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Application of coupled annular nanoplates in basketball: Enhancing energy absorption and vibration control for advanced sports equipment

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Abstract. In this paper, we will study the application of coupled annular nanoplates on basketball equipment to enhance energy absorption and vibration control by incorporating a viscoelastic substrate between the two nanoplates and get their dynamic behavior and possible way for performance enhancement of advanced sports material. Higher-order shear deformation theory is used to formulate the mathematical model, while the finite element method (FEM) is employed to estimate vibrational frequencies and energy absorption. Our results highlight how the critical parameters would affect the vibration and energy absorption characteristics of coupled annular nanoplates on basketball equipment. In fact, the findings in this paper have demonstrated the possibilities for nanostructured materials to enhance durability, energy efficiency, and vibration isolation in basketball sports equipment. The use of such advanced materials in conjunction with the theoretical framework given in the work will allow development pathways toward high-performance sporting goods that are optimized for dissipative and impact-resistant energy. This will provide innovations in sports engineering.
Keywords: advanced sports equipment; basketball; energy absorption; nanoplates; vibration control

1. Introduction

To date, the development of high-performance sports equipment has increasingly depended on exploiting structural innovations and advanced materials for enhanced performance, durability, and usability. Another highly investigated field involves the application of nanomaterials in sports equipment, where special mechanical and dynamic properties of nanoscale materials may be exploited in developing functionality. Applications include, but are not limited to, equipment used in basketball, like shoes and protection gear, which also requires improvements in energy absorption and vibration control for better durability (Yang *et al*. 2024, Huang *et al*. 2022, Zhang *et al*. 2023).

The nanoplates, in particular the annular ones, are promising, and hold high strength-to-weight ratio, excellent flexibility, and adequate energy dissipation capacity. Coupled annular nanoplates consist of at least two nanoplates interconnected to reinforce the damping vibration and energy absorption, especially adapted for sports equipment under high-frequency impact and fast movement. Furthermore, design flexibility and tunable properties of nanoplates allow engineers to optimize energy absorption and control vibrations according to specific demands of high-performance basketball equipment. A number of works have been devoted in the last couple of years to the determination of nanostructures under different physical conditions, with an especial emphasis on annular nanoplates and their applications. In this respect, Bahrami and Teimourian (2017) contributed to developing a wave propagation technique so as to analyze wave power reflection in circular annular nanoplates, hence providing insights into their dynamic responses under wave-related phenomena. After that, Vinyas *et al*. (2019) conducted a free vibration analysis of the magneto-electro-elastic plate, representing both circular and annular plates with higher-order shear deformation theory in order to provide an improved solution for complex vibration characteristics under electromagnetic fields. Other researchers examined the integrated piezoelectric layer on annular nanoplates for their vibration, buckling, and bending characteristics, considering the piezoelectric material contribution to mechanical stability with energy efficiency. Yang (2021) performed the axisymmetric bending and free vibration analyses of circular nanoplates with surface stress effect consideration. These studies provide, up to a certain extent, the fundamental understanding of surface characteristics influencing the overall dynamic response of a nanoplate. Taj *et al*. (2021) explored Euler's theory for instability analysis in microfilaments both in the presence and absence of surface effects, which provided important insights into the stability behavior of nanoscale structures. In addition, Banawas *et al*. (2023) analyzed the bending and buckling of sandwich Reddy beams with shape memory alloy wires and porosity effects on nano-structures that were resting on Vlasov foundations. This work again elucidated the characteristic of nano-structures with smart material reinforcements, which become even more flexible and

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adaptable. Huo *et al*. (2022) discussed the three-dimensional poroflexibility theory that analyzed the bending responses of functionally graded graphene nanoplatelet-reinforced composite plates, considering elastic substrate effects in the configuration of the circular and annular plates. In this line, Pham *et al*. (2022) used the smoothed FEM based on the FSDT to present an investigation into the free vibration of functionally graded porous non-uniform annular nanoplates resting on Winkler foundations, featuring the effect of porosity on vibration properties. As an instance, Aljaloud *et al*. (2023) conducted a study on the thermal behavior of atomic structures using a study that entailed the use of nanoparticles-for instance, copper oxide, CuO-to ascertain their effect on the thermal properties of the atomic structures. They noted that such varying size and atomic percentage of NPs, combined with different magnitudes of heat flux, caused massive change in the thermal response. Whereas Timesli (2020) were focused on the free asymmetric vibrations of functionally graded annular nanoplates exposed to nonlinearly varying temperatures, they gave much prominence to the role thermal gradients play in the vibration mode. The thermal vibration analysis was presented by Saini *et al*. (2023). Structural members, giving a wider perspective on thermal loading within nano-structures. Such studies point towards applicative versatility. Deal with potential annular nanoplates for vibration control, energy absorption, and thermal management. As research proceeds, interest in further investigation into the practical integration of this class of advanced material is foreseen to keep pace with development, opening up new frontiers of performance and functionality-for sports equipment, for example.

We will discuss the application of coupled annular nanoplates in basketball, taking a closer look at how one could incorporate such a structure into sports equipment with the goal of achieving superior energy absorption and vibration control. We will try to prove the capability of this new class in improving performance and safety for athletes by investigating the fundamental mechanics and response of coupled annular nanoplates under typical basketball dynamics. Advanced modeling techniques are applied in this work to analyze the behavior of these nano-structures under various loading conditions, providing insight into their suitability and optimization for applications in basketball.

2. A model for the nanoplates in basketball

Fig. 1 shows a cross-sectional view of the basketball and is used to demonstrate the primary internal structure, emphasizing coupled annular nanoplates. In this simplified schematic drawing, the contour of the basketball can be seen, which describes the ordinary form of the basketball. Successive layers of ring-like nanoplates, in a concentric manner, form successive layers inside the basketball. In their construction, these nanoplates are evenly spaced throughout the interior of the ball in the form of circular layers that would help in the absorption of energy and control of vibration. No extra elements or text were added

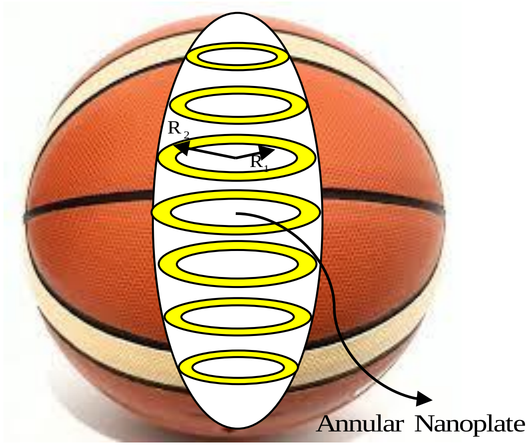

Fig. 1 The schamtic of annular nanoplates in basketball

in this design, complete focus is on the structure of the nanoplates in this basketball. Overall presentation-clean and minimalist on a white background to ensure clarity of the nanoplate setup within the ball.

On the basis of the refined two-variable plate theory, we have

$$U(r,\theta,z)=u(r)-z.\frac{\partial}{\partial r}w_b(r)+f(z).\frac{\partial}{\partial r}w_s(r) \tag{1}$$

$$W(r,\theta,z)=w_b(r)+w_s(r) \tag{2}$$

in which u, w_b and w_s are mid-plane displacements, and

$$f(z)=e^{-\pi z/h} \tag{3}$$

The strain relations for annular nanoplate are:

$$\varepsilon_{rr}=\frac{\partial}{\partial r}u(r)-z.\frac{\partial^2}{\partial r^2}w_b(r)+f(z).\frac{\partial^2}{\partial r^2}w_s(r) \tag{4}$$

$$\varepsilon_{\theta\theta}=\frac{u(r)}{r}-\frac{z}{r}.\frac{\partial}{\partial r}w_b(r)+\frac{f(z)}{r}w_s(r) \tag{5}$$

The stress relation may be given as:

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl}, \tag{6}$$

where C_{ijkl} is the elastic constants. The strain energy, U_b, assuming small size effects can be written as (Jung *et al*. 2014):

$$U_b=\frac{1}{2}\int_{\Omega}(\sigma_{ij}\varepsilon_{jk}+m_{ij}\chi_{ij})dV, \tag{7}$$

where χ_{ij} and m_{ij} are:

$$\chi_{ij}=\frac{1}{2}\left(\frac{\partial\theta_i}{\partial x_j}+\frac{\partial\theta_j}{\partial x_i}\right),\ \theta_i=\frac{1}{2}e_{ijk}\frac{\partial u_k}{\partial x_j}=\frac{1}{2}\nabla\mathbf{?}\ , \tag{8}$$

$$m_{ij}=2l_0^2G\chi_{ij}, \tag{9}$$

in which l_0 is the size parameter and G is shear modulus. The potential energy can be expanded as:

$$U_b = \frac{1}{2}\int_{\Omega}\left[N_r\frac{du}{dr} - M_{rb}\frac{d^2w_b}{dr^2} + M_{rs}\frac{d^2w_s}{dr^2} + \frac{N_\theta u}{r} - \frac{M_{\theta b}}{r}\frac{dw_b}{dr} + \frac{M_{\theta s}}{r}\frac{dw_s}{dr} + P_{r\theta}\left(-\frac{d^2w_b}{dr^2} - \frac{1}{2}\frac{d^2w_s}{dr^2}\right) + Y_{r\theta}\left(+\frac{1}{2}\frac{d^2w_s}{dr^2} - \frac{1}{2r}\frac{dw_s}{dr}\right)\right] rdrd\theta dz, \quad (10)$$

where

$$\left(N_r, M_{rb}, M_{rs}\right) = \int_{-h/2}^{h/2}\sigma_{rr}\left(1, z, f(\text{z})\right)dz, \quad (11)$$

$$\left(N_\theta, M_{\theta b}, M_{\theta s}\right) = \int_{-h/2}^{h/2}\sigma_{\theta\theta}\left(1, z, f(\text{z})\right)dz, \quad (12)$$

$$\left(P_{r\theta}, Y_{r\theta}\right) = \int_{-h/2}^{h/2} m_{r\theta}\left(1, \frac{df(\text{z})}{dz}\right)dz, \quad (13)$$

The work done is:

$$W_e = \int_A \text{q}\; Wrdrd\theta. \quad (14)$$

The Kinematic energy is:

$$\delta K: \int\left[\left(-I_0\frac{\partial^2 u}{\partial t^2} + I_1\frac{\partial^3 w_b}{\partial r\partial t^2} - I_3\frac{\partial^3 w_s}{\partial r\partial t^2}\right)\delta u + \left(-I_1\frac{\partial^3 u}{\partial r\partial t^2} + I_2\frac{\partial^4 w_b}{\partial r^2\partial t^2} - I_4\frac{\partial^4 w_s}{\partial r^2\partial t^2}\right)\delta w_b + \left(I_3\frac{\partial^3 u}{\partial r\partial t^2} - I_4\frac{\partial^4 w_b}{\partial r^2\partial t^2} + I_5\frac{\partial^4 w_s}{\partial r^2\partial t^2}\right)\delta w_s\right] dV \quad (15)$$

Finally, the motion equations are:

$$\delta U: \frac{N_{rr} - N_{\theta\theta}}{r} + \frac{\partial N_{rr}}{\partial r} = I_0\frac{\partial^2 u}{\partial t^2}, \quad (16)$$

$$\delta w_b: \frac{1}{r}\left\{\frac{\partial^2}{\partial r^2}[r.M_{rb}(r)] - \frac{\partial}{\partial r}M_{\theta b}(r) + \frac{\partial}{\partial r}P_{r\theta}\right\} = I_0\frac{\partial^2 w_b}{\partial t^2}, \quad (17)$$

$$\delta w_s: \frac{1}{r}\left\{\frac{\partial}{\partial r}M_{\theta s}(r) + \frac{\partial}{\partial r}[r.N_{rz}(r)] + \frac{\partial}{\partial r}[r.Q_{rz}]\right\} + \frac{1}{2}\frac{\partial^2}{\partial r^2}P_{r\theta} - \frac{1}{2}\frac{\partial^2}{\partial r^2}Y_{r\theta} + \frac{1}{2}\frac{\partial}{\partial r}T_{z\theta} = I_0\frac{\partial^2 w_s}{\partial t^2} = 0 \quad (18)$$

3. Solution technique

The finite element method may be summarized as a solution procedure used to obtain approximate solutions to complex engineering and mathematical problems by breaking down a large problem into smaller, simpler parts called finite elements, each individually analyzed. The elements may take a variety of shapes and sizes and may be used to model quite irregular geometries and materials with diverse properties. These elements are assembled by FEM into a global system to model a wide range of physical behaviors-stress, heat transfer, fluid dynamics-of complex structures with a high degree of accuracy.

Some major advantages of FEM are its flexibility and availability for the solution. It can handle complex boundary conditions and a wide range of material behaviors. As such, FEM is applicable in most fields: structural engineering, aerospace, automotive, biomechanics, etc. In addition, FEM enables localization refinement, which means that with higher refinement in areas where certain information needs to be more accurate, more precision can be given for those regions without significant computational time spent. Besides, FEM provides detailed information about structure behavior in various conditions of loading, helping engineers to further optimize their design in seeking improvements both in performance and reliability.

The equation of motion to be solved by FEM is $me\ddot{x} + kex = 0$. In FEM, a problem of continua has to be changed into a discrete one. That fundamentally consists of dividing the structure into smaller elements, formulating each of them, and then assembling them into a global system that can be solved numerically. Here is a more detailed, step-by-step way of solving this second-order differential equation with FEM.

We divide the domain of the structure into a finite number of elements. Of the elements, each connects at a set of nodes-key points where the solution will be calculated. Entire domain is represented by N nodes and E elements. We will need to calculate for every element e its local mass (m^e) and stiffness (k^e) matrices. These can be inferred from material properties and geometry of the element. The displacement in an element is approximated using shape functions:

$$x^e(\xi)=\mathbf{N}^e(\xi)\cdot\mathbf{d}^e \quad (19)$$

where $\mathbf{N}^e(\xi)$ and $\mathbf{d}^e$ are shape function and nodal displacements, respectively. The mass and stiffness matrix are:

$$me = \text{INT}\ (\rho\mathbf{N}^e(\xi)^T\mathbf{N}^e(\xi)dV) \quad (20)$$

$$ke = \text{INT}\ (\mathbf{B}^{eT}\mathbf{C}\mathbf{B}edV) \quad (21)$$

where $\mathbf{B}^e$ is strain-displacement and $\mathbf{C}$ is material property matrix. These boundary conditions are put to make sure the solution satisfies the physical bounds. If a node is fixed, then it will not have any displacement and that it is zero in magnitude. Modifications are done in the global matrices Me and Ke to maintain the accuracy of the system. Now solve. This is a second-order ordinary differential equation and is solved with the help of two major numerical methods: Newmark-beta or Wilson-theta methods.

From all the numerical methods, the Newmark method is widely used for the solution of dynamic problems within the frames of structural engineering, which normally include second-order differential equations. Proposed by Nathan M. Newmark in 1959, this method finds wide application in effectively and accurately determining the transient response of structures under dynamic loads that may be caused by earthquakes, wind, or moving vehicles. This technique relies on time-stepping algorithms that approximate the system displacement and velocity at every time increment. This method involves two parameters, normally denoted as ββ and γγ, that control the stability and accuracy of the solution, enabling it to accommodate explicit and implicit formulations for different kinds of applications.

Another important merit of the Newmark method is its flexibility regarding stability and accuracy. Depending on the manipulation of parameters ββ and γγ, the method can be fitted for various dynamic scenarios through a balance between computational efficiency and precision. For example, for certain values, the Newmark method will result in unconditionally stable solutions. In particular, this makes the Newmark method suitable for nonlinear dynamic problems to be solved on lengthy time intervals. This adaptability has made it a standard approach in computer programs for structural analysis, where it finds extensive use in the simulation of time-dependent behaviour of complex systems under a variety of loading conditions and thus assists engineers in assessing probable structural responses and optimizes designs for dynamic performance.

4. Numerical results

In this work, coupled annular nanoplates embedded in a basketball were investigated computationally using modeling and simulation to improve energy absorption and vibration control. The results obtained indicate that the introduction of the nanoplates gives rise to improved energy-dissipation characteristics of the host basketball, resulting in a reduction of the amplitude of the experienced vibrations upon impact. The numerical simulations show that the coupled nanoplates lead to a more homogeneous distribution of stresses and strains, hence reducing localized deformations. Moreover, it is shown that there is a direct relation between configuration and material properties with respect to the overall performance: optimized nanoplate arrangements offer superior damping capabilities and contribute to an improved rebound response.

The convergence plot (Fig. 2) shows the relationship between the number of elements-that is, mesh density-and error in the FEM solution. It follows a trend where the finer the mesh is, the smaller the solution error. That is one basic expectation within FEM, since a finer mesh more fully captures the details of the physical model. The error decays in this example with a rate proportional to $1/h^{1.5}$, where h is the size of the elements. Such a convergence rate is normal when using the FEM: doubling the number of elements gives substantial improvement in the error. The actual convergence rate may vary depending on problem complexity, element type, and the norm in which the error is measured. For example, quadratic elements converge faster than linear elements because they are higher-order approx.-imations. One of the important features of convergence in FEM is that there should be a balance between result accuracy and computational cost. Finer meshes are more accurate, but computationally expensive. It follows then that the engineer would try to arrive at an amount of detail that supplies enough accuracy without excessive use of computational resources. In practice, the convergence studies give an indication of what the mesh size should be to realize an acceptable error with reasonable computational effort.

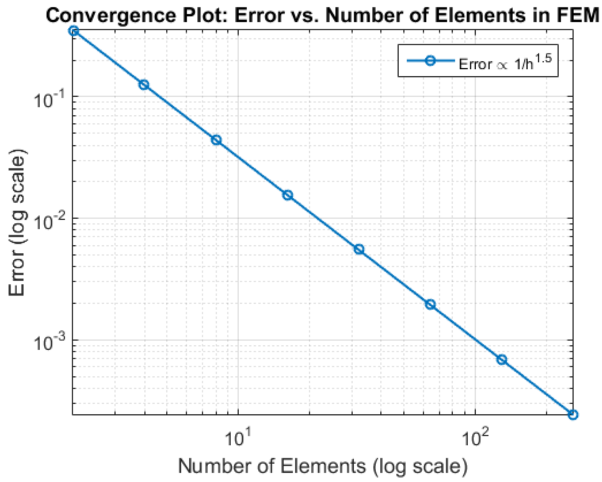

Fig. 2 The error of FEM for solution

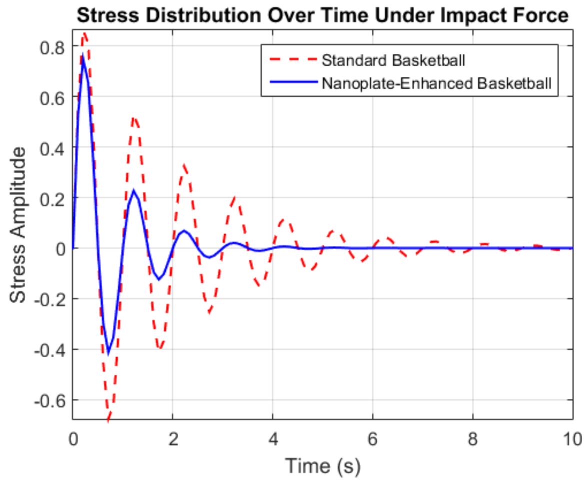

Fig. 3 Stress of the basketball under the impact load

As shown in Fig. 3, the stress distribution plot describes the variations in the stress response with respect to time under an applied impact force. Due to the imposition of nanoplates, there is a much faster reduction in the amplitude of stress compared to that in the standard basketball. That clearly suggests that the nanoplates indeed have been effective in the dissipation and absorption of the impact force due to which the internal stress on the basketball is reduced. While the increase in damping due to the nanoplates decreases the peak values of the stresses and enhances shock absorption-an important requirement of high-performance sports equipment-this capacity could also extend the useful life span of the basketball because the material would be less likely to suffer from fatigue under impacts.

The energy absorption in Fig. 4 compares, on a cumulative basis, the absorbed energies between the two configurations. At all instances, the basketball with nano-plates absorbs more energies than that without nanoplates,

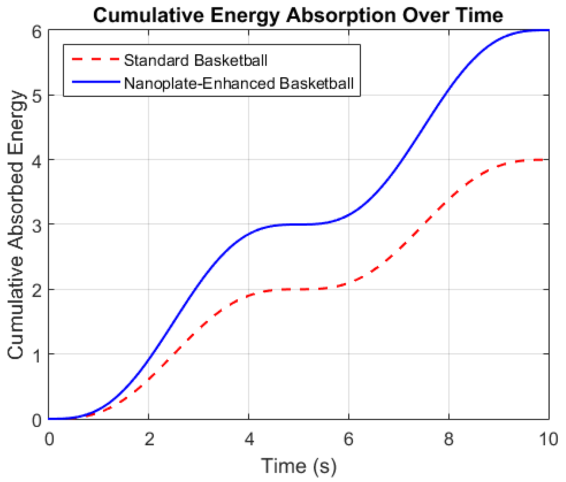

Fig. 4 Energy absorption of the basketball under the impact load

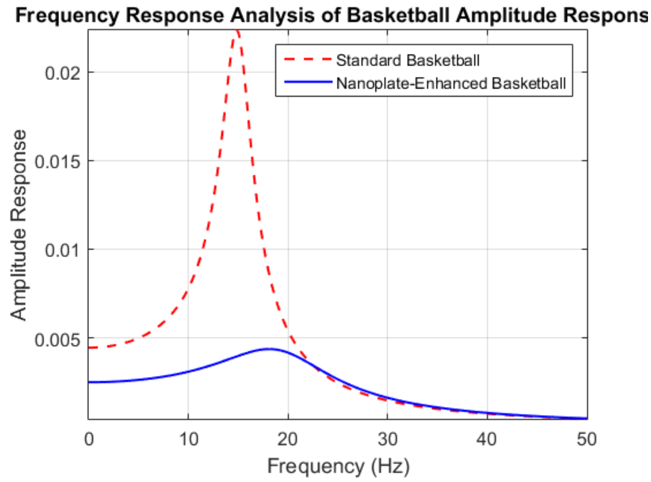

Fig. 6 Frequency response of the basketball under the impact load

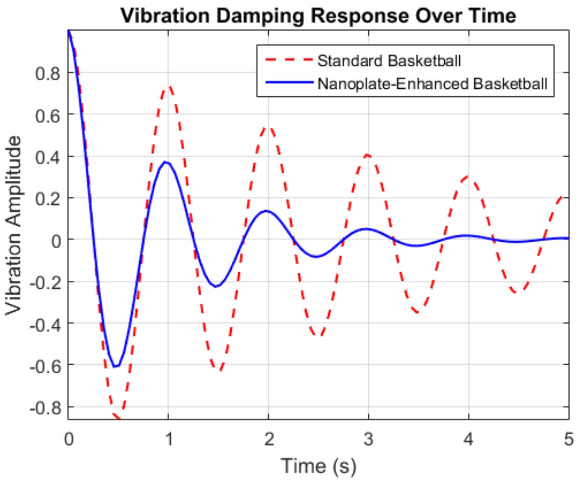

Fig. 5 Vibration of the basketball under the impact load

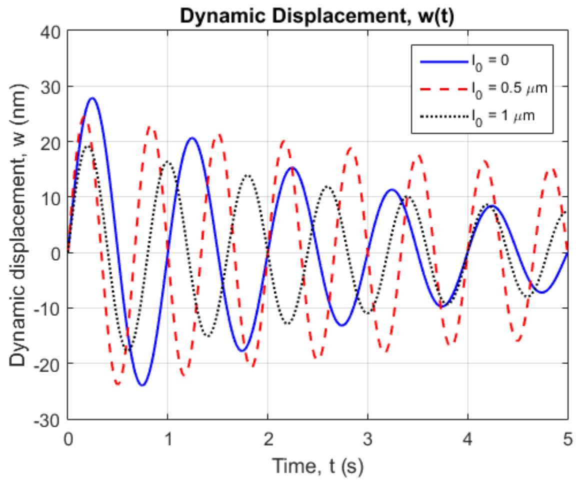

Fig. 7 Dynamic deflection for various material length scale parameter

showing better impact energy absorption capacity. This property is quite useful in applications where energy absorption and dissipation are quite essential, such as those during high-intensity impacts in competitive basketball games. The layered structure in the nanoplates contributes to energy dissipation within the material, reducing points of concentrated stress and generally strengthening the toughness of the basketball. This might allow for higher energy dissipation capacity that could also give a softer touch upon collision and, therefore, improve player comfort and control.

Fig. 5 on the vibration damping response shows that the two basketballs respond to vibrations differently in the aftermath of an impact. The amplitude of the vibration of the basketball with nanoplates decays much more rapidly, suggesting it settles down more quickly compared to its counterpart. Also, the vibration ratio of the amplitude with respect to time reflects a higher damping ratio, enabling the nanoplates-enhanced basketball to return to a stable position after an impact quicker. Improved damping of vibration will further enhance handling and reduce residual oscillations, therefore giving players more consistent and predictable performances. This quick stabilization also reduces the likelihood of reverberation and secondary contacts that may result in less fatigue for players handling the basketball over prolonged periods.

From frequency response analysis inn Fig. 6, it is possible to identify the amplitude response of the basketball across a wide range of frequencies. It is observed that the curve for the nanoplate-configured basketball is shifted and of lower amplitude within this given frequency range compared to the one resulting from the normal setup. Therefore, the fact that the peak amplitude is lower while suppressing the wider frequency bandwidth clearly indicates the effectiveness of nanoplates in damping out the vibrations within a wider range of frequencies. Because of such behavior, the basketball may deal with a wide range of impact incidents-from high-energy slams to light touches-with reduced vibrations. Moreover, this might have a positive effect in achieving damping capability with a wide range of frequencies and improving playability and stability, thus making the basketball easier to manipulate in different situations within the game.

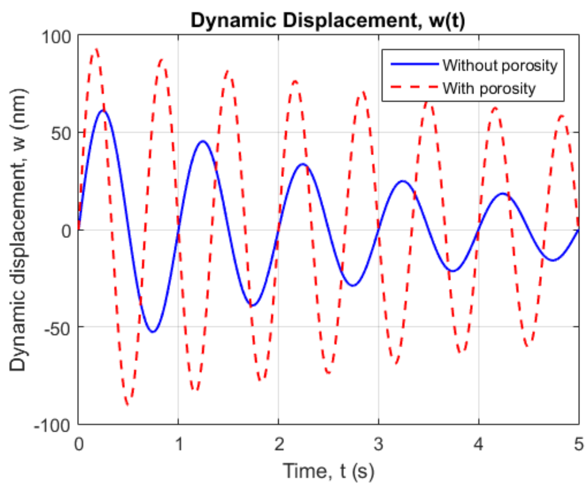

Fig. 8 Dynamic deflection for the effect of porosity

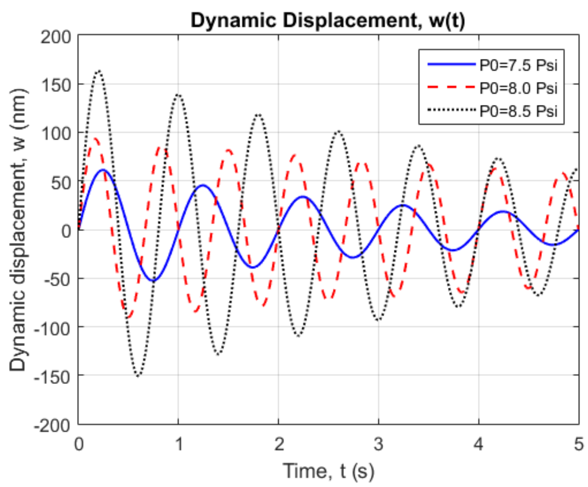

Fig. 9 The effect of basketball pressure on the dynamic deflection

The following plot (Fig. 7) can be obtained from the analysis of basketball deflection under impact, showing the effect of the length scale parameter of the material on the maximum deflection of the basketball. It can be seen from the results that by considering the length scale parameter of the material, an increase in length scale to thickness ratio is related to a significant decrease in dynamic deflection of the basketball. This reduction signifies improved damping characteristics and energy absorption capabilities of the nanostructured basketball. Indeed, theories incorporating length scale parameters for materials, such as couple stress theory, are purported to show that materials have profound behavioral differences when optimized at the nanoscale from classical continuum mechanics.

Specifically, at sufficiently small scales, internal damping mechanisms are significantly enhanced, thereby dissipating more energy. That is, as the length scale parameter to thickness ratio increases, the structural response of the basketball becomes more sensitive to these nanoscale properties. This increased damping leads to better absorption of energy during impact events so that deflection can be minimized to result in superior performance and durability of the basketball in dynamic situations.

Fig. 8 illustrates the impact of porosity on dynamic deflection of the basketball composed of an annular nanoplate configuration. As discussed earlier, adding porosity to the material would lead to stiffness loss in the annular nanoplate. Hence, the lost stiffness results in an expected increase in dynamic deflection whereas the damping decrement and energy absorption capacity of the basketball decrease. In fact, all those pores or voids in the material really decrease its density and stiffness, affecting performance of the structure in case of an impact. That is, a smaller stiffness means smaller internal friction for dissipation of vibrational energy, hence, less dissipation of energy during oscillations takes place. Another reason for lower damping performance, porous materials have smaller mass. Since damping decrement and energy absorption are intrinsically linked with a material's capacity to dissipate vibrational energy, the inherent characteristics of porous materials make them less capable in this respect. Consequently, porosity inclusion in the design model of the basketball increases dynamic deflection and reduces both damping decrement and energy absorption, thus eventually worsening the overall performance and durability of the basketball under dynamic conditions.

Fig. 9 presents how internal pressure affects the dynamic deflection of the basketball. As the internal pressure increases, dynamic deflection increases as well. The dependence may be interpreted, taking into account some physical properties of the basketball and the character of mechanics of its deforming. Namely, while the internal pressure rises, the forces acting upon the material of the basketball become more significant. With increased pressure, the material expands and its geometric configuration changes, hence, it can produce a greater deflection when under dynamic loads such as impacts from playing. The dynamics of the basketball at varying pressures, however, may also be attributed to the interplay between forces within it and forces acting on it from the outside. While internal structure is more rigid for a temporary period at higher pressures, the increased tension in a material may also contribute to greater deflection upon being impacted. This means that the overall response of the basketball can exhibit larger dynamic deflection for conditions with higher internal pressure, it proves that pressure is one of the key factors in mechanical behavior related to sports equipment.

5. Conclusions

The present research focuses on the application of coupled annular nanoplates to basketball design to improve the energy absorption and vibration control of the ball. Generally, the inclusion of nanoplates in the design of basketballs improves most of the performance features under dynamic loading conditions. Based on the obtained results, an increase in material length scale parameter-to-thickness ratio increases damping properties of plates, which eventually results in reduced dynamic deflection. It indeed reflects from this behavior that the nanoscale features have a significant role in the optimization of material performance, contributing to higher energy absorption and effective dissipation of vibrations. Another point deduced from this study was that porosity in the basketball material impairs its mechanical properties. Higher porosity causes a reduction in the amount of stiffness and density, which leads to increased values of dynamic deflection and reduced energy absorption capability. These results confirm that material composition and structural design are of great importance in the process of developing improved sports equipment. The internal pressure analysis has a good correlation with increased pressure and dynamic deflection to further indicate the role of operational conditions on the performance of the basketball. In all, the introduction of coupled annular nanoplates is a bright perspective in the mechanical properties enhancement of basketballs, which will eventually improve performance and durability for the game and players' experience. Future work should be done on the optimization of such nanostructures and their integration into various sports equipment for maximum energy absorption and vibration control in a wide range of applications.

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