) \\ &+\frac{1}{2}\rho\left(\frac{\partial w}{\partial t}\right)^2 + \frac{1}{2}\rho\xi^2\left(\frac{\partial \varphi}{\partial t}\right)^2 - \rho z\xi\left(\frac{\partial^2 w}{\partial x \partial t}\right)\left(\frac{\partial \varphi}{\partial t}\right) \\ &+\rho\xi\left(\frac{\partial u}{\partial t}\right)\left(\frac{\partial \varphi}{\partial t}\right) + \frac{1}{2}\rho\xi^2\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 + \rho\xi\left(\frac{\partial u}{\partial t}\right)\left(\frac{\partial^2 w}{\partial x \partial t}\right) \\ &+\frac{1}{2}\rho z^2\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 - \rho z\xi\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 \end{aligned} \tag{10}$$

The nonlocal strain gradient theory is a size-dependent continuum mechanics framework that improves classical theories by including the effects of material length scales, especially significant at the nanoscale. This theory integrates nonlocal interactions and higher-order strain gradients, accurately depicting the influence of neighboring material sites on a structure's mechanical behavior. The nonlocal strain gradient theory alters the governing equations of traditional beam and tube theories by including supplementary variables that address small-scale effects. This produces a more extensive differential equation for beams, including bending moments and higher-order stress resultants related to strain gradients. To account for variations in thickness and curvature caused by size-dependent parameters, the theory revises the stress-strain equations for tubes. These modifications enhance the theory's ability to predict nanostructure deflection, vibration, and buckling behavior. This is essential for developing complex materials and structures that conventional theories cannot adequately describe due to size effects. The general mathematical formulation of nonlocal strain gradient theory is formulated as follows.

$$(1 - (ea)^2\nabla^2)\sigma_{ij} + (l^2\nabla^2 - 1)C:\varepsilon_{ij} = 0 \tag{11}$$

where 'ea' is the nonlocal parameter, 'l' is the strain gradient parameter, '∇' is the Laplace operator, and 'C' refers to the elastic modulus tensor. Applying the nonlocal strain gradient theory to the stress-strain relation will generate the following size-dependent governing equation and boundary conditions.

'$\delta(u)$'

$$\Psi_1\frac{\partial^2 u}{\partial x^2} - l^2\Psi_1\frac{\partial^4 u}{\partial x^4} + (ea)^2\Theta_1\frac{\partial^4 u}{\partial x^2 \partial t^2} - \Theta_1\frac{\partial^2 u}{\partial t^2} = 0 \tag{12a}$$

'$\delta(\varphi)$'

$$\begin{aligned} &l^2\Psi_2\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) - \Psi_2\left(\varphi + \frac{\partial w}{\partial x}\right) + l^2\Psi_3\frac{\partial^5 w}{\partial x^5} \\ &+\Xi_4\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) - l^2\Psi_4\left(\frac{\partial^4 \varphi}{\partial x^4} + \frac{\partial^5 w}{\partial x^5}\right) - \Psi_3\frac{\partial^3 w}{\partial x^3} = \\ &(ea)^2\left(\Theta_3\frac{\partial^5 w}{\partial x^3 \partial t^2} - \Theta_2\frac{\partial^4 \varphi}{\partial x^2 \partial t^2}\right) + \Theta_2\frac{\partial^2 \varphi}{\partial t^2} - \Theta_3\frac{\partial^3 w}{\partial x \partial t^2} \end{aligned} \tag{12b}$$

'$\delta(w)$'

$$\begin{aligned} &l^2\Psi_5\frac{\partial^6 w}{\partial x^6} - \Psi_5\frac{\partial^4 w}{\partial x^4} + l^2\Psi_2\left(\frac{\partial^3 \varphi}{\partial x^3} + \frac{\partial^4 w}{\partial x^4}\right) \\ &-\Psi_2\left(\frac{\partial \varphi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - \Psi_3\frac{\partial^3 \varphi}{\partial x^3} + l^2\Psi_3\frac{\partial^5 \varphi}{\partial x^5} \\ &+3(ea)^2\frac{d^2 C}{dx^2}\frac{\partial^2 w}{\partial x^2} + 3(ea)^2\frac{dC}{dx}\frac{\partial^3 w}{\partial x^3} \\ &+(ea)^2 C\frac{\partial^4 w}{\partial x^4} + (ea)^2\frac{d^3 C}{dx^3}\frac{\partial w}{\partial x} - \frac{dC}{dx}\frac{\partial w}{\partial x} - F + C\frac{\partial^2 w}{\partial x^2} \\ &+(ea^2)\frac{d^2 F}{dx^2} = (ea)^2\Theta_3\frac{\partial^5 \varphi}{\partial x^3 \partial t^2} - \Theta_3\frac{\partial^3 \varphi}{\partial x \partial t^2} - \Theta_1\frac{\partial^2 w}{\partial t^2} + \\ &(ea)^2\Theta_4\frac{\partial^6 w}{\partial x^4 \partial t^2} + (ea)^2\Theta_1\frac{\partial^4 w}{\partial x^2 \partial t^2} - \Theta_4\frac{\partial^4 w}{\partial x^2 \partial t^2} \end{aligned} \tag{12c}$$

Also, the size-dependent boundary conditions are:

'$u = 0$' *Or*

$$\Psi_1\frac{\partial u}{\partial x} - l^2\Psi_1\frac{\partial^3 u}{\partial x^3} - (ea)^2\Theta_1\frac{\partial^3 u}{\partial x \partial t^2} = 0 \tag{13a}$$

'$\varphi = 0$' *Or* (13b)

$$l^2\Psi_3\frac{\partial^4 w}{\partial x^4}+\Psi_4\left(\frac{\partial\varphi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)-l^2\Psi_4\left(\frac{\partial^3\varphi}{\partial x^3}+\frac{\partial^4 w}{\partial x^4}\right)$$
$$-\Psi_3\frac{\partial^2 w}{\partial x^2}=\Theta_3(ea)^2\frac{\partial^4 w}{\partial x^2\partial t^2}-(ea)^2\Theta_2\left(\frac{\partial^3\varphi}{\partial x\partial t^2}\right)$$

'$w=0$' Or

$$\Psi_2\left(\varphi+\frac{\partial w}{\partial x}\right)-l^2\Psi_3\frac{\partial^4\varphi}{\partial x^4}-(ea)^2\frac{d\bar{R}}{dx}\frac{\partial w}{\partial x}-(ea)^2\bar{R}\frac{\partial^2 w}{\partial x^2}$$
$$+\Psi_5\frac{\partial^3 w}{\partial x^3}+\Psi_3\frac{\partial^2\varphi}{\partial x^2}-l^2\Psi_5\frac{\partial^5 w}{\partial x^5}-l^2\Psi_2\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^3 w}{\partial x^3}\right)+ \quad (13c)$$
$$\bar{R}\frac{\partial w}{\partial x}+(ea)^2\left(\Theta_1\frac{\partial^3 w}{\partial x\partial t^2}+\Theta_3\frac{\partial^4\varphi}{\partial x^2\partial t^2}+\Theta_4\frac{\partial^5 w}{\partial x^3\partial t^2}\right)=0$$

'$\partial w/\partial x=0$' Or

$$\Psi_3\frac{\partial^3\varphi}{\partial x^3}+l^2\Psi_5\frac{\partial^4 w}{\partial x^4}-\Psi_3\frac{\partial\varphi}{\partial x}-\Psi_5\frac{\partial^2 w}{\partial x^2} \quad (13d)$$
$$+(ea)^2\left(\Theta_4\frac{\partial^4 w}{\partial x^2\partial t^2}+\Theta_3\frac{\partial^3\varphi}{\partial x\partial t^2}+\Theta_1\frac{\partial^3 w}{\partial x\partial t^2}\right)=0$$

where

$$\Theta_1=\iint\rho r dr d\theta \quad (14a)$$

$$\Theta_2=\iint\rho\xi^2 r dr d\theta \quad (14b)$$

$$\Theta_3=\iint\rho\xi r^2\sin(\theta)\,dr d\theta-\iint\rho\xi^2 r dr d\theta \quad (14c)$$

$$\Theta_4=2\iint\rho\xi r^2\sin(\theta)\,dr d\theta-\iint\rho\xi^2 r dr d\theta-\iint\rho r^3\sin^2(\theta)\,dr d\theta \quad (14d)$$

$$\Psi_1=\iint E r dr d\theta \quad (14e)$$

$$\Psi_2=\iint G\left[\left(\frac{\partial\xi}{\partial y}\right)^2+\left(\frac{\partial\xi}{\partial z}\right)^2\right]r dr d\theta \quad (14f)$$

$$\Psi_3=\iint E\xi r^2\sin(\theta)\,dr d\theta-\iint E\xi^2 r dr d\theta \quad (14g)$$

$$\Psi_4=\iint E\xi^2 r dr d\theta \quad (14h)$$

$$\Psi_5=2\iint E\xi r^2\sin(\theta)\,dr d\theta-\iint E\xi^2 r dr d\theta-\iint E r^3\sin^2(\theta)\,dr d\theta \quad (14i)$$

3. Numerical solution methodology

This section presents the numerical solution methods used to address the size-dependent governing equations of spinning tube constructions. The equations formulated in Section 2, using high-order tube theory in conjunction with nonlocal strain gradient theory, are resolved by the Generalized Differential Quadrature Method (GDQM) integrated with the Newmark Beta approach. This methodology guarantees precise and effective solutions for the dynamic behavior of nanoscale objects subjected to rotating loading conditions.

3.1 Overview of the generalized differential quadrature method (GDQM)

The Generalized Differential Quadrature Method is a powerful numerical technique widely utilized for solving partial differential equations in structural mechanics and engineering applications. It approximates the derivatives of a function at specific points by gathering all functional values at those points in a weighted linear form. At any given point 'x_i' within its domain, the first derivative of the function '$u(x)$' can be expressed as:

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_i}=\sum_{j=1}^{n}\Lambda_{ij}u(x_j) \quad (15a)$$

where 'Λ_{ij}' is weighting coefficients determined based on the choice of basis functions, and 'n' is discrete points in the domain. Likewise, higher-order derivatives may be computed iteratively with the same technique. The GDQM discretizes the spatial domain, transforming the governing PDEs into a system of algebraic equations, hence enhancing computing efficiency for problems with complex geometries and boundary conditions.

3.2 Application of GDQM to size-dependent governing equations

The governing equations in Section 2 include the nonlocal parameter 'ea' and the strain gradient parameter 'l', leading to higher-order differential equations that consider size-dependent effects. The spatial domain of the spinning tube construction is segmented into 'n' grid points for the execution of the GDQM. The displacement fields (u, w, and φ) and its derivatives are evaluated at these locations using the GDQM formulation. The second and m^{th} spatial derivatives of '$w(x,t)$' are represented as follows:

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=x_i}=\sum_{j=1}^{n}\Lambda_{ij}^{(2)}w(x_j) \quad (15b)$$

$$\left.\frac{\partial^m w}{\partial x^m}\right|_{x=x_i}=\sum_{j=1}^{n}\Lambda_{ij}^{(m)}w(x_j) \quad (15c)$$

'$\Lambda_{ij}^{(2)}$' and '$\Lambda_{ij}^{(m)}$' are the weighting coefficients associated with the second-order and m^{th}-order derivatives, respectively. The coefficients are determined by employing suitable basis functions, such as Lagrange or Chebyshev polynomials, that are customized to satisfy the specific needs of the task.

3.3 Temporal discretization using the newmark beta technique

For the purpose of solving the time-dependent governing equations, the Newmark Beta approach is used for the

Table 1 validation of calculated numerical natural frequency ($\omega^2 L^4\, \Theta_1/(4\Psi_1)$) of cantilever nanotube versus the nonlocal parameter for the nonlocal tube

	Current study	Man (2022)
Local Boundary conditions		
'ea = 0'	6.990001018	6.983018
'ea = 0.1L'	7.020705549	7.012291
'ea = 0.2L'	7.113785915	7.10455
'ea = 0.3L'	7.285833507	7.275648
'ea = 0.4L'	7.575762924	7.564416
'ea = 0.5L'	8.09455489	8.080818
Non-Local Boundary conditions		
'ea = 0'	6.990001018	6.983018
'ea = 0.1L'	6.864675137	6.8564474
'ea = 0.2L'	6.413598753	6.4052719
'ea = 0.3L'	5.822674548	5.8145342
'ea = 0.4L'	5.213262582	5.2054544
'ea = 0.5L'	4.652212239	4.6443169

purpose of making temporal discretization. Because of its unconditional stability and second-order precision, this approach is especially well-suited for solving problems that include dynamic conditions. The lateral displacement '$w(x,t)$', velocity '$\dot{w}(x,t)$', and acceleration '$\ddot{w}(x,t)$' at discrete time steps are estimated in the following manner:

$$w^{\tau+1} = w^{\tau} + \Delta t.\dot{w}^{\tau} + \frac{\Delta t^2}{2}[(1-2\beta)\ddot{w}^{\tau} + 2\beta\ddot{w}^{\tau+1}] \quad (16a)$$

$$\dot{w}^{\tau+1} = \dot{w}^{\tau} + \Delta t.\dot{w}^{\tau} + \Delta t[(1-\gamma)\ddot{w}^{\tau} + \gamma\ddot{w}^{\tau+1}] \quad (16b)$$

where 'τ' is the time step, 'Δt' is time step size, 'β' and 'γ' are algorithm parameters (typically chosen as $\beta = 0.25$ and $\gamma = 0.5$ for optimal accuracy and stability). These approximations may be included in the system's governing equations to create a set of nonlinear algebraic equations for each discrete time period. The equations are solved using numerical techniques, such as the Newton-Raphson method, in an iterative process.

3.4 Implementation of boundary conditions

The physical validity of the numerical solution is mainly dependent on the boundary conditions, which play a significant influence in this regard. In the case of the spinning tube construction, the usual boundary conditions consist of clamped ends, free ends, or mixtures of the two. The boundary conditions that have been derived can be integrated into the discretized equations through the adjustment of the rows in the stiffness matrix that correspond to the specified boundary conditions. This ensures that the equations conform to the physical constraints of the scenario.

3.5 Solution procedure

The solution procedure involves the following steps:

• Discretization of Spatial Domain: Divide the spatial domain into 'n' grid points and compute the weighting coefficients '$\Lambda_{ij}^{(m)}$' using the GDQM.

• Temporal Discretization: Apply the Newmark Beta technique to discretize the governing equations in the time domain, reducing them to a system of algebraic equations.

• Assembly of System Equations: Combine the spatial and temporal discretization to form the global system of equations for the rotating tube structure.

• Application of Boundary Conditions: Incorporate the specified boundary conditions into the system of equations by modifying the stiffness matrix.

• Iterative Solution: Solve the resulting system of equations iteratively for each time step using the Newton-Raphson method or similar techniques.

• Validation of Results: Compare the numerical results with analytical solutions or experimental data from previous studies to ensure accuracy and reliability.

3.6 Convergence and accuracy

The convergence and accuracy of the numerical solution are influenced by the quantity of grid points 'n' employed in the spatial discretization, as well as the time step size 'Δt'. A sufficient number of grid points must be selected to accurately capture the higher-order derivatives introduced by the strain gradient parameter 'l' and the nonlocal parameter 'ea'. The time step size 'Δt' must be sufficiently small to accurately capture the dynamic behavior of the rotating tube structure. Convergence studies are performed to identify the optimal values of 'n', 'τ' and 'Δt' for the specified problem.

4. Discussion of the numerical results

This study aims to improve the efficacy of sports equipment through the incorporation of spinning small-scale structures, such as nanobeams and carbon nanotubes (CNTs), to elevate the quality of athletic implements. The characteristics of these structures reveal distinct mechanical properties, such as heightened stiffness, effective vibration damping, and superior fatigue resistance, rendering them exceptionally suitable for improving the efficiency, control, and responsiveness of sports equipment when subjected to dynamic loading conditions. This section presents quantitative data intended to substantiate the mathematical model formulated in section 2, which integrates the nonlocal strain gradient theory to address size-dependent phenomena and material heterogeneity at the nanoscale. The proposed methodologies demonstrate their accuracy and reliability through a comparative analysis of simulation outcomes against analytical solutions and empirical data from prior research. The validated results undergo a comprehensive analysis, emphasizing their significance for particular athletic equipment, including golf clubs, tennis rackets, and skis. This discourse illustrates the capacity of rotating small-scale structures to augment athletic performance and underscores the necessity of integrating modern computational methodologies into the design processes of sports engineering.

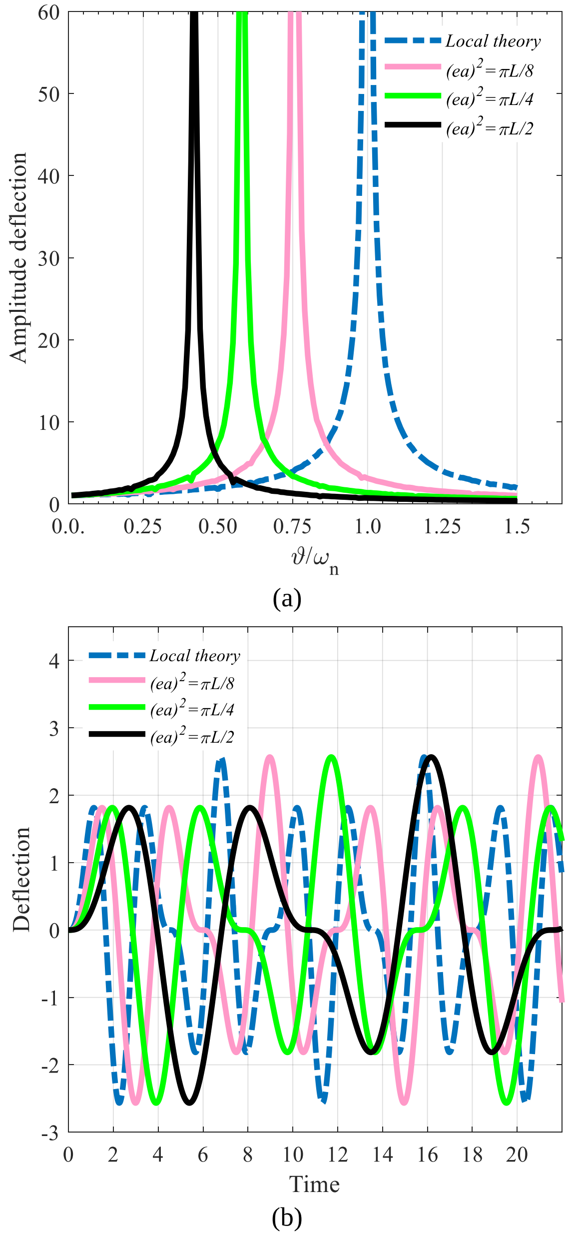

Fig. 3 Illustration of the influence of the nonlocal parameter (ea) on the dynamic response of a clamped-free rotating structure. Part A depicts the relationship between amplitude deflection and the excitation frequency ratio ($\vartheta\ /\omega_n$), while Part B presents the time-dependent deflection at the free end via $\vartheta = 0.6\omega_n$.

The calculated numerical natural frequency ($\omega^2 L^4 \Theta_1 / (4\Psi_1)$) for a cantilever nanotube across different values of the nonlocal parameter (ea) is presented in Table 1. This table serves as a validation tool by comparing the results with those published by Man (2022), who employed an alternative approach to examine rotating small-scale structures, thereby confirming the accuracy and reliability of the findings. The nonlocal parameter gauges the impact of long-range interactions on the mechanical properties of nanotubes. By altering 'ea' we can examine how size-dependent effects affect the structure's natural frequency. This study establishes a benchmark for 'ea' in relation to nanotube length (L), where 'ea = 0' indicates no nonlocal effects, and 'ea = 0.5L' represents a significant nonlocal influence. The table has two sections: Local Boundary Conditions and Nonlocal Boundary Conditions. Each row represents a distinct value of 'ea', with the first column displaying the natural frequency determined in this research and the second column offering the matching values from Man (2022). The notable relationship observed between the two data sets reinforces the theoretical framework and quantitative approach utilized in this research. The validation process is essential for confirming the reliability of the proposed model, especially in its capacity to predict the dynamic behavior of spinning nanoscale structures under real-world conditions. The study employs nonlocal strain gradient theory to adeptly tackle size-dependent phenomena, thus improving the model's predictive precision. The design of sporting equipment, including golf clubs, tennis rackets, and skis, carries significant implications, particularly where the accurate modeling of nanoscale behavior is crucial. In the following presentation, the nonlocal boundary condition will be employed.

This study proposes carbon nanotubes (CNTs) as the primary material for rotating small-scale structures, attributed to their exceptional mechanical properties that enhance the performance of sports equipment. Carbon nanotubes demonstrate a remarkably high elastic modulus, generally between 1 TPa and 5 TPa, facilitating effective energy transfer and minimal deformation during high-impact occurrences, such as striking a golf ball or bending a ski (Lu 1997). The Poisson ratio of carbon nanotubes, which reflects the relationship between lateral strain and axial strain, generally falls between 0.1 and 0.4, thus providing dimensional stability under dynamic loading conditions. The density of carbon nanotubes varies between 1.3 and 2.1 g/cm^3, enabling the development of lightweight and strong structures that improve agility and speed in athletic equipment while ensuring durability. The unique characteristics of high stiffness, favorable Poisson ratio, and low density are essential for improving the performance of sports equipment (Kanagaraj *et al*. 2007, Ghorbanzadeh Ahangari *et al*. 2013).

Fig. 3 shows how the nonlocal parameter (ea) affects the dynamic deflection of a clamped-free structure during external excitation. In Part A, the amplitude-deflection versus the excitation frequency ratio (ϑ/ω_n) shows that increasing 'ea' decreases the resonant frequency at which maximum oscillation occurs. Increased 'ea' values, which consider long-range interatomic interactions, shift resonance to lower frequencies, leading to reduced structural stiffness. Conversely, under local boundary conditions that exclude nonlocal effects, the opposite trend emerges, emphasizing the significance of NSGT for precise nanoscale assessment. In Part B, the time-dependent deflection $(-100w \iint Er^3 sin^2(\theta)/FL^4\pi)$ of the free end shows that 'ea' primarily affects the temporal response, extending the time period while keeping the domain deflection relatively stable. This implies that nonlocal effects dominate the damping and energy dissipation mechanisms, rather than static deformation. Designing sports equipment, such as golf clubs or skis, requires taking nonlocality into account,

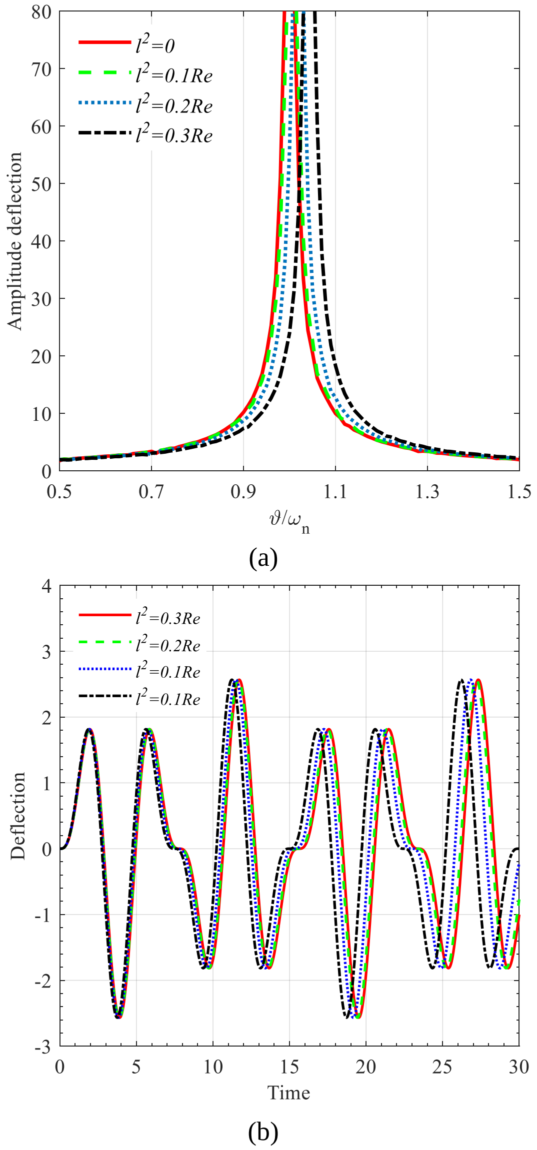

Fig. 4 Representation of the effect of strain gradient parameter (l) on the dynamic deflection of a rotating structure. Part A shows the variation of amplitude deflection with respect to the excitation frequency ratio (ϑ /ω_n, and Part B illustrates the time-dependent deflection ($-100w \iint Er^3 sin^2(\theta)/FL^4\pi$) when $\vartheta = 0.6\omega_n$, $(ea)^2 = \pi L/4$

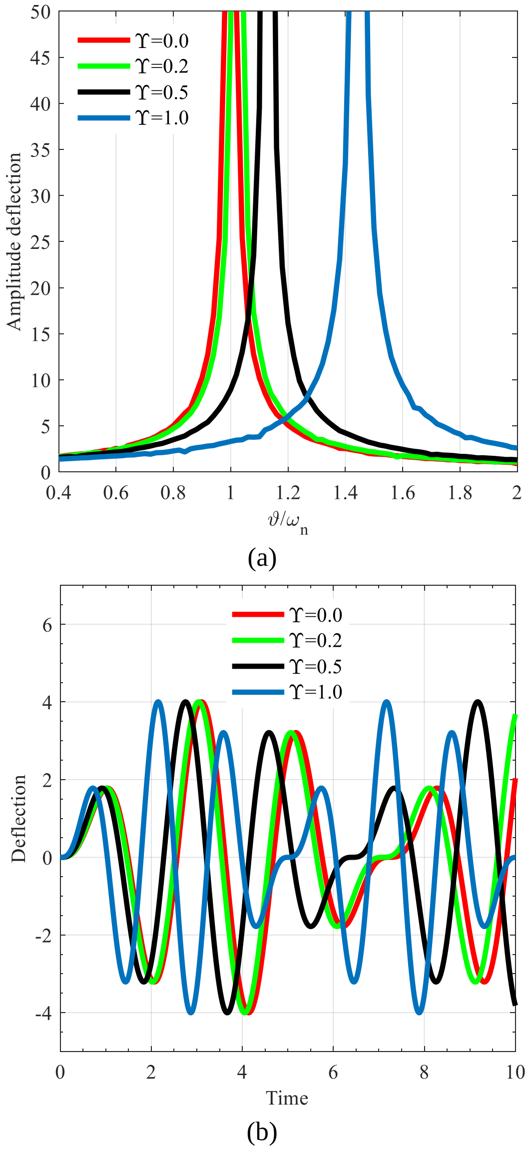

Fig. 5 Visualization of the impact of rotational speed (ϒ) on the dynamic response of a cantilever rotating structure. Part A demonstrates the relationship between amplitude deflection and the excitation frequency ratio (ϑ /ω_n), and Part B presents the time-dependent deflection ($-100w \iint Er^3 sin^2(\theta)/FL^4\pi$) when $\vartheta = 0.5\omega_n$, $l^2 0.1Re$, $(ea)^2 = \pi L/4$, χ=0.15L

as stiffness and resonant behavior affect energy transfer and athlete control. Optimizing 'ea' can create structures that reduce vibrations and maintain integrity during high-impact events.

Fig. 4 examines the impact of the strain gradient parameter (l) on the dynamic behavior of the rotating structure. Part A shows that increasing 'l' raises the resonant frequency, indicating enhanced stiffness due to the incorporation of size-dependent strain gradients. This aligns with the theoretical expectation that higher strain gradient effects stabilize structures at the nanoscale. In contrast to the nonlocal parameter, 'l' directly counteracts softening effects, a phenomenon critical for applications requiring high-frequency stability, such as tennis rackets or diving boards. Engineers can enhance energy transfer efficiency and minimize deflection in rotational movements, such as bending skis, by modifying 'l'. Part B illustrates that a larger 'l' leads to shorter oscillation periods, thereby increasing the natural frequency. This finding strengthens the strain gradient theory, making it more effective in accounting for surface and gradient effects, which are vital for predicting the behavior of nanotube-reinforced composites under dynamic loads.

Fig. 5 evaluates the impact of rotational speed (ψ), expressed dimensionlessly as 'ϒ' ($\Upsilon^2 = \pi L^4 \psi^2 \Theta_1 / \iint Er^3 sin^2(\theta)$) on the dynamic response of a cantilever structure. Part A demonstrates that increasing 'ϒ' elevates the resonant frequency, consistent with the centrifugal stiffening effect. This phenomenon, where rotational inertia induces additional stiffness, is critical for sports equipment subjected to high-speed motions, such as golf club shafts or pole vault poles. The delay in maximum oscillation at higher 'Y' values reflects enhanced resistance to deformation, directly improving energy retention and responsiveness during athletic activities. Part B demonstrates that rotational speed mainly influences temporal dynamics, shortening the time period and enhancing the natural frequency while leaving domain deflection unchanged. This underscores the significance of considering rotational dynamics in the design of sports equipment, as overlooking centrifugal forces may result in imprecise predictions of structural performance in real-world scenarios.

The interplay of 'ea', 'l', and 'ψ' highlights how rotating small-scale structures can enhance athletic tools. For example:

• Golf Clubs: The effect of stiffness-enhancing agents and centrifugal stiffening can improve energy transfer upon impact, resulting in more efficient swings and reduced unwanted vibrations.

• Skis: The nonlocal parameter contributes to damping and frequency modulation, potentially increasing stability in high-speed maneuvers. Meanwhile, strain gradient effects bolster bending resistance.

• Protective Gear: The shorter time frame indicated by 'l' and 'ψ' points to enhanced shock absorption abilities, which are essential for helmets and pads.

This study closes a significant gap in the literature by incorporating rotational dynamics and size-dependent effects (via NSGT) into the analysis of sports equipment. Traditional models frequently overlook these factors, leading to inaccurate predictions of performance in real-world scenarios. The observed phenomena, such as the contrasting effects of 'ea' and 'l', highlight the significance of NSGT for nanoscale design, particularly in materials like carbon nanotubes, where surface effects are critical. The study emphasizes the need to account for environmental and operational factors, including angular velocity, which are integral to sports activities like swinging or bending. This is consistent with earlier research that highlighted the importance of dynamic, multidisciplinary analyses in sports engineering. The study shows that nanoscale parameters and operational conditions can be optimized to enhance performance metrics such as stiffness, resonant frequency, and energy transfer. Engineers can employ rotating small-scale structures to develop athletic tools that are lighter, stronger, and more responsive, fulfilling the study's objective of enhancing sports equipment through nano-technology. CNT-reinforced composites with optimized properties can improve the durability of tennis rackets and the damping of ski poles. The observed phenomena provide actionable recommendations for optimizing materials in sporting equipment.

5. Conclusions

This study analyzed the mechanical properties of rotating small-scale structures, such as nanobeams and carbon nanotubes, under dynamic loading conditions to improve sports equipment performance. A mathematical model was created using nonlocal strain gradient theory to explore size-dependent effects and material heterogeneity. We formulated equations that describe the rotational dynamics of structures like golf clubs, skis, and protective gear. The model's validity was established through numerical simulations employing the generalized differential quadrature method (GDQM) and the Newmark Beta technique, showing substantial agreement with previous analytical and experimental data. The results demonstrate that the nonlocal parameter diminishes structural stiffness by lowering the natural frequency, while the strain gradient parameter enhances stiffness and raises resonant frequencies. Additionally, rotational speed induced centrifugal stiffening, leading to elevated natural frequencies and a delay in maximum oscillation during dynamic excitation. he phenomena underscore the significance of size-dependent effects and rotational dynamics in improving energy transfer, vibration damping, and dimensional stability in athletic equipment.

The study addressed research gaps by combining NSGT with high-order tube theory to analyze sports equipment, an area frequently overlooked by previous literature, which favored static or quasi-static approaches. The study emphasizes nanomaterials' ability to improve athletic performance by connecting theoretical insights with practical applications such as increasing energy efficiency in golf clubs and reducing vibrations in tennis rackets. alidation with experimental data and case studies, such as diving boards and baseball bats, improves the model's reliability for real-world design applications. This study helps to create lighter, stronger, and more efficient sports equipment by providing a structured method for optimizing nanoscale properties and rotational dynamics. Incorporating insights from materials science, mechanical engineering, and sports technology paves the way for future research into the environmental and thermal impacts of nanomaterials used in athletic gear. Upcoming research should focus on experimentally validating these parameters in real-world equipment and investigating their interactions under various loading conditions, thereby closing the gap between theoretical models and practical industry applications.

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