Advances in Nano Research, *Vol. 17, No.4 (2024) 351-367*
https://doi.org/10.12989/anr.2024.17.4.351

Control the stability of small-scale non-uniform structures via neural networks applied to partial differential equations

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(*Received July 16, 2022, Revised October 1, 2024, Accepted October 2, 2024*)

Abstract. This research uses a numerical technique and a neural network process to investigate the stability management of non-uniform cylindrical constructions with varying sizes. The non-uniform or truncated conical shapes vary in axial length. This complicated geometry results in partial differential equations in the mathematical explanation of stability performance. Furthermore, material distributions vary in the radial direction in functionally graded materials such as metal and ceramic. The governing equations are obtained from beam theory using the energy technique and non-classical size-dependent theory, respectively. These equations are then solved using both a numerical and neural network methodology. This research can potentially be utilized in nanotechnology to build and evaluate size-dependent non-uniform cylindrical structures. As a consequence, it will help to develop sophisticated nanoscale materials and architectures.
Keywords: functionally graded materials; neural network procedure; non-uniform cylindrical structures; numerical approach; stability control

1. Introduction

1.1 Application of non-uniform and truncated conical small-scale structures

Nanotechnology has transformed the design and use of small-scale structures, including non-uniform and truncated cones. These structures are often employed in a variety of engineering and technology sectors, including MEMS, NEMS, biosensors, and new materials. The distinctive geometry of non-uniform and truncated conical structures, together with the outstanding qualities of nanoscale materials, give various benefits in terms of mechanical stability, strength, and use (Liu *et al*. 2024, Wang *et al*. 2024c, Wu *et al*. 2024b).

1.1.1 Mechanical and structural advantages

Non-uniform and truncated conical shapes have intrinsic mechanical benefits when developed at small sizes. Their tapered form increases buckling and mechanical instability resistance, which is critical in applications that need high strength-to-weight ratios. For example, nanostructures such as conical nanowires or nanopillars are often used in systems that need mechanical stability under variable stresses. The gradation in diameter over the length of these structures distributes stress more uniformly, lowering the chance of failure during compression or bending (Wu *et al*. 2023). Furthermore, including nanoparticles in these structures improves stiffness and strength owing to size-dependent material features, including surface area-to-volume ratio (Zhang *et al*. 2017, 2022a, 2024).

1.1.2 Functionally graded materials (FGMs) in conical structures

Functionally graded materials (FGMs), which vary in composition and properties across their volume, are often integrated into non-uniform and truncated conical structures at the nanoscale. Engineers may increase device performance for certain applications by changing the material properties across the conical structure. For example, cylindrical nanosensors use FGMs to need different types of material in different parts of the structure, which makes them more sensitive and useful (Nami *et al*. 2015). This gradation lets the structure change with the seasons or mechanical conditions while keeping the structure strong and efficient (Zhang *et al*. 2022b, 2024).

1.1.3 Applications in MEMS and NEMS

Nanostructures that are not regular and are truncated cone-shaped are very important in the development of sensors, motors, and emitters in the fields of MEMS and NEMS. For uses where accuracy and small size are important, these designs are perfect (Fu *et al*. 2023b, Cao *et al*. 2024, Chen *et al*. 2024). For instance, conical nano-structures are used in nanoprobe technologies. Their tapered shape improves the sensitivity and spatial precision of scanning imaging and nanoscale material handling. Similarly, the uneven shape of nano-actuators makes it easier to control how they work mechanically, which leads to better performance in things like micro-mirrors, pressure sensors, and accelerometers (Torkashvand *et al*. 2024).

1.1.4 Biosensing and biomedical applications

Biosensing and biological devices are two other important uses for short circular shapes on a small scale.

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http://www.techno-press.org/?journal=journal=anr&subpage=5
ISSN: 2287-237X (Print), 2287-2388 (Online)

These structures that find biological and chemical toxins are often made with nanomaterials. For instance, DNA sequencing technologies use cylindrical nanopores because their form makes it easy to separate and identify parts. Nanotechnology also makes it possible to make safe, non-uniform cone-shaped drug delivery systems. In cases where controlled release of healing agents is needed for treatment to work, these systems are very useful (Narmani *et al*. 2023). Being able to change the surface features at the nanoscale level makes it easier for these materials to connect with living things, which makes them more useful in medical settings (Han *et al*. 2018, 2020, Li *et al*. 2024).

1.1.5 Energy harvesting and nanoelectronics

Nanogenerators and solar cells are two examples of devices that make use of non-uniform and truncated conical forms. These structures are used in the field of Energy Harvesting. Their shape makes it possible to gather and convert energy in an effective manner, notably in piezoelectric nanogenerators, which are devices that transform mechanical energy into electrical energy. The conical form improves the concentration of strain, which in turn leads to an increase in energy production. The cylindrical form makes it easier for the pressure to be spread out, which increases the energy output. Truncated cone-shaped structures are used in field-effect transistors (FETs) and capacitors in nanoelectronics because their shape makes it easier to control the electrical properties at the nanoscale (Zhang *et al*. 2020).

Non-uniform and truncated conical small-scale structures are critical components in many modern technologies made possible by nanotechnology. Their distinct shape, along with the improved characteristics of nanoscale materials, gives considerable mechanical, functional, and operational benefits in a variety of applications, including MEMS, NEMS, biosensing, and energy harvesting. As nanotechnology advances, the relevance of these structures in the creation of novel gadgets will grow, opening up new avenues for scientific progress (Liu *et al*. 2022, Liu *et al*. 2023, Shu *et al*. 2025).

1.2 Stability analysis of small-scale structures

The stability analysis of small-scale structures, especially at the micro- and nanoscale, has evolved during the past decade as a result of advances in nanotechnology and material science. Traditional stability criteria, such as Euler's buckling theory, have been adjusted to account for the effects of size, material inhomogeneity, and external surroundings, which are important at tiny scales. As reported by Zheng *et al*. (2024), these advancements have led to the development of innovative analytical procedures and computational approaches that are appropriate for applications on the nanoscale and the microscale. In addition, nonlocal elasticity theories and modified couple stress theories have gained popularity in the field of analyzing the stability of small-scale structures. This is due to the fact that these methodologies take into consideration the size-dependent behavior of materials at lower scales (Attar *et al*. 2021).

Recent studies have mostly focused on the application of nonlocal elasticity theories, which change classical continuum mechanics by include scale factors considering the consequences of small-scale events (Sun *et al*. 2024, Van *et al*. 2024, Sun *et al*. 2025). In nanostructures, where traditional theories typically fail to precisely anticipate behavior, this method has proven very helpful for examining buckling and vibration properties (Barretta *et al*. 2023). Often employed for numerical simulations in both two- and three-dimensional structures, nonlocal elasticity is coupled with finite element techniques (FEM) and the Generalized Differential Quadrature Method (GDQM).

A major factor under consideration in the stability study of small-scale structures is also material heterogeneity at the nanoscale (Yang *et al*. 2021, Wang *et al*. 2024b, Zhou *et al*. 2024b). Because of their adjustable mechanical qualities, functionally graded materials (FGMs) and composite materials have become very popular for use in nanoengineering. These materials show varied reactions to external loads and their stability under various boundary conditions has been thoroughly explored using modified high-order shear deformation and nonlocal theories (Islam *et al*. 2020). These models enable the buckling behavior under many thermal, mechanical, and electromagnetic settings to be predicted.

Moreover, stability models for nanoscale systems have been progressively include surface energy effects. Surface-to----volume ratios become important at tiny dimensions, surface energy affects the general stability of constructions, especially in thin films and nanowires (Narendar *et al*. 2012). More precise predictions for the beginning of instability in nanoscale systems under external loads have come from recent research using surface elasticity and residual stresses in stability equations (Andrievski 2003). Particularly with regard to nonlocal elasticity, modified pair stress theories, and surface energy effects, stability analysis methods for small-scale structures have evolved remarkably in the previous decade. These developments have made stability models more applicable to many different materials and external conditions, therefore facilitating the design and research of increasingly complex nanoscale devices. As the need for nanotechnology continues growing, further research on multiscale modeling and experimental validation will be absolutely essential to raise these stability assessments (Lim *et al*. 2015).

1.3 Controlling the stability of small-scale non-uniform structures via neural networks

The stability management of small-scale non-uniform structures has received more interest as nanotechnology and its applications in engineering and material science develop. Because of their susceptibility to external events and material inhomogeneities, traditional stability analysis approaches including those based on classical mechanics often find it difficult to forecast and manage the complex behaviors of small-scale, non-uniform systems. Offering the capacity to model, forecast, and manage the dynamic stability of such systems with enhanced precision and flexibility, neural networks (NNs) have become more promising in recent years to solve this difficulty.

Particularly deep learning models, neural networks can manage nonlinear and complicated systems. In the framework of small-scale constructions, NNs may be taught to understand the link between many factors, including material qualities, geometrical configurations, and external loading conditions, and their influence on structural stability. Once trained, the network may be used to detect possible instabilities, forecast important buckling stresses, and provide ideal design configurations improving stability. NNs especially help to control non-uniform structures where traditional analytical approaches might be inadequate because of their predictive capacity (Wang *et al*. 2023b).

Furthermore, in systems whose environmental variables are always changing—such as in nanodevices or micro-electromechanical systems (MEMS—real-time stability management may be accomplished with NNs. Feedback loops in the control system enable NNs to dynamically change load distribution or boundary conditions, hence preserving stability. With this real-time adjustment capacity, one has a major advantage over static approaches in that it allows one to consider time-dependent fluctuations (Wang *et al*. 2023c).

Apart from prediction and real-time control, neural networks find use in design optimization of non-uniform structures for maximum stability. Engineers may investigate a large design space automatically looking for combinations that enhance stability while decreasing material use or weight by applying machine learning methods. From nanowires and thin films to sophisticated composite materials, this optimization technique may be used to a broad spectrum of small-scale structures therefore allowing the construction of more dependable and efficient systems (Raiaan *et al*. 2024).

1.4 Shortcomings, gaps and novelty of the study

Computational intelligence and mechanical engineering have a strong synergy shown by the inclusion of neural networks into the stability analysis and control of small-scale non-uniform structures. The use of NNs in this field is projected to develop as computer resources keep expanding and machine learning methods become more advanced, therefore offering more solid solutions for managing stability in ever sensitive and complicated surroundings (Cheng *et al*. 2023, Dai *et al*. 2023, Fu *et al*. 2023a, Guan 2023, He and Deng 2023, Jia *et al*. 2023, Jin *et al*. 2023, Lau and Li 2023, Li *et al*. 2023a, b, c, d, Li 2023, Ma *et al*. 2023, Song *et al*. 2023, Su *et al*. 2023, Wang *et al*. 2023a, Yang and Mao 2023, Ye *et al*. 2023, Zhang and Huang 2023, Zhang *et al*. 2023a, b, c).

The present work offers a fresh method based on neural networks applied to partial differential equations to manage the stability of small-scale non-uniform structures. Although earlier studies have made significant progress in studying small-scale structures like truncated conical forms and non-uniform shapes, some flaws still exist. Particularly when material inhomogeneities and external dynamic forces are involved, traditional approaches like classical mechanics and nonlocal elasticity theories typically find it challenging to forecast these systems' complicated, nonlinear behavior. Furthermore, these traditional methods provide little flexibility in real-time situations—precisely what micro and nanoscale devices need.

This approach closes these gaps by incorporating neural networks (NNs) to improve stability's prediction and control. Learning complicated correlations between many parameters—e.g, material qualities, geometrical changes, and external loads—and their consequences on structural stability is good for neural networks. Furthermore, absent in static techniques is the capacity of the NN model to dynamically change control tactics in real-time. Applications such as MEMS, NEMS, and nano-devices—where ambient conditions vary quickly—dependent on this real-time feedback mechanism are very vital.

This work introduces a unique approach to solving partial differential equations controlling non-uniform structures by combining energy-based size-dependent theories with sophisticated computational intelligence tools such as NNs. This study represents significant progress in designing and managing innovative nanoscale materials and gadgets as it not only forecasts critical buckling loads and possible instability sites but also optimizes structural topologies. The study creates fresh opportunities for building more dependable, efficient, and flexible systems in nanotechnology, biosensing, and energy harvesting.

2. Mathematical modelling

2.1 Material distributions

Within the scope of this investigation, the cylindrical beam under consideration is constructed out of a functionally graded material (FGM) that incorporates both ceramic and metal components. Metal is used for the internal surface, which results in increased toughness and ductility. Ceramic is used for the coating of the exterior surface, which results in increased heat resistance and stiffness. The characteristics of the material change in a continuous manner over the thickness of the beam, transitioning in a seamless manner from the metal in the center to the ceramic on the outside layer. This progressive distribution is governed by a power-law function, which guarantees that the beneficial characteristics of both materials are preserved in a state of equilibrium. Table 1 and Eq.(1) provide a full explanation of the mathematical formulation that governs the material distribution and the mechanical characteristics of the FGM construction that correspond to it. Based on the following mathematical equations, the distribution of mechanical properties, including the elastic modulus (E), density (ρ), and Poisson ratio (ν), is as follows:

$$E(r,x) = E_{Metak} - (E_{Metal} - E_{Ceramic})\left(\frac{r - \frac{Di}{2}}{\frac{De(x)}{2} - \frac{Di}{2}}\right)^{\eta} \quad (1a)$$

$$\rho(r,x) = \rho_{Metak} - (\rho_{Metal} - \rho_{Ceramic})\left(\frac{r - \frac{Di}{2}}{\frac{De(x)}{2} - \frac{Di}{2}}\right)^{\eta} \quad (1b)$$

$$v(r,x) = v_{Metak} - (v_{Metal} - v_{Ceramic})\left(\frac{r - \frac{Di}{2}}{\frac{De(x)}{2} - \frac{Di}{2}}\right)^{\eta} \quad (1c)$$

where ‘η’ is the functionally graded parameter to control the volume fraction between the ceramic and metal, ‘*Di*’ and ‘*De*’ are the internal and external diameters. It is noted that the mechanical characteristics are influenced by both ‘*r*’ and ‘*x*’. This indicates that the mechanical properties of the cylindrical structures change in both radial and axial directions. In the previous explanation, it was discussed how metal and ceramics, representing different phases of functionally graded structures involving Nickel and Aluminum Oxide (Al_2O_3), were used to create the functionally graded structures listed in the table below.

Table 1 Mechanical properties of Nickle and Aluminum Oxide (Yang and Shen 2002)

	Nickel	Aluminum oxide
Mass density	8900 (Kg/m^3)	3800 (Kg/m^3)
Poisson ratio	0.31	0.24
Young’s modulus	223.95 (GPa)	323.393 (GPa)

2.2 Geometry

An important contribution to the mechanical and thermal behavior of a cylindrical beam with nonuniform geometry in its axial direction is investigated in this work (See Fig. 1). Variations in stress and deformation patterns under applied loads occur from changes in the cross-sectional area and moment of inertia resulting from changing geometry. Accurate evaluation of the structural performance of the beam depends on accounting for this nonuniformity as it confounds estimates of buckling, vibration, and general stability. Furthermore, influencing the distribution of the functionally graded material (FGM), geometric variation modulates the material characteristics change along the beam. Development of precise theoretical models and improvement of the actual performance of the beam depend on exactly describing this nonuniform geometry. Fig. 1 plots the cylindrical beam via internal Diameter, Di, and external Diameter, De, where the internal diameter is constant, and the external diameter is dependent on the ‘x’ as the independent axial component, and its mathematic function is as follows.

$$De(x) = De_0\left(1 - \beta\frac{x}{L}\right) \quad (2)$$

where ‘De_0’ is the initial external diameter, and ‘β’ is the aspect ratio.

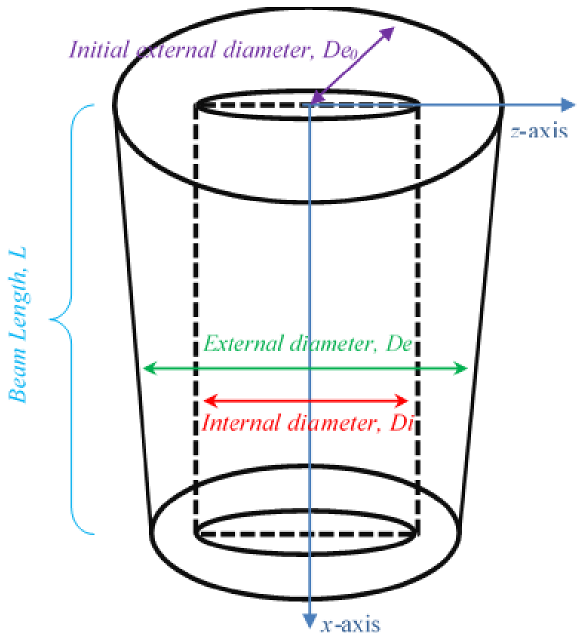

Fig. 1 Geometry of cylindrical nonuniform beam

2.3 Energy method

The investigation of buckling behavior in nanocylindrical beams is critical in structural and materials engineering. As materials decrease to the nanoscale, their stability and mechanical characteristics change dramatically owing to surface effects and size-dependent phenomena. Analyzing stability, particularly in terms of buckling, is critical for ensuring the safety and performance of nano-engineered structures utilized in disciplines like as aeronautical engineering and medicinal equipment (Omidi *et al*. 2013, Ebrahimi and Shafiei 2016, Ghadiri and Shafiei 2016a, Ghadiri *et al*. 2016b, d, 2017e, Shafiei *et al*. 2016a, b, c, e, 2017c, 2019, Ebrahimi and Shafiei 2017, Ehyaei *et al*. 2017, Mirjavadi *et al*. 2017c, d, Mousavi *et al*. 2017, Shafiei and Kazemi 2017a, b, Shivanian *et al*. 2017).

The energy technique is one of the most effective ways to investigate buckling behavior. It enables researchers to develop the governing equations of a system by taking into account the total potential energy, resulting in a better understanding of how external loads influence stability. By integrating this technique with variational principles, researchers may create a trustworthy framework for evaluating nano-cylindrical beam buckling (Azimi *et al*. 2016, 2018, Ghadiri and Shafiei 2016b, c, Ghadiri *et al*. 2016a, c, 2017a, b, c, d, e, Shafiei *et al*. 2016d, f, g, 2017a, b, d, 2020, Ebrahimi *et al*. 2017, Mirjavadi *et al*. 2017a, b, Shafiei and She 2018).

In this study, the energy method to obtaining the governing equations that are relevant to the stability analysis of nano-cylindrical beams is investigated, with a particular focus placed on the behavior of buckling. Engineers and researchers have the capacity to improve the design of nanostructures by gaining an understanding of theoretical concepts and using advanced mathematical methodologies. This will result in an increase in the dependability and functioning of the nanostructures.

Minimum potential energy is the foundation of the energy technique. This concept suggests that the condition in which the total potential energy is minimal corresponds to a stable equilibrium configuration of a system. This idea is used to construct the governing equations for structural stability analysis by computing the total potential energy of the system and applying the equilibrium conditions (Habibi *et al*. 2017, 2019, 2020, Safarpour *et al*. 2018, 2020, Ebrahimi *et al*. 2019a, b, 2020, Hashemi *et al*. 2019, Moayedi *et al*. 2019, 2020a, b, Mohammadgholiha *et al*. 2019, Mohammadi *et al*. 2019, Ghazanfari *et al*. 2020, Oyarhossein *et al*. 2020, Shariati *et al*. 2020a, b, Shokrgozar *et al*. 2020).

Usually, a beam system consists of two components that together represent its total potential energy (Π):

• The internal energy kept in the structure via deformation is known as strain energy (U).

• Potential Energy of External Loads (V): The energy related to outside forces influencing the construction.

Making sure the initial variation of the total potential energy is zero helps one to derive the governing equation: that is,

$$\delta(\Pi) = 0 \tag{3}$$

A nano-cylindrical beam's strain energy varies with its deformation under compressive stresses. The strain energy for a tiny deformation may be written as:

$$U = \frac{\iiint \sigma_{ij} : \varepsilon_{ij} dv}{2} \tag{4}$$

where 'ε' and 'σ' are the strain and stress tensors, and they defined as follows:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

Here '*u*' is the displacement field, and the sinusoidal shear deformation theory assumes that the displacement field is sinusoidal, allowing for a more realistic depiction of shear deformation over the beam's thickness. The displacement field is commonly stated as follows (Zhang *et al*. 2023b, Wang *et al*. 2024a):

$$\begin{gathered} u_1 = u - z\frac{\partial w}{\partial x} + \\ \frac{(De - Di)}{2\pi} sin\left(\frac{2\pi z}{(De - Di)}\right)(\psi + w_{,x}) \\ u_2(x, y, z, t) = 0 \\ u_3(x, y, z, t) = w(x, t) \end{gathered} \tag{6}$$

The axial movement is represented by '*u*', the lateral movement by '*w*', and the rotation component by 'ψ'. Then the strain and stress tensors are calculated as follows:

$$\begin{gathered} \varepsilon_{xx} = u_{,x} - z w_{,xx} + \\ \frac{(De(x) - Di)}{2\pi} sin\left(\frac{2\pi z}{(De(x) - Di)}\right)(w_{,xx} + \psi_{,x}) \\ \varepsilon_{xy} = \\ \frac{(w_{,x} + \psi)}{2}\frac{\partial}{\partial y}\left[\frac{(De(x) - Di)}{2\pi} sin\left(\frac{2\pi z}{(De(x) - Di)}\right)\right] \\ \varepsilon_{xz} = \\ \frac{(w_{,x} + \psi)}{2}\frac{\partial}{\partial z}\left[\frac{(De(x) - Di)}{2\pi} sin\left(\frac{2\pi z}{(De(x) - Di)}\right)\right] \end{gathered} \tag{7a}$$

$$\begin{gathered} \sigma_{ij} = G\varepsilon_{ij}, i \neq j \\ \sigma_{ij} = E\varepsilon_{ij} i = j \\ G = E\left(\frac{1 + \nu}{2}\right) \end{gathered} \tag{7b}$$

External loads in buckling analysis are often compressive forces applied along the longitudinal axis of the beam. External mechanical loads have potential energy—that is, the energy kept in a system by means of external forces or loads. In mechanics, particularly in stability analysis such in buckling problems, this energy is integrated into the total energy computation of a system. One may represent the possible energy connected with these loads as follows:

$$\delta V = \iiint \vartheta(w_{,x})\delta(w_{,x})dv \tag{8}$$

where 'ϑ' is the buckling load. In solid mechanics, the nonlocal strain gradient theory is an advanced model used to explain the behavior of materials at small-scale, where standard theories (such as local elasticity) may fail to reflect size-dependent phenomena. This theory is better appropriate for examining nanostructures and micro-structures when these impacts become important as it combines nonlocal effects with strain gradient influences.

Incorporating both nonlocal and gradient elements, the nonlocal strain gradient theory alters the traditional stress-strain interaction. One may get the constitutive equation by:

$$\sigma_{ij} = \int_{\Omega} \alpha(|x - x'|) C_{ijkl} \varepsilon_{kl} d\,\Omega' + \gamma(\nabla\nabla\varepsilon ij) \tag{9}$$

where, 'C' is the material stiffness tensor, '$\alpha(|x - x'|)$' is the nonlocal kernel function that determines the influence of strain at point '*x*'' on point '*x*', '$\gamma\nabla\nabla\varepsilon_{ij}$' is the strain gradient term, where '$\nabla\nabla\varepsilon_{ij}$' represents the second-order derivatives of strain, and 'Ω' is the domain of the body being analyzed. Also, In nonlocal elasticity theory, the stress-strain relationship for a beam may be expressed as (Wang *et al*. 2022):

$$(1 - l^2\nabla^2)C : \varepsilon_{ij} = (1 - (ea)^2\nabla^2)\sigma_{ij} \tag{10}$$

In this context, '*ea*' represents the nonlocal parameter, while '*l*' denotes the strain gradient. When the nonlocal strain gradient theory is applied to both the strain energy and the energy attributable to the external work, the following Euler-Lagrange equation will be found (Wu *et al*. 2024a).

$$\begin{gathered} \delta\psi\!: \ \Theta_{21}\frac{\partial^2\psi}{\partial x^2} + l^2\Theta_{12}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^2\psi}{\partial x^2}\right) - l^2\Theta_{21}\frac{\partial^4\psi}{\partial x^4} - \\ l^2\Theta_{23}\frac{\partial^5 w}{\partial x^5} + \Theta_{23}\frac{\partial^3 w}{\partial x^3} - \Theta_{12}\left(\frac{\partial w}{\partial x} + \psi\right) = 0 \end{gathered} \tag{11a}$$

$$\begin{gathered} \delta w\!: \ +\Theta_{23}\frac{\partial^3\psi}{\partial x^3} - \Theta_{12}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial\psi}{\partial x}\right) - l^2\Theta_{12}\left(\frac{\partial^4 w}{\partial x^4} + \right. \\ \left. \frac{\partial^3\psi^3}{\partial x}\right) + \Theta_{22}\frac{\partial^4 w}{\partial x^4} + (ea)^2\vartheta\frac{\partial^4 w}{\partial x^4} + 3(ea)^2\vartheta_{,xx}\frac{\partial^2 w}{\partial x^2} - \\ \vartheta_{,x}\frac{\partial w}{\partial x} - l^2\Theta_{22}\frac{\partial^6 w}{\partial x^6} - \vartheta\frac{\partial^2 w}{\partial x^2} + 3(ea)^2\vartheta_{,x}\frac{\partial^3 w}{\partial x^3} + \\ (ea)^2\vartheta_{,xxx}\frac{\partial w}{\partial x} - l^2\Theta_{23}\frac{\partial^5\psi}{\partial x^5} = 0 \end{gathered} \tag{11b}$$

$$\delta u\!: \ -\Theta_{11} l^2\frac{\partial^4 u}{\partial x^4} + \Theta_{11}\frac{\partial^2 u}{\partial x^2} = 0 \tag{11c}$$

The boundary conditions are as follows (Thai and Vo 2012):

$$\delta\psi:\ \Theta_{21}\frac{\partial\psi}{\partial x}-l^2\Theta_{21}\frac{\partial\psi}{\partial x}+\Theta_{23}\frac{\partial^2 w}{\partial x^2}-l^2\Theta_{23}\frac{\partial^4 w}{\partial x^4}=0 \quad (11d)$$

$$\delta w:\ \Theta_{12}\left(\frac{\partial w}{\partial x}+\psi\right)-\Theta_{23}\frac{\partial^2\psi}{\partial x^2}+\vartheta\frac{\partial w}{\partial x}-\Theta_{22}\frac{\partial^3 w}{\partial x^3}+ l^2\Theta_{23}\frac{\partial^4\psi}{\partial x^4}+l^2\Theta_{22}\frac{\partial^5 w}{\partial x^5}-l^2\Theta_{12}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^2\psi}{\partial x^2}\right)-(ea)^2\vartheta\frac{\partial^2 w}{\partial x^2}-(ea)^2\vartheta_{,x}\frac{\partial w}{\partial x}=0 \quad (11e)$$

$$\delta u:\ -l^2\Theta_{11}\frac{\partial^3 u}{\partial x^3}+\Theta_{11}\frac{\partial u}{\partial x}=0 \quad (11f)$$

$$\delta w_{,x}:\ \Theta_{23}\frac{\partial\psi}{\partial x}-l^2\Theta_{22}\frac{\partial^4 w}{\partial x^4}+\Theta_{22}\frac{\partial^2 w}{\partial x^2}-l^2\Theta_{23}\frac{\partial^3\psi}{\partial x^3}=0 \quad (11g)$$

where

$$\begin{pmatrix}\Theta_{11}\\ \Theta_{12}\\ \Theta_{21}\\ \Theta_{22}\\ \Theta_{23}\end{pmatrix}=\int\begin{pmatrix}E\\ GK_S\left({\varsigma_{,y}}^2+{\varsigma_{,z}}^2\right)\\ E\varsigma^2\\ E(\varsigma^2+(r\,sin(\theta))^2-2\varsigma r\,sin(\theta))\\ E\varsigma(\varsigma-r\,sin(\theta))\end{pmatrix}rdrd\theta \quad (12a)$$

In this context, 'K_S' represents the shear correction factor (Wu *et al*. 2024a), while 'ς' is defined as follows.

$$\varsigma=\frac{De-Di}{2\pi}sin\left(\frac{2\pi r\,sin(\theta)}{De-Di}\right) \quad (12b)$$

3. Numerical procedure

The Generalized Differential Quadrature Method (GDQM) is a sophisticated numerical approach that is commonly used to solve partial differential equations (PDEs), notably in the study of the stability and mechanical behavior of small-scale structures, such as functionally graded materials. These materials have non-uniform distributions of material characteristics across their volume, making typical stability analysis approaches less efficient. GDQM has developed as an effective tool for modeling and solving complicated governing equations in such systems (Wang *et al*. 2022, Jia *et al*. 2023, Zhang *et al*. 2023a, b, c).

3.1 Overview of GDQM

GDQM is a numerical method for estimating the derivatives of functions at discrete locations by weighting linear sums of function values at those places. Compared to traditional finite difference techniques, GDQM provides improved accuracy by needing fewer grid points while still producing trustworthy results, making it excellent for solving PDEs regulating small-scale non-uniform structures. In this context, GDQM can effectively address issues combining nonlocal elasticity theories and complicated boundary conditions, which are typical in small-scale structures such as nanowires, thin films, and microbeams (Chen *et al*. 2000, Wang 2015).

3.2 Application of GDQM to small-scale FGMs

The PDEs regulating mechanical behavior (such as bending, buckling, and vibration) in small-scale functionally graded non-uniform structures become extremely nonlinear owing to material gradation and non-uniformity. To account for the effects of tiny dimensions, these structures are often described using high-order shear deformation and nonlocal elasticity theories. GDQM discretizes the governing PDEs by converting them into algebraic equations that are then numerically solved. The benefit of GDQM is its ability to accurately capture the impacts of both material non-uniformity and size-dependency, particularly when working with small-scale objects with complicated behaviors (Shanab *et al*. 2017).

3.3 Stability analysis using GDQM

For stability analysis, such as buckling or post-buckling behavior, GDQM is used to solve differential stability equations obtained from the governing equations of a small-scale structure. These constructions are often subjected to a variety of boundary conditions and external stresses, affecting their stability. Engineers may use GDQM to correctly calculate critical buckling loads and analyze stability margins under various loading circumstances. This is especially relevant for FGMs, because material characteristics vary continually throughout the structure, influencing its stability response (Wei and Qing 2022).

GDQM may also be used to analyze dynamic stability in small-scale non-uniform structures under time-dependent loads or environmental variables. In such circumstances, GDQM may be used with time-stepping techniques to solve time-dependent PDEs, giving information about the structure's static and dynamic stability characteristics.

3.4 Numerical accuracy and efficiency

GDQM's numerical efficiency is one of the advantages for stability analysis. Less grid points and processing resources are needed here than in conventional finite element methods (FEM) or finite difference techniques (FDM). GDQM maintains great accuracy even with fewer discretization points, especially for issues with complicated boundary conditions or non-uniform material distribution. This makes it particularly helpful for studying the stability of small-scale FGMs, where the size and material complexity rapidly raise computing costs (Rafiee *et al*. 2017, Punera and Mukherjee 2022, Zhou *et al*. 2024a). For the solution of PDEs connected to the stability study of small-scale functionally graded non-uniform structures, GDQM presents a strong and effective numerical approach. Its perfect fit for investigating the stability of FGMs is derived from its capacity to faithfully replicate complicated boundary conditions, nonlocal effects, and material inhomogeneity. Engineers and researchers working on the mechanical examination of sophisticated materials at tiny sizes will find great value in the method's numerical precision and efficiency. Using this approach, the issue domain is separated into multiple sub-domains or components, which are then discretized using GDQM. To accomplish so, examine the solution technique listed below. The n^{th} order derivative of '$f(x_i)$' may be defined as follows (Shu 2012):

$$\left.\frac{\partial^n f(x)}{\partial x^n}\right|_{x=x_m}=\sum_{j=1}^{m}\Upsilon_{ij}^{(n)}f(x_i) \quad (13)$$

'm' denotes the grid points number in the tube's length direction, they are computed as follows:

$$x_i = \frac{1}{2}L\left(1 - cos\left(\frac{i-1}{n-1}\pi\right)\right) \tag{14}$$

Furthermore, computed as follows mathematical matrices based on the GDQM is the derivative n^{th}-order function ($\frac{\partial^n}{\partial x^n}$).

$$\Upsilon_{ij}^{(n)} = n\left(\frac{\Upsilon_{ij}^{(1)}}{\Upsilon_{ij}^{-(1-n)}} + (x_j - x_i)\Upsilon_{ij}^{(n-1)}\right) \tag{15}$$

where

$$\Upsilon_{ij}^{(1)} = (x_i - x_j)\frac{\Phi(x_i)}{\Phi(x_j)} \tag{16}$$

In which

$$\Phi(x_i) = \prod_{j=1, j\neq i}^{n}(x_i - x_j) \tag{17}$$

The following mathematical connection is supposed to be true for the buckling analysis, as shown by the eigenvalue analysis which was performed.

$$\vartheta = \frac{\begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & K_{23} \\ 0 & K_{32} & K_{33} \end{bmatrix}}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}}\begin{Bmatrix} u \\ w \\ \psi \end{Bmatrix} \tag{18}$$

where

$$K_{11} = \Theta_{11}\left(-l^2\sum_{j=1}^{n}\Upsilon_{ij}^{(4)} + \sum_{j=1}^{n}\Upsilon_{ij}^{(2)}\right) \tag{19a}$$

$$\begin{aligned} K_{22} &= 3(ea)^2\vartheta_{,xx}\sum_{j=1}^{n}\Upsilon_{ij}^{(2)} - \vartheta\sum_{j=1}^{n}\Upsilon_{ij}^{(2)} \\ &+(ea)^2\vartheta\sum_{j=1}^{n}\Upsilon_{ij}^{(4)} + (ea)^2\vartheta_{,xxx}\sum_{j=1}^{n}\Upsilon_{ij}^{(1)} + \Theta_{22}\sum_{j=1}^{n}\Upsilon_{ij}^{(4)} \\ &+3(ea)^2\vartheta_{,x}\sum_{j=1}^{n}\Upsilon_{ij}^{(3)} - l^2\Theta_{12}\sum_{j=1}^{n}\Upsilon_{ij}^{(4)} - l^2\Theta_{22}\sum_{j=1}^{n}\Upsilon_{ij}^{(6)} \\ &-\vartheta_{,x}\sum_{j=1}^{n}\Upsilon_{ij}^{(1)} - \Theta_{12}\sum_{j=1}^{n}\Upsilon_{ij}^{(2)} \end{aligned} \tag{19b}$$

$$\begin{aligned} K_{23} &= l^2\Theta_{12}\sum_{j=1}^{n}\Upsilon_{ij}^{(3)} - \Theta_{12}\sum_{j=1}^{n}\Upsilon_{ij}^{(1)} - l^2\Theta_{23}\sum_{j=1}^{n}\Upsilon_{ij}^{(5)} \\ &+ \Theta_{23}\sum_{j=1}^{n}\Upsilon_{ij}^{(3)} \end{aligned} \tag{19c}$$

$$\begin{aligned} K_{32} &= \Theta_{23}\sum_{j=1}^{n}\Upsilon_{ij}^{(3)} - \Theta_{12}\sum_{j=1}^{n}\Upsilon_{ij}^{(1)} + l^2\Theta_{12}\sum_{j=1}^{n}\Upsilon_{ij}^{(3)}\left(\frac{\partial^2\psi}{\partial x^2}\right) \\ &- l^2\Theta_{23}\sum_{j=1}^{n}\Upsilon_{ij}^{(5)} \end{aligned} \tag{19d}$$

$$K_{33} = l^2\Theta_{12}\sum_{j=1}^{n}\Upsilon_{ij}^{(2)} - \Theta_{12} - l^2\Theta_{21}\sum_{j=1}^{n}\Upsilon_{ij}^{(4)} + \Theta_{21}\sum_{j=1}^{n}\Upsilon_{ij}^{(2)} \tag{19e}$$

$$M_{22} = -(ea)^2\sum_{j=1}^{n}\Upsilon_{ij}^{(4)} + \sum_{j=1}^{n}\Upsilon_{ij}^{(2)} \tag{19f}$$

To ensure accurate analysis, the boundary conditions must be included into the reformulated governing equations using the GDQM. By doing so, we may get the eigenvalues of Eq. (18), which provide vital information on the critical buckling load for our present study.

4. Neural network training and stability prediction

The neural network method is used to make it easier to guess the stable factors of complicated buildings with different shapes and amounts of materials. The system can guess the limits of stability under different loads and material arrangements by training the neural network with data from numerical simulations (Liang *et al*. 2024).

4.1 Steps in neural network training:

4.1.1 Collecting data:

Numerical models are run using the Generalized Differential Quadrature Method (GDQM) for a number of different non-uniform cylinder shapes, taking into account differences in size, material distribution, and boundary conditions. It is the results of these simulations—like the key bending loads and displacement patterns—that are used to train new people.

4.1.2 Picking out features:

Axis length, radial material distribution, physical parameters (like taper angle for cone-shaped structures), and size-dependent effects (recorded by nonlocal parameters) are some of the most important factors that affect stability. These things are used as factors that the neural network can read.

4.1.3 Train:

The neural network is taught with supervised learning, and the output variables are the critical bending load and the deformation patterns. During training, the network changes its weights to reduce the amount of wrong predictions. This is usually done with backpropagation.

4.1.4 Validation and testing:

After being taught, the network is checked for accuracy and usefulness on new data. This step makes sure that the network can accurately guess how stability will work in a lot of different structure configurations.

4.2 Stability management using neural networks

The neural network removes the need for time-consuming numerical calculations after it has been trained as it can rapidly evaluate the stability of non-uniform cylindrical

Table 1 The mathematical form of the CNT volume fraction distribution configuration

	$L = 10\frac{De_0 - Di}{2}$	$L = 20\frac{De_0 - Di}{2}$	$L = 100\frac{De_0 - Di}{2}$
First-order shear deformation theory, Wang *et al*. (2024a)	9.5145	9.6903	9.7677
High-order tube theory, Zhang *et al*. (2023a)	9.6106	9.7882	9.8664
Sinusoidal shear deformation beam theory, present study	9.45332154	9.65123214	9.75123648

constructions. This is so as the neural network can assess the structural stability fast. Using this strategy may help one get many advantages:

4.2.1 Real-time stability monitoring:

Combining the neural network with engineering tools would allow one to create real-time forecasts on construction stability while under design and monitoring. This real-time observation has the ability to provide vital understanding of the structural behavior under different conditions, thereby allowing proactive changes and improvements to be done.

4.2.2 Optimization:

The neural network might be very important in optimizing the shape of the construction and the distribution of materials to get perfect stability while reducing the weight or material consumption. This optimization is highly helpful in complex aeronautical and civil engineering applications where the efficient use of materials and the reduction of weight are major components in design and performance. Engineers may develop products with more stability that concurrently reduce the needed weight and material consumption, therefore enhancing general efficiency and performance. This is made feasible by taking use of neural network capabilities.

The paper presents an advanced paradigm for investigating the stability of non-uniform cylindrical structures that combines classical beam theory, size-dependent effects, and functionally graded material distributions. The Generalized Differential Quadrature Method (GDQM) assures that complicated partial differential equations resulting from these setups may be solved effectively. Furthermore, the use of neural networks offers a strong tool for maintaining and forecasting stability, enabling engineers to swiftly and precisely experiment with different design configurations. This hybrid technique improves understanding and practical application of stability analysis in advanced engineered structures using functionally graded materials (Qi *et al*. 2024).

5. Results and discussion

The findings of this research demonstrate significant patterns and behaviors in the buckling and stability analysis of the shear deformation beam under varied loading circumstances. By including nonlocal strain gradient theory, the results highlight the importance of both nonlocal effects and strain gradients in effectively forecasting beam response, particularly at smaller scales. The observed disparities between conventional and nonlocal models show that disregarding these elements might result in underestimating of critical buckling loads and deflection patterns. These results not only confirm the theoretical model used, but also emphasize the need of factoring in size-dependent effects in nano-structural stability studies.

5.1 Validation of numerical results by GDQM

The data in Table 2 compare the buckling load predictions from this study with those of Wang *et al*. (2024a) and Zhang *et al*. (2023a), utilizing various theoretical approaches. The buckling load $(\frac{1}{2}\pi\vartheta L^2/\Theta_{11})$ values exhibit a reasonable level of agreement across the various aspect ratios $(L/\left(\frac{De_0}{2} - \frac{Di}{2}\right))$ considered, albeit with some variations. The observed differences indicate the impact of the distinct assumptions and formulations inherent in each theoretical approach. The comparison indicates that the method employed in this study produces results that align with established theories, thereby affirming the model’s reliability for stability analysis.

5.2 Artificial neural network (ANN) training and validation

5.2.1 Overview of ANN Implementation

The use of an artificial neural network (ANN) model aiming at predicting and evaluating the stability properties of non-uniform cylindrical constructions is shown in this part. Training for the artificial neural network (ANN) used a dataset generated from numerical findings obtained by the Generalized Differential Quadrature Method (GDQM). Using this ANN model primarily aims to provide a precise prediction tool able to estimate buckling loads and assess structural stability under many boundary conditions and loading scenarios.

5.2.2 Structure of the Neural Network

Axial length, material distribution, and size-dependent variables derived from the governing partial differential equations fit in an input layer of the artificial neural network (ANN). The output layer generates forecasts on several stability performance metrics and important buckling loads. To improve the learning efficiency of the network, many hidden layers with the suitable number of neurons are integrated between the input and output layers. By using a backpropagation technique, the training process helps the model to change its weights therefore lowering prediction errors. A mean squared error (MSE) loss function helps to track these mistakes.

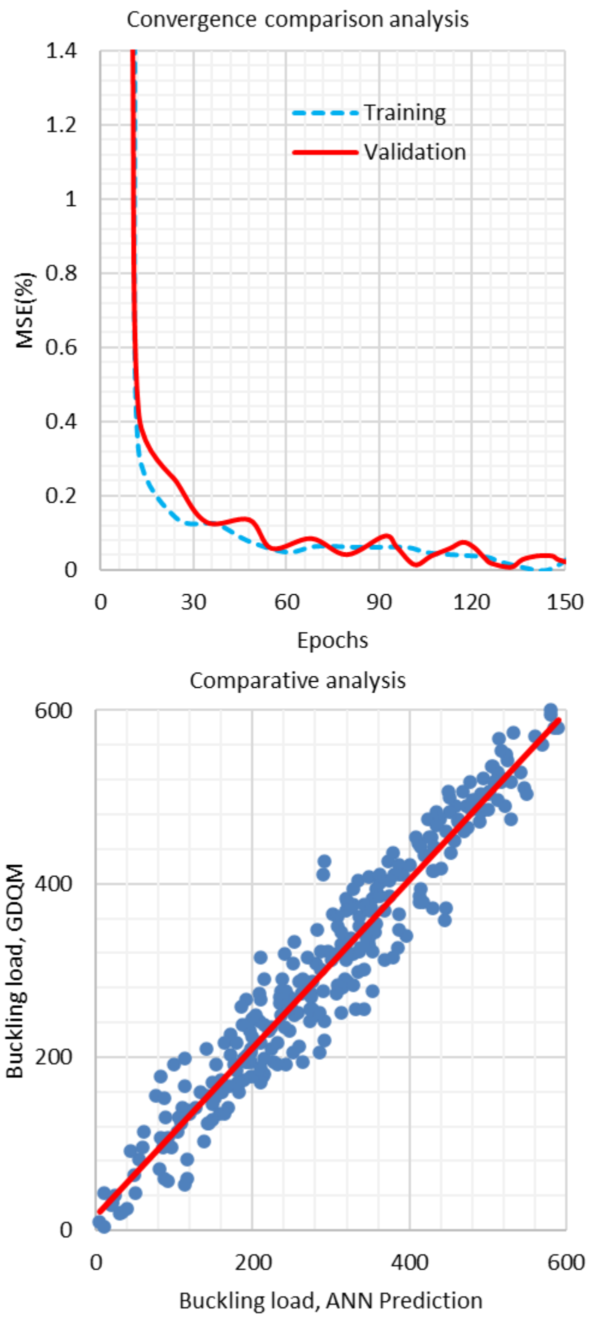

Fig. 2 Training, convergence, and comparative analysis between presented GDQM and artificial neural network

5.2.3 Incorporating particle swarm optimization (PSO)

Particle Swarm Optimization (PSO) has been used as an optimization approach to increase the performance of ANNs. PSO is an algorithm inspired by natural occurrences, notably the social behaviors of flocking birds and schooling fish. Each particle in the swarm represents a possible solution, and they explore the issue space by adjusting their placements in response to individual and collective experiences. Within the context of an artificial neural network (ANN), particle swarm optimization (PSO) is used to improve the network weights, boosting the model's ability to minimize error and achieve quicker convergence. PSO improves the efficiency of the training process,

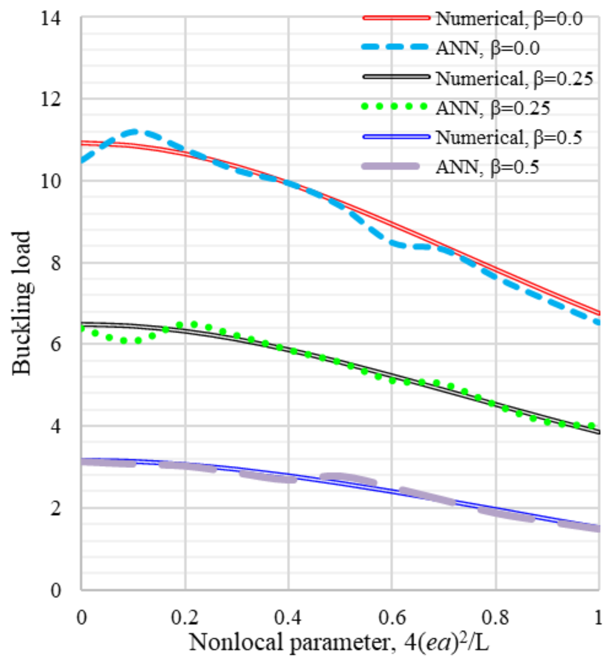

Fig. 3 Buckling load ($\pi\vartheta L^2/\int E_{Nickle}\, r^2 \sin^2(\theta)\, dA$) Simply-supported FGM tube versus the nonlocal parameter (ea) and various rate of section change (β) for both ANN and GDQM, $l^2 = \frac{L}{500}$, η=1

allowing the ANN to locate optimum solutions more efficiently than traditional techniques.

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5.2.4 Procedure of Training and Validation

Seventy percent of the dataset was used for training, thirty percent was put aside for testing and validation. The training procedure continued until the network reached convergence, therefore preserving the error rate within reasonable limits during validation. After that, the results of the ANN model were examined with respect to the GDQM solution and showed a noteworthy relationship. The efficiency of the ANN in faithfully forecasting the stability behavior of non-uniform cylindrical structures was proven by the comparison.

The convergence comparison analysis for the ANN's training and validation stages is shown in Fig. 2, along with a comparison of the ANN's trained results with the numerical findings from GDQM. A significant point was found when the training and validation curves started to diverge, suggesting that overfitting may have occurred, in order to assess the neural network's accuracy. This variance serves as a crucial indicator of the correctness and efficaciousness of the ANN's generalization to new data. The results unequivocally reveal the remarkable accuracy of the ANN and indicate that it can almost equal the accuracy of GDQM numerical solutions. The capacity of the ANN to provide accurate predictions with a large reduction in time and computational effort is one of its most notable advantages, especially after undergoing extensive training. This demonstrates the ANN's effectiveness and establishes it as a strong contender for replacing conventional numerical approaches, particularly when it comes to buckling load prediction. It is noted Mean Squared Error (MSE) is calculated as follows:

$$MSE = \frac{1}{X}\sum_{i=1}^{X}\left(\vartheta_i^{ANN} - \vartheta_i^{GDQM}\right)^2 \tag{20}$$

'X' represents the quantity of numerical datasets utilized for training, 'ϑ^{ANN}' denotes the buckling load forecasted by the Artificial Neural Network, and 'ϑ^{GDQM}' indicates the numerical buckling load derived from the Generalized Differential Quadrature Method.

5.3 Presentation of results

Fig. 3 shows the buckling loads calculated using the Generalized Differential Quadrature Method (GDQM) and the Artificial Neural Network (ANN) for both uniform and non-uniform sections. In the uniform portion, the outputs from GDQM and ANN are almost identical, providing parallel values for buckling loads as the nonlocal parameter, '*ea*', varies. As '*ea*' increases from 0 to 1.0, both techniques provide a constant decrease in buckling load. This significant connection demonstrates the ANN's accuracy in forecasting buckling loads, with small error margins. The non-uniform section with β = 0.25 has lower buckling load values compared to the uniform counterpart, indicating that the geometric arrangement has a major influence on the structure's stability. Increasing β to 0.5 results in the lowest buckling loads across all scenarios. Specifically, the GDQM predicts a buckling load as low as 1.50 at '*ea*' = 1.0, whereas the ANN predicts a close number of about 1.49. This enhances ANN's capacity to handle complex geometric transformations, making it extremely competent in estimating key buckling loads even in more demanding conditions.

The ANN typically outperforms the GDQM in terms of accuracy, with just slight exceptions, particularly at high nonlocal parameter values. However, these differences are minor and have no significant impact on the total forecasts. Furthermore, the ANN's capacity to provide quick and accurate predictions with modest computing demands highlights its efficiency and applicability for large-scale structural assessments. The results show that when it comes to predicting the stability of cylindrical constructions, ANN provides a competitive option to standard numerical approaches such as GDQM. The strong alignment between the two approaches, notably in the uniform and substantially non-uniform portions, demonstrates ANN's ability to accurately predict complicated stability behaviors under a variety of situations. Its agility and computational efficiency make it useful in current engineering processes, providing a dynamic, dependable instrument for assessing structural stability.

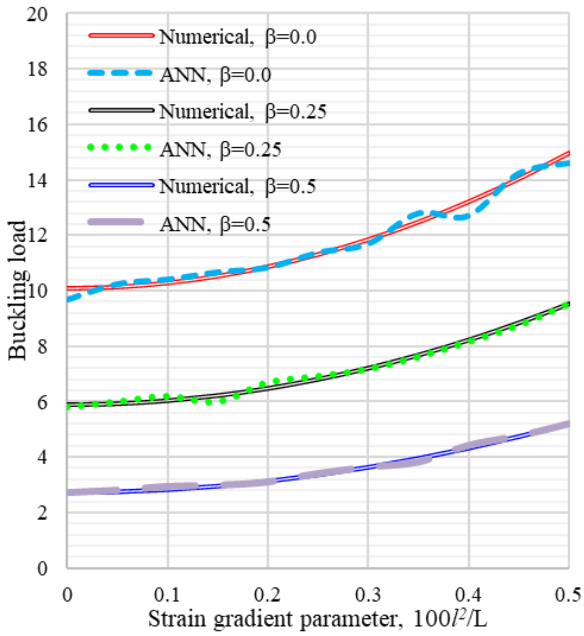

Fig. 4 Buckling load ($\pi\vartheta L^2/\int E_{Nickle}\, r^2 \sin^2(\theta)\, dA$) Simply-supported FGM nano beam versus the strain gradient parameter (l^2) and various rate of section change (β) for both ANN and GDQM, $(ea)^2 = \frac{L}{40}$, η=1

In Fig. 4, buckling loads are calculated for different strain gradient parameters (l^2) and β. Results from the GDQM and ANN are compared for uniform and non-uniform sections (β = 0, 0.25, and 0.5). For the uniform section (β = 0), both the GDQM and ANN findings reveal a continuous increase in buckling loads as the strain gradient parameter (l^2) climb. This happens because the strain gradient parameter improves beam stability by boosting the hardening effect. The tight alignment of the techniques implies that the ANN is accurate in forecasting buckling stresses for uniform sections, with little departure from numerical values. Buckling loads in non-uniform sections are considerably lower than in uniform sections, indicating that variations in section geometry have an influence on structural stability. As the strain gradient parameter (l^2) grows, both GDQM and ANN indicate a rise in buckling loads. This demonstrates the ANN's capacity to properly capture the impacts of nonlinearity and strain gradient factors on structural stability. The non-uniform segment with β = 0.5 exhibits the lowest buckling stresses, indicating its complicated stability characteristics. These results show that the ANN is precise in handling complicated, non-uniform sections.

When comparing the two approaches, the ANN outperforms the other in all circumstances, including

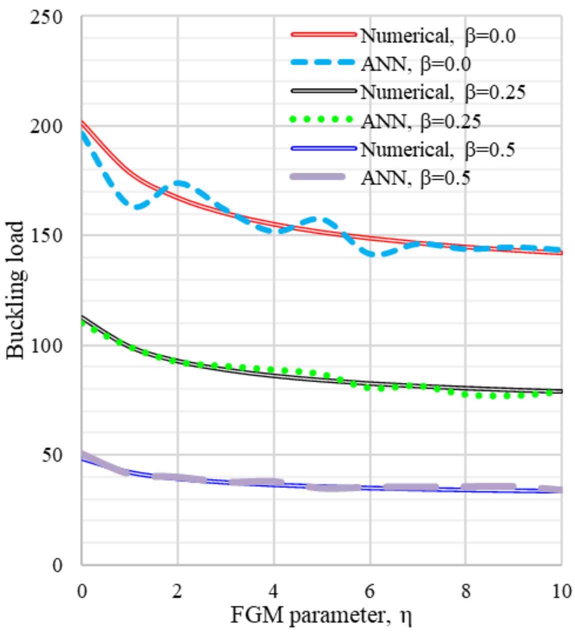

Fig. 5 Comparison of Buckling Loads ($\pi\vartheta L^2/\int E_{Nickle}\, r^2 \sin^2(\theta)\, dA$) for Uniform and Non-Uniform Sections with Varying FGM Parameter (η) Using GDQM and ANN $(ea)^2 = \frac{L}{40}$, $l^2 = \frac{L}{1000}$

uniform and non-uniform sections. As 'l^2' grows, modest variations arise, particularly in non-uniform portions, although these changes are minimal and within acceptable limits. The ANN not only performs quickly but also with consistent accuracy, making it a great tool for forecasting the stability of cylindrical structures. The findings demonstrate that the ANN is a reliable and efficient alternative to existing numerical approaches such as GDQM for calculating buckling loads in cylindrical structures. The high correlation between the two techniques, notably for uniform and substantially non-uniform sections, demonstrates the ANN's capacity to mimic complicated stability behavior with different strain gradient parameters. The ANN's ability to provide accurate forecasts while significantly lowering processing time makes it a potential option for stability analysis in current engineering applications.

Fig. 5 illustrates the projected buckling loads for uniform and non-uniform cylindrical sections using both the GDQM and ANN Values are shown for three configurations: uniform section (β = 0), non-uniform section with β = 0.25, and non-uniform section with β = 0.5. Results for varying FGM parameter η (from 0 to 10) show how material gradation affects structural stability. The FGM parameter (η) affects the stiffness of the beam by controlling the material gradation between stiffer and softer components. As η rises, the fraction of stronger material (usually Aluminum Oxide) in the functionally graded material drops, resulting in a reduction in beam stiffness. The GDQM and ANN findings demonstrate that lower η values make the beam more rigid, allowing it to bear higher loads before buckling occurs. The reduction in buckling loads with increasing η is similar across regular and non-uniform sections, highlighting the importance of material composition in stability performance. The section change rate (β) is important for the structural stability of non-uniform beams. As β grows, the beam's shape becomes more varied, and the effective thickness decreases. This results in a "softening" effect, in which the stiffness of the beam is lessened as the sections shrink. Higher β values lead to lower buckling loads, since the beam's resistance to deformation decreases under load. The softening effect is most noticeable when comparing uniform parts (β = 0) to non-uniform sections with higher β values. Uniform beams have the maximum buckling loads owing to their continuous thickness and structural integrity. Non-uniform beams, particularly those with higher β values, have much lower stability. Buckling loads in non-uniform sections decrease as section variability rises, indicating that beams with larger β values are more likely to buckle under lower loads. The GDQM and ANN approaches provide a consistent knowledge of how changes in η and β impact the stability of cylindrical structures. The ANN properly predicts the decrease in buckling loads with β and η, which is consistent with the GDQM findings. The minor differences between the two methodologies have little impact on the overall results concerning the link between material composition, section shape, and structural stability.

The investigation indicates that the FGM parameter η and section change rate β significantly impact the buckling behavior of cylindrical constructions. Increasing β and η diminishes stability by softening the beam, reducing its effective thickness and structural stiffness. Both GDQM and ANN show the same underlying tendencies, with ANN providing a faster and more reliable option for forecasting stability in complicated systems. This makes the ANN an important tool for engineering applications in which the balance of material qualities and geometry is vital in structural design.

Fig. 6 demonstrates the anticipated buckling loads for various FGM parameter (η) values for fully simply-supported, clamped-simply supported, and fully clamped situations. The fully clamped state forecasts the greatest buckling loads, while the fully simply supported condition predicts the lowest buckling loads and has the worst stability performance. The findings from both techniques show a similar pattern, demonstrating that the ANN can properly predict this boundary condition. Increasing η reduces beam stiffness, leading to a decrease in buckling stresses. As η increases, the functionally graded material contains more Nickel and less Aluminum Oxide. Nickel's lesser rigidity relative to Aluminum Oxide results in a less stiff beam and lower critical buckling stresses with increasing η. Buckling loads in the clamped-simply supported example are greater than in the fully simply supported case, indicating that clamping one end provides extra rigidity. The maximum buckling loads are recorded in the completely clamped situation owing to the structure's enhanced stiffness. While the ANN sometimes overestimates buckling loads, it consistently captures the general trend.

The fully clamped state forecasts larger buckling stresses due to the increased structural stiffness supplied by

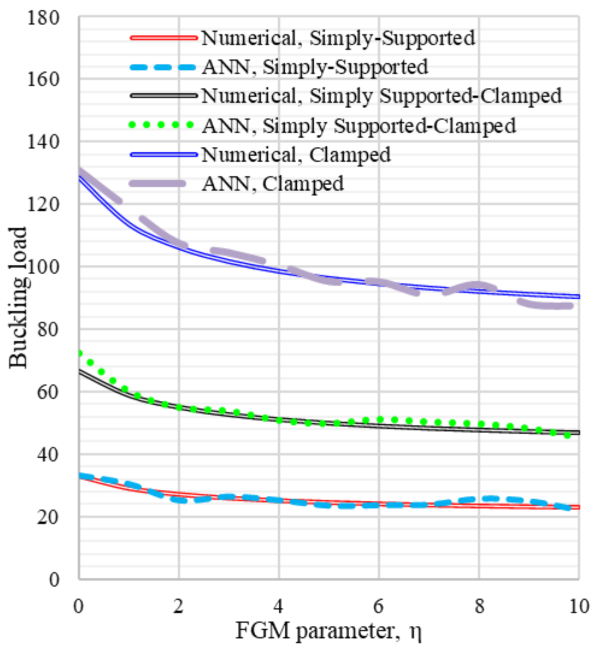

Fig. 6 Buckling Load ($\pi\vartheta L^2/\int \mathrm{E}_{Nickle}\, \mathrm{r}^2\sin^2(\theta)\, \mathrm{dA}$) Predictions for Different Boundary Conditions with Varying FGM Parameter (η) Using GDQM and ANN, $(ea)^2 = \frac{L}{40}$, $l^2 = \frac{L}{1000}$, β=0.2

clamping both ends of the beam. Clamping limits translational and rotational motions at the ends, resulting in a more rigid boundary condition. This rigidity considerably improves the beam's stiffness and resistance to deformation under compression loads, enabling it to sustain higher forces before buckling. The fully clamped boundary condition increases stability by severely restricting the beam's motions, resulting in larger critical buckling stresses. In contrast, the fully simply-supported condition places the beam under little limitation since it is solely supported at both ends and has no rotating constraint. This results in decreased stiffness and structural resistance to buckling. The simply supported beam may spin at its ends, making it more susceptible to deformation under load. As a consequence, the beam achieves its buckling point at substantially lower stresses than the clamped examples. The decreased stiffness and absence of rotational constraint in the simply supported state are directly related to lower anticipated buckling loads and poor stability performance. The ANN and GDQM findings clearly show a difference in stability, with both approaches demonstrating that boundary conditions play an important role in determining a structure's capacity to withstand buckling. Fully clamped beams have the most stability owing to their rigid support, while merely supported beams have the lowest stability and resistance to buckling. The ANN correctly predicts these patterns, demonstrating its ability to capture complicated structural processes.

The ANN is effective in predicting buckling loads under all boundary conditions. The ANN successfully catches overall patterns, despite modest changes in individual values, especially at lower η values. The ANN's computational efficiency, paired with its near approximation to GDQM findings, makes it an effective tool for studying stability in cylindrical structures. The findings show that the ANN is a dependable and efficient alternative to classic numerical approaches like GDQM for estimating buckling loads under completely simply-supported, clamped-simply supported, and fully clamped circumstances. The ANN accurately predicts GDQM outcomes, especially for larger values of the FGM parameter η. The minimal discrepancies found are within acceptable bounds, proving the ANN's suitability for use in engineering applications requiring speedy and accurate predictions.

6. Conclusions

This paper investigated the stability of small-scale non-uniform cylindrical constructions by use of neural network approaches in conjunction with numerical methods. With specific focus on geometric design and material distribution, the buckling loads of uniform and non-uniform cylindrical sections were predicted using the Generalized Differential Quadrature Method (GDQM) and Artificial Neural Networks (ANN). Particularly for functionally graded materials (FGM) under different boundary conditions, the research showed that both techniques produced consistent and accurate forecasts of buckling loads.

With only slight differences that had no appreciable effect on the general accuracy of the predictions, the study found that the ANN very faithfully reflected the findings acquired using GDQM. Capture of the effects of the nonlocal parameter, section change rate (β), and FGM parameter (η) on the stability of cylindrical constructions was very successful for the ANN. Increasing the nonlocal parameter (*ea*), FGM parameter, and β produced a decrease in buckling loads as predicted, hence underlining the negative influence of stiffness and geometric non-uniformity on structural stability. On the other hand, raising the Strain gradient parameter (l^2) increased stability because the structural rigidity and buckling resistance were strengthened by the change toward stiffer constructions.

Furthermore, the investigation verified that the stability performance of the cylindrical constructions was much impacted by boundary circumstances. The maximum buckling loads were from totally clamped situations, the lowest from entirely simple-supported ones. By effectively modeling the impact of boundary conditions on structural stability, the ANN proved able to forecast these patterns. For general buckling load prediction of cylindrical constructions, the ANN turned out to be a rather dependable and effective instrument. Its precision combined with low computing requirements makes it a reasonable substitute for conventional numerical approaches such GDQM. The findings of this work provide important new perspectives on the stability management of small-scale non-uniform structures, thereby laying a basis for next developments in nanoscale material and structural design. Future research might investigate the use of this method to other forms of non-uniform geometries and materials, therefore increasing the usefulness of this methodology in many other engineering disciplines.

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