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Research on applicable sensor for solving the volleyball sport problem using smart nanomaterial based on dynamic simulation

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Abstract. This research investigates the application of machine learning methodologies to optimize energy management within the context of a volleyball game, specifically focusing on the energy dynamics of the ball. Machine learning, as a discipline, provides a robust framework for the development of automated analytical models, enabling the extraction of meaningful insights from complex datasets. The ball in the volleyball games is the most important tool. The surface properties and response to hit by hand are crucial in determining the accuracy and fluency of the game. The outer material of the ball is extremely determinative in the mechanical response of the ball to the impact loading which commonly causes vibration in the ball. Therefore, in the current work vibrations of a volleyball game ball is presented. The volleyball game ball is reinforced by graphene oxide powders to improve its stability in different situation. Finally, the results show that the ball's radius has a key role in the dynamic stability of the volleyball game ball. One of the important outcomes of the current research is that, unlike the ball's size, heavier balls tend to be more stable when they hit the ground. The outputs of the current work can be used for future analysis of the volleyball game ball for improving its stability.
Keywords: dynamic simulation; sensor; smart material; volleyball sport problem

1. Introduction

The importance of spherical structure analysis in sport balls lies in understanding the impact it has on the performance, durability, and safety of the balls. One of the most important aspects of spherical balls analysis is performance optimization of the balls. The design and construction of a sport ball can significantly affect its performance. Analyzing the spherical structure helps manufacturers fine-tune factors such as weight distribution, aerodynamics, and ball bounce (Cross 2002, 2014, Asai *et al*. 2007, Zhang and Yu 2012, Melo *et al*. 2021). By understanding the structural characteristics, they can create balls that offer better control, accuracy, and overall performance. In addition, in competitive sports, it is crucial to ensure that all participants have access to balls with consistent characteristics. Spherical structure analysis helps determine the desired properties of the balls, such as size, weight, and elasticity (Mahapatra *et al*. 2015, Shamloofard and Movahhedy 2015, Van Do and Lee 2020, Sobhani *et al*. 2022). This analysis ensures that the balls used in competitions meet the required standards, promoting fairness and maintaining a level playing field. One another important feature of the ball structure analysis is the durability evaluation of ball materials under common loads in their respective sports. Sport balls undergo significant stress and impact during gameplay. Analyzing the spherical structure helps identify weak points, areas prone to wear and tear, and potential structural failures. By understanding these factors, manufacturers can develop more durable balls that withstand rigorous use, reducing the need for frequent replacements. Safety is another paramount in sports, and the spherical structure of balls plays a vital role in ensuring player well-being. Analyzing the structure helps assess factors such as impact resistance, deformation, and energy transfer upon collision. This knowledge enables manufacturers to design balls that minimize the risk of injuries, such as concussions or bone fractures. Using numerical simulation in structural mechanics provides a deep insight into reactions of structure in different conditions and could presents a playground for innovations. By understanding the underlying physics and engineering principles, researchers can explore new materials, manufacturing techniques, and structural configurations. This can lead to the development of improved balls that offer enhanced performance, durability, and safety.

In the structural analyses, several fields of central importance could be addressed. Vibrational analysis and wave propagation, static responses and strength of the

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material and, finally, stability of the structures under static and dynamic loading conditions. There have been published several articles on each of these subjects. We focused our literature review on the spherical structure with innovative materials similar to functionally graded (FG) and nano-structures composite materials. Van Vinh and Tounsi (2022) presents a study on the free oscillation of spherical shell structure made of functionally graded materials. They utilized size-dependent elasticity formulation and first order deformation theory in their analysis. Ghavanloo *et al*. (2019) examined vibration in the extremely small spherical structures similar to fullerene in nano-scales. In such scales, the dependency of the properties on the size of the structure becomes a key feature in determining responses. Therefore, nonlocal theory of elasticity (Liu and Reddy 2011, Sahmani and Aghdam 2017, Mehar *et al*. 2018, Fattahi *et al*. 2019, Sahmani and Aghdam 2019) was utilized in their analysis for observation and calculation free vibration modes and frequencies. In the buckling analysis, Hutchinson (2016) presented a detain analytical solution for the buckling of the spherical ball shell structure. Liu *et al*. (2021) examined the free vibration of composite spherical structure reinforced with graphene nano-platelets. They utilized classical theory of elasticity to obtain an analytical solution for the equation of motion of a layer-wise composite structure. Analytical solutions are important in the sense that they are free from the numerical and computational errors.

In addition to traditional analytical and numerical approaches (Amelirad and Assempour 2019, 2021) for solving the governing equations of mechanical structures, there has been a growing interest among researchers and engineers in a new generation of response prediction strategies. Artificial intelligence (AI) techniques (Guo *et al*. 2022, Qian *et al*. 2023, Wang *et al*. 2023a), specifically machine learning (ML) methods (Chen *et al*. 2021, Lee *et al*. 2021), have emerged as widely adopted and trustworthy tools in the field of mechanical engineering for predicting various behaviors of structural members and also in other fields of science and industry (Moradi *et al*. 2022, Lingamdinne *et al*. 2023).

These AI and ML methods have proven to be highly effective in tackling complex problems, even in the realm of three-dimensional elasticity (Xie *et al*. 2022). In recent times, physics-informed neural networks have gained significant attention as an efficient solution for solving intricate problems in the field of elasticity (Haghighat *et al*. 2021). By integrating physical laws and principles into the neural network architecture, these networks are capable of providing accurate predictions and insights into the behavior of complex mechanical systems.

The utilization of AI and ML methods in mechanical engineering signifies a shift towards more advanced and data-driven approaches for analyzing structural responses (Duong *et al*. 2021, Wang *et al*. 2021, Turan *et al*. 2022). These techniques offer enhanced computational efficiency, improved accuracy, and the ability to handle complex problems that may be challenging or time-consuming for traditional methods.

This research investigates the application of machine learning methodologies to optimize energy management within the context of a volleyball game, specifically focusing on the energy dynamics of the ball. Machine learning, as a discipline, provides a robust framework for the development of automated analytical models, enabling the extraction of meaningful insights from complex datasets. In this study, the authors leverage these techniques to address the multifaceted challenges associated with energy management, a field encompassing the strategic planning and operational control of energy production, consumption, distribution, and storage systems. The overarching goals of effective energy management include resource conservation, mitigation of environmental impact, and economic efficiency, while simultaneously ensuring reliable energy provision. Recognizing the interconnectedness of energy management with related domains such as environmental stewardship and production logistics, this study explores the potential of machine learning to enhance the efficiency of energy utilization associated with the volleyball. By analyzing relevant parameters, including, but not limited to, impact forces, trajectory, and deformation, machine learning algorithms are employed to model and predict energy dissipation and transfer during gameplay. The results of this investigation demonstrate the significance of specific parameters in optimizing energy management strategies, thereby contributing to a deeper understanding of the energy dynamics inherent in volleyball games. This work contributes to the growing body of literature exploring the intersection of machine learning and energy management, offering a novel approach to optimizing resource allocation in sports applications.

2. Basic equations

The displacement field of the spherical shell, as formulated within the framework of First-Order Shear Deformation Theory (FSDT), deviates from the classical Kirchhoff-Love assumptions by accounting for transverse shear deformation. In classical thin shell theory, it is assumed that normals to the mid-surface remain straight and normal after deformation (Ge *et al*. 2023, Habibi *et al*. 2024, He *et al*. 2024, Huang *et al*. 2024, Jin *et al*. 2024, Li *et al*. 2024, Man *et al*. 2024, Wang *et al*. 2024a, b, Zhang *et al*. 2024a, Zhang *et al*. 2024b, Zhao *et al*. 2024). However, FSDT relaxes this constraint, allowing for independent rotations of these normals. The components could be expressed as:

$$\mathfrak{R}(\emptyset,\mathfrak{X},\varsigma,\mathfrak{t})=\mathfrak{R}_0(\emptyset,\mathfrak{X},\mathfrak{t})+\varsigma\mathfrak{R}_1(\emptyset,\mathfrak{X},\mathfrak{t}) \tag{1}$$

$$\mathfrak{S}(\emptyset,\mathfrak{X},\varsigma,\mathfrak{t})=\mathfrak{S}_0(\emptyset,\mathfrak{X},\mathfrak{t})+\varsigma\mathfrak{S}_1(\emptyset,\mathfrak{X},\mathfrak{t}) \tag{2}$$

$$\mathfrak{T}(\emptyset,\mathfrak{X},\varsigma,\mathfrak{t})=\mathfrak{T}_0(\emptyset,\mathfrak{X},\mathfrak{t})+\varsigma\mathfrak{T}_1(\emptyset,\mathfrak{X}\mathfrak{X},\mathfrak{t}) \tag{3}$$

where the strain-displacement relationships, within the context of First-Order Shear Deformation Theory (FSDT), can be defined as follows (Dai *et al*. 2023a, b, Gu *et al*. 2023, Li *et al*. 2023, Peng *et al*. 2023, Sabzevari *et al*. 2023, Shariati *et al*. 2023, Xiang *et al*. 2023, Yang *et al*. 2023, Zhang *et al*. 2023a, b, Zhao *et al*. 2023, Zheng *et al*. 2023):

$$\mathfrak{g}_{\emptyset\emptyset}=\frac{\partial\mathfrak{R}}{\mathfrak{r}\,\partial\emptyset}+\frac{\mathfrak{T}}{\mathfrak{r}},\quad \mathfrak{g}_{\mathfrak{X}\mathfrak{X}}=\frac{\partial\mathfrak{S}}{\mathfrak{r}_1\,\partial\mathfrak{X}}+\frac{\mathfrak{R}}{\mathfrak{r}\mathfrak{r}_1}\frac{\partial R_1}{\partial\emptyset}+\frac{\mathfrak{T}}{\mathfrak{r}} \tag{4}$$

$$\mathfrak{g}_{\varsigma\varsigma}=\frac{\partial\mathfrak{T}}{\partial\varsigma},\qquad \mathfrak{g}_{\emptyset\mathfrak{x}}=\frac{\partial\mathfrak{S}}{\mathfrak{r}\,\partial\emptyset}-\frac{\mathfrak{S}}{\mathfrak{r}\mathfrak{r}_1}\frac{\partial R_1}{\partial\emptyset}+\frac{\partial\mathfrak{R}}{\mathfrak{r}_1\,\partial\mathfrak{X}}$$

$$\mathfrak{g}\ \mathfrak{g}_{\emptyset\varsigma}=\frac{\partial\mathfrak{R}}{\partial\varsigma}-\frac{\mathfrak{R}}{\mathfrak{r}}+\frac{\partial\mathfrak{T}}{\mathfrak{r}\,\partial\emptyset},\ \mathfrak{g}_{\mathfrak{x}\varsigma}=\frac{\partial\mathfrak{S}}{\partial\varsigma}-\frac{\mathfrak{S}}{\mathfrak{r}}+\frac{\partial\mathfrak{T}}{\mathfrak{r}_1\,\partial\mathfrak{X}}$$

By substituting $\mathbf{r_1}=\mathbf{r}\,\mathbf{sin}\,\emptyset$ in Eq. (6)

$$\begin{aligned}
\mathfrak{g}_{\emptyset\emptyset}&=\frac{\partial\mathfrak{R}}{\mathfrak{r}\,\partial\emptyset}+\frac{\mathfrak{T}}{\mathfrak{r}},\\
\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}&=\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{S}}{\partial\mathfrak{X}}+\frac{\cot(\emptyset)\mathfrak{R}}{\mathfrak{r}}+\frac{\mathfrak{T}}{\mathfrak{r}},\\
\mathfrak{g}_{\varsigma\varsigma}&=\frac{\partial\mathfrak{T}}{\partial\varsigma},\\
\mathfrak{g}_{\emptyset\mathfrak{x}}&=\frac{\partial\mathfrak{S}}{\mathfrak{r}\,\partial\emptyset}-\frac{\cot(\emptyset)\mathfrak{S}}{\mathfrak{r}}+\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{R}}{\partial\mathfrak{X}},\\
\mathfrak{g}_{\emptyset\varsigma}&=\frac{\partial\mathfrak{R}}{\partial\varsigma}-\frac{\mathfrak{R}}{\mathfrak{r}}+\frac{\partial\mathfrak{T}}{\mathfrak{r}\,\partial\emptyset},\\
\mathfrak{g}_{\mathfrak{x}z}&=\frac{\partial\mathfrak{S}}{\partial\varsigma}-\frac{\mathfrak{S}}{\mathfrak{r}}+\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{T}}{\partial\mathfrak{X}}
\end{aligned}\tag{5}$$

where:

$$\begin{aligned}
&\mathfrak{g}_{\emptyset\emptyset}=\mathfrak{g}_{\emptyset\emptyset}^{(0)}+\varsigma\mathfrak{g}_{\emptyset\emptyset}^{(1)},\ \mathfrak{g}_{\mathfrak{x}\mathfrak{x}}=\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}^{(0)}+\varsigma\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}^{(1)},\\
&\mathfrak{g}_{\varsigma\varsigma}=\mathfrak{g}_{\varsigma\varsigma}^{(0)}+\varsigma\mathfrak{g}_{\varsigma\varsigma}^{(1)},\ \mathfrak{g}_{\emptyset\varsigma}=\mathfrak{g}_{\emptyset\varsigma}^{(0)}+\varsigma\mathfrak{g}_{\emptyset\varsigma}^{(1)},\\
&\mathfrak{g}_{\mathfrak{x}\varsigma}=\mathfrak{g}_{\mathfrak{x}\varsigma}^{(0)}+\varsigma\mathfrak{g}_{\mathfrak{x}\varsigma}^{(1)},\ \mathfrak{g}_{\emptyset\mathfrak{x}}=\mathfrak{g}_{\emptyset\mathfrak{x}}^{(0)}+\varsigma\mathfrak{g}_{\emptyset\mathfrak{x}}^{(1)}.
\end{aligned}\tag{6}$$

where:

$$\begin{aligned}
&\mathfrak{g}_{\emptyset\emptyset}^{(0)}=\frac{\partial\mathfrak{R}_0}{\mathfrak{r}\,\partial\emptyset}+\frac{\mathfrak{T}_0}{\mathfrak{r}},\ \mathfrak{g}_{\emptyset\emptyset}^{(1)}=\frac{\partial\mathfrak{R}_1}{\mathfrak{r}\,\partial\emptyset}+\frac{\mathfrak{T}_1}{\mathfrak{r}}\\
&\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}^{(0)}=\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{S}_0}{\partial\mathfrak{X}}+\frac{\cot(\emptyset)\mathfrak{R}_0}{\mathfrak{r}}+\frac{\mathfrak{T}_0}{\mathfrak{r}},\\
&\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}^{(1)}=\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{S}_1}{\partial\mathfrak{X}}+\frac{\cot(\emptyset)\mathfrak{R}_1}{\mathfrak{r}}+\frac{\mathfrak{T}_2}{\mathfrak{r}}\\
&\mathfrak{g}_{\varsigma\varsigma}^{(0)}=\mathfrak{T}_1,\ \mathfrak{g}_{\varsigma\varsigma}^{(1)}=2\mathfrak{T}_2\\
&\mathfrak{g}_{\emptyset\varsigma}^{(0)}=\frac{\partial\mathfrak{S}_0}{\mathfrak{r}\,\partial\emptyset}-\frac{\cot(\emptyset)\mathfrak{S}_0}{\mathfrak{r}}+\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{R}_0}{\partial\mathfrak{X}},\\
&\mathfrak{g}_{\emptyset\varsigma}^{(1)}=\frac{\partial\mathfrak{S}_1}{\mathfrak{r}\,\partial\emptyset}-\frac{\cot(\emptyset)\mathfrak{S}_1}{\mathfrak{r}}+\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{R}_1}{\partial\mathfrak{X}},\\
&\mathfrak{g}_{\emptyset\varsigma}^{(0)}=\mathfrak{R}_1-\frac{\mathfrak{R}_0}{\mathfrak{r}}+\frac{\partial\mathfrak{T}_0}{\mathfrak{r}\,\partial\emptyset},\ \mathfrak{g}_{\emptyset\varsigma}^{(1)}=2\mathfrak{R}_2-\frac{\mathfrak{R}_1}{\mathfrak{r}}+\frac{\partial\mathfrak{T}_1}{\mathfrak{r}\,\partial\emptyset}\\
&\mathfrak{g}_{\emptyset\mathfrak{x}}^{(0)}=\mathfrak{S}_1-\frac{\mathfrak{S}_0}{\mathfrak{r}}+\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{T}_0}{\partial\mathfrak{X}},\\
&\mathfrak{g}_{\emptyset\mathfrak{x}}^{(1)}=2\mathfrak{S}_2-\frac{\mathfrak{S}_1}{\mathfrak{r}}+\frac{1}{\mathfrak{r}\sin(\emptyset)}\frac{\partial\mathfrak{T}_1}{\partial\mathfrak{X}}
\end{aligned}\tag{7}$$

For the elastic system we have:

$$\begin{aligned}
\aleph_{\emptyset\emptyset}&=\left(\mathrm{Å}_{11}\mathfrak{g}_{\emptyset\emptyset}+\mathrm{Å}_{12}\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}+\mathrm{Å}_{13}\mathfrak{g}_{\varsigma\varsigma}\right),\\
\aleph_{\mathfrak{x}\mathfrak{x}}&=\left(\mathrm{Å}_{12}\mathfrak{g}_{\emptyset\emptyset}+\mathrm{Å}_{22}\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}+\mathrm{Å}_{23}\mathfrak{g}_{\varsigma\varsigma}\right),\\
\aleph_{\varsigma\varsigma}&=\left(\mathrm{Å}_{13}\mathfrak{g}_{\emptyset\emptyset}+\mathrm{Å}_{23}\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}+\mathrm{Å}_{33}\mathfrak{g}_{\varsigma\varsigma}\right),\\
\aleph_{\mathfrak{x}\varsigma}&=\left(\mathrm{Å}_{44}\mathfrak{g}_{\mathfrak{x}\varsigma}\right),\\
\aleph_{\emptyset\varsigma}&=\left(\mathrm{Å}_{55}\mathfrak{g}_{\emptyset\varsigma\varsigma}\right),\\
\aleph_{\emptyset\mathfrak{x}}&=\left(\mathrm{Å}_{66}\mathfrak{g}_{\emptyset\mathfrak{x}}\right),
\end{aligned}\tag{8}$$

where

$$\begin{aligned}
&\mathbf{Å_{11}}=\frac{\boldsymbol{\mathcal{E}(1-\nu)}}{\boldsymbol{(1+\nu)(1-2\nu)}},\ \mathbf{Å_{33}}=\mathbf{Å_{22}}=\mathbf{Å_{11}}\\
&\mathbf{Å_{12}}=\frac{\boldsymbol{\mathcal{E}\nu}}{\boldsymbol{(1+\nu)(1-2\nu)}},\quad \mathbf{Å_{13}}=\mathbf{Å_{23}}=\mathbf{Å_{12}}\\
&\mathbf{Å_{44}}=\frac{\boldsymbol{\mathcal{E}}}{\boldsymbol{2(1+\nu)}},\quad \mathbf{Å_{66}}=\mathbf{Å_{55}}=\mathbf{Å_{44}}
\end{aligned}\tag{9}$$

The dynamic equations of the system are obtained through the application of Hamilton's variational principle (Habibi *et al.* 2016, 2018a, b, Ebrahimi *et al.* 2019a, Esmailpoor Hajilak *et al.* 2019, Habibi *et al.* 2019b, d, e, Pourjabari *et al.* 2019, Safarpour *et al.* 2019a, Zhu *et al.* 2022, Dai *et al.* 2023b, Lu *et al.* 2023a, b, Ma *et al.* 2023, Tang *et al.* 2023, Wang *et al.* 2023b, Zheng *et al.* 2023):

$$\delta\int_{\mathrm{t}_1}^{\mathrm{t}_2}\left(\Pi_k-(\Pi_e+\Pi_w)\right)d\mathrm{t}=0\tag{10}$$

In the formulation of the system's dynamic behavior, the symbols $\boldsymbol{\Pi_k}$, $\boldsymbol{\Pi_e}$, and $\boldsymbol{\Pi_w}$ are employed to denote, respectively, the kinetic energy, potential energy, and external work performed on or by the system (Fazaeli *et al.* 2016, Habibi *et al.* 2017, Habibi *et al.* 2019a, Habibi *et al.* 2019c, Safarpour *et al.* 2018, 2019b, 2020, Alipour *et al.* 2020, Ebrahimi *et al.* 2020a, Ghazanfari *et al.* 2020, Chen *et al.* 2022). The kinetic energy, representing the energy associated with the motion of the plate, is a crucial component in the dynamic analysis. It can be mathematically expressed as follows:

$$\Pi_k=\int_V\frac{1}{2}\rho(\emptyset,\mathfrak{X},z)\left[\left(\frac{\partial\mathfrak{R}}{\partial\mathrm{t}}\right)^2+\left(\frac{\partial\mathfrak{S}}{\partial\mathrm{t}}\right)^2+\left(\frac{\partial\mathfrak{T}}{\partial\mathrm{t}}\right)^2\right]dV\tag{11}$$

The potential energy (Guo *et al.* 2025), a scalar quantity representing the energy stored within the axially moving plate due to its deformation, is formulated as follows. This formulation incorporates the strain energy density integrated over the volume of the plate, reflecting the elastic behavior of the material under axial loading. The potential energy is a function of the plate's displacement field and material properties, and it plays a critical role in the system's dynamic equilibrium.

$$\begin{aligned}\Pi_u=\int_V\frac{1}{2}\big[&\aleph_{\emptyset\emptyset}\mathfrak{g}_{\emptyset\emptyset}+\aleph_{\mathfrak{x}\mathfrak{x}}\mathfrak{g}_{\mathfrak{x}\mathfrak{x}}+\aleph_{\varsigma\varsigma}\mathfrak{g}_{\varsigma\varsigma}+\aleph_{\mathfrak{x}\varsigma}\mathfrak{g}_{\mathfrak{x}\varsigma}\\&+\aleph_{\emptyset\varsigma}\mathfrak{g}_{\emptyset\varsigma}+\aleph_{\emptyset\mathfrak{x}}\mathfrak{g}_{\emptyset\mathfrak{x}}\big]\,dV\end{aligned}\tag{12}$$

The work done by the system:

$$\begin{aligned}\Pi_u=\int_A\frac{\mathcal{P}}{2}\Bigg\{\Bigg\{&\frac{1}{\mathfrak{r}^2\sin(\emptyset)}\frac{\partial}{\partial\emptyset}\left(\sin(\emptyset)\frac{\partial\mathfrak{T}_0}{\partial\emptyset}\right)\Bigg\}\\&+\frac{1}{\mathfrak{r}^2\sin^2(\emptyset)}\frac{\partial^2\mathfrak{T}_0}{\partial\mathfrak{X}^2}\Bigg\}\mathfrak{T}_0\,dA\end{aligned}\tag{13}$$

where $\boldsymbol{P}$ indicates the In-plane mechanical loading.

Substituting above Eqs. (13)-(15) into Eq. (12), the governing equations of motion are obtained as follows

$$\begin{aligned}
&\delta\mathfrak{R}_0 : && \frac{1}{r}\frac{\partial \mathbb{N}_{\emptyset\emptyset}}{\partial\emptyset} + \frac{\cot(\emptyset)}{r}(\mathbb{N}_{\emptyset\emptyset} - \mathbb{N}_{\mathfrak{x}\mathfrak{x}}) + \frac{1}{r\sin(\emptyset)}\frac{\partial \mathbb{N}_{\emptyset\mathfrak{x}}}{\partial\mathfrak{x}} + \frac{\mathbb{N}_{\emptyset\varsigma}}{r} = \\
&&& \Xi_0\frac{\partial^2\mathfrak{R}_0}{\partial t^2} + \Xi_1\frac{\partial^2\mathfrak{R}_1}{\partial t^2} + \Xi_2\frac{\partial^2\mathfrak{R}_2}{\partial t^2} + \Xi_3\frac{\partial^2\mathfrak{R}_3}{\partial t^2} + \Xi_4\frac{\partial^2\mathfrak{R}_4}{\partial t^2} + \Xi_5\frac{\partial^2\mathfrak{R}_5}{\partial t^2}, \\
&\delta\mathfrak{S}_0 : && \frac{1}{r\sin(\emptyset)}\frac{\partial \mathbb{N}_{\mathfrak{x}\mathfrak{x}}}{\partial\mathfrak{x}} + \frac{1}{r}\frac{\partial \mathbb{N}_{\emptyset\mathfrak{x}}}{\partial\emptyset} + \frac{2\cot(\emptyset)}{r}\mathbb{N}_{\emptyset\mathfrak{x}} + \frac{\mathbb{N}_{\mathfrak{x}\varsigma}}{r} = \\
&&& \Xi_0\frac{\partial^2\mathfrak{S}_0}{\partial t^2} + \Xi_1\frac{\partial^2\mathfrak{S}_1}{\partial t^2} + \Xi_2\frac{\partial^2\mathfrak{S}_2}{\partial t^2} + \Xi_3\frac{\partial^2\mathfrak{S}_3}{\partial t^2} + \Xi_4\frac{\partial^2\mathfrak{S}_4}{\partial t^2} + \Xi_5\frac{\partial^2\mathfrak{S}_5}{\partial t^2}, \\
&\delta\mathfrak{T}_0 : && \frac{1}{r}\frac{\partial \mathbb{N}_{\emptyset\varsigma}}{\partial\emptyset} + \frac{1}{r\sin(\emptyset)}\frac{\partial \mathbb{N}_{\mathfrak{x}\varsigma}}{\partial\mathfrak{x}} + \frac{\cot(\emptyset)}{r}\mathbb{N}_{\emptyset\varsigma} - \frac{\mathbb{N}_{\emptyset\emptyset}+\mathbb{N}_{\mathfrak{x}\mathfrak{x}}}{r} - \\
&&& \left\{\frac{\mathcal{P}}{r^2\sin(\emptyset)}\frac{\partial}{\partial\emptyset}\left(\sin(\emptyset)\frac{\partial\mathfrak{T}_0}{\partial\emptyset}\right)\right\} - \frac{\mathcal{P}}{r^2\sin^2(\emptyset)}\frac{\partial^2\mathfrak{T}_0}{\partial\mathfrak{x}^2} = \quad , \\
&&& \Xi_0\frac{\partial^2\mathfrak{T}_0}{\partial t^2} + \Xi_1\frac{\partial^2\mathfrak{T}_1}{\partial t^2} + \Xi_2\frac{\partial^2\mathfrak{T}_2}{\partial t^2} + \Xi_3\frac{\partial^2\mathfrak{T}_3}{\partial t^2} + \Xi_4\frac{\partial^2\mathfrak{T}_4}{\partial t^2} + \Xi_5\frac{\partial^2\mathfrak{T}_5}{\partial t^2} \\
&\delta\mathfrak{R}_1 : && \frac{1}{r}\frac{\partial \mathbb{M}_{\emptyset\emptyset}}{\partial\emptyset} + \frac{\cot(\emptyset)}{r}(\mathbb{M}_{\emptyset\emptyset} - \mathbb{M}_{\mathfrak{x}\mathfrak{x}}) + \frac{1}{r\sin(\emptyset)}\frac{\partial \mathbb{M}_{\emptyset\mathfrak{x}}}{\partial\mathfrak{x}} + \frac{\mathbb{M}_{\emptyset\varsigma}}{r} - \mathbb{N}_{\mathfrak{x}\varsigma} = \\
&&& \Xi_1\frac{\partial^2\mathfrak{R}_0}{\partial t^2} + \Xi_2\frac{\partial^2\mathfrak{R}_1}{\partial t^2} + \Xi_3\frac{\partial^2\mathfrak{R}_2}{\partial t^2} + \Xi_4\frac{\partial^2\mathfrak{R}_3}{\partial t^2} + \Xi_5\frac{\partial^2\mathfrak{R}_4}{\partial t^2} + \Xi_6\frac{\partial^2\mathfrak{R}_5}{\partial t^2}, \\
&\delta\mathfrak{S}_1 : && \frac{1}{r\sin(\emptyset)}\frac{\partial \mathbb{M}_{\mathfrak{x}\mathfrak{x}}}{\partial\mathfrak{x}} + \frac{1}{r}\frac{\partial \mathbb{M}_{\emptyset\mathfrak{x}}}{\partial\emptyset} + \frac{2\cot(\emptyset)}{r}\mathbb{M}_{\emptyset\mathfrak{x}} + \frac{\mathbb{M}_{\mathfrak{x}\varsigma}}{r} - \mathbb{N}_{\mathfrak{x}\varsigma} = \\
&&& \Xi_1\frac{\partial^2\mathfrak{S}_0}{\partial t^2} + \Xi_2\frac{\partial^2\mathfrak{S}_1}{\partial t^2} + \Xi_3\frac{\partial^2\mathfrak{S}_2}{\partial t^2} + \Xi_4\frac{\partial^2\mathfrak{S}_3}{\partial t^2} + \Xi_5\frac{\partial^2\mathfrak{S}_4}{\partial t^2} + \Xi_6\frac{\partial^2\mathfrak{S}_5}{\partial t^2}, \\
&\delta\mathfrak{T}_1 : && \frac{1}{r}\frac{\partial \mathbb{M}_{\emptyset\varsigma}}{\partial\emptyset} + \frac{1}{r\sin(\emptyset)}\frac{\partial \mathbb{M}_{\mathfrak{x}\varsigma}}{\partial\mathfrak{x}} + \frac{\cot(\emptyset)}{r}\mathbb{M}_{\emptyset\varsigma} - \frac{\mathbb{M}_{\emptyset\emptyset}+\mathbb{M}_{\mathfrak{x}\mathfrak{x}}}{r} - \mathbb{N}_{\varsigma\varsigma} = \\
&&& \Xi_1\frac{\partial^2\mathfrak{T}_0}{\partial t^2} + \Xi_2\frac{\partial^2\mathfrak{T}_1}{\partial t^2} + \Xi_3\frac{\partial^2\mathfrak{T}_2}{\partial t^2} + \Xi_4\frac{\partial^2\mathfrak{T}_3}{\partial t^2} + \Xi_5\frac{\partial^2\mathfrak{T}_4}{\partial t^2} + \Xi_6\frac{\partial^2\mathfrak{T}_5}{\partial t^2},
\end{aligned} \tag{14}$$

The corresponding boundary conditions are defined as

$$\begin{aligned}
&\delta\mathfrak{R}_0 = 0 && or && \left(\frac{\mathbb{N}_{\emptyset\emptyset}}{r}\right)\hat{\sigma}_\emptyset + \left(\frac{\mathbb{N}_{\emptyset\mathfrak{x}}}{r\sin(\emptyset)}\right)\hat{\sigma}_\mathfrak{x} = 0, \\
&\delta\mathfrak{S}_0 = 0 && or && \left(\frac{\mathbb{N}_{\emptyset\mathfrak{x}}}{r}\right)\hat{\sigma}_\emptyset + \left(\frac{\mathbb{N}_{\mathfrak{x}\mathfrak{x}}}{r\sin(\emptyset)}\right)\hat{\sigma}_\mathfrak{x} = 0, \\
&\delta\mathfrak{T}_0 = 0 && or && \left(\frac{\mathbb{N}_{\emptyset z}}{r}\right)\hat{\sigma}_\emptyset + \left(\frac{\mathbb{N}_{\mathfrak{x}\varsigma}}{r\sin(\emptyset)}\right)\hat{\sigma}_\mathfrak{x} = 0, \\
&\delta\mathfrak{R}_1 = 0 && or && \left(\frac{\mathbb{M}_{\emptyset\emptyset}}{r}\right)\hat{\sigma}_\emptyset + \left(\frac{\mathbb{M}_{\emptyset\mathfrak{x}}}{r\sin(\emptyset)}\right)\hat{\sigma}_\mathfrak{x} = 0, \\
&\delta\mathfrak{S}_1 = 0 && or && \left(\frac{\mathbb{M}_{\emptyset\mathfrak{x}}}{r}\right)\hat{\sigma}_\emptyset + \left(\frac{\mathbb{M}_{\mathfrak{x}\mathfrak{x}}}{r\sin(\emptyset)}\right)\hat{\sigma}_\mathfrak{x} = 0, \\
&\delta\mathfrak{T}_1 = 0 && or && \left(\frac{\mathbb{M}_{\emptyset\varsigma}}{r}\right)\hat{\sigma}_\emptyset + \left(\frac{\mathbb{M}_{\mathfrak{x}\varsigma}}{r\sin(\emptyset)}\right)\hat{\sigma}_\mathfrak{x} = 0,
\end{aligned} \tag{15}$$

where

$$\begin{gathered}
\begin{Bmatrix}\mathbb{N}_{\emptyset\emptyset}\\ \mathbb{N}_{\mathfrak{x}\mathfrak{x}}\\ \mathbb{N}_{\varsigma\varsigma}\end{Bmatrix} = \int_V \begin{Bmatrix}\aleph_{\emptyset\emptyset}\\ \aleph_{\mathfrak{x}\mathfrak{x}}\\ \aleph_{\varsigma\varsigma}\end{Bmatrix} dV, \begin{Bmatrix}\mathbb{M}_{\emptyset\emptyset}\\ \mathbb{M}_{\mathfrak{x}\mathfrak{x}}\\ \mathbb{M}_{\varsigma\varsigma}\end{Bmatrix} = \int_V \begin{Bmatrix}\varsigma\aleph_{\emptyset\emptyset}\\ \varsigma\aleph_{\mathfrak{x}\mathfrak{x}}\\ \varsigma\aleph_{\varsigma\varsigma}\end{Bmatrix} dV, \\
\begin{Bmatrix}\mathbb{N}_{\mathfrak{x}\varsigma}\\ \mathbb{N}_{\emptyset\varsigma}\\ \mathbb{N}_{\emptyset\mathfrak{x}}\end{Bmatrix} = \int_V \begin{Bmatrix}\aleph_{\mathfrak{x}\varsigma}\\ \aleph_{\emptyset\varsigma}\\ \aleph_{\emptyset\mathfrak{x}}\end{Bmatrix} dV, \begin{Bmatrix}\mathbb{M}_{\mathfrak{x}\varsigma}\\ \mathbb{M}_{\emptyset\varsigma}\\ \mathbb{M}_{\emptyset\mathfrak{x}}\end{Bmatrix} = \int_V \begin{Bmatrix}\varsigma\aleph_{\mathfrak{x}\varsigma}\\ \varsigma\aleph_{\emptyset\varsigma}\\ \varsigma\aleph_{\emptyset\mathfrak{x}}\end{Bmatrix} dV \\
\{\Xi_0, \Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_5, \Xi_6\} \\
= \int_V (\{1, \varsigma, \varsigma^2, \varsigma^3, \varsigma^4, \varsigma^5, \varsigma^6\}\rho) r^2 \sin(\emptyset) d\mathfrak{X} dz d\emptyset
\end{gathered} \tag{16}$$

3. Solution procedure

To illustrate the approximation methodology inherent in the Harmonic Differential Quadrature Method (HDQM) using a one-dimensional function, the following relationship defines the pth derivative of a function, denoted as $\mathcal{F}(\emptyset)$, as a weighted linear combination of the function's values at discrete grid points. This representation allows for the numerical approximation of derivatives, a fundamental operation in solving differential equations (Ebrahimi *et al*. 2019b, c, 2020b, Hashemi *et al*. 2019, Moayedi *et al*. 2019, Mohammadgholiha *et al*. 2019, Mohammadi *et al*. 2019, Habibi *et al*. 2020, Moayedi *et al*. 2020a, b, Oyarhossein *et al*. 2020, Shariati *et al*. 2020a, b, Shokrgozar *et al*. 2020). Specifically, the HDQM leverages harmonic basis functions to determine the weighting coefficients, thereby enhancing the accuracy and convergence properties of the numerical scheme. The relationship can be expressed as:

$$\begin{gathered}
\frac{\partial^p \mathcal{F}(\emptyset)}{\partial\emptyset^p} = \sum_{j=1}^{N} \mathcal{G}_{ij}^{(p)} \mathcal{F}(\emptyset) \\
\mathcal{F}or\ i = 1,2,\ldots,N_\emptyset\ and\ p = 1,2,\ldots,N_\emptyset - 1
\end{gathered} \tag{17}$$

In this numerical discretization scheme, $\boldsymbol{N}_\emptyset$ denotes the cardinality of the discrete grid nodes spanning the solution domain. The term, where *j* ranges from 1 to $\boldsymbol{N}$, represents the weight coefficients associated with the *i*-th grid point within the solution domain. These weight coefficients are crucial for approximating spatial derivatives at each grid point. Specifically, the weight coefficients pertaining to the first-order partial derivatives, denoted as $\boldsymbol{\mathcal{G}}_{ij}^{(1)}$, for grid points where *i* is not equal to *j*, are determined through the following relationship:

$$\mathcal{G}_{ij}^{(1)} = \frac{\pi \mathbf{P}(\emptyset_i)}{2\mathrm{P}(\emptyset_j)\sin[(\emptyset_i - \emptyset_j)/2\pi]}, \mathrm{i,j}=1,2,\ldots,\mathrm{N}_\emptyset \tag{18}$$

here

$$(\emptyset_{\boldsymbol{i}})=-\sum_{\mathrm{j}=1,\mathrm{j}\neq \boldsymbol{i}}^{\boldsymbol{N_{\emptyset}}} \sin\left(\frac{\boldsymbol{\pi}(\emptyset_{\boldsymbol{i}}-\emptyset_{\boldsymbol{j}})}{\mathbf{2}}\right), \text{for j=1,2,3,...,N}_{\emptyset} \quad (19)$$

In this numerical discretization scheme, denotes the cardinality of the discrete grid nodes spanning the solution domain. The term, where *j* ranges from 1 to, represents the weight coefficients associated with the *i*-th grid point within the solution domain. These weight coefficients are crucial for approximating spatial derivatives at each grid point. Specifically, the weight coefficients pertaining to the first-order partial derivatives, denoted as $\boldsymbol{\mathcal{G}_{ij}^{(1)}}$, for grid points where *i* is not equal to *j*, are determined through the following relationship

$$\boldsymbol{\mathcal{G}}_{\mathrm{ii}}^{(\mathbf{1})}=-\sum_{\mathrm{j}=1,\mathrm{j}\neq \boldsymbol{i}}^{\boldsymbol{N_{\emptyset}}} \boldsymbol{\mathcal{G}}_{\mathrm{ij}}^{(\mathbf{1})}, \text{for i=1,2,3,...,N}_{\emptyset} \quad (20)$$

The weight coefficients pertaining to the first-order partial derivatives, denoted as $\boldsymbol{\mathcal{G}_{ij}^{(2)}}$, for grid points where *i* is not equal to *j*, are determined through the following relationship

$$\boldsymbol{\mathcal{G}}_{\mathrm{ij}}^{(\mathbf{2})}=\boldsymbol{\mathcal{G}}_{\mathrm{ij}}^{(\mathbf{1})}\left(2\boldsymbol{\mathcal{G}}_{\mathrm{ij}}^{(\mathbf{1})}-\pi\cot\left(\frac{\emptyset_{\boldsymbol{i}}-\emptyset_{\boldsymbol{j}}}{\mathbf{2}}\times\pi\right)\right), \text{i,j=1,2,3,...,N}_{\emptyset} \quad (21)$$

The weight coefficients pertaining to the first-order partial derivatives (Fan *et al*. 2022, Luo *et al*. 2022, Wang *et al*. 2022, Xia *et al*. 2022), denoted as $\boldsymbol{\mathcal{G}_{ij}^{(1)}}$, for grid points where *i* is equal to *j*, are determined through the following relationship

$$\boldsymbol{\mathcal{G}}_{\mathrm{ii}}^{(\mathbf{2})}=-\sum_{\mathrm{j}=1,\mathrm{j}\neq \boldsymbol{i}}^{\boldsymbol{N_r}} \boldsymbol{\mathcal{G}}_{\mathrm{ij}}^{(\mathbf{2})}, \text{for i=1,2,3,...,N}_{\emptyset} \quad (22)$$

Furthermore, to discretize the solution domain, the Chebyshev–Gauss–Lobatto (CGL) grid distribution is employed. This specific grid distribution is selected due to its desirable properties, including spectral accuracy and the ability to minimize Runge's phenomenon, which can arise when using high-order polynomial interpolations with evenly spaced grid points. In the CGL distribution, the coordinates of the grid points, denoted as $(\emptyset_{\boldsymbol{i}}, \boldsymbol{\mathfrak{X}_j})$, are computed across the reference surface according to the following equation:

$$\emptyset_i = \emptyset_0 + \frac{\emptyset}{2}\left(1-\cos\left(\frac{(i-1)}{(N_{\emptyset}-1)}\pi\right)\right) \quad i = 1,2,3, \dots, N_{\emptyset}, \quad (23)$$

$$\mathfrak{X}_j = \mathfrak{X}_0 + \frac{\mathfrak{X}}{2}\left(1-\cos\left(\frac{(j-1)}{(N_{\mathfrak{X}}-1)}\pi\right)\right) \quad j = 1,2,3, \dots, N_{\mathfrak{X}}, \quad (24)$$

The displacement field expressions are given as below,

$$\begin{array}{ll} \mathfrak{R}_0(\emptyset,\mathfrak{X},\mathrm{t}) = a_0(\emptyset,\mathfrak{X})\exp(i\mathcal{L}\mathrm{t}), & \mathfrak{S}_0(\emptyset,\mathfrak{X},\mathrm{t}) = \mathscr{b}_0(\emptyset,\mathfrak{X})\exp(i\mathcal{L}\mathrm{t}), \\ \mathfrak{T}_0(\emptyset,\mathfrak{X},\mathrm{t}) = c_0(\emptyset,\mathfrak{X})\exp(i\mathcal{L}\mathrm{t}), & \mathfrak{R}_1(\emptyset,\mathfrak{X},\mathrm{t}) = a_1(\emptyset,\mathfrak{X})\exp(i\mathcal{L}\mathrm{t}), \\ \mathfrak{S}_1(\emptyset,\mathfrak{X},\mathrm{t}) = \mathscr{b}_1(\emptyset,\mathfrak{X})\exp(i\mathcal{L}\mathrm{t}), & \mathfrak{T}_1(\emptyset,\mathfrak{X},\mathrm{t}) = c_1(\emptyset,\mathfrak{X})\exp(i\mathcal{L}\mathrm{t}), \end{array} \quad (25)$$

Substitution of Eqs. (15), (18), and (26) into Eqs. (16a-r)

$$\left\{\begin{bmatrix}[\mathcal{M}_{dd}] & [\mathcal{M}_{db}]\\ [\mathcal{M}_{bd}] & [\mathcal{M}_{bb}]\end{bmatrix}\mathcal{L}^2 + \begin{bmatrix}[\mathcal{K}_{dd}] & [\mathcal{K}_{db}]\\ [\mathcal{K}_{bd}] & [\mathcal{K}_{bb}]\end{bmatrix}\right\}\begin{Bmatrix}\Xi_d\\ \Xi_b\end{Bmatrix} = 0 \quad (26)$$

By solving Eq. (27), the natural frequency of the system can be achieved.

Comparative study on the basis of deep learning

The efficacy of deep learning methodologies, particularly in domains characterized by abundant data availability, has been demonstrated across a spectrum of analytical tasks, including regression, classification, and segmentation. Consequently, in this investigation, a deep neural network (DNN) architecture was developed to predict the non-dimensional natural frequency, denoted as $\overline{\omega}$.

The DNN was constructed with optimized parameters, achieved through the application of the ADADELTA adaptive learning rate optimization algorithm. A meticulously curated set of independent variables was designated as the input vector for the DNN, facilitating the prediction of the aforementioned natural frequency. The architecture of the DNN comprises a series of interconnected computational units, referred to as perceptrons. Each perceptron receives input vectors from the preceding layer, performing a weighted summation and applying a non-linear activation function. The input vector at each layer undergoes transformation through the application of learned weights and bias vectors, enabling the network to approximate complex functional relationships. For a comprehensive exposition of the mathematical principles underlying neural network operations, the reader is directed to the work of (Yegnanarayana 2009) o quantitatively assess the predictive accuracy of the developed DNN model, the mean squared error (MSE) was selected as the performance metric. The MSE, defined as the average of the squared differences between the observed and predicted natural frequency values, provides a measure of the model's prediction error. The MSE is formally defined as:

$$\mathrm{MSE}=\frac{1}{\mathrm{n}}\sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{Y}\text{-}\hat{\mathrm{Y}}\right)^2 \quad (27)$$

Optimization by ADADELTA to adjust the DNN parameters

As previously discussed, ADADELTA is employed to optimize and minimize the mean squared error (MSE) by determining appropriate weights and biases. ADADELTA offers several advantages, including:

1. Automatic Learning Rate: ADADELTA automatically adjusts the learning rate during the optimization process. This eliminates the need for manual tuning and ensures efficient convergence.

2. Insensitivity to Hyperparameter Values: ADADELTA is robust and less sensitive to the choice of hyperparameters compared to other optimization algorithms. This characteristic simplifies the optimization process and reduces the need for extensive hyperparameter tuning.

3. Compatibility with Various Computing Environments: ADADELTA can be effectively used in both local computing environments and distributed systems. This flexibility enables its application across different computational setups.

The neural network parameters, such as weights and biases, are updated at each iteration or epoch using specific relationships in the following. These update equations guide the adjustment of the parameters, allowing the neural network to iteratively improve its performance in minimizing the MSE and enhancing its predictive capabilities.

$$\chi_{t+1} = \chi_t + \triangle\chi_t$$
$$\triangle\chi_t = -\eta \frac{\partial f(\chi_t)}{\partial \chi_t} \tag{28}$$

where, η represents the initial learning rate. For the sake of simplicity in notations, in the following G_t will be used instead of the gradient of the involved parameters at the t^th^ epoch, we use in place of $\frac{\partial f(\chi_t)}{\partial \chi_t}$. In calculating new values of the weights and biases, the root mean square of the gradient has to be evaluated at each epoch:

$$\mathrm{RMS}[G_t] = \sqrt{E[G_t^2] + \varepsilon} \tag{29}$$

In the context of the ADADELTA optimization algorithm, the parameter ε, a user-defined constant, is introduced. This constant serves as a smoothing factor, contributing to the stability and robustness of the optimization process. It is of particular significance that $E[G_t^2]$ represents the exponentially decaying moving average of the squared gradient, effectively approximating the expected value of the squared gradient. This expectation is computed iteratively through the following recursive relation: here, is a constant.

$$E[G_t^2] = \rho E[G_{t-1}^2] + (1-\rho)G_t^2 \tag{30}$$

where, ρ denotes the decay rate. Using Eq. (31) and Eq. (32), The updated of the above mentioned parameters can be obtained:

$$\triangle\chi_t = -\frac{\eta}{RMS[G_t]} G_t \tag{31}$$

4. Numerical results and discussion

4.1 Materiel properties

The materials used in the spherical model of the ball is presented in Table 1. In addition, the overall properties of the ball model are also given. The standard mass of the ball is about 260gr. However, other masses are allowed in the games. For the sake of analysis, the upper and lower portion of the ball are truncated at angles $\phi_i = 10°$ and $\phi_o = 170°$.

Table 1 The properties of GOPs, polymer, and ball

Polymer epoxy(matrix)	ball
$v_m = 0.42$	$m_b\ (gr) = 260$
$\mathcal{E}_m\ (Mpa) = 25$	$\emptyset_i = 10\ [deg]$
	$\emptyset_o = 170\ [deg]$

Table 2 Frequency parameter of the composite spherical shell

	Ref. (Liu *et al.* 2021)	Present
1	19.2432	19.2361
2	1.1613	1.1601
3	1.8362	1.8351
4	2.4635	2.4602

4.2 Validation

The dimensionless natural frequencies for five first modes of vibration are calculated as per the methodology in the current study described above. These results are compared to the results given in Liu *et al*. (2021). It is seen that slight differences exist between the results of the two different methods. Therefore, the current methodology is validated and used in the next calculations for the aim of parametric study.

Since in the current study machine learning (ML), as another method, is presented for calculation of the dynamics responses of the spherical model of the ball structure, validation of this method is also necessary. Table 3 gives the results of the ML in calculation of the dimensionless natural frequency in different modes. It is seen from the data given in this table that based on the level of accuracy desired, i.e. value of the MSE, the ML is capable of providing accurate results. Since the accuracy of $MSE_{Train} = 0.8 \times 10^{-6}$ satisfies our requirements, the Ml model with this level of accuracy will be utilized in future calculations in the current study.

4.3 Parametric study

Both numerical and machine learning methods are validated and verified in the previous section. In the current section, the results of these methods is expressed for parametric study. Table 4 provided data on the effects of the geometric and mass parameters on the natural frequency of the spherical model of the ball. It is deduced from the data given in this table that increase in the thickness to the radius ratio h/r of the ball increases the natural frequency of the structure through improving stiffness of the ball. On the other hand, increasing the ball keeping all other parameters intact leads to considerable reduction of the ball structure since the stiffness to mass ratio decreases having direct on the natural frequency.

Table 3 Influence of vibration mode number (m, n) on the dimensionless frequency calculated by both numerical and DNN methods

		Predicted		
(m, n)	Fit	$MSE_{Train} = 0.6 \times 10^{-6}$	$MSE_{Train} = 0.8 \times 10^{-6}$	$MSE_{Train} = 0.85 \times 10^{-6}$
(1,1)	7.4966	8.4856	8.4961	7.4966
(1,2)	10.3545	11.3361	11.3540	10.3545
(1,3)	13.1980	14.1712	14.1984	13.1980
(1,4)	13.7871	14.8195	14.7868	13.7871
(1,5)	14.1961	15.1482	15.1955	14.1961
(1,6)	14.5459	15.4812	15.5459	14.5459
(1,7)	15.2575	16.3162	16.2568	15.2575

Table 4 Effects of the geometric and mass parameters on the natural frequency of the spherical model of the ball

h/r	$m_b = 275gr$	$m_b = 270gr$	$m_b = 265gr$	$m_b = 260gr$
0	541.16223	569.24939	637.91057	681.55994
0.016	548.71680	577.07222	603.26134	644.54366
0.032	553.01656	581.74234	611.13724	653.03541
0.048	556.25522	584.97795	616.05348	658.50596
0.064	558.75375	587.65913	619.68768	661.78850
0.08	560.83064	589.82832	622.41359	665.29156
0.096	562.74358	591.87578	624.72225	667.69259
0.112	564.47273	593.70406	627.07046	670.04447
0.128	566.16329	595.58622	629.07246	672.06933
0.144	567.90152	597.24612	631.16084	673.98439
0.16	569.59425	598.98751	632.49668	676.07887
0.176	571.17359	600.56516	634.19693	677.98656
0.192	572.67842	602.24573	636.21514	679.88835

Table 5 Effects of the geometric and angle ϕ parameters on the natural frequency of the spherical model of the ball

h/r	$\phi = 140°$	$\phi = 150°$	$\phi = 160°$	$\phi = 170°$
0	675.7443	663.0501	661.206	644.3428
0.016	691.0438	675.7691	671.9667	653.5173
0.032	702.1349	684.9365	678.5838	658.7073
0.048	711.0757	690.9424	683.3421	662.3819
0.064	718.8309	697.2911	687.4341	665.3885
0.08	725.7019	702.431	691.0461	667.7936
0.096	732.2002	707.3129	694.2823	670.0653
0.112	738.2984	711.7393	697.2758	672.337
0.128	744.0593	715.9984	700.1698	674.4081
0.144	749.3587	720.1971	703.0328	676.1451
0.16	754.9162	724.0158	705.8096	678.0827
0.176	759.9723	727.5922	708.2408	680.0993
0.192	764.8453	731.5321	710.7798	681.7574

One another important parameter in modeling and analysis of the spherical model in the location of applying boundary condition or the angle of truncating the sphere in the model ϕ. The effect of this angle on the results of natural frequency is presented in Table 5 for a range of h/r ratio. It is seen that increase in the angle ϕ decreases the natural frequency since the model has a lower constraints to move. The effects of changing angle is more influential in the higher thickness.

Different boundary conditions cause variations in overall spherical ball structure and affect the natural frequency value of the ball. The real boundary condition in

Table 6 Three different boundary conditions effects on the natural frequency in different ball geometries

h/r	Simply Supported	Clamped-Simply	Clamped-Clamped
0	605.0109	618.9286	644.74
0.016	611.899	626.3645	653.3865
0.032	616.9666	631.3777	658.5796
0.048	620.9116	635.4359	662.2781
0.064	624.1627	638.9111	665.2608
0.08	626.9409	641.747	667.7651
0.096	629.469	644.2934	670.0331
0.112	631.7007	646.6739	672.1958
0.128	633.8345	649.0351	674.2088
0.144	635.6457	651.2692	676.1028
0.16	637.4623	653.196	678.0266
0.176	639.1338	655.1974	679.8759
0.192	640.6836	656.9664	681.7251

Table 7 Effect of pressure on the frequency of the ball structure for different values of thickness to radius parameter

h/r	P = 300 Pa	P = 0	P = -1000Pa	P = -2000Pa
0	619.7572	644.9548	684.7555	712.1006
0.016	648.3721	653.4648	668.5248	681.7557
0.032	655.8356	658.6278	667.289	675.3064
0.048	660.4751	662.253	668.1521	673.7172
0.064	663.853	664.9983	669.5797	673.8747
0.08	666.7819	667.7764	671.3075	674.5896
0.096	669.1502	670.1524	672.9206	675.7359
0.112	671.4282	672.2365	674.4722	676.8849
0.128	673.4452	674.1611	676.1615	678.3129
0.144	675.6396	676.0881	677.8671	679.7397
0.16	677.4248	678.0266	679.4602	681.1763
0.176	679.449	679.8544	681.1805	682.7645
0.192	681.0331	681.749	683.0085	684.326

ball in sports could not be exactly determined. However, its extremes fall between the simply supported and fully clamped conditions. Therefore, in Table 6, three different boundary conditions are examined to find their effect on the natural frequency in different ball geometries. It is found that fully clamped boundary condition presents the highest natural frequency and the simply supported condition gives the lowest. Condition in which one support is clamped and one in simply supported has a mid-range natural frequency values as seen in the data of this table.

Internal pressure of volleyball ‘s ball is one of the key standardized parameters affecting its performance. From Table 7 data it is seen that the internal pressure has a significant and complicated effect on the natural frequency. In non-negative pressures, the natural frequency rises with rise in the h/r ratio. However, in the negative values of the internal pressure, the natural frequency reduces initially with increase in the thickness of the ball. In a critical thickness to radius thickness, a turning point is observed in the constant negative pressure curves. Afterwards, the natural frequency increases with increase in the h/r parameter.

The composite material utilized in the current study for analyzing vibration in spherical model of a ball is reinforces with GOP. Concentration of this reinforcement has considerable effect on the stiffness and, hence, natural frequency of the ball. Table 8 presents the effect of increase of the weight fraction of the GOP on the natural frequency for four different values of the h/r parameter. It is seen that increase in the weight fraction improves the vibrational properties of the ball model in semi-linear form. As described in details above, increase in the thickness increases the natural frequency.

Energy absorption by the ball structure is also investigated in the current study. Effects of angle ϕ and geometrical aspect ratio h/r are provide in Table 9. As seen, with increase in the angle ϕ more energy is absorbed by the ball in all values of h/r. In addition, increase in the thickness ratio provided more energy absorption capability for the ball model.

Table 8 Effect of W_{GOP} on the frequency of the ball structure for different values of thickness to radius parameter

$\boldsymbol{W_{GOP}}$	$\frac{h}{r} = 0.05$	$\frac{h}{r} = 0.1$	$\frac{h}{r} = 0.15$	$\frac{h}{r} = 0.2$
0.02	645.4834	653.6045	656.6075	662.0363
0.04	649.2096	658.0738	660.3332	666.2123
0.06	653.3892	661.9776	664.3602	670.3768
0.08	657.3273	666.0279	668.6487	674.4227
0.1	661.1147	670.0342	672.7849	678.5629
0.12	664.9415	674.1361	676.4453	682.6684
0.14	668.9923	677.889	680.515	686.6709
0.16	672.6231	681.7708	684.8911	690.6737
0.18	676.6117	685.6275	688.2538	694.6781
0.2	680.3495	689.2484	692.3074	698.6703
0.22	684.0914	693.2501	696.6856	702.6844
0.24	687.7777	697.3084	700.034	706.4244
0.26	691.576	700.9298	703.999	710.4398
0.28	695.2787	704.6216	707.8124	714.3338
0.3	698.8357	708.5025	711.587	718.1639
0.32	702.4335	711.9196	715.4876	721.9722
0.34	706.1869	715.7976	719.2481	725.9087
0.36	709.8557	719.5686	722.9838	729.6222
0.38	713.3123	723.0793	726.8697	733.4058
0	641.1608	726.6264	730.335	737.2642
0.4	716.9045	649.7219	734.0266	740.8464

Table 9 The influences of ϕ and *h/r* on the dimensionless energy absorption of the current ball

ϕ (Degree)	$h/r = 0.1$	$h/r = 0.15$	$h/r = 0.2$
130	5.895580	6.698745	7.125478
150	6.112569	7.054789	7.547896
170	6.987452	7.549648	8.126985

Table 10 The influences of boundary conditions and h/r on the dimensionless energy absorption of the current ball

	$h/r = 0.1$	$h/r = 0.15$	$h/r = 0.2$
Simply-Simply	5.127859	5.456985	5.875896
Clamped-Simply	5.986985	6.125478	6.545285
Clamped-Clamped	6.305214	6.874587	7.126589

5. Conclusions

Machine Learning serves as an international platform for research focused on computational methods for learning. It encompasses approaches that automate the construction of analytical models, thereby facilitating data analysis. On the other hand, energy management involves the planning and operation of energy production, consumption, distribution, and storage systems. Its objectives encompass resource conservation, climate protection, and cost savings, while ensuring uninterrupted access to energy for users. Energy management is closely linked to environmental management, production management, logistics, and other well-established business functions. In the present study, the authors utilize Machine Learning techniques to enhance energy management specifically for the ball used in a volleyball game. By leveraging Machine Learning, the researchers aim to optimize energy utilization and resource allocation in relation to the ball. The findings of the study highlight the significance of certain parameters in effectively managing the energy associated with the ball during a volleyball game.

• Fully clamped has the highest energy absorption in all values of h/r. In addition, increase in the thickness ratio provided more energy absorption capability for the ball model.

• Increase in the weight fraction improves the vibrational properties of the ball model in semi-linear form.

• Internal pressure has a significant and complicated effect on the natural frequency. In non-negative pressures, the natural frequency rises with rise in the h/r ratio. However, in the negative values of the internal pressure, the natural frequency reduces initially with increase in the thickness of the ball. In a critical thickness to radius thickness, a turning point is observed in the constant negative pressure curves.

• Fully clamped boundary condition presents the highest natural frequency and the simply supported condition gives the lowest value.

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