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On application of machine learning techniques for predicting the bending and buckling behavior of FGM nanobeams

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Abstract. The present article aims to carry out a comparative study between various machine learning based algorithms, which can predict the bending and buckling behavior of functionally graded (FG) nanobeams accurately. The algorithm has been developed in the framework of two regression machine learning models namely, Gaussian Process Regression (GPR), and Random Forest (RF). Geometric and material properties are taken as the variables including length-to-thickness ratio, power-law index, and nonlocal parameter. For having random non-biased input dataset, the Sobol sequence has been used. Using these values, maximum deflections and critical buckling loads are obtained. These values along with the corresponding input variables, surrogate models were formulated. It has been observed that the GPR model is able to predict the behavior of FG nanobeams more accurately as compared to the behavior predicted by RF surrogate model even for an unseen dataset.
Keywords: concrete disk; instability; nanocomposite reinforcement; non-classical boundary conditions; stability

1. Introduction

Functionally Graded Materials (FGM) are cross-disciplinary used materials (civil, aerospace, automobile, biomedical industry, etc.) which bypass the laminates and possess high strength-to- weight ratio (Zhang *et al*. 2023), withstand high temperature and don does not have interfaces (Garg *et al*. 2021a). The FGMs are not only used for constructing the structures at macroscopic levels but are also used for making up the structures at nano or micro levels (Sabherwal *et al*. 2024, Garg *et al*. 2024a). Nano FGMs are used for making light weight sensors, drug delivery, high frequency nano resonators, nano electro-mechanical devices etc. (Ates *et al*. 2020, El-sherbiny *et al*. 2013, Sharma *et al*. 2023).

Several beam theories are available in the literature for the analysis of FGM beams. These theories are broadly classified as Equivalent Single Layer (ESL) theory, Layer-Wise Theory (LWT) and 3D Elasticity theories (Wang *et al*. 2023). However, these theories cannot be used directly for the analysis of FGM beams at nano level because, at nano scale, interatomic forces are also present and plays a significant role in controlling the behavior of the structure (McFarland and Colton 2005). Therefore, scale- based theories are incorporated in ESL, LWT or elasticity theories in contemplation of engulfing the nano effects (Garg *et al*. 2023a). Nonlocal elasticity theory, surface elasticity theory, nonlocal strain gradient theory, couple stress theory, Cosserat theory etc. are among the popular theories for analyzing the nano FGM structures (Garg *et al*. 2024b). Among the stated theories, nonlocal elasticity theory is used by most of the researchers for incorporating nano effects (Garg *et al*. 2021b).

Classical Beam Theory (CBT), the simplest among ESL theories, is incompetent in picturizing the response of beams reliably as this theory took no notice of transverse shear strains (Nejad *et al*. 2018, Belarbi *et al*. 2024). First-order shear deformation theory (FSDT) presume constant transverse displacement field across the thickness of the beam and therefore forecast uniform value of transverse shear stress across the thickness of the beam (Hakima *et al*. 2020). Azandariani *et al*. (2022) carried out the analysis of bidirectional FG Timoshenko beams using generalized differential quadrature method. This theory also incorporates shear correction factor, which depends on material properties, geometry of beam etc. (Garg and Chalak 2019). Higher-order shear deformation theories (HSDTs) adds higher order terms to the FSDTs and able to predict parabolic or higher-order variation of transverse shear stresses across the thickness of the beam (Hadi *et al*. 2018). This theory also bypasses the requirement of shear

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correction factor. Using nonlocal sinusoidal shear deformation theory (SSDT), Thai and Vo (2012) carried out vibration, bending and buckling analysis of nano beams. Tounsi *et al*. (2013) published results for bending and dynamic analysis of FG nanobeams obtained using analytical solutions based SSDT. Chaht *et al*. (2014) included thickness stretching terms in SSDT for studying bending and buckling behavior of FGM nanobeams in framework on Eringen's nonlocal theory. Li *et al*. (2023) demonstrated the concept of diameter adjustable mandrel for adapting the different diameter tube bending process. Xu *et al*. (2023, 2024) predicted the bending behavior of plates made up of variable stiffness fabrics. Li *et al*. (2024) predicted the bearing capacity of ice by employing anisotropic beam theory. Zhang *et al*. (2024) studied the flexural behavior of fiber reinforced beams.

Rahmani and Jandaghian (2015) employed third-order shear deformation theory (TSDT) for buckling analysis of FG nanobeams. Semi-analytical solutions for vibration analysis of power and sigmoid FG nano beams were published by Ehyaei *et al*. (2016). Ebrahimi and Barati (2016a) analyzed piezo-electric FG nano beams under buckling conditions under piezo-electro-magnetic conditions. In another work, Ebrahimi and Barati (2016b) carried out thermal based buckling analysis of FG nanobeams using TSDT. With the help of nonlocal strain gradient theory based HSDT, Ebrahimi and Barati (2017) carried out buckling analysis of porous curved FG nano beams. Using nonlocal strain gradient based quasi-3D theory, Houari *et al*. (2018) performed buckling analysis of FG nanobeams with different scale parameters. Closed form solutions for FG nanobeams under bending conditions were given by Barretta *et al*. (2018). Benahmed *et al*. (2019) employed nonlocal HSDT for evaluating critical buckling loads for FG nanobeams. Ebrahimi *et al*. (2020) carried out buckling analysis of porous FG nanobeams under thermal conditions. Belarbi *et al*. (2021) employed parabolic shear deformation theory for carrying out bending and buckling analysis of FG nano beams using finite element method. Ebrahimi *et al*. (2021) predicted the thermoelastoplastic response of FGM linearly hardening rotating thick cylindrical pressure vessels. Zerrouki *et al*. (2021) carried out bending analysis of FG carbon nanotubes reinforced nano beams using HSDT. Singh and Azam (2021) predicted the vibration behavior of nano FGM plates under hygrothermal conditions. Using quasi-3D shear deformation theory, Bouhadra *et al*. (2021) predicted the free vibration behavior of nano FG beams. Using two-step perturbation technique, Shen and Xiang (2021) studied the thermal-based post-buckling behavior of laminated shells. Belarbi *et al*. (2022) employed parabolic theory for the analysis of FG nanoplates.

A detailed summary on the analysis of FG nanobeams is published in the review works of Eltaher *et al*. (2016), Ghayesh, and Farajpour (2019), and Garg *et al*. (2021b). In all the available models in the literature, solutions are obtained using analytical, semi-analytical, or approximate methods such as finite element method, isogeometric analysis etc. Computing the behavior of FG nanobeams with these models take some computational efforts (Garg *et al*. 2024c).

Machine learning (ML) techniques helps in building the relationship between set of input variables and the output and thus the surrogate model will be able to predict the output for other set of unseen variables without any large computational efforts (Wang *et al*. 2024). Several ML techniques are available in the literature, which can be used for establishing the relationship between the variables and output (Huang *et al*. 2022). Support vector (SV) regression-based ML model was proposed by Wang *et al*. (2019) for free vibration analysis of FG bars. Banh *et al*. (2021) optimized the topology of bi-direction FGM plates by employing multigrid preconditioned conjugate gradient technique. Vaishali *et al*. (2020) carried out analysis of FG shells using finite element assisted support vector machine model. SV based model was put in by Wang *et al*. (2020) for studying the behavior of FG frames. Madenci and Ozkili (2021) employed artificial neural network (ANN) for free vibration analysis of open cell FG porous beams. Vibration analysis of FG beams was carried out by Trinh and Jun (2021) using ANN models. Ming *et al*. (2021) proposed the application of deep neural network for the analysis of sandwich structures. With the help of deep learning network, Zhou *et al*. (2022) studied the response of graphene platelet reinforced plates under thermo-electrical conditions. The optimization processes can also be carried out using machine learning techniques (Wang and Sigmund 2023, 2024).

Several works are reported in the literature for the analysis of FG beams using different ML models. However, no work is available in the literature for the analysis of FG nanobeams using ML techniques. In present work, bending and buckling behavior of FG nanobeams is studied using two regression-based ML techniques, namely, Gaussian process regression (GPR), and Random Forest (RF). The performances of these two modeling techniques on predicting the behavior of FG nanobeams has been studied. For generating equally spaced non-biased data set of input variables, Sobol sequences have been used. It has been observed that GPR-based surrogate model is found to predict the bending and buckling behavior of FG nanobeams more precisely as compared to RF- based surrogate model even for an unseen dataset.

2. Mathematical and material modeling

2.1 Machine learning model

The present work carries out bending and buckling analysis of FG nanobeams using two regression-based machine learning (ML) algorithms, namely, GPR, and RF.

2.1.1 Gaussian Process Regression (GPR)

It is a non-parametric probabilistic kernel-based ML model. Assuming a training dataset taken

from an unknown distribution of n values $\{(x_i, y_i), i = 1,2,3,\ldots,n\}$, where $x_i \in R_d$ and $y_i \in R$. The value of y as an output for an unseen x can be obtained from the linear regression model as (Garg and Li 2024)

$$y = x^T\beta + n \tag{1}$$

where $n \sim N(0, \varrho^2)$. The values for the coefficient β and variance ϱ^2 are obtained from the dataset. If $\{f(x), x \in R_d\}$ is GP for observations $x_1, x_2, x_3, \ldots, x_n$ then, $f(x_1)$, $f(x_2)$, $f(x_3)$,…, $f(x_n)$ is Gaussian. A mean function $m(x)$ and covariance function $k(x, x')$. Since $\{f(x), x \in R_d\}$ is a Gaussian then $E\big(f(x)\big) = m(x)$ and $cov[f(x), f(x')] = E[\{f(x) - m(x)\}\{f(x') - m(x')\}] = k(x, x')$.

Thus, $hx^T\beta + f(x)$ $h(x)T\beta$ + $f(x)$ represents model for zero mean $f(x)$. This represents the GPR model. The output y can be estimated as (Garg *et al*. 2022)

$$P(y|f(x_i), x_i) \sim N(y_i|h(x_i)^T)\beta + f(x_i), \varrho^2 \tag{2}$$

2.1.2 Random forest

Random forest is an upgraded version of decision trees method in which several decision trees are constructed for predicting the output. The output from all the trees is then combined or mean the prediction for regression. Each decision tree calculates Scikit-learn using Gini Importance, assuming only two child nodes (binary tree)

$$ni_j = w_j C_j - w_{left(j)} C_{left(j)} - w_{right(j)} C_{right(j)} \tag{3}$$

where ni_j is importance of node j, w_j is weighted number of samples approaching node j, C_j represents impurity value of node j, $left(j)$ and $right(j)$ stands for child node from left and right split on node j, respectively. Importance of each variable on decision tree is determined as (Garg *et al*. 2023b, c)

$$fi_i = \frac{\sum_{j:node\ j\ splits\ on\ feature\ i} ni_j}{\sum_{k \in all\ nodes} ni_k} \tag{4}$$

which is then normalized between 0 and 1 as

$$\text{norm } fi_i = \frac{fi_i}{\sum_{j \in all\ nodes} fi_j} \tag{5}$$

2.2 Theoretical formulation of HSDT

In the present work, finite element (FE) method based on the HSDT proposed recently by Belarbi *et al*. (2021) has been used for preparing the dataset for training the ML techniques discussed in previous sub-section. The cross-section of the beam analyzed is shown in Fig. 1. Nonlocal continuum theory proposed by Eringen (1983) has been employed for incorporating the scale-based effects as

$$(1 - \mu \nabla^2)\sigma_{ij} = \sigma_{ij}^l \tag{6}$$

where ∇^2 is Laplacian operator, μ is nonlocal parameter, σL is the classical (local) stress tensor, and σ_{ij} denotes the nonlocal stress tensor.

The displacement fields adopted are given as (Belarbi *et al*. 2021)

$$u_x(x, z) = u_0(x) - z\frac{\partial w_0}{\partial x} + f(z)\phi_x \qquad u_z(x, z) = w_0(x) \tag{7}$$

where u_0 and w_0 are the mid-plane displacements, ϕx is the cross-sectional rotation of the beam about neutral axis,

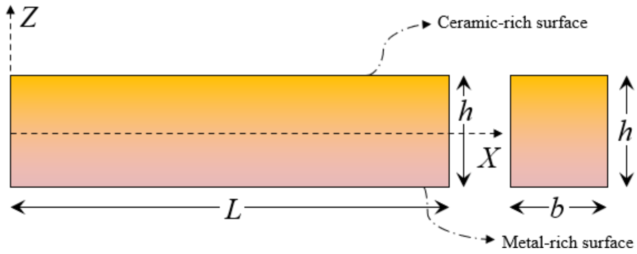


Fig. 1 Geometry of a functionally graded (FG) nanobeam

and $f(z)$ is the shear function assumed parabolic, defined as

$$f(z) = z\left(1 - \frac{3}{2}\left(\frac{z}{h}\right)^2 + \frac{2}{5}\left(\frac{z}{h}\right)^4\right) \tag{8}$$

The principal of minimum potential energy can be explained as

$$\delta\Pi = \delta(U - V) = 0 \tag{9}$$

where Π is the total potential energy, and $\delta(U - V)$ is the variation in the difference between the strain energy and work done by the external forces.

Substituting the energy terms, the stress resultants can be obtained. Two noded C^1 FE having four degrees of freedom per node is used. Using the general FE procedure, the governing equations can be obtained for bending and buckling studies, respectively, as (Garg *et al*. 2024d)

$$[K]\{d\} = \{F\} \tag{10}$$

$$\big([K] - N_0[K_g]\big)\{q\} = \{0\} \tag{11}$$

where $[K]$ is the global stiffness matrix, $\{d\}$ is the nodal displacement vector, $\{F\}$ is the load vector, $[K_g]$ represents global geometric stiffness matrix, and $\{q\}$ stands for the global degrees of freedom vector.

For detailed formulation, one can refer to the work published by Belarbi *et al*. (2021). For modeling the material property variation across the thickness of the beam, simple power law is adopted. The GPR and RF based surrogate models were formed using the inbuilt code available in MATLAB.

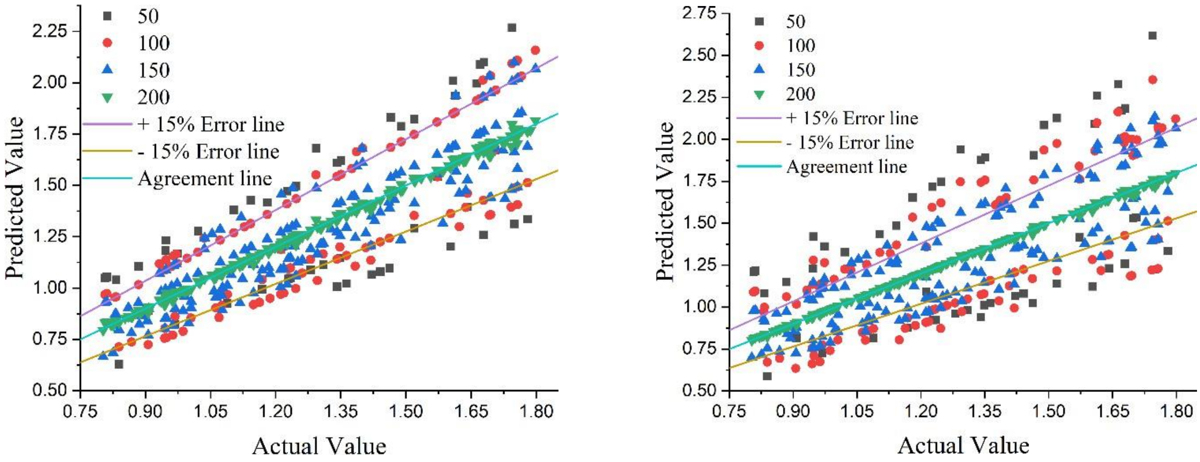
2.3 Material property modeling: Power law

For modeling the variation of material property across the thickness of FG nanobeam, simple power law is used, which is defined mathematically as

$$P(z) = \left(P_{top} - P_{bottom}\right)\left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{11}$$

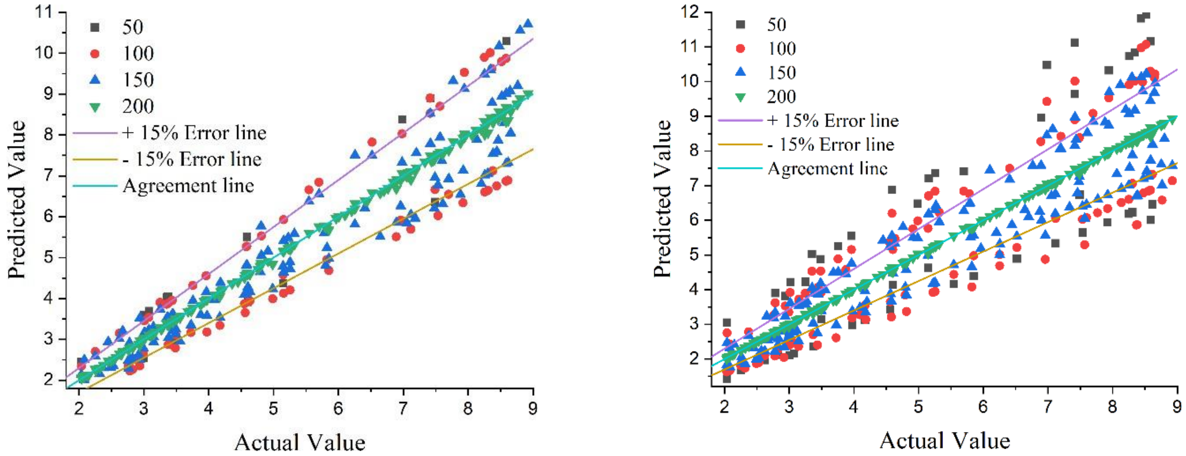
where P represents the engineering property of the material, n is the power-law index as reported in Fig. 1, P_{top} and P_{bottom} denote the properties at top and bottom pure materials, respectively.

Note that the size of micro-structures in FGM ranges typically over several orders of magnitude, and therefore the FGM may not be enough modeled by employing the pure classical continuum theory of elasticity alone. It is probably more accurately modeled using non-classical mechanics (such as nonlocal theory of elasticity) in



(a) GPR (b) RF

Fig. 2 Actual v/s predicted values for non-dimensional central transverse displacement predicted using GPR and RF models for different count of training datasets



(a) GPR (b) RF

Fig. 3 Actual v/s predicted values for non-dimensional buckling load predicted using GPR and RF models for different count of training datasets

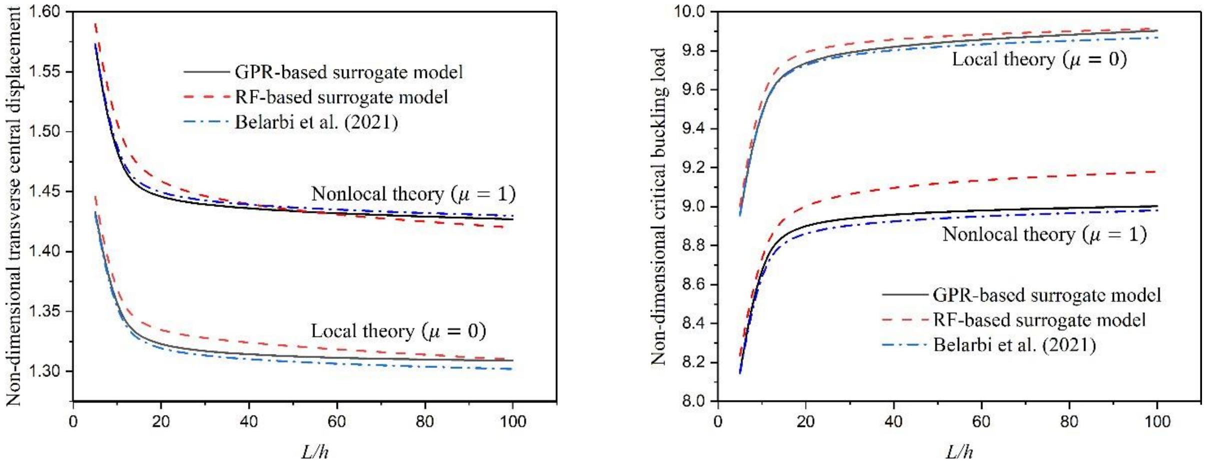
conjunction with spatial variation of material properties (Li and Hu 2016).

The engineering constants for the material from which the beam is assumed to be made up of are taken as E_1 = 1 *TPa*, E_2 = 0.25 *TPa*, $v_1 = v_2$ = 0.30, otherwise stated (Amine *et al*. 2015).

3. Results and Discussion

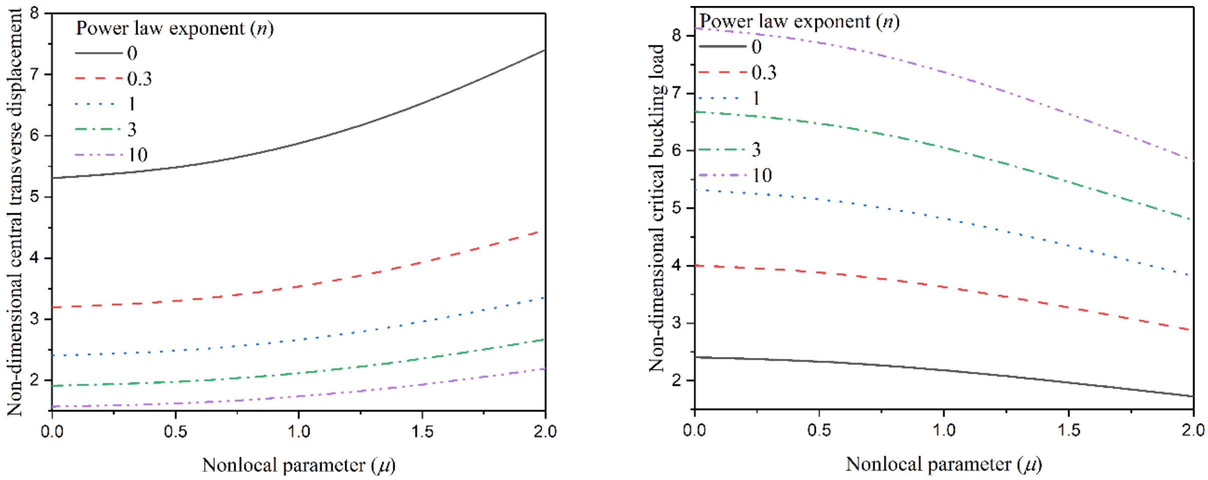
The efficiency and the accuracy of the shear deformation theory adopted have already been presented in the work by Belarbi *et al*. (2021). For preparing the dataset to train the techniques used in present work, the same theory has been adopted. Material properties of ceramic and metal, power- law exponent, L/h, end condition, and nonlocal parameter are adopted as the variables, whereas non- dimensional deflection, and non-dimensional critical buckling loads are the predictions. Separate machine learning models are modelled for both bending and buckling conditions.

Figs. 2 and 3 show the convergence study related to the amount of dataset required for predicting the bending and buckling behavior of FGM nanobeams, respectively using different ML models as stated in the form of agreement line diagram. Nonlocal parameter, power law exponent, L/h, material properties, and load are considered as variables, whereas non-dimensional central transverse displacement ($\bar{w} = 100wE_1I/(qL^4)$) and non-dimensional critical buckling load ($\bar{P}_{cr} = P_{cr}L^2/E_1I$) are the output variables. Results for the actual values v/s predicted values for $\bar{w}$ and $\bar{P}_{cr}$ obtained using GPR and RF based surrogate models for different count of training datasets are reported in Figs. 2 and 3 respectively. When the training dataset count reaches 150, most of the values lies in the range of ± 15%. However, for training dataset equals 200, approximately all the values lie on the agreement line i.e., predicted value is equal to the actual value. Hence, the surrogate model trained using 200 dataset is used for the further studies. Also, the accuracy of the GPR surrogate model is found to be more as compared to the accuracy of the RF surrogate model.



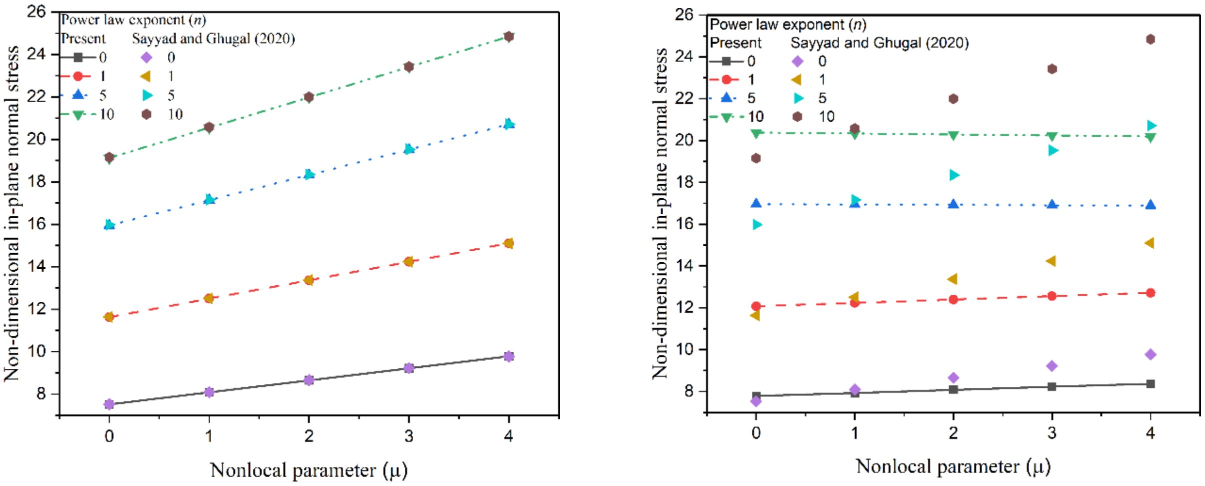
(a) Non-dimensional transverse central displacement (b) Non-dimensional critical buckling load

Fig. 4 Values for (a) non-dimensional transverse central displacement, and (b) non-dimensional critical buckling load predicted using surrogate GPR and RF models for different values of *L/h* and μ for simply supported homogenous nanobeams



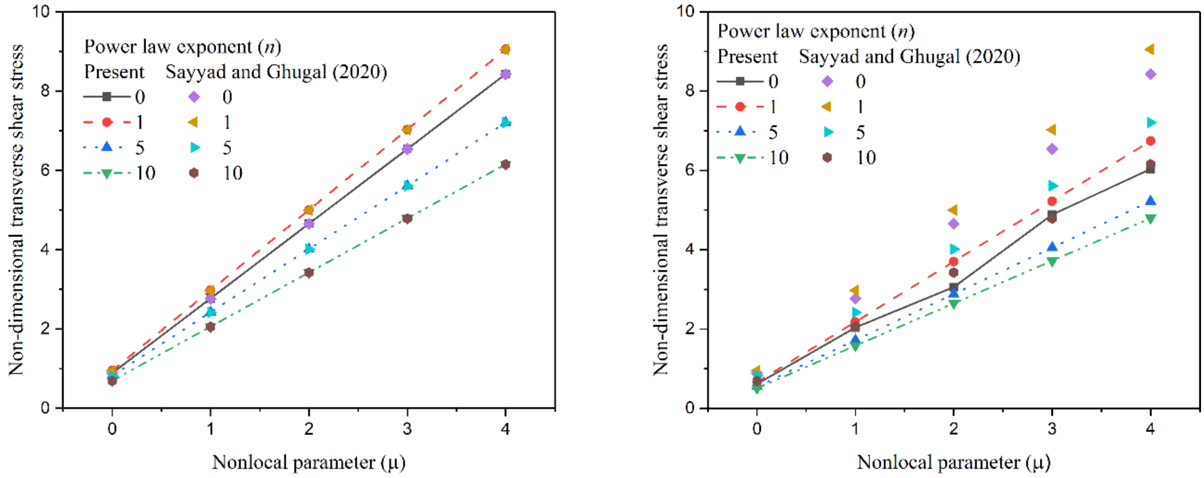
(a) Non-dimensional transverse central displacement (b) Non-dimensional critical buckling load

Fig. 5 Values for (a) non-dimensional transverse central displacement, and (b) non-dimensional critical buckling load predicted using surrogate GPR models for different values of μ for simply supported FG nanobeams *L/h* = 10



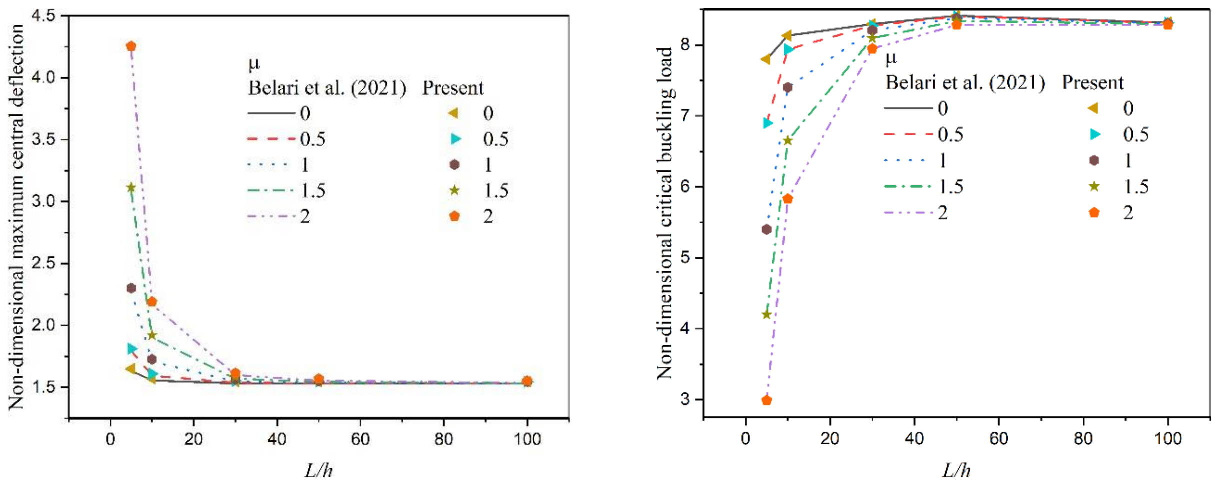
(a) GPR (b) RF

Fig. 6 Values for non-dimensional in-plane normal stress ($\bar{\sigma}_{xx}$) obtained using (a) GPR, and (b) RF-based surrogate models for different values of μ and *n* for simply supported FG nanobeams *L/h* = 10



(a) GPR (b) RF

Fig. 7 Values for non-dimensional transverse shear stress $(\bar{\sigma}_{xz})$ obtained using (a) GPR, and (b) RF-based surrogate models for different values of μ and n for simply supported FG nanobeams L/h = 10



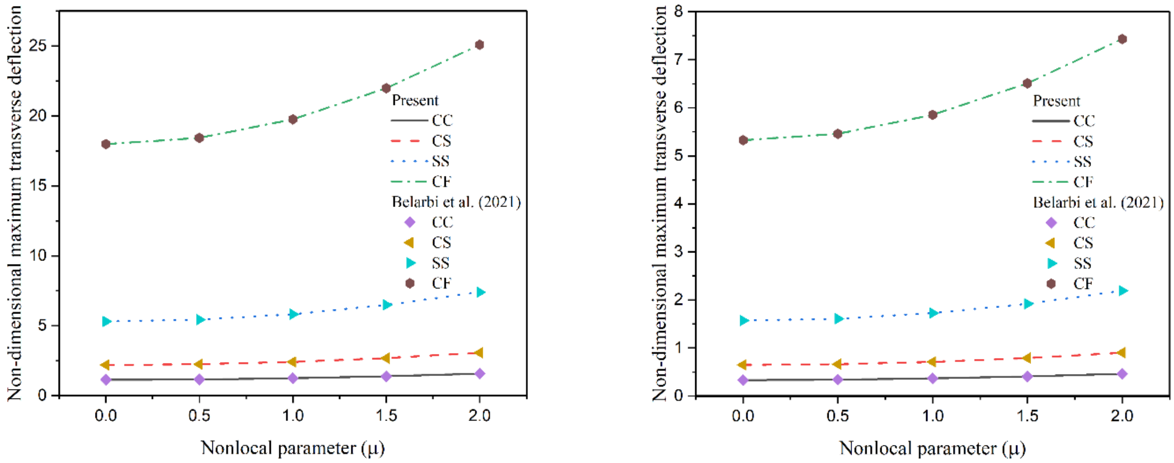
(a) Non-dimensional transverse central displacement (b) Non-dimensional critical buckling load

Fig. 8 Values for (a) non-dimensional transverse central displacement, and (b) non-dimensional critical buckling load predicted using surrogate GPR models for different values of L/h and μ for simply supported FG nanobeams

Fig. 4 exhibits the values for non-dimensional transverse central displacement and critical buckling loads for two different values of nonlocal parameters μ i.e., considering and without considering the nonlocal effects obtained using GPR and RF-based surrogate models for homogenous nanobeams. Material properties adopted are as: E = 1 GPa, v = 0.30. The present results are compared with the results presented by Belarbi *et al*. (2021) using FE based HSDT and are found to be in good agreement. It can be observed that with an increase in L/h value, the nonlocal effects diminish. GPR-based surrogate model can predict the behavior of FG nano beams more accurately as compared to the RF-based surrogate model. Incorporating the nonlocal effects, non-dimensional central displacement increases whereas non-dimensional critical buckling load decreases.

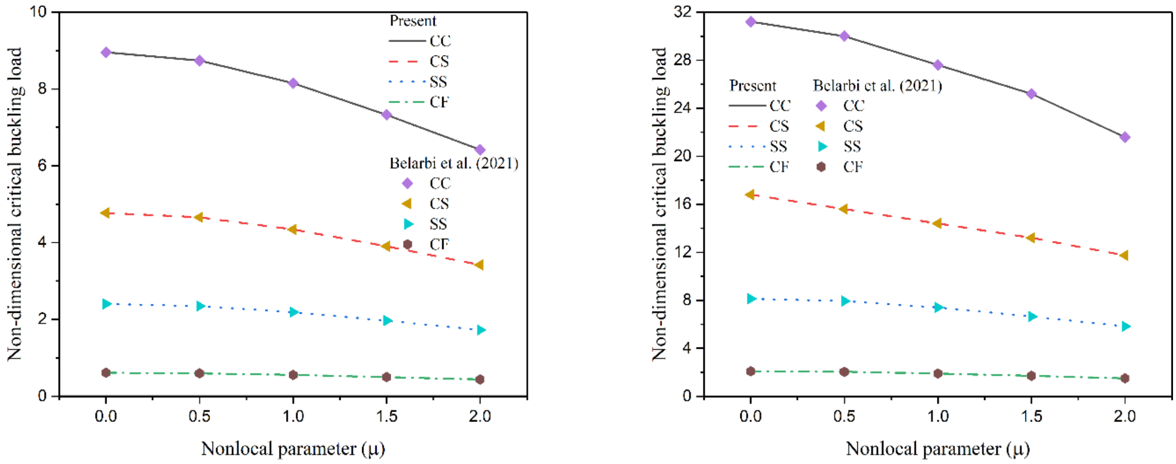
After noticing that the GPR-based surrogate model predicts the bending and buckling behavior of nanobeams more effectively as compared to the RF-based surrogate, therefore, GPR-based surrogate model is employed to study the bending and buckling behavior of FG nanobeams for different values of nonlocal parameters (μ), and power law exponent (n). Results for the same cases are reported in Fig. 5 for simply supported beam with L/h = 10. As the value for n increases, the value for $\bar{w}$ decreases whereas the value for $\bar{P}_{cr}$ increases. Similar type of behavior is also observed when the value of μ increases.

Figs. 6 (a) and (b) shows the values for non-dimensional bending normal stress $(\bar{\sigma}_{xx} = \sigma_{xx}h/qL)$ obtained at the top surface of the FG nanobeam subjected to uniform load over its entire length obtained using GPR and RF-based surrogate models, respectively. The beam is assumed to be made up of Alumina (E = 380 GPa) and Aluminium (E = 70 GPa) (Garg *et al*. 2021c). The present results are compared with those given by Sayyad and Ghugal (2020). It can be inferred that the GPR-based results are closer to the results published by Sayyad and Ghugal (2020) as compared to the behavior predicted by RF-based model. Figs. 7 (a) and (b) shows the variation of non-dimensional transverse shear stress $(\bar{\sigma}_{xz} = \sigma_{xz}h/qL)$ at the centre of FG nanobeam for different values of μ and n. For transverse shear stresses



(a) $n = 0$ (b) $n = 10$

Fig. 9 Values for non-dimensional maximum transverse displacement predicted using surrogate GPR models for FG nanobeams made up of different end conditions



(a) $n = 0$ (b) $n = 10$

Fig. 10 Values for non-dimensional critical buckling load predicted using surrogate GPR models for FG nanobeams made up of different end conditions

also, RF failed to predict the values accurately. This highlights the dataset count required for training to predict the transverse displacement of the FG nanobeam is not enough to predict the stresses. For same, more data count is required for training the model.

4. Conclusions

In the present work, bending and buckling analysis of FG nanobeams is carried out using regression-based machine learning techniques, namely Gaussian process regression (GPR) and Random Forest (RF) methods. For training the machine learning models, transverse central displacement and critical buckling loads are obtained from the parabolic higher-order shear deformation theory recently proposed in the literature. Material properties, L/h, power law exponent, end condition, and nonlocal parameter are considered as the input variables on which the values for transverse displacement and buckling load depend. The following are the points noted down

- The accuracy of the surrogate model in predicting the behavior of FG nanobeams is widely affected by the count of dataset used for training the model.
- GPR-based surrogate model can predict the bending and buckling behavior of FG nanobeams more effectively as compared to the RF-based surrogate model.
- For predicting stresses accurately, RF model requires more dataset count for training itself as compared to the dataset required for training GPR model.
- Nonlocal effects are more pronounced for beams having lower value of L/h.
- With an increase in the metallic content in the beam, the non-dimensional transverse displacement of the beam decreases, whereas non-dimensional critical buckling load increases. This indicates an increase in the stiffness of the FG nanobeam.
- The value for nonlocal parameter adopted affects the behavior of short FG nanobeams to a greater extent as compared to the large FG nanobeams.
- The end conditions widely affect the bending and buckling behavior of FG nanobeams.

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