

The effect of a nonlocal stress-strain elasticity theory on the vibration analysis of Timoshenko sandwich beam theory

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Abstract. In this article, a nonlocal stress-strain elasticity theory on the vibration analysis of Timoshenko sandwich beam theory with symmetric and asymmetric distributions of porous core and functionally graded material facesheets is introduced. According to nonlocal elasticity Eringen's theory (nonlocal stress elasticity theory), the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body; while, according to a new nonlocal strain elasticity theory, the strain at a reference point in the body is dependent not only on the stress state at that point, but also on the stress state at all of the points throughout the body. Also, with combinations of two concepts, the nonlocal stress-strain elasticity theory is defined that can be actual at micro/nano scales. It is concluded that the natural frequency decreases with an increase in the nonlocal stress parameter; while, this effect is vice versa for nonlocal strain elasticity, because the stiffness of Timoshenko sandwich beam decreases with increasing of the nonlocal stress parameter; in which, the nonlocal strain parameter leads to increase the stiffness of structures at micro/nano scale. It is seen that the natural frequency by considering both nonlocal stress parameter and nonlocal strain parameter is higher than the nonlocal stress parameter only and lower for a nonlocal strain parameter only.

Keywords: functionally graded materials; nonlocal stress-strain elasticity theory; sandwich Timoshenko beam; symmetric and asymmetric distribution of porous core; vibration analysis

1. Introduction

Sandwich structures are widely used in various industries because high ratio of strength-to-weight, good energy and sound absorption capability, and good thermal insulation. These structures made of three layers including flexible core layer and two facesheets layers. Many studies about the behavior of the sandwich structures are investigated.

The classical theory (Hooke's law or law of elasticity) discovered by the English scientist Robert Hooke in 1660, which states that, for relatively small deformations of an object, the stress at a reference point in the body is dependent to the strain state at that point. The previous nonlocal elasticity theory was first presented by Eringen's (1972, 1983). According to nonlocal stress elasticity theory, the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body. The nonlocal theory is used by some researchers as follows:

Polizzotto (2001) refined the Eringen's model by assuming an attenuation function depending on the 'geodetical distance' between material particles, such that in the diffusion processes of the nonlocality effects certain obstacles as holes or cracks existing in the domain can be circumvented. Also, He considered a suitable thermo-

dynamic framework with nonlocality as a firm basis of the model. Lazar *et al.* (2006) studied a theory of nonlocal elasticity of bi-Helmholtz type. They employed Eringen's model of nonlocal elasticity, with bi-Helmholtz type kernels, to study dispersion relations, screw and edge dislocations. Also, they derived the nonlocal kernels analytically as Green functions of partial differential equations of fourth order. This continuum model of nonlocal elasticity involves two material length scales which may be derived from atomistics. The new nonlocal kernels are nonsingular in one-, two- and threedimensions. Reddy (2007) presented nonlocal theories for bending, buckling and vibration of beams. Ghorbanpour Arani *et al.* (2008) illustrated buckling analysis of multi-walled carbon nanotubes under combined loading considering the effect of small length scale. Mohammadimehr *et al.* (2010) considered torsional buckling of a DWCNT embedded on winkler and pasternak foundations using nonlocal theory. They concluded that with increasing the nonlocal parameter, the torsional buckling decreases. Narendar (2011) studied buckling analysis of micro-/nano-scale plates based on two-variable refined plate theory incorporating nonlocal scale effects. Ansari *et al.* (2015) investigated size-dependent nonlinear forced vibration analysis of magneto-electro-thermo-elastic Timoshenko nanobeams based upon the nonlocal elasticity theory. Karami *et al.* (2019) depicted static analysis of functionally graded anisotropic nanoplates using nonlocal strain gradient theory. Zheng *et al.* (2020) presented a meshless collocation method for band structure simulation of nanoscale phononic crystals based on nonlocal elasticity theory. Taj *et al.* (2020) studied the

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nonlocal orthotropic elastic shell model for vibration analysis of protein microtubules. Hussain *et al.* (2020) presented Eringen's nonlocal model sandwich with Kelvin's theory for vibration of DWCNT.

Another size dependent effect such as modified couple stress theory (MCST) is studied based on Yang *et al.* (2002) for first time. Ma *et al.* (2008) presented a microstructure-dependent Timoshenko beam model based on a modified couple stress theory. Ke and Wang (2011) illustrated size effect on dynamic stability of functionally graded microbeams based on a modified couple stress theory. Mahmudian *et al.* (2013) investigated bending and free vibration analysis of nonlocal functionally graded nanocomposite Timoshenko beam model reinforced by SWBNT based on modified coupled stress theory. Jung *et al.* (2014) presented a modified couple stress theory for buckling analysis of S-FGM nanoplates embedded in Pasternak elastic medium. Ghorbanpour Arani *et al.* (2015) considered the vibration of bioliquid-filled microtubules embedded in cytoplasm including surface effects using modified couple stress theory. Nguyen *et al.* (2017) depicted a refined quasi-3D isogeometric analysis for functionally graded microplates based on the modified couple stress theory. Babaei and Eslami (2019) presented thermally induced large deflection of FGM shallow microarches with integrated surface piezoelectric layers based on modified couple stress theory. Alizadeh Hamidi *et al.* (2020) considered an exact solution on gold microbeam with thermoelastic damping via generalized Green-Naghdi and modified couple stress theories.

There are various theories to consider the size dependent effect such as modified strain gradient theory (MSGT) (proposed by Lam *et al.* (2003) for first time, Thai *et al.* (2018) considered isogeometric of sandwich FG microplates based on MSGT, Farzam and Hassani, (2019)) presented FG microplates with temperature-dependent material properties using MSGT and most general strain gradient theory (MGSGT), Ansari *et al.* (2013) and Mohammadimehr *et al.* (2018c)) considered bending, buckling and vibration of FG Timoshenko micro beams based on MGSGT and nonlocal strain gradient theory (NSGT) (Ebrahimi *et al.* 2016, Li and Hu 2019, Karami *et al.* 2020, Mohammadimehr 2022).

Some researchers worked about the size dependent effect on the vibration analysis of micro and nano structures as follows:

By decoupling the field equations of Eringen theory, Jomehzadeh and Saidi (2011) considered three dimensional vibration analysis of nano-plates. At first, they obtained three decoupled equations in terms of displacement components and three decoupled equations in terms of rotation components. Then, by exact solutions, they find the natural frequencies of nano-plates. Danesh *et al.* (2012) studied the small scale effect on the axial vibration of a tapered nanorod based on nonlocal elasticity theory. They applied the differential quadrature method to solve the governing equations of the nanorod for clamped-clamped, clamped-free and fixed-attached spring boundary conditions. Rahmati *et al.* (2014) illustrated the electro-thermo-mechanical vibration analysis of non-uniform and non-

homogeneous boron nitride nanorod (BNNR) embedded in elastic medium. They solved the equations based on differential quadrature method. Monajemi and coworkers (2016) studied the nonlinear free vibration analysis of boron-nitride micro ribbon on the Pasternak elastic foundation under electrical, mechanical and thermal loadings using modified strain gradient theory. Employing the von Kármán nonlinear geometry theory, they obtained the nonlinear equations of motion for the graphene micro ribbon with considering attached mass and size effects based on Hamilton's principle. Their equations are converted into the nonlinear ordinary differential equations by elimination of the time variable using Kantorovich time-averaging method and to determine nonlinear frequency of GMR under various boundary conditions, and considering mass effect, differential quadrature element method (DQEM). Karličić *et al.* (2017) presented by using the nonlocal theory, the influence of in-plane magnetic field on the natural frequency of viscoelastic orthotropic multi-nanoplate system embedded in a viscoelastic medium. They solved their governing equations of motion based on closed form solutions for complex natural frequencies. Based on first order shear deformation theory, Mohammadimehr *et al.* (2018a) presented free vibration analysis of magneto-electro-elastic cylindrical composite panel reinforced by various distributions of CNTs including uniform distribution (UD), FG-V, FG-A, FG-X and FG-O in Poly-vinylidene fluoride (PVDF) matrix with considering open and closed circuits boundary conditions. In the other work, based on modified couple stress theory, they (2018b) considered stress and free vibration analysis of piezoelectric hollow circular functionally graded boron nitride nanotubes reinforced nanocomposite plate. Chu *et al.* (2018) defined Flexoelectric effect as the coupling between strain gradient and electric polarization. Their present work is to study the flexoelectric effect in functionally graded composite nanostructure with a volume fraction distribution function. Bamdad *et al.* (2019) illustrated magneto-electro-elastic vibration and buckling analyses of sandwich Timoshenko porous beam with temperature-dependent material properties. Using the modified couple stress theory, Tadi Beni and Mehralian (2019) depicted the size-dependent free vibration of magneto-electro-elastic nanobeams in thermal environment. Mousavi *et al.* (2019) presented analytical solution for buckling analysis of micro sandwich hollow circular plate. Nematzadeh and Baradaran-Nasiri (2019) considered mechanical performance of fiber-reinforced recycled refractory brick concrete exposed to elevated temperatures. Yazdani *et al.* (2019) illustrated Wave propagation analysis of double bonded Cooper-Naghdi micro sandwich cylindrical shells with porous core and CNTRC face sheets. Timesli (2020) presented buckling analysis of double walled carbon nanotubes embedded in Kerr elastic medium under axial compression using the nonlocal Donnell shell theory. Civalek *et al.* (2020) considered frequency, bending and buckling loads of nanobeams with different cross sections. Shariati *et al.* (2020a, b) investigated vibrational behavior of graphene sheets under linearly varying in-plane bending load based on the nonlocal strain gradient theory and nonlocal couple

stress theory, respectively. Raad *et al.* (2020) illustrated scale-dependent thermal vibration analysis of FG beams having porosities based on DQM. Anvari *et al.* (2020) presented vibration behavior of a micro cylindrical sandwich panel reinforced by graphene platelet. Sobhy and Zenkour (2020) investigated the effects of hygrothermal conditions on bending, free vibration, and thermal buckling, of exponentially graded microplates. They employed the trigonometric four-variable plate theory incorporated to the modified couple stress theory (MCST) to derive the equations of motion. Some researchers about space fractional non-local models (Sumelka 2014, Pang *et al.* 2015, Lazopoulos 2016, Rahimi *et al.* 2017, Hei *et al.* 2018).

Some researchers worked about numerical methods to solve Partial Differential Equations (PDEs) that is added in the revised manuscript in introduction part as follows:

Anitescu *et al.* (2019) and Guo *et al.* (2019) worked about artificial and deep neural networks that are a topic of great interest in the machine and deep learning community due to their ability to solve very difficult problems, respectively. Thus, Samaniego *et al.* (2020) presented an energy approach to the solution of partial differential equations in computational mechanics via machine learning including concepts, implementation and applications. The uncertainties method consists of three ingredients: (1) sampling method, (2) surrogate models, (3) sensitivity analysis (SA) method are usually used for data collection, for example, to measure temperature and pressure, Vu-Bac *et al.* (2016) investigated this effect on the efficiency of thousands of data devices for temperature and pressure. It has been said that using the available methods to check the uncertainty of data accuracy, the degree of uncertainty and the effect of each of the parameters on the output of the system is also examined. However, in the present work, the input data for finding system frequencies has not been measured by the authors and has been considered as a nominal value in the analysis (as well as other references from the same nominal values for their analysis and its effect on behavior of the system is used). The present work also provides a theoretical method for examining the behavior of the system, and the construction process is not performed to address the uncertainty issues regarding the dimensional measurements of the structure. Therefore, in the end, it can be said that due to the theoretical nature of the analysis and the lack of measured data, it is not possible to check the uncertainty.

The novelty of this research can be introducing a nonlocal stress-strain elasticity theory on the vibration analysis of Timoshenko sandwich beam theory with symmetric and asymmetric distributions of porous core and functionally graded material facesheets. It may seem that the nonlocal strain gradient theory is similar to a nonlocal stress-strain elasticity theory, but they are different in concept and proof of process. According to nonlocal elasticity Eringen's theory (nonlocal stress elasticity theory), the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body; while, according to a new nonlocal strain elasticity theory,

the strain at a reference point in the body is dependent not only on the stress state at that point, but also on the stress state at all of the points throughout the body. Also, with combinations of two concepts, the nonlocal stress-strain elasticity theory is defined that can be actual in micro/nano scales. At the practice in the other work, the authors are obtained that with adding the carbon nanotube to composite material for facesheets layers, the stiffness of structures increases, thus it is concluded that the natural frequency for a micro/nano scales enhances. Thus, the researchers follow the best theory to close with the practice application that is the nonlocal stress- strain theory. On the other hands, in this paper, the defining a physical phenomenon of the resent research is the development of nonlocal stress and strain parameters on the porous sandwich beam with functionally graded materials in the top and bottomn face sheets. It is noted that by considering only nonlocal stress parameter $((e_0a)_{stress})$, the stiffness of structures decreases, also if you consider only nonlocal strain parameter $((e_0a)_{strain})$, the stiffness of structures increases, but when both $((e_0a)_{stress}$ and $((e_0a)_{strain}$ consider the stiffness of structures increases while the value of stiffness in this case $((e_0a)_{stress}$ and $((e_0a)_{strain}$ simultaneously) is lower than nonlocal strain parameter $((e_0a)_{strain})$ only that can be more actual at micro/nano scales.

2. A nonlocal stress-strain elasticity theory:

The previous nonlocal elasticity theory (the nonlocal stress elasticity theory) was first presented by Eringen (1972 and 1983). According to nonlocal stress elasticity theory, the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body. Thus, the constitutive equation of the nonlocal stress elasticity theory can be written as follows Eringen (1983):

$$(1 - (e_0a)_{stress}^2 \nabla^2) \sigma_{ij} = (\lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}) \quad (1)$$

where σ_{ij} and ε_{ij} are second order stress and strain tensors, respectively. λ and μ are Lamé's constants. Moreover, $((e_0a)_{stress})$ denotes the small scale effect based on the nonlocal stress elasticity theory. δ_{ij} is Kronecker delta.

The limitations and assumptions of the present model have been considered as follows

(1) The constitutive equations of this work are assumed linear in the elastic region. Also, it is assumed that the equations of the equilibrium become linear.

(2) The nonlocal stress and strain parameters has been used to consider the size-dependent effect on the governing equation of equilibrium of the beam.

(3) The sandwich beam is assumed to be as a continuous body that no delamination occurs between the layers of the sandwich beam.

(4) There are no debonding between functionally graded materials.

According to a nonlocal strain elasticity theory, the strain at a reference point in the body is dependent not only on the stress state at that point, but also on the stress state at all of the points throughout the body. Thus, the constitutive

equation of a nonlocal strain elasticity theory can be stated as follows:

$$\sigma_{ij} = (1 - (e_0 a)_{strain}^2 \nabla^2) (\lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}) \quad (2)$$

where $(e_0 a)_{strain}$ denotes the small scale effect based on strain gradient theory. Using superposition principle for linear elastic theory, we can combine equations (1) and (2), thus the nonlocal stress-strain elasticity theory can be considered as follows:

$$\begin{aligned} (1 - (e_0 a)_{stress}^2 \nabla^2) \sigma_{ij} \\ = (1 - (e_0 a)_{strain}^2 \nabla^2) (\lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}) \end{aligned} \quad (3)$$

where the Lamé's constant are defined as follows:

$$\lambda = \frac{Ev}{(1 + \nu)(1 - 2\nu)}, \mu = \frac{E}{2(1 + \nu)} \quad (4)$$

In Eq. (3), with combinations of two concepts (Eqs. (1) and (2)), the nonlocal stress-strain elasticity theory is defined that can be actual in micro/nano scales.

The displacement fields for Euler-Bernoulli and Timoshenko beam theories are considered as follows:

$$\begin{aligned} u(x, y, z, t) &= u_0(x, t) + z\psi(x, t) \\ v(x, y, z, t) &= 0 \\ w(x, y, z, t) &= w(x, t) \end{aligned} \quad (5)$$

where u, v, w denote the displacements in $x, y,$ and z directions, respectively. $u_0(x, t), \psi(x, t)$ and $w(x, t)$ are the axial displacement, slope and transverse displacement for Timoshenko beam model, respectively.

Using Eq. (5), the kinematic equations for Timoshenko beam are written as:

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + z \frac{\partial \psi}{\partial x}, \gamma_{xz} = 2\varepsilon_{xz} = \psi + \frac{\partial w}{\partial x} \quad (6)$$

Based on the variational method (energy method), we have:

$$\delta U = \int \int_A (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dA dz \quad (7)$$

where δU is the variational of strain energy for Timoshenko beam model.

The resultant force and moment are defined as follows:

$$\begin{aligned} N_x &= \int_{-0.5H}^{0.5H} \sigma_{xx} dz \\ M_x &= \int_{-0.5H}^{0.5H} \sigma_{xx} z dz \\ Q_x &= \int_{-0.5H}^{0.5H} \tau_{xz} dz \end{aligned} \quad (8)$$

where H is the total height of sandwich beam. Also, N_x and Q_x are the resultant axial and shear forces and M_x shows the bending moment for Timoshenko beam model.

Substituting Eq. (8) into Eq. (7) yields:

$$\begin{aligned} \delta U = \int_A \left(-\frac{\partial N_x}{\partial x} \delta u_0(x, t) - \frac{\partial M_x}{\partial x} \delta \psi(x, t) + Q_x \delta \psi(x, t) \right. \\ \left. - \frac{\partial Q_x}{\partial x} \delta w(x, t) \right) dA \end{aligned} \quad (9)$$

Based on equation (5), the kinetic energy is written as follows:

$$T = \frac{1}{2} \int_A \rho \left(\left(\frac{\partial u_0(x, t)}{\partial t} + z \frac{\partial \psi(x, t)}{\partial t} \right)^2 + \left(\frac{\partial w(x, y, z, t)}{\partial t} \right)^2 \right) dA \quad (10)$$

where T and ρ show the kinetic energy and density for Timoshenko beam model, respectively. Thus, the coefficients based on Eq. (10) are defined as

$$(I_0, I_1, I_2) = \int_{-0.5H}^{+0.5H} \rho (1, z, z^2) dz \quad (11)$$

By substituting Eq. (11) into Eq. (10) and variational method, we have:

$$\begin{aligned} \delta T \\ = \int_A \left(-I_0 \frac{\partial^2 u_0(x, t)}{\partial t^2} \delta u_0(x, t) - I_1 \frac{\partial^2 u_0(x, t)}{\partial t^2} \delta \psi(x, t) \right. \\ \left. - I_1 \frac{\partial^2 \psi(x, t)}{\partial t^2} \delta u_0(x, t) \right. \\ \left. - I_2 \frac{\partial^2 \psi(x, t)}{\partial t^2} \delta \psi(x, t) - I_0 \frac{\partial^2 w(x, t)}{\partial t^2} \delta w(x, t) \right) dA \end{aligned} \quad (12)$$

The Hamilton's principle is considered as follows:

$$\int_{t_1}^{t_2} \delta \Pi dt = 0 \Rightarrow \int_{t_1}^{t_2} (\delta T - \delta U) dt = 0 \quad (13)$$

Substituting Eqs. (9), (12) into Eq. (13), the equations of motion for Timoshenko sandwich beam theory are obtained as:

$$\delta u_0(x, t): \frac{\partial N_x}{\partial x} = I_0 \frac{\partial^2 u_0(x, t)}{\partial t^2} + I_1 \frac{\partial^2 \psi(x, t)}{\partial t^2} \quad (14a)$$

$$\delta w(x, t): \frac{\partial Q_x}{\partial x} = I_0 \frac{\partial^2 w(x, t)}{\partial t^2} \quad (14b)$$

$$\delta \psi(x, t): + \frac{\partial M_x}{\partial x} - Q_x = I_1 \frac{\partial^2 u_0(x, t)}{\partial t^2} + I_2 \frac{\partial^2 \psi(x, t)}{\partial t^2} \quad (14c)$$

Substituting Eq. (3) into Eq. (8) and based on a new nonlocal stress-strain elasticity theory, we have:

$$\begin{aligned} (1 - (e_0 a)_{stress}^2 \nabla^2) N_x \\ = \left(1 - (e_0 a)_{strain}^2 \nabla^2 \right) \int_{-0.5H}^{0.5H} (\lambda + 2\mu) \left(\frac{\partial u_0(x, t)}{\partial x} \right. \\ \left. + z \frac{\partial \psi(x, t)}{\partial x} \right) dz \\ (1 - (e_0 a)_{stress}^2 \nabla^2) M_x \\ = \left(1 - (e_0 a)_{strain}^2 \nabla^2 \right) \int_{-0.5H}^{0.5H} (\lambda + 2\mu) \left(\frac{\partial u_0(x, t)}{\partial x} \right. \\ \left. + z \frac{\partial \psi(x, t)}{\partial x} \right) z dz \end{aligned} \quad (15)$$

$$\begin{aligned} (1 - (e_0 a)_{stress}^2 \nabla^2) Q_x \\ = k_s (1 - (e_0 a)_{strain}^2 \nabla^2) \int_{-0.5H}^{0.5H} \mu (\psi(x, t) \\ + \frac{\partial w(x, t)}{\partial x}) dz \end{aligned}$$

The following parameter based on Eq. (15) can be

considered as:

$$\begin{cases} C_{11}^{(0)} \\ C_{11}^{(1)} \\ C_{11}^{(2)} \\ C_{11}^{(0)} \end{cases} = \int_{-0.5H}^{0.5H} (\lambda + 2\mu) \begin{Bmatrix} 1 \\ z \\ z^2 \end{Bmatrix} dz \quad (16)$$

$$C_{12}^{(0)} = k_s \int_{-0.5H}^{0.5H} \mu dz$$

Substituting Eq. (16) into Eq. (15) yields the following equation:

$$\begin{aligned} & (1 - (e_0 a)^2_{stress} \nabla^2) N_x \\ &= (1 - (e_0 a)^2_{strain} \nabla^2) (C_{11}^{(0)} \frac{\partial u_0(x, t)}{\partial x} + C_{11}^{(1)} \frac{\partial \psi(x, t)}{\partial x}) \\ & (1 - (e_0 a)^2_{stress} \nabla^2) M_x \\ &= (1 - (e_0 a)^2_{strain} \nabla^2) (C_{11}^{(1)} \frac{\partial u_0(x, t)}{\partial x} + C_{11}^{(2)} \frac{\partial \psi(x, t)}{\partial x}) \quad (17) \\ & (1 - (e_0 a)^2_{stress} \nabla^2) Q_x \\ &= (1 - (e_0 a)^2_{strain} \nabla^2) (C_{12}^{(0)} \psi(x, t) + C_{12}^{(0)} \frac{\partial w(x, t)}{\partial x}) \end{aligned}$$

Based on a new nonlocal stress-strain elasticity theory, and by substituting Eq. (17) into Eqs. (14a)-(14c), the governing equations of motion for Timoshenko sandwich beam theory with porous core and graphene platelets reinforced composite facesheets are derived as follows:

$$\begin{aligned} & \delta u_0(x, t) (1 - (e_0 a)^2_{strain} \nabla^2) (C_{11}^{(0)} \frac{\partial^2 u_0(x, t)}{\partial x^2} \\ & \quad + C_{11}^{(1)} \frac{\partial^2 \psi(x, t)}{\partial x^2}) \quad (18a) \\ &= (1 - (e_0 a)^2_{stress} \nabla^2) (I_0 \frac{\partial^2 u_0(x, t)}{\partial t^2} + I_1 \frac{\partial^2 \psi(x, t)}{\partial t^2}) \end{aligned}$$

$$\begin{aligned} & \delta w(x, t) (1 - (e_0 a)^2_{strain} \nabla^2) (C_{12}^{(0)} \frac{\partial \psi(x, t)}{\partial x} \\ & \quad + C_{12}^{(0)} \frac{\partial^2 w(x, t)}{\partial x^2}) \quad (18b) \\ &= (1 - (e_0 a)^2_{stress} \nabla^2) I_0 \frac{\partial^2 w(x, t)}{\partial t^2} \end{aligned}$$

$$\begin{aligned} & \delta \psi(x, t) (1 - (e_0 a)^2_{strain} \nabla^2) (C_{11}^{(1)} \frac{\partial^2 u_0(x, t)}{\partial x^2} \\ & \quad + C_{11}^{(2)} \frac{\partial^2 \psi(x, t)}{\partial x^2}) \quad (18c) \\ & - (1 - (e_0 a)^2_{strain} \nabla^2) (C_{12}^{(0)} \psi(x, t) + C_{12}^{(0)} \frac{\partial w(x, t)}{\partial x}) \\ &= (1 - (e_0 a)^2_{stress} \nabla^2) (I_1 \frac{\partial^2 u_0(x, t)}{\partial t^2} + I_2 \frac{\partial^2 \psi(x, t)}{\partial t^2}) \end{aligned}$$

The displacements for Timoshenko sandwich beam theory based on Navier's method can define as follows:

$$\begin{aligned} u_0(x, t) &= \sum_{m=1}^{\infty} U_m \cos(\frac{m\pi x}{L}) e^{i\omega t} \\ \psi(x, t) &= \sum_{m=1}^{\infty} \psi_m \cos(\frac{m\pi x}{L}) e^{i\omega t} \end{aligned} \quad (19)$$

$$w(x, t) = \sum_{m=1}^{\infty} W_m \sin(\frac{m\pi x}{L}) e^{i\omega t}$$

where ω , m and L show the natural frequency, axial wave number, and the length of Timoshenko beam model, respectively. Substituting Eq. (19) into Eqs. (18a)-(18c), The matrix form for Timoshenko sandwich beam theory is obtained as

$$([k] - \omega^2 [M]) \{X\} = \{0\}, \quad (20)$$

where the stiffness and mass matrices are obtained as follows:

$$\begin{aligned} [K] &= (1 + (e_0 a)^2_{strain} \alpha^2) \begin{bmatrix} C_{11}^{(0)} \alpha^2 & 0 & C_{11}^{(1)} \alpha^2 \\ 0 & C_{12}^{(0)} \alpha^2 & C_{12}^{(0)} \alpha \\ C_{11}^{(1)} \alpha^2 & C_{12}^{(0)} \alpha & C_{11}^{(1)} \alpha^2 + C_{12}^{(0)} \end{bmatrix} \\ [M] &= (1 + (e_0 a)^2_{stress} \alpha^2) \begin{bmatrix} I_0 & 0 & I_1 \\ 0 & I_0 & 0 \\ I_1 & 0 & I_2 \end{bmatrix} \\ \alpha &= \frac{m\pi}{L} \\ \{x\} &= \begin{Bmatrix} U_m \\ W_m \\ \psi_m \end{Bmatrix} \end{aligned} \quad (21)$$

The Young's modulus (E_c) and density (ρ_c) of porous core for two types of different distributions is defined as follows (Chen *et al.* 2017):

$$E_c = E_{0c} \left(1 - e_0 \cos\left(\frac{\pi z}{2h_c} + \frac{\pi}{4}\right) \right) \quad \text{Asymmetric type} \quad (22)$$

$$\rho_c = \rho_{0c} \left(1 - e_m \cos\left(\frac{\pi z}{2h_c} + \frac{\pi}{4}\right) \right)$$

$$E_c = E_{0c} \left(1 - e_0 \cos\left(\frac{\pi z}{h_c}\right) \right) \quad \text{Symmetric type} \quad (23)$$

$$e_m = 1 - \sqrt{1 - e_0}$$

Based on functionally graded material properties, the Young's (E_{11}^t and E_{11}^b) moduli and density (ρ^t and ρ^b) for top and bottom facesheets of sandwich beam are written as:

$$E_{11}^t = E_{Al} + (E_{st} - E_{Al}) \times \left(\frac{1}{2} + \frac{z_t}{h_t}\right)^n \quad (24)$$

$$E_{11}^b = E_{Al} + (E_{st} - E_{Al}) \times \left(\frac{1}{2} + \frac{z_b}{h_b}\right)^n$$

$$\rho^t = \rho_{Al} + (\rho_{st} - \rho_{Al}) \times \left(\frac{1}{2} + \frac{z_t}{h_t}\right)^n \quad (25)$$

$$\rho^b = \rho_{Al} + (\rho_{st} - \rho_{Al}) \times \left(\frac{1}{2} + \frac{z_b}{h_b}\right)^n$$

where n is the power law index.

Table 1 The mechanical and geometrical parameters for sandwich beam

The geometric of sandwich beam	$H = 10\mu m, h_c = 0.8H, k_s = 5/6, m = 1,$
The properties of facesheets layers	$E_{Al} = 70GPa, E_{st} = 200GPa, n = 2;$ $\nu_{Al} = 0.33, \nu_{st} = 0.3,$ $\rho_{Al} = 2700kg/m^3, \rho_{st} = 8760kg/m^3,$
The properties of core	$E_{0c} = 120GPa, \rho_{0c} = \frac{1130kg}{m^3},$ $\nu_c = 0.34, e_0 = 0.3,$
Nonlocal stress-strain parameters	$(e_0a)_{strain} = 1\mu m, (e_0a)_{stress} = 1\mu m$

3. Numerical results and discussions:

In this research, a new nonlocal stress-strain elasticity theory on the vibration analysis of Timoshenko sandwich beam theory with symmetric and asymmetric distributions of porous core and graphene platelets reinforced composite facesheets is introduced.

The mechanical and geometrical parameters for sandwich beam with porous cores and graphene platelets are considered in Table 1.

Figs. (1) and (2) show the three first natural frequencies for $m=1$ versus core thickness to total thickness ratio (h_c/H) for different values of nonlocal strain parameter ($(e_0a)_{strain}$) and nonlocal stress parameter ($(e_0a)_{stress}$) based on a new nonlocal strain elasticity theory, nonlocal stress elasticity theory (Eringen’s theory), respectively. It can be seen that with increasing of $(e_0a)_{strain}$, the stiffness of a micro sandwich beam enhances, and then the natural frequencies increases; while, it is vice versa for $(e_0a)_{stress}$, because with an increase the nonlocal stress parameter, the sandwich structure becomes softer and then the natural frequency reduces.

Figs. (3) and (4) depict the effect of nonlocal stress and strain parameters on the natural frequency based on a new nonlocal stress-strain elasticity theory. It is concluded that with an increase in the nonlocal stress parameter, the natural frequency reduces; while, this point is vice versa for nonlocal strain elasticity, because the stiffness of Timoshenko sandwich beam decreases with increasing of the nonlocal stress parameter; in which, the nonlocal strain parameter leads to increase the stiffness of structures at micro/nano scale. It is seen from Fig. (4) that the natural frequency by considering both nonlocal stress parameter and nonlocal strain parameter is higher than the nonlocal stress parameter only and lower for a new nonlocal strain parameter. On the other hands, we name the nonlocal stress parameter, nonlocal strain parameter, and the nonlocal stress-strain parameter as Eringen’s parameter (small scale effect), ideal state, and actual state, respectively, because adding graphene platelets in matrix leads to increase the stiffness of structure, while the agglomeration phenomenon occurs actually due to adding graphene platelets in matrix, thus in actual case, we must consider simultaneously the nonlocal stress and strain parameters. At the micro/nano scale, the size-dependent effects such as the material length scale parameter (nonlocal strain parameter) based on nonlocal strain theory increase the stiffness of structures

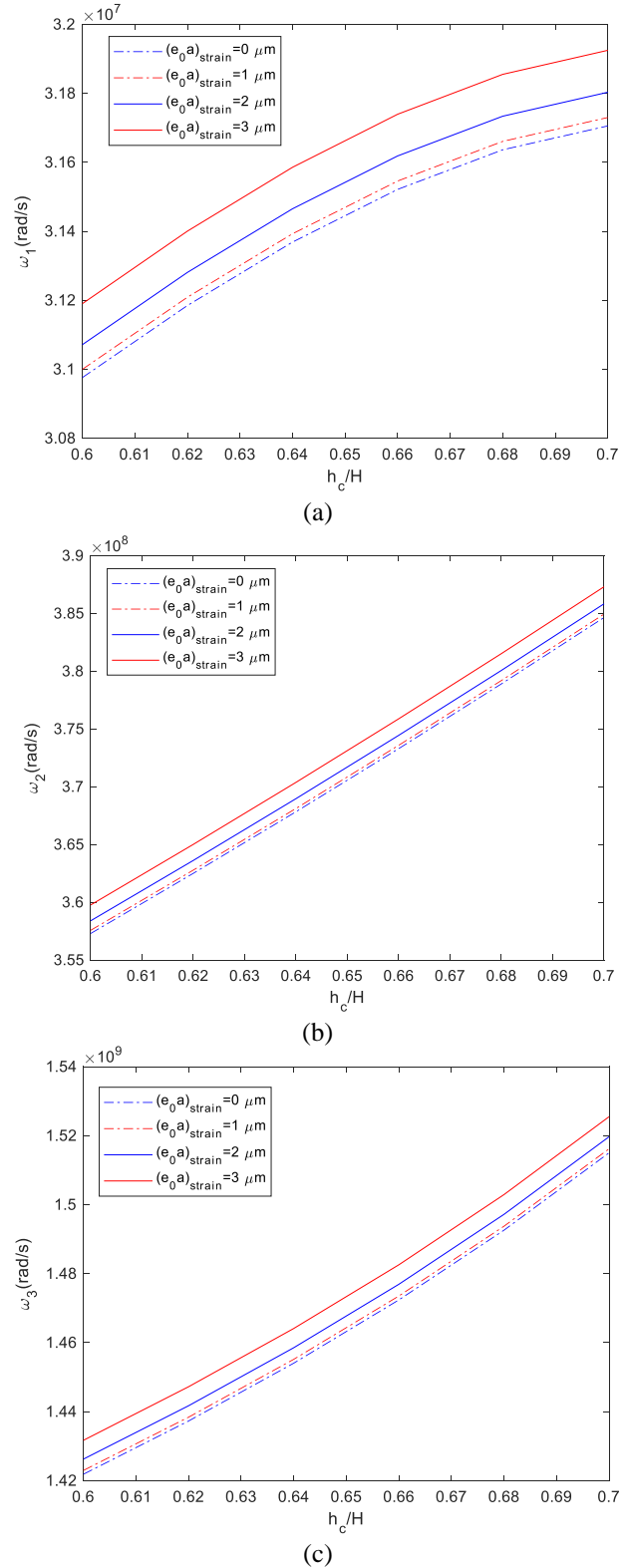
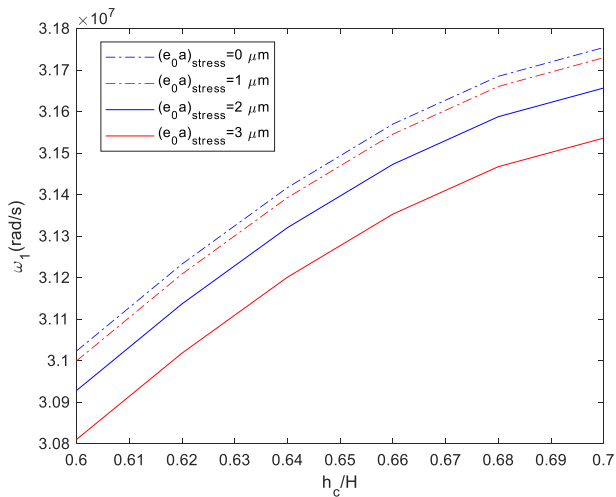
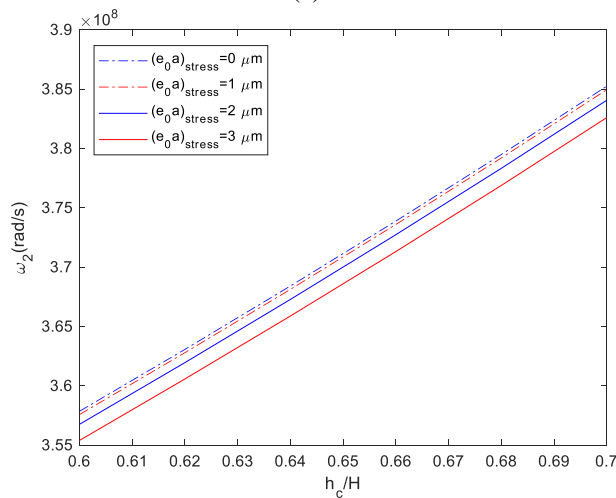


Fig. 1 Three natural frequencies versus core thickness to total thickness ratio for various values of nonlocal strain parameter (a) axial amplitude (b) transverse amplitude (c) bending slope

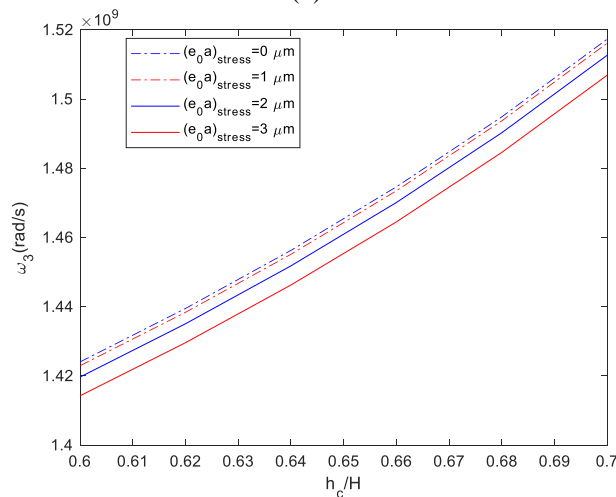
then the natural frequency enhances, while the effect of nonlocal parameter (nonlocal stress parameter) on the natural frequency is vice versa. Basically, the use of nano in



(a)



(b)



(c)

Fig. 2 Three natural frequencies versus core thickness to total thickness ratio for various values of nonlocal stress parameter (a) axial amplitude (b) transverse amplitude (c) bending slope

structures leads to increasing the flexural rigidity which is predicted by the both nonlocal stress parameter and nonlocal strain parameter as well, while Eringen's nonlocal

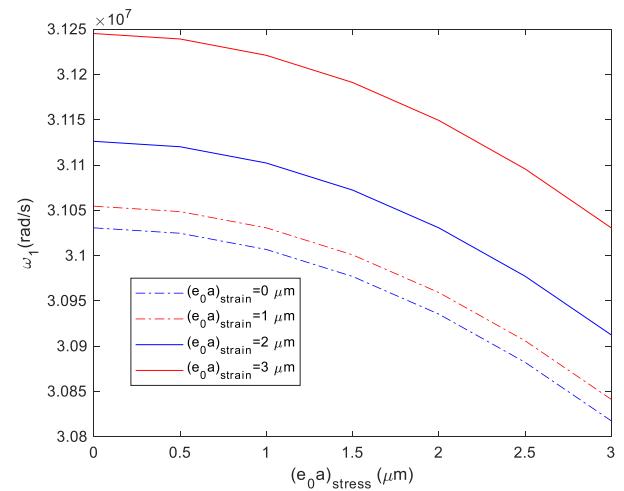


Fig. 3 The natural frequency versus nonlocal stress parameter for various values of nonlocal strain parameter

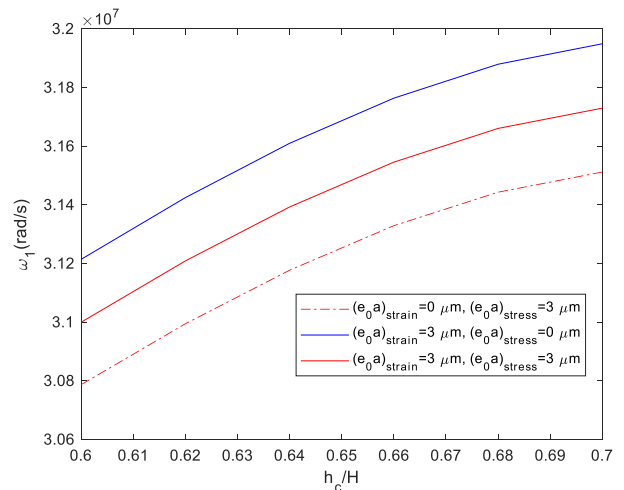


Fig. 4 The effect of nonlocal stress and strain parameters on the natural frequency of Timoshenko sandwich beam

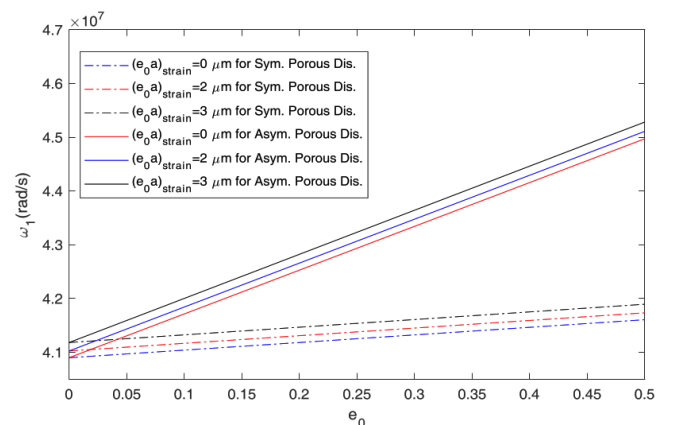


Fig. 5 The effect of porous parameter on the natural frequency of Timoshenko sandwich beam for symmetric and asymmetric distributions of porous core

elasticity theory using nonlocal stress parameter reduces the stiffness of structures and does not anticipate, thus the considering both nonlocal stress parameter and nonlocal

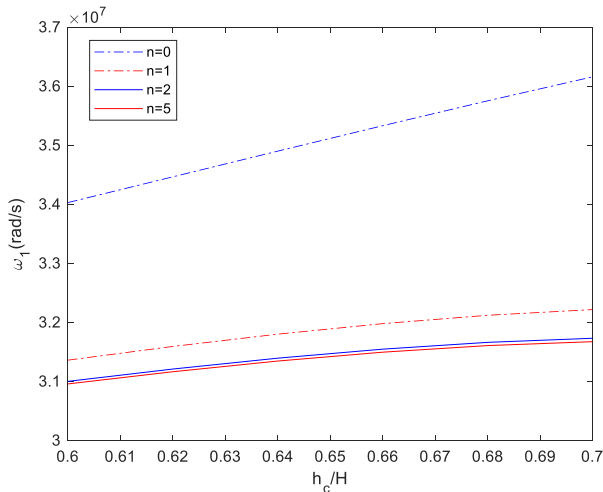


Fig. 6 The effect of power law index on the natural frequency of Timoshenko sandwich beam

strain parameter is lower than nonlocal strain parameter only. Thus, the nonlocal stress and strain parameters can be more actual at micro/nano scales.

Fig. 5 illustrates the effect of porous parameter on the natural frequency of Timoshenko sandwich beam for symmetric and asymmetric distributions of porous core based on a new nonlocal stress-strain elasticity theory. It is seen that with an increase in the porous parameter, the natural frequency increases. Also, the obtained results for asymmetric distribution is higher than for symmetric distributions of porous core.

Fig. 6 shows the effect of power law index on the natural frequency of Timoshenko sandwich beam. It is seen that the natural frequency decreases with an enhance in power law index because the stiffness of structure decreases. It is stated that the curve slope of power law index is lower, on the other hands, the higher power law index is noticeable.

4. Conclusions

In this paper, a nonlocal stress-strain elasticity theory was considered on the vibration behavior of Timoshenko sandwich beam theory with two type distributions of porous core and functionally graded material facesheets. According to nonlocal elasticity Eringen's theory, the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body; while, according to a nonlocal strain elasticity theory, the strain at a reference point in the body is dependent not only on the stress state at that point, but also on the stress state at all of the points throughout the body. It is concluded that the natural frequency decreases with an increase in the nonlocal stress parameter; while, this effect is vice versa for nonlocal strain elasticity, because the stiffness of Timoshenko sandwich beam decreases with increasing of the nonlocal stress parameter; in which, the nonlocal strain parameter leads to increase the stiffness of structures at micro/nano scale. It is seen that the natural frequency by considering both nonlocal

stress parameter and nonlocal strain parameter (a nonlocal stress-strain elasticity theory) is higher than the nonlocal stress parameter only and lower for a nonlocal strain parameter. On the other hands, we know that the nonlocal stress parameter, nonlocal strain parameter, and the nonlocal stress-strain parameter are as Eringen's parameter (small scale effect), ideal state, and actual state, respectively. Moreover, with combinations of two concepts, the nonlocal stress-strain elasticity theory is defined that can be actual at micro/nano scales. Also, it is seen that the natural frequency decreases with an enhance in the power law index, besides, the higher power law index is noticeable.

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