

# The concentration profile behavior for chemical reaction and energy activation parameter in Casson nanofluid flow: Via Darcy-Forchheimer Medium

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**Abstract.** Examining the viscous incompressible bioconvection Casson type nanoliquid flow with Darcy-Forchheimer resistance across a non-linear stretching surface is the primary goal of this study. Non-dimensional variables are used to decrease non-linear expressions. The shooting technique is used to solve the simplified ordinary differential equations. The behavior of several influential parameters is examined graphically and the concentration profile is examined. Thermophoresis parameter, Prandtl number, energy activation, chemical reaction factor, and concentration slip parameter all improve the concentration profile. By contrasting the current findings with earlier published research, the validity of the current study is validated.

**Keywords:** chemical reaction factor; concentration slip parameter; energy activation; Prandtl number; thermophoresis parameter

## 1. Introduction

Due to its numerous applications in geothermal reservoirs, chemical engineering, nuclear reacting cooling, thermal oil recovery, and more, the concept of mass transfer with chemical reaction and Arrhenius energy activation has received a lot of attention. Chemical reaction relations involving mass transfer are sometimes very difficult to study in applications involving production and reactant species at various speeds in the mass transport of nanoliquids (Tan *et al.* 2025, Hang *et al.* 2023). We achieved the best thermal properties at the lowest concentrations by employing nanofluids. Because of their high heat conductivity, nanofluids have recently replaced base liquids as working liquids. Choi (1995) coined the term “nanofluids” after examining and investigating nanoparticles. A novel way to raise the heat exchanger coefficient of conventional liquids has been rapidly emerging: nano liquid, a new generation of superoperative medium. Particularly in recent years, the development of nanofluids has emerged as an impetuous direction. There is a lot of somatic assets, mass exchangers, steaming heat exchangers, ongoing research, and some national model growth in the field of nanofluids analysis. Nanoparticles or strong nanofibres with diameters ranging from 1 to 100 nm are found in nanofluids. Nanomaterials can transfer without

obstructing the outflow system and have a stable suspension strength due to the moderate size of nanofluids. Particularly in home refrigerators and freezers, nuclear reactors, machineries, and electronic devices, nanoliquids are recognized as good coolants due to their superior thermal efficiency compared to the base fluid. Many researchers have developed a strategy to use nanoliquids as an elective technique to improve the adequacy of energy transfer by taking into account the aforementioned applications. The non-homogeneous equilibrium structure, which includes convective transport in nanoliquids, was used by Buongiorno (2006). By taking into account the impact of the nanomaterial fractions, (Tiwari and Das 2007) suggested a homogenous structure. Through a circular cavity, Sheikholeslami *et al.* (2019) examined the impact of fluctuating magnetic forces on a magnetizable hybrid nano liquid heat exchanger. The impact of MHD on a nanoliquid heat exchanger in a baffled U-shaped enclosure was investigated numerically by Yuan *et al.* (2019).

They found that the addition of nanoliquid to heat exchangers improved the Hartmann number while decreasing the Rayleigh number. Using the lattice Boltzmann technique, (Zhixiong *et al.* 2019) demonstrated the nanoliquid transmission of heat in a porous duct in the presence of the Lorentz forces parameter. They observed that the LBM was used to explain how the Reynolds (Re), Darcy (Da), and magnetic (Ha) parameters affected the behavior of the nanoliquid. The references (Anwar *et al.* 2019, Chaudhary and Kanika 2019, Ambreen and Kim 2018, Shafee *et al.* 2019, Sheikholeslami 2018) provide a range of relevant material. The Lorentz forces (the total of

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the electric and magnetic fields) interact with the nanofluids. The hydrothermal analysis of MHD nanoliquid flow between two radiative stretching and spinning disks was observed by Zangoee *et al.* (2019). The MHD Viscoelastic Nanoliquid Flow of Thin Film across an Unsteady Vertical Stretchable Surface with Entropy Generation was investigated by Ullah *et al.* (2019). One can view the researchers' reports on magnetohydrodynamic flows in the references (Izadi *et al.* 2019, Abbasi *et al.* 2019, Shehzad *et al.* 2019, Turkyilmazoglu 2017, Hsiao 2017). (Rashid and Liang 2020) observed the analysis of nanomaterials shape influence on MHD nanoliquid flow and heat exchanger over a rotating stretchable disk through porous media. A collection of literature considering the model of flow can be found in Refs. (Nadeem *et al.* 2013, Zhang *et al.* 2025, Shah *et al.* 2018, Malik *et al.* 2016, Bilal *et al.* 2018, Yi *et al.* 2024).

The properties of the momentum boundary layer that formed in the liquid flow across a flat surface were reported by Blasius in 1950. An amazing study on Williamson nanoliquid flow past a cone was conducted by Khan *et al.* (2017). The plate was used to report the specific case. According to the investigation, the temperature field decreases as Prandtl increases, but it also decreases in the direction of a long-lasting thermophoretic force. A magneto-hydrodynamic analysis of a Williamson liquid across a nonlinear variable surface was studied by Hayat *et al.* (2016). Zou *et al.* (2024) and Xu *et al.* (2023) investigated the biological swarm and high order analysis for of Boltzmann model.

In their 2013 study, Nadeem *et al.* (2013) investigated the mass, heat, and liquid outflow mechanisms across a stretchy surface using a Williamson liquid as their subject. While Hayat *et al.* (2015) used convective boundary conditions to investigate the impact of Dufour and Soret on Williamson liquid flow, Salahuddin *et al.* (2016) used the Cattaneo–Christov theory for mass and heat transport advancements to discuss flow of Williamson liquid over stretchable area. The results show that both thermal and solute Biot numbers are factors that enhance temperature distribution. In the articles, (Hill *et al.* 1989, Pedley and Kessler 1992) different kinds of bio-convective structure was investigated. In bio-medical process, bio-convection has an immense scope (Kuznetsov 2010). Process of bio-convection and nanofluid are relatively attractive for modern micro-fluidic devices. Siddiqi *et al.* (2016) reported on heat and mass exchange in bio-convection flow of water-based nanoliquids with microorganism. Bég *et al.* (2015) numerically examined the non-Newtonian nanoliquids through a permeable media in the presence of gyrotactic microorganism. Mutuku and Makinde (2014) reported the MHD nanoliquid flow with mass and heat exchanger over a vertical plate containing Bioconvection phenomenon. Further, new literature on flow of bio-convection can be seen in Refs. (Khan *et al.* 2013, Xun *et al.* 2017, Zuhra *et al.* 2018, Waqas *et al.* 2019). Recently, some researcher used different methods for nonlinear modeling (Eltaher *et al.* 2019, Ebrahimi *et al.* 2019, Safaei *et al.* 2019, Shahsavari *et al.* 2019, Benmansour *et al.* 2019, Yao *et al.* 2025, Qianbin *et al.* 2023).

Here, the steady incompressible Darcy-Forchheimer

boundary layer flow of MHD bioconvection Casson type nanofluid by nonlinear stretchable surface with slip conditions. The present results have been related with the previous related literatures to examine the fluctuation in physical quantities of interest. The non-linear mathematical problem is solved via numerical scheme by utilizing shooting method. The prominent parameters are sketched by graphs to analyze the variation in concentration profile of physical quantities along with justification in discussion section. The results are discussed for Prandtl number, energy activation, chemical reaction factor, nanoparticles concentration slip parameter against concentration profile.

## 2. Theoretical formulation

We assume viscous incompressible Darcy-Forchheimer Casson type nanofluid flow saturating the permeable media by a nonlinear stretching surface. Moreover, the concentration equation is modified by including chemical reaction and Arrhenius energy activation with fitted constant rate  $m$  and reaction rate  $K_r$ .<sup>2</sup> The cluster of nanosize particles is ignored, the nanoparticles suspension is assumed to be stable compound that is essential for the presence of motile microorganisms. The surface having stretchable velocity  $U_w$  with  $x$ -axis and  $n$  denotes the non-linearity in surface stretching rate i.e.  $u_w(x) = cx^n$ . At free surface zero velocity is observed. To emphasize the thermo-physical characteristics, a uniform magnetic field is considered for the fluid. To decline the impact of induced magnetic effect, a small Reynolds number is involved. The  $x$ -axis is directed towards the nonlinear stretching surface and  $y$ -axis is taken normal to the  $x$ -axis. Attributes of Brownian dissemination and thermophoresis are additionally attended. For an incompressible and isotropic flow the rheological equation of state for Casson type nanofluid is exhibit by (Eldabe and Salwa 1995):

$$\tau_{ij} = \begin{cases} 2 \left( \mu_B + \frac{P_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left( \mu_B + \frac{P_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi < \pi_c \end{cases} \quad (1)$$

where  $\pi$  denotes the component product of the deformation rate with itself,  $\mu_B$  is dynamic viscosity of fluid,  $P_y$  is the yield fluid stress,  $\pi_c$  is critical value of component product of tensor rate of strain with itself. The governing equations under approximation of boundary layer can be described as (Buongiorno 2006, Haq *et al.* 2014).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \left[ \frac{\sigma B_0^2}{\rho} - \frac{\nu}{K'} \right] u - \frac{\bar{C}_b}{\sqrt{k}} u^2 \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau^* \left[ \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} D_{Br} + \left( \frac{\partial T}{\partial y} \right)^2 \frac{D_{Th}}{T_\infty} \right] \quad (4)$$

$$\begin{aligned}
 & + \frac{\mu}{\rho C_p} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)^2 \\
 & u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D_{Th}}{T_\infty} \frac{\partial^2 T}{\partial y^2} + D_{Br} \frac{\partial^2 C}{\partial y^2} \\
 & - K_r^2 (C - C_\infty) \left(\frac{T}{T_\infty}\right)^m \exp\left(\frac{-E_a}{k_B T}\right)
 \end{aligned} \tag{5}$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{bW_c}{(C_w - C_\infty)} \left[\frac{\partial}{\partial y} \left(N \frac{\partial C}{\partial y}\right)\right] = D_n \left(\frac{\partial^2 N}{\partial y^2}\right) \tag{6}$$

Here,  $(u, v)$  are velocities component in  $(x, y)$  directions, respectively. The slip boundary conditions are:

$$\begin{aligned}
 u &= U_w = cx^n + M_1 \left(1 + \frac{1}{\beta}\right) \frac{\partial u}{\partial y}, \\
 T &= T_w + M_2 \frac{\partial T}{\partial y}, \quad C = C_w + M_3 \frac{\partial C}{\partial y}, \\
 N &= N_w + M_4 \frac{\partial N}{\partial y}, \quad v = 0, \text{ as } y = 0
 \end{aligned} \tag{7}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, N \rightarrow N_\infty, \text{ as } y \rightarrow \infty \tag{8}$$

where  $M_1(x) = M_0 x^{-(\frac{n-1}{2})}$ ,  $M_2(x) = N_0 x^{-(\frac{n-1}{2})}$ ,  $M_3(x) = L_0 x^{-(\frac{n-1}{2})}$ ,  $M_4(x) = P_0 x^{-(\frac{n-1}{2})}$  are slip parameters for velocity, temperature, nanoparticles concentration and microorganism's distributions.  $M_0, N_0, L_0$  and  $P_0$  are constants.  $\mu$  is dynamic viscosity,  $\nu$  is kinematic viscosity,  $\sigma$  is electrical conductivity,  $\rho$  is base fluid density,  $F = \frac{Cb}{\sqrt{k}}$  is inertial factor,  $K'$  is permeability coefficient of porous media,  $\tau$  denotes ratio of nanosize particles heat capacity and heat capacity of base fluid,  $D_{Br}$  is Brownian diffusion,  $\alpha$  is temperature diffusivity,  $D_{Th}$  is thermophoretic effect,  $c > 0$  is stretchable rate,  $N_\infty, T_\infty, C_\infty$  are ambient microorganisms, thermal and concentration distributions,  $B_0$  is magnetic field effect. We considered the following transformation variables:

$$\begin{aligned}
 \eta &= \frac{1}{2} \sqrt{\frac{2\rho c(n+1)}{\mu}} x^{\frac{n-1}{2}} y \\
 v &= -\frac{1}{2} \sqrt{2cv(n+1)} x^{\frac{n-1}{2}} \left[\frac{(n+1)f + (n-1)f'\eta}{n+1}\right] \\
 u &= cx^n f', \quad \theta(\eta)(T_w - T_\infty) = T - T_\infty \\
 \phi(\eta)(C_w - C_\infty) &= C - C_\infty \\
 N(\eta)(N_w - N_\infty) &= N - N_\infty
 \end{aligned} \tag{9}$$

By using the above mentioned transformation in Eqs. (2) to (8), we get:

$$\begin{aligned}
 f''' &+ \left(\frac{\beta}{\beta+1}\right) f f'' - Fr \left(\frac{2}{n+1}\right) \left(\frac{\beta}{\beta+1}\right) f'^2 \\
 &- M \left(\frac{2}{n+1}\right) \left(\frac{\beta}{\beta+1}\right) f' + \lambda \left(\frac{2}{n+1}\right) \left(\frac{\beta}{\beta+1}\right) f' \\
 &- \left(\frac{2n}{n+1}\right) \left(\frac{\beta}{\beta+1}\right) f'^2 = 0
 \end{aligned} \tag{10}$$

$$\theta'' + Nb Pr \theta' \phi' + Nt Pr \theta'^2 \tag{11}$$

$$+ Pr f \theta' + \left(1 + \frac{1}{\beta}\right) Ec f'^2 = 0$$

$$\begin{aligned}
 \phi'' &+ \frac{Nt}{Nb} \theta'' + Le Pr f \phi' \\
 &- (Le) Pr \left(\frac{2}{n+1}\right) \gamma^* (1 + \Omega^* \theta)^m \exp\left(\frac{-E}{1 + \Omega^* \theta}\right) \phi = 0
 \end{aligned} \tag{12}$$

$$N'' + Lb f N' - Pe[\phi''(N + \delta_1) + N' \phi'] = 0 \tag{13}$$

$$\begin{aligned}
 f(0) &= 0, \quad f'(0) = 1 + \alpha \left(1 + \frac{1}{\beta}\right) f''(0), \\
 \theta(0) &= 1 + \gamma \theta'(0), \quad \phi(0) = 1 + \sigma \phi'(0), \\
 N(0) &= 1 + \delta N'(0), \quad \text{at } \eta = 0
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 f'(\infty) &\rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0, \\
 N(\infty) &\rightarrow 0, \text{ at } \eta \rightarrow \infty
 \end{aligned} \tag{15}$$

Here,  $\alpha$  is velocity slip parameter,  $\gamma$  is temperature slip parameter,  $\sigma$  is nanoparticles concentration slip parameter,  $\delta$  is microorganisms slip parameter,  $M$  is magnetic parameter,  $Fr$  is Forchheimer parameter,  $\lambda$  is porosity parameter,  $Le$  is Lewis number,  $Pr$  denotes the Prandtl number,  $Nt$  is thermophoretic parameter,  $Nb$  denotes the Brownian factor,  $Ec$  is Eckert number,  $\gamma^*$  is chemical reaction factor,  $E$  denotes energy activation,  $Lb$  is bioconvected Lewis number,  $Pe$  is bioconvected Peclet number. Mathematically,

$$\begin{aligned}
 \alpha &= M_1 x^{\frac{n-1}{2}} \sqrt{\frac{c(1+n)}{2\nu}}, \quad \gamma = M_2 x^{\frac{n-1}{2}} \sqrt{\frac{c(1+n)}{2\nu}}, \\
 \sigma &= M_3 x^{\frac{n-1}{2}} \sqrt{\frac{c(1+n)}{2\nu}}, \quad \delta = M_4 x^{\frac{n-1}{2}} \sqrt{\frac{c(1+n)}{2\nu}}, \\
 M &= \frac{\sigma B_0^2}{c \rho x^{n-1}}, \quad Ec = \frac{u_w^2}{c_p (T_w - T_\infty)}, \quad Fr = \frac{x C_b}{\sqrt{k}}, \\
 \lambda &= \frac{\nu}{K' c x^{n-1}}, \quad Le = \frac{\nu}{D_{Br}}, \quad E = \frac{E_a}{k T_\infty}, \\
 Pr &= \frac{\nu}{\alpha}, \quad Nb = \tau D_{Br} (\nu^{-1} C_w - \nu^{-1} C_\infty), \\
 Nt &= \frac{D_{Th} (\tau T_w - \tau T_\infty)}{\nu T_\infty}, \quad \gamma^* = \frac{k_r^2}{c x^{n-1}}, \\
 Lb &= \frac{\nu}{D_m}, \quad Pe = \frac{bW_c}{D_m}, \quad \delta_1 = \frac{N_\infty}{N_w - N_\infty},
 \end{aligned}$$

The physical quantities are local motile number, Sherwood number, Nusselt number and skin friction coefficient expressions are defined as:

$$\begin{aligned}
 N_n &= \frac{-x}{(N_w - N_\infty)} \left(\frac{\partial N}{\partial y}\right)_{y=0}, \quad Shr = \frac{-x}{(C_w - C_\infty)} \left(\frac{\partial C}{\partial y}\right)_{y=0}, \\
 Nu &= \frac{-x}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial y}\right)_{y=0}, \\
 Cf_x &= \frac{1}{\rho u_w^2} \left(\mu \beta \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}\right)
 \end{aligned} \tag{16}$$

In terms of non-dimensional variables reduced density number of microorganisms, reduced Sherwood number, reduced Nusselt number and skin friction coefficient are as follows:

$$\begin{aligned}
 Cf_x Re_x^{\frac{1}{2}} &= \left(\frac{n+1}{2}\right)^{\frac{1}{2}} \left(1 + \frac{1}{\beta}\right) f''(0) \\
 Nu_x Re_x^{-\frac{1}{2}} &= -\left(\frac{n+1}{2}\right)^{\frac{1}{2}} \theta'(0)
 \end{aligned} \tag{17}$$

$$Sh_x Re_x^{-\frac{1}{2}} = -\left(\frac{n+1}{2}\right)^{\frac{1}{2}} \phi'(0)$$

$$Nn_x Re_x^{-\frac{1}{2}} = -\left(\frac{n+1}{2}\right)^{\frac{1}{2}} N'(0)$$

where  $Re_x = \frac{xu_w}{\nu}$  denotes the Reynolds number.

The RK (Runge-Kutta) method is widely used for determining the initial- value problems. RK method is very stable, self-starting and very easy to implement. The nonlinear differential Eqs. (10)-(15) along boundary conditions constitute two points BVP (Boundary value problem) are tackled numerically by applying RK method, in this technique the system of Eqs. (10)-(15) is reduced to first order ODE (Ordinary differential equations).

$$\left. \begin{aligned} y_1 &= y_1 \\ y_1' &= y_2 \\ y_1'' &= y_2' = y_3 \\ y_3' &= \left(\frac{\beta}{\beta+1}\right) \left[ Fr \left(\frac{2}{n+1}\right) y_2^2 - y_1 y_3 + M \left(\frac{2}{n+1}\right) y_2 + \lambda \left(\frac{2}{n+1}\right) y_2 + \left(\frac{2n}{n+1}\right) y_2^2 \right] \\ y_4 &= y_4 \\ y_4' &= y_5 \\ y_5' &= -Pr \left[ y_1 y_5 + Nb y_5 y_7 + Nt y_5^2 \right] - \left(1 + \frac{1}{\beta}\right) Ec y_3^2 \\ y_6 &= y_6 \\ y_6' &= y_7 \\ y_7' &= -Le Pr y_1 y_7 - \left(\frac{Nt}{Nb}\right) y_5^2 + Le Pr \left(\frac{2}{n+1}\right) \gamma^* (1 + \Omega^* y_4)^m \exp\left(\frac{-E}{1 + \Omega^* y_4}\right) y_6 \\ y_8 &= y_8 \\ y_8' &= y_9 \\ y_9' &= -Lb y_1 y_9 + Pe \left[ (y_8 + \delta_1) y_7' + y_7 y_9 \right] \end{aligned} \right\}$$

With the transformed conditions (14) and (15):

$$y_1(0) = 0, y_2(0) = 1 + \alpha \left(1 + \frac{1}{\beta}\right) y_3(0),$$

$$y_4(0) = 1 + \gamma y_5(0), y_6(0) = 1 + \sigma y_7(0),$$

$$y_8(0) = 1 + \delta y_9(0), \text{ as } \eta \rightarrow 0$$

$$y_2(\infty) \rightarrow 0, y_4(\infty) \rightarrow 0, y_6(\infty) \rightarrow 0,$$

$$y_8(\infty) \rightarrow 0, \text{ as } \eta \rightarrow \infty$$

For the residual of continuous outcomes erratum control and mesh section provided for all the calculations.

### 3. Result and discussion

In this section, the effects of chemical reaction and Arrhenius energy activation are also scrutinized. Finally, the present results have been related with the previous related literatures to examine the fluctuation in physical quantities of interest. Table 1 displayed to validate the current solutions with previous published literature in specific cases. We noted that present results have excellent agreement with previous results by (Reddy Gorla and Sidawi 1994) in specific case. The numerical values of Sherwood number for distinct parameters are shown in Table 2. Fig. 1 depicted

Table 1 Comparison of results for  $-\theta'(0)$  with distinct values of  $Pr$ , when  $\beta \rightarrow \infty, n = 1, Fr = \lambda = M = Nb = Nt = \alpha = \gamma = \sigma = \delta = Le = Pe = Lb = Ec = 0$ .

$Pr$	Reddy Gorla and Sidawi (1994)	Present results
0.20	0.1691	0.1691
0.70	0.4539	0.4539
2.00	0.9114	0.9113
7.00	1.8905	1.8954
20.00	3.3539	3.3539
70.00	6.4621	6.4621

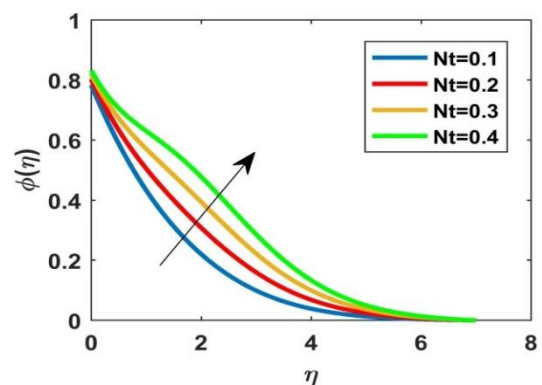


Fig. 1 Concentration profile for variation of  $Nt$

the effect of thermophoresis parameter  $Nt$  on concentration profile  $\phi$ . In is found that for progressive values of  $Nt$ , nanoparticles concentration increases. Physically, an increment in  $Nt$ , leads to raise the thermophoretic force inside a liquid region, which are responsible an enhancement in nanoparticles concentration. The effect of Prandtl number  $Pr$  on concentration profile  $\phi$  is displayed in Fig. 2.

Table 2 Numerical values of Sherwood number for different values of parameters

$\alpha$	$\gamma$	$\sigma$	$\lambda$	$Nt$	$Nb$	$Pr$	$Fr$	$M$	$Ec$	$E$	$-\phi'(0)$
0.3											0.8492
0.6	0.5	0.5	0.2	0.1	0.2	2.0	0.1	0.2	0.3	0.2	0.7521
0.9											0.7017
	0.3										0.7603
	0.6										0.7841
	0.9										0.8029
		0.3									0.8520
		0.6									0.7440
		0.9									0.6602
			0.3								0.7698
			0.6								0.7515
			0.9								0.7366
				0.1							0.7768
				0.3							0.5452
				0.5							0.4095
					0.2						0.7768
					0.4						0.8734
					0.6						0.9048
						2.5					0.7624
						3.0					0.7513
						3.5					0.7427
							0.3				0.7726
							0.6				0.7667
							0.9				0.7614
								0.3			0.7698
								0.6			0.7515
								0.9			0.7366
									0.3		0.7768
									0.6		0.8375
									0.9		0.8983
										0.2	0.7768
										0.5	0.7437
										0.7	0.7259

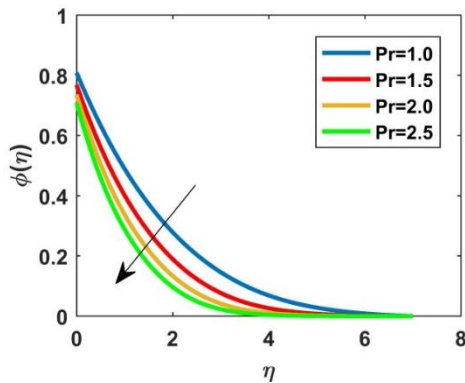


Fig. 2 Concentration profile for variation of  $Pr$

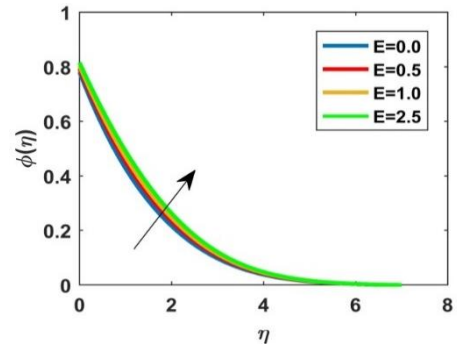


Fig. 3 Concentration profile for variation of  $E$

Concentration profile  $\phi$  decreases for higher values of  $Pr$ . The impact of energy activation  $E$  on concentration profile  $\phi$  is seen in Fig. 3. It is observed that concentration profile  $\phi$  increases for maximum values of  $E$ . Enhancing dimensionless energy activation  $E$  along reduces temperature, as a result constant reaction rate decreases and finally modest reaction is practiced. This increases the concentration solute. Chemical reaction parameter  $\gamma^*$  on concentration profile  $\phi$  is shown in Fig. 4. Nanoparticles concentration is decelerated on augment of  $\gamma^*$ . This behavior shows the weak impact of buoyancy impact due to concentration

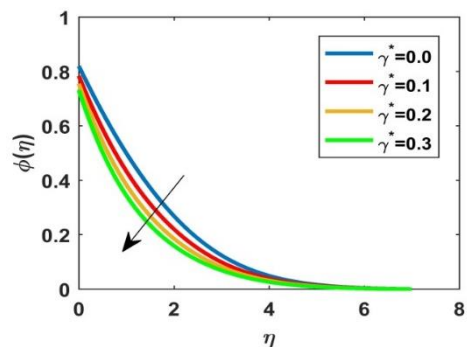


Fig. 4 Concentration profile for variation of  $\gamma^*$

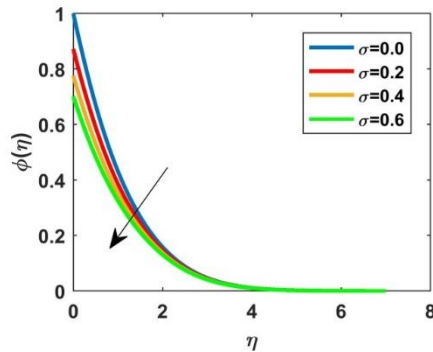


Fig. 5 Concentration profile for variation of  $\sigma$

gradient which causes the concentration profile decreases. Fig. 5 shows the influence of concentration slip parameter  $\sigma$  on nanoparticles concentration  $\phi$ . It is found that nanoparticles concentration  $\phi$  retarded with an increment in  $\sigma$ .

#### 4. Conclusions

The shooting technique is used in this work to solve the simplified ordinary differential equations. Thermophoresis parameter, Prandtl number, energy activation, chemical reaction factor, and concentration slip parameter versus the concentration profile is investigated. The behavior of several influential parameters is examined graphically and the concentration profile is examined. An acceptable agreement is reached when the numerical technique is compared to the body of existing literature. The concentration profile declines as the Prandtl number rises. The concentration curves exhibit reversal behavior when the energy activation and chemical reaction coefficients rise.

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