

Closed form interaction for safety assessment of DWCNTs: Mechanical vibration

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(Received March 1, 2022, Revised August 31, 2024, Accepted September 5, 2024)

Abstract. Here, vibration of double walled carbon nanotubes is evaluated using Euler-Bernoulli beam model. These tubes are placed on Winkler elastic foundation. A simple Galerkin's approach is presented to solve the tube governing equations and for extracting of vibration eigen-frequencies of double walled carbon nanotubes. The procedure is easy for computer programming with various combinations of boundary conditions. The frequency influence is observed with different parameters. Effects of Winkler foundation versus frequencies with varying lengths is examined for a number of boundary conditions. It is noticed that the frequencies are lower for higher length on increasing the Winkler foundation. The frequencies of clamped-clamped are higher than that of clamped simply supported end condition. The obtained results are compared with some experimental ones.

Keywords: boundary conditions; computer programming; double walled carbon nanotubes; Winkler elastic foundation

1. Introduction

While designing these tubes, it is important to know their resonant frequencies because fatigue could be caused by excessive vibrations. This problem is generated while investigating vibrations of waves produced in water, noise and fluid flow in pipes. These tubes are designed and structured to meet particular applications and properly amended to study their vibratory response. Free vibration analysis of carbon nanotubes has a wide range of research study in mechanical field. Vibrational characteristics of various nano-structures are widely investigated based on nonlocal beam model.

The nonlocal theory mostly focused on the free vibrational analysis of the nano-structure, especially, carbon nanotubes. Furthermore, one significant device type with several applications in a range of scientific and technical domains is nanostructures (Tserpes and Papanikos 2005, Soltani *et al.* 2012). The nonlinear forced vibration of carbon nanotubes has seldom been observed (Das *et al.* 2013, Bocko and Lengvarský 2014, Reddy and Pang 2008). However, because induced nonlinear vibration carbon nanotubes are widely used in many useful devices, this issue is quite important. The triply periodic minimum

surface (TPMS) was published by Li *et al.* (2024) due to its remarkable mechanical qualities and mathematically adjustable geometric topology. Notable characteristics of this structure include high porosity, stiffness, specific strength, and energy absorption capacity. This means that in order to examine the nano-size structure, a new model is needed. The higher order elasticity theories were examined by a few researchers (Demir and Civalek 2016, Yayli 2013, Narendar and Gopalakrishnan 2011, Civalek *et al.* 2009, Murmu and Pradhan 2009). The researcher has focused on many non-classical elasticity theories, such as nonlocal theory (Fleck and Hutchinson 1993, Eringen and Edelen 1972) and stress and strain theories (Mindlin and Tiersten 1962, Toupin 1964). For the interpretation of vibrational influence of single-walled carbon nanotubes (SWCNTs), the nonlocal elasticity theory and Timoshenko beam model are used (Mindlin and Tiersten 1962, Toupin 1964, Fleck and Hutchinson 1993). The electro-hydraulic-driven fluid-power transmission and control technology, invented by Yuan *et al.* (2024), may greatly increase the stabilization platform stiffness, load-carrying capacity, and control precision. Using a nonlocal parameter, Chawis *et al.* (2013) developed a nonlocal theory with scale length to conduct vibration of SWCNTs with Euler beam theory. The outcomes of FEM are contrasted with those of classical solutions. They stated that as length, diameter, and atomic arrangement increased, distinct variations in frequency were noticed. Additionally, a novel frequency pattern was noticed when the nonlocal parameter was increased. Li *et al.* (2024) analysed of SPM machines with respect for magnet shaping

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approach, but because of its division and superposition theory-based mechanism. Wang *et al.* (2013) investigated the nonlinear vibration of embedded SWCNTs under harmonic stresses using nonlocal shell theory. Furthermore, for the first time, the dynamic response of material lengths, elastic constants, and nonlocal factors is shown. Numerous influences have been clearly shown to affect SWCNT behavior. Zhang and Ma (2024) research indicates that the excessive vibrations resulting from intrinsic internal friction in spline joints have a substantial impact on the dependability and security of high-speed rotating machinery. By using FEM to study the vibration of SWCNTs, Swain *et al.* (2013) discovered good agreement in the literature. Three SWCNT categories' vibrations are shown, with the frequency of the vibrations being found to be significant. The natural frequency falls as the length-to-diameter ratio rises. Because of the chiral index, this study has a significant impact on the vibration of nanotubes. Kong *et al.* (2021) investigated the dynamic response of ballastless track X-section cast-in-place concrete (XCC) pile-raft foundations under different axle loads.

Moghadam *et al.* (2014) used a molecular mechanics approach using a range of parameters to examine the frequency of SWCNTs. The frequency results showed that the frequencies varied according to the orientation of the carbon-carbon bond. The adaptive Active Noise Cancellation (ANC) technique was studied by Liu *et al.* (2023) in order to solve the problem of wheel-track contact noise interfering with the bearing sound signal in the TADS. Murmu and Pradhan (2009) used nonlocal small scale effects to study the vibrational frequencies with various modes accompanying temperature change. Conversely, the nonlocal frequencies were significantly reduced for the length scale coefficient and the soft elastic medium containing embedded carbon nanotubes. It was discovered that the nonlocal model's frequencies at various temperature stages are higher than those of the nonlocal model at the same temperature. With various boundary conditions, the vibration frequencies of the zigzag SWCNTs (5, 0), (8, 0), (9, 0), and (11, 0) were taken into consideration and numerically computed using MD simulation. A thorough analysis was conducted on the impact of nonlocal parameters on both height and radius. The nonlocal theories for the in-extensional vibration of SWCNTs were investigated by Das *et al.* (2013). Using the circumferential wave number and positive strain gradient theory, the in-extensional mode frequency was addressed. According to Du *et al.* (2024), there are new difficulties in deicing railway contact wire due to the frequent occurrence of extremely cold weather. Nowadays, the majority of the time, breaking off the frozen ice still requires people to stand on the train. Ansari and Arash (2013) used the differential quadrature method (DQM) to study the vibrations of DWCNTs. A thorough investigation is conducted into the mechanical behavior of DWCNTs with geometrical parameters, layer-wise boundary conditions, and small scale factors. Li *et al.* (2024) examined hybrid reluctance actuators, which are superior than piezoelectric stack or voice coil actuators due to their exceptional motor constant and bidirectional noncontact force. According to

Bocko and Lengvarký (2014), the nonlocal theory computed the fundamental natural frequency (FNF) with two separate diameters and a continuously changing length. They have demonstrated that nonlocal characteristics have a significant impact on a carbon nanotube's bending vibration. It was demonstrated that boundary conditions containing nonlocal parameters had a greater effect on the vibration of nanotubes. Furthermore, it has been noted that as the length of the CNT increases, the frequencies fall as the nonlocal parameter increases. According to Zhang *et al.* (2023) research, nanoparticles stand out from traditional materials due to their exceptional physical, chemical, and electrical capabilities. Zhang (2023) looked into the various control systems that engineers and researchers from around the world have developed and used. Soltani (2016) used Karman's geometric non-linearity theory and the theory of nonlocal elasticity to study the nonlinear vibrational properties of SWCNTs. Galerkin's method was used to transform partial differential equations into differential equations, and the governing equation was obtained using Donnell's shell theory. Aspect ratios, nonlocal, nonlinear, and circumferential characteristics were also examined for their potential effects. Recently, research on vibration of SWCNTs has been done by many material researchers (Wang *et al.* 2007, Fatahi-Vajari *et al.* 2019, Gupta *et al.* 2010, Swain *et al.* 2013). Recently some researcher used different methods for nonlinear modeling (Eltaher *et al.* 2019, Banoqitah *et al.* 2022, Ebrahimi *et al.* 2019, Safaei *et al.* 2019, Hussain and Naeem 2019, Shahsavari *et al.* 2019, Benmansour *et al.* 2019, Akbaş, 2016a, b, 2017a, b, Fattahi *et al.* 2021, Zhang *et al.* 2021, Salmai *et al.* 2021, Timesil, 2021, Asghar *et al.* 2020, Khadimallah *et al.* 2020a, b, Hussain 2022, Qazaq *et al.* 2022, Muzamal 2022, Arshad *et al.* 2024, Hussain *et al.* 2024, Hussain *et al.* 2020a, b).

The current problem is solved using Galerkin's approach, and the frequency equation for the vibration of double-walled carbon nanotubes is obtained in the form of an eigenvalue. The governing formula for the vibrations of carbon nanotubes with two walls has been derived, taking into account boundary conditions that are both clamped-clamped and clamped-free. MATLAB software is utilized for programming in order to examine the vibration properties of tubes. The outcomes are displayed in tabular form. On the Winkler base, the tube is positioned with various boundary conditions. For two distinct lengths, the natural frequency increases as the Winkler foundation grows. By contrasting them with other ones discovered in the literature, their validity is confirmed.

2. Governing equations of CNTs

The Governing differential equation for vibration of carbon nanotubes is modeled as Euler-Bernoulli beam (Elishakoff and Pentaras 2009).

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = p \quad (1)$$

where x , t denoted the space and time variable. $w(x, t)$, A represents for deflection of CNTs and area of cross section,

respectively. E, ρ and I are the young's modulus, mass density and moment of inertia of CNTs, respectively. When thickness of outer and inner tubes is supposed to be constant then the Eq. 1 can be enhanced to outer and inner nanotube layers of double walled carbon nanotubes. The transverse force applied on the carbon nanotubes is denoted by p . The pressure on the outer and inner tubes is due to Vander Waals (vdW) forces. The deflection of interlayer interactions between the double-walled carbon nanotube tubes is taken into account by the suggested vdW model.

So the coupled equation is written with the interaction of outer and inner layers in the form of PDE, which is called governing equation of double walled carbon nanotubes (DWCNTs).

$$\frac{p_1}{\rho A_1} = \frac{EI_1}{\rho A_1} \frac{\partial^4 w_1}{\partial x^4} + \frac{\partial^2 w_1}{\partial t^2} \tag{2}$$

$$\frac{p_2}{\rho A_2} = \frac{EI_2}{\rho A_2} \frac{\partial^4 w_2}{\partial x^4} + \frac{\partial^2 w_2}{\partial t^2} \tag{3}$$

The pressure at any point between the nested tubes is a linear function and can be written as the difference of the deflection at prescribed point.

$$p_1 = c(w_2 - w_1) \tag{4a}$$

$$p_2 = -c(w_2 - w_1) \tag{4b}$$

The subscript 1 and 2 are for the designation of outer and inner tubes. The interaction coefficient between the outer and inner layers of nanotubes is called vdW coefficient and denoted by term c .

The surrounding elastic medium with Winkler model is applied to control the pressure of outer layer of nanotube.

$$p_2 = p_w - c(w_2 - w_1) \text{ where } p_w = -Kw_2 \tag{5}$$

The spring constant is denoted by K and negative sign shows that indicate that p_w is in the direction opposite to the deflection of nanotubes.

He *et al.* (2006) presented the energy potential through Vander Waals (vdW) forces is given as

$$c = \frac{\pi \epsilon R_1 R_2 \sigma^6}{a^4} \left[\frac{1001 \sigma^6}{3} H^{13} - \frac{1120}{9} H^7 \right] \tag{6}$$

where H^m be as elliptic integral which is given as

$$H^m = (R_1 + R_2)^{-m} \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1 - K \cos^2 \theta)^{\frac{m}{2}}} \tag{7}$$

$$(m = 7, 13)$$

and

$$K = \frac{4R_1 R_2}{(R_1 + R_2)^2} \tag{8}$$

Here C-C bond length is given by $a = 0.142 \text{ nm}$, R_1, R_2 as radius of inner and outer tubes. The depth of potential is denoted by ϵ , is so called Lennard-Jones potential and σ as parameter concluded by equilibrium distance.

A physical problem is mathematically framed by considering the expressions for quantities which are

involved in a system. Usually the unknown functions are expressed in differential equations. These functions depend on some physical variables like space and time etc. Here the governing equations of motion for a rotating circular cylindrical shell are obtained in a set of three partial differential equations. The Galerkin method (Elishakoff and Pentaras 2009) has been used for the frequencies calculations of double walled CNTs problem. Hence, the vibrational response of the system can be expressed in terms of the first Eigen function of a clamped – clamped beam $\varphi(x)$ and the generalized coordinate as follows.

So the solution for vibration of DWCNTs in the displacement form is written as

$$w_j = \varphi_j(x) e^{i\omega t} \tag{9}$$

$$(j = 1, 2)$$

For vibration purpose, various vibration modes $\varphi_j(x), j = 1, 2$ generated by the inner and outer tubes of DWCNTs.

$\varphi(x)$ signifies the unknown axial function that fulfills the end conditions stated at two shell ends.

$$\varphi_j(x) = C_j \text{Sin} \left(\frac{2m+1}{L} \pi x \right) \tag{10}$$

$$(j = 1, 2)$$

Substituting Eq. (9) into Eqs. (2) and (3), multiplying both sides by eigen function $\varphi(x)$ and integrating over the interval $(0, L)$ yields the following equation, we obtain the following coupled frequency equation:

$$\begin{pmatrix} c + EI_1 \left(\frac{2m+1}{L} \pi \right)^4 & -c \\ -c & c + EI_2 \left(\frac{2m+1}{L} \pi \right)^4 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \tag{11}$$

$$= \omega^2 \begin{pmatrix} \rho A_1 & 0 \\ 0 & \rho A_2 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

By varying the axial wave number value, a single MATLAB application can gain frequencies for various boundary circumstances. This eigenvalue Eq. (11) is solved by using the MATLAB program. In Eq. (8), the unknown axial function is solved for simply supported edge condition. For more conditions, the values are given in Table 1.

3. Results and discussion

The present analysis is associated with vibration characteristics of double walled carbon nanotubes using Euler-Bernoulli beam model with Winkler effect. Since there are very few cases of results for vibration of DWCNTs, their comparison are made with ones existing literature. The comparison has been done for frequencies calculated by Khan, 2016, Xu *et al.* 2006) as shown in Tables 2 and 3. In Table 2, the frequencies of simply supported and clamped-clamped are well matched with the results of Khan (2016). Khan used dynamic stiffness method to obtain the frequencies of carbon nanotubes. A good agreement has been noted between two sets of the

Table 1 Different boundary conditions for axial wave number are:

Boundary Conditions	Simply supported	Clamped-Clamped	Clamped-Free	Clamped-Simply supported
Wave Number	$\frac{m\pi}{L}$	$\frac{(2m+1)\pi}{2L}$	$\frac{(2m-1)\pi}{2L}$	$\frac{(4m+1)\pi}{4L}$

Table 2 Frequencies of present model with simply supported boundary and clamped conditions in THz

Method	Frequencies	
	Simply supported	Clamped-clamped
Dynamic stiffness matrix (Khan2016)	4.6E+11	1.04E+12
Present	4.3E+11	1.00E+12

Table 3 Frequencies of present model with simply supported boundary and clamped conditions in THz

Clamped-Clamped			
Aspect ratio		10	20
Natural Frequencies (THz)	Xu <i>et al.</i> (2006)	1.06367	0.2660
	Present	1.06210	0.2328

frequency parameters. An excellent consistency between the natural frequencies has been noticed. In Table 3, a comparison of present method for carbon nanotubes is done with Xu *et al.* (2006). Both ends of the tube, clamped end condition is applied. It has been seen good coherence between two sets of frequencies.

In Fig. 1, natural frequencies (THz) for double walled carbon nanotubes versus Winkler foundation (K) is examined for clamped –clamped boundary conditions with Values of geometrical parameter ($E = 1.0$ TPa, $R_1 = 0.35$ nm, $R_2 = 0.7$ nm, $\rho = 2.3$ g/cm³). The frequency is different for two different lengths ($L = 5, 10$ nm). It has been noticed that the frequencies at length $L = 5$ nm are higher than that for length $L = 10$ nm. It has also been noticed that the frequency increases as the value of Winkler foundation increases. The frequency gap for increasing the length $L = 5$ to 10 nm is much visible. In Fig. 2, the natural frequencies are examined for the double walled carbon nanotubes based on elastic foundations of Winkler bases. This foundation may affect the natural frequency (THz) of the double walled carbon nanotubes on increasing the length from $L = 5 \sim 10$ nm with same geometrical parameters. The with clamped simply supported of DWCNTs end conditions are used. The natural frequencies of double walled carbon nanotubes increase with the variation of K . The frequencies formed as then curve on increasing the values of Winkler base. It is noticed that the frequencies are lower for higher length on increasing the Winkler base. The frequencies of clamped-clamped are higher than that of clamped simply supported end condition. In this figure, the frequency pattern is slightly different on increasing the length and the gape is less than in the case of Fig. 1. In Fig. 3, the behavior of Winkler foundation versus

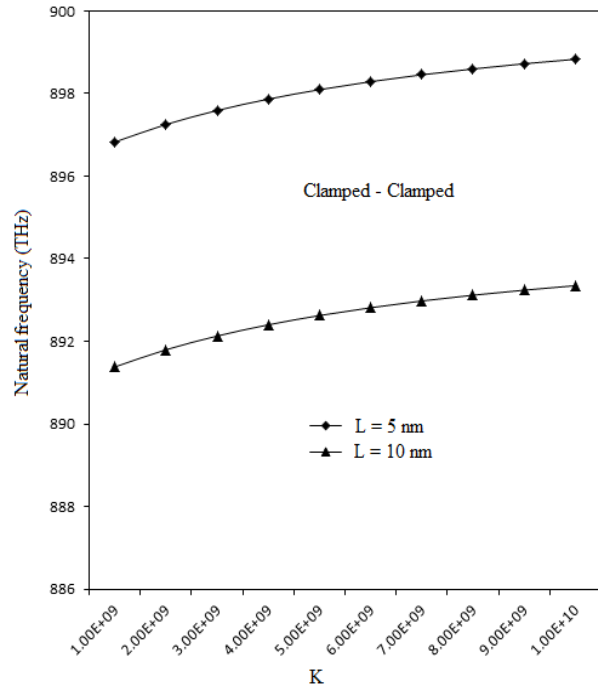


Fig. 1 Frequency variation of natural frequencies (THz) for C-C DWCNTs with K ($E = 1.0$ TPa, $R_1 = 0.35$ nm, $R_2 = 0.7$ nm, $\rho = 2.3$ g/cm³)

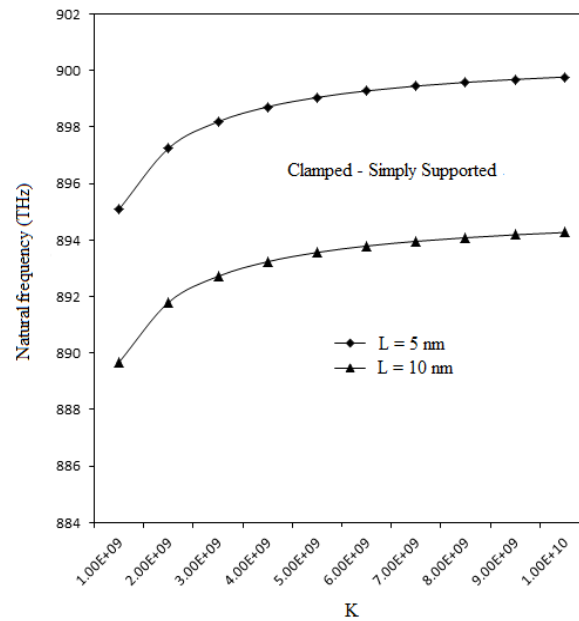


Fig. 2 Frequency variation of natural frequencies (THz) for C-SS DWCNTs with K ($E = 1.0$ TPa, $R_1 = 0.35$ nm, $R_2 = 0.7$ nm, $\rho = 2.3$ g/cm³)

frequency of double walled carbon nanotubes is demonstrated for simply supported. Both the ends are fixed as simply supported. The values of Winkler base is placed from $K = 1 \times 10^9 \sim 1 \times 10^{10}$. As the Values of K is increases from $K = 1 \times 10^9$ to $K = 3 \times 10^9$, the frequencies in increasing form and from $K = 3 \times 10^9$ to $K = 1 \times 10^{10}$ is observed as linear. The frequencies at length $L = 10$ nm is lowest than

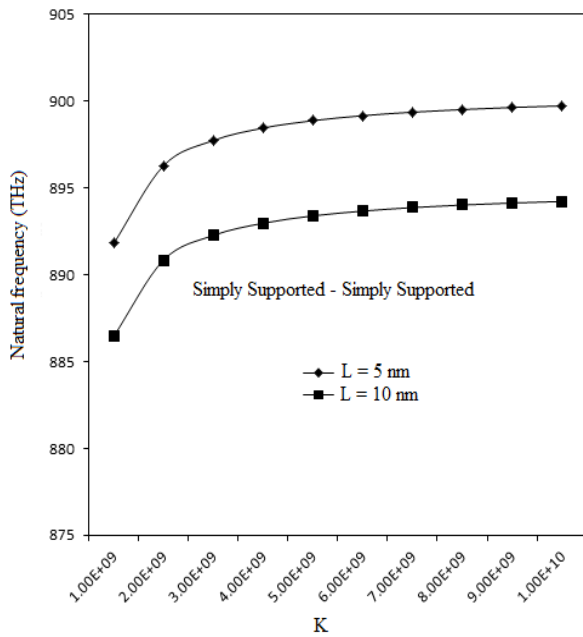


Fig. 3 Frequency variation of natural frequencies (THz) for SS-SS DWCNTs with K ($E = 1.0$ TPa, $R_1 = 0.35$ nm, $R_2 = 0.7$ nm, $\rho = 2.3$ g/cm³)

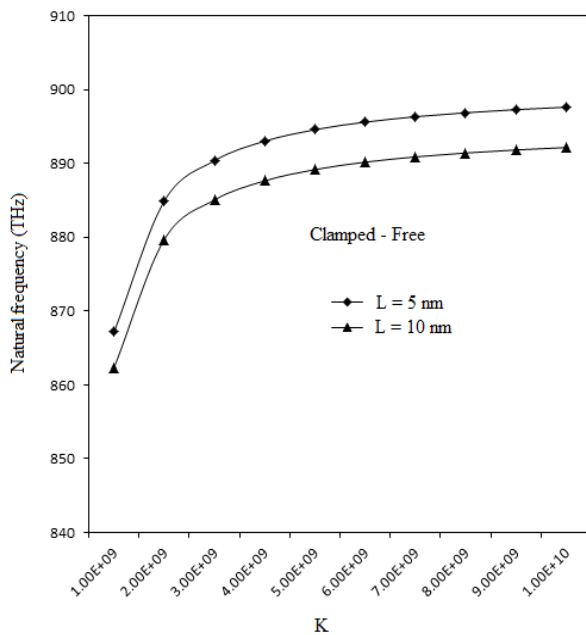


Fig. 4 Frequency variation of natural frequencies (THz) for C-F DWCNTs with K ($E = 1.0$ TPa, $R_1 = 0.35$ nm, $R_2 = 0.7$ nm, $\rho = 2.3$ g/cm³)

that of length $L = 5$ nm. The gap between the frequency curves is small as compared to Figs. 1 and 2. Fig. 4 shows natural frequency of double walled carbon nanotubes with the variation of K . The tube is placed on Winkler base with clamped free boundary conditions. The natural frequency increase as K is increases for two different lengths. It is interesting that the gap of frequencies in this figure is much smaller than that of clamped-clamped, simply supported and clamped-simply supported case.

4. Conclusions

Here, Galerkin's method is employed to obtain the frequency equation of double walled carbon nanotubes placed on Winkler foundation. The Euler-Bernoulli beam model is adopted for accurate vibration of carbon nanotubes. MATLAB software is utilized for the extraction of frequencies in numerical form. The results are verified with the open literature. The influence of the elastic foundation with two different values of length is investigated with different boundary conditions. The frequencies of Winkler base and length are counter part of each other. The natural frequencies (THz) for double walled carbon nanotubes versus Winkler foundation (K) is examined for clamped-clamped boundary conditions. The frequency is different for two different lengths ($L = 5, 10$ nm). It has been noticed that the frequencies at length $L = 5$ nm are higher than that for length $L = 10$ nm. It has also been noticed that the frequency increases as the value of Winkler foundation increases. The natural frequency increase as K is increases for two different lengths. The frequency first increases on increasing the Winkler constant.

Acknowledgment

The authors extend their appreciation to the Deanship of Research and Graduate Studies at King Khalid University for funding this work through Large Research Project under grant number RGP2/210/45.

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