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On the thermo-mechanical vibration of an embedded short-fiber-reinforced nanobeam

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Abstract. This work investigates the thermo-mechanical vibration frequencies of an embedded composite nano-beam restrained with elastic springs at both ends. Composite nanobeam consists of a matrix and short fibers as reinforcement elements placed inside the matrix. An approach based on Fourier sine series and Stokes' transform is adopted to present a general solution that can examine the elastic boundary conditions of the short-fiber-reinforced nanobeam considered with the Halpin-Tsai model. In addition to the elastic medium effect considered by the Winkler model, the size effect is also considered on the basis of non-local strain gradient theory. After creating an eigenvalue problem that includes all the mentioned parameters, this problem is solved to examine the effects of fiber and matrix properties, size parameters, Winkler stiffness and temperature change. The numerical results obtained at the end of the study show that increasing the rigidity of the Winkler foundation, the ratio of fiber length to diameter and the ratio of fiber Young's modulus to matrix Young's modulus increase the frequencies. However, thermal loads acting in the positive direction and an increase in the ratio of fiber mass density to matrix mass density lead to a decrease in frequencies. In this study, it is clear from the eigenvalue solution calculating the frequencies of thermally loaded embbeded short-fiber-reinforced nanobeams that changing the stiffness of the deformable springs provides frequency control while keeping the other properties of the nanobeam constant.

Keywords: elastic boundaries; non-local strain gradient theory; short-fiber-reinforced nanobeam; thermo-mechanical vibration; Winkler foundation

1. Introduction

In recent years, studies on composites have increased the interest in composite materials. Due to their unique properties, these materials are widely used in industry. Composites have high specific strength, high specific modulus and easy adaptability. For this reason, these materials are often seen in the construction industry (Frigione and Lettieri 2018, Zaman *et al.* 2013), the sports equipment (Sreejith and Rajeev 2021) and the automotive industry (Lin *et al.* 2020). Fibers are present in composites and these fibers can be short, long and continuous. Conventional composite materials have problems such as discontinuity of stress distributions at interfaces and low resistance to high temperature gradient. Therefore, it is essential to investigate the static and mechanic properties of these composite materials before their application in various fields. In the construction industry, the free vibration behavior of beam structures constructed from composite or functionally graded material (FGM) is often investigated. In recent years, the number of studies dealing with dynamic analysis of sandwich beams, composite beams and functionally graded (FG) beams has increased considerably. Murín *et al.* (2010) studied the derivation of a fourth-order differential equation for beam bending of an FGM with changing material properties. Wattanasakulpong and Ungbhakorn (2014) studied the linear and non-linear dynamic analysis of porous beams constructed from FGMs. In another research, Ladmek *et al.* (2023b) conducted studies investigating the mechanical behavior of composite beams reinforced with FG carbon nanotube (FG-CNT) in a polymeric matrix.

Moreover, composite materials are also used in modelling nano- and microstructures. In this context, studies examining the mechanical characteristics of composite micro/nano beams have been conducted in the literature. Attia *et al.* (2024) examined the mechanical response of a double-cracked nanobeam made of a bidirectional FG material. Uzun and Yaylı (2024) demonstrated the dynamic behavior of FG nanobeams with pores in the cross sections. Ahmadi *et al.* (2024) analyzed the free vibration of a two-directional (2D) FG coarse curved nano/micro beam for several boundary conditions. Marinca *et al.* (2023) proposed an investigation on the non-linear dynamic behavior of a FG nanobeam depending on the non-local strain gradient theory (NSGT). Herisanu *et al.* (2023) studied the non-linear forced vibration analysis of the FG nano/micro beam exposed to mechanical impact and electromagnetic motion. Lovisi *et al.* (2024) studied the long-range interaction and the synergistic effect of surface energy on high vibrational modes through a mathematical model of Bernoulli-Euler nanobeams made of FGM. Uzun *et al.* (2023) aimed to research the vibration behavior of FG nanorods employing

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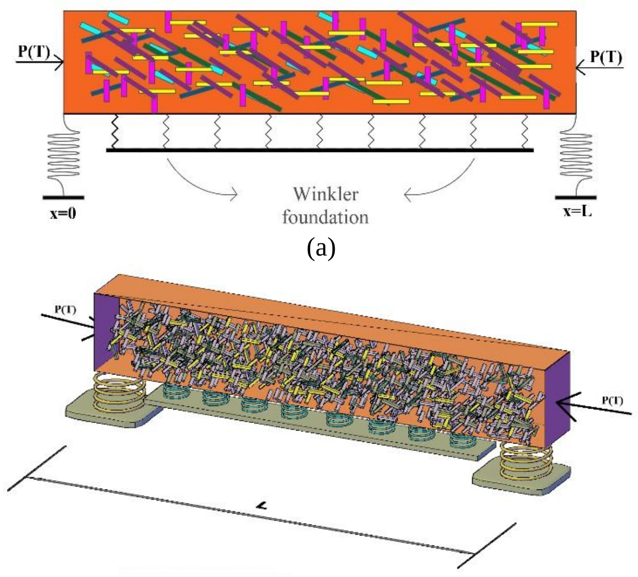
the hardening non-local elasticity theory (NET). In their paper, Yuan *et al*. (2023) explored the vibration behavior of viscoelastic FG-CNTs reinforced nanobeams by taking into account the impact of thickness stress under magneto-hygro-thermo loading. Feo *et al*. (2023) investigated the synergistic effect of long-range interactions and surface energy on the mechanical behaviour of metal-ceramic FG nanobeams. Hosseini and Beni (2023) investigated the dynamic behaviour of a spinning piezoelectric micro/nano beam under the presence of piezoelectric effects. Mojahedi (2024) investigated the vibration of a nonlinear nano/micro beam using NSGT. It is possible to find more papers in the literature investigating the mechanical properties of composite nanobeams under different conditions (Alfred *et al*. 2024, Barati and Zenkour 2018, Chen *et al*. 2024, El-Shahrany and Zenkour 2023, Lal and Dangi 2020, Modanloo *et al*. 2023, Moradi *et al*. 2023, Nalbant *et al*. 2023, Penna *et al*. 2023, Tuyen and Du 2023, Vahidi-Moghaddam *et al*. 2023, Yang *et al*. 2023, Zhao *et al*. 2022).

As can be understood, nanostructures have been investigated in different states and conditions using different methods. The effect of thermal environment, which is one of these different conditions, has been studied by many researchers. For example, using a thermo-mechanical method, Ebrahimi *et al*. (2019) studied the vibration behavior of porous and curved FG nanobeams based on NSGT. In another study on the vibration behavior of FG porous beams in a thermal environment, Ebrahimi and Jafari (2016) used a semi-analytical differential transformation method in the analysis. Besides thermal effects, Abdullah *et al*. (2020) also considered the effects of magnetic forces on the vibrational behavior of the nano-beam based on the NET. Li and Qing (2024) performed the thermo-mechanical analysis of the FG viscoelastic Timoshenko microbeam based on NSGT. Besides these studies, there are also studies on thermo-mechanic and -static analysis of flexoelectric (Beni 2023, Ebnali Samani and Beni 2018, Ghobadi *et al*. 2020, 2021) and various (Mohammadi *et al*. 2023, Qing 2023, Semmah *et al*. 2019, Shafiei *et al*. 2019, Sobhy 2017, Tounsi *et al*. 2013, Wei *et al*. 2024) nanostructures.

The studies on the production of fiber-reinforced nanostructures and the investigation of their properties are also of great interest to researchers (Dean *et al*. 2006, Junaedi *et al*. 2020). The ability to add short-fiber into composite materials (Abrate 1986) has created a need in the literature to study the mechanical characteristics of short-fiber reinforced (SFR) composite materials. In the literature, studies on the vibrations of nanocomposites containing multi-walled CNTs and carbon fiber reinforced polymer composites (Khan *et al*. 2011), unidirectional SFR composites (Lei *et al*. 2012), SFR non-circular micro-wires (Civalek *et al*. 2023), SFR composite nanobeams (Gul 2024), embedded SFR reinforced nanocomposite rods (Civalek *et al*. 2022), FG-CNT reinforced composite micro-beams embedded in elastic media (Ghorbani Shenas *et al*. 2019), have been investigated. Dynamic analysis of CNT reinforced composite Timoshenko microbeams (Dehkordi and Beni 2023), FG-CNT reinforced beams with shear deformation theory (Ladmek *et al*. 2023a), CNT reinforced dynamic loading (Abdelrahman *et al*. 2023) and viscous damped SFR nanocomposite rods are also studied (Gul and Aydogdu 2023).



(a)

(b)

Fig. 1 (a) Forced boundary conditions with Winkler foundation (b) Short-fiber-reinforced nanobeam restrained by elastic springs subjected to thermal forces

Composite materials are important forms that attract attention at both macro and small scale dimensions and whose mechanical responses are investigated. Recently, composite materials reinforced with short fibers have attracted attention. In this study, free vibration analysis of nanoscale beams formed from short fibers dispersed in a matrix under the influence of thermal loads, an elastic environment and size parameters is discussed the purpose of this study is to provide an effective solution procedure that can investigate the frequencies of SFR nanobeams for both elastic and rigid boundary conditions. For this purpose, the SFR nanobeams, whose material properties are defined using the Halpin-Tsai model, is modeled by elastic springs allowing deflection at each ends. A solution method depending on Fourier sine series and Stokes' transform is applied to investigate the boundary conditions as desired (rigid and non-rigid). Thus, with a single eigenvalue problem obtained, the problem in question has the opportunity to be analyzed in a wide scope.

2. Short-fiber reinforced material

In this study, the thermo-mechanical vibration of SFR nanobeams on a Winkler foundation is presented. SFR composite materials are produced by reinforcing short fibers into a matrix in various distributions. In this study, the short fiber reinforcements are randomly aligned in the rectangular nanobeam. It is much easier to fabricate nanobeams with randomly distributed fibers rather than aligned and/or functionally graded reinforcing elements along the beam length. Therefore, in this study, short-fiber-reinforced nanobeams are analyzed. The NSGT is employed to find the thermo-mechanical vibration of SFR nano/ microbeam. In this study, the Halpin-Tsai equations for SFR composite nanobeams are considered. Using the Halpin-Tsai equations, the material characteristics of SFR composites are derived as follows (Guzmán de Villoria and Miravete 2007):

$$E_L = E_m \left(\frac{1 + \left(\frac{2l}{d}\right) \eta_L V_f}{1 - \eta_L V_f} \right) \tag{1}$$

$$E_T = E_m \left(\frac{1 + 2\eta_T V_f}{1 - \eta_T V_f} \right) \tag{2}$$

where, E_T is the transverse Young's modulus (YM) of the composite and E_L is the longitudinal YM of the composite. Also E_m is the YM of the matrix, while l, d and V_f are the length, diameter and volume fraction of the fiber, respectively. Furthermore, η_L and η_T are derived as follows (Guzmán de Villoria and Miravete 2007):

$$\eta_L = \left(\frac{\frac{E_f}{E_m} - 1}{\frac{E_f}{E_m} + 2\left(\frac{l}{d}\right)} \right) \tag{3}$$

$$\eta_T = \left(\frac{\frac{E_f}{E_m} - 1}{\frac{E_f}{E_m} + 2} \right) \tag{4}$$

Here, E_f is the YM of the fiber material. Using the transverse (E_T) and longitudinal (E_L) components, the YM of a randomly rotated SFR composite material (E_c) can be derived as (Guzmán de Villoria and Miravete 2007):

$$E_c = E_{random} = \frac{3}{8} E_L + \frac{5}{8} E_T \tag{5}$$

The material density of SFR composite (ρ_c) is calculated as follows (Pan 1993):

$$\rho_c = \rho_m (1 - V_f) + \rho_f V_f \tag{6}$$

Here, ρ_m defines the density of matrix and ρ_f denotes the density of fiber.

3. Thermo-mechanical vibration behaviour of SFR nanobeam

In this paper, the thermal vibration response of a SFR nanobeam with elastic support conditions is investigated using beam theory of Euler-Bernoulli. In this theory, the displacement fields at any point are expressed as (Jena *et al.* 2019):

$$v_1 = -z \partial_x \mathrm{w}(\mathrm{x}, \mathrm{t}) \tag{7}$$

$$v_2 = 0 \tag{8}$$

$$v_3 = \mathrm{w}(\mathrm{x}, \mathrm{t}) \tag{9}$$

In which, v_1 is the displacement component of the nanobeam in the axial direction, v_2 is displacement in the width direction and v_3 is displacement in the transverse (lateral) direction. Also $\mathrm{w}(\mathrm{x}, \mathrm{t})$ is the lateral displacement of any point of the nanobeam and t is time. The strain energy term of the slender nano-scaled beam is denoted by S and is shown as (Jena *et al.* 2019):

$$S = \frac{1}{2} \int_0^L \int_A \sigma_{xx} \epsilon_{xx} dA dx \tag{10}$$

Here, L and A denotes the length and the cross-sectional area of the nanobeam respectively. σ_{xx} represents the non-zero stress of the SFR nanobeam, while ϵ_{xx} represents the non-zero strain of the SFR nanobeam. ϵ_{xx} and the bending moment (M) are shown as follows, respectively (Jena *et al.* 2019):

$$\epsilon_{xx} = -z \partial_{xx} \mathrm{w}(\mathrm{x}, \mathrm{t}) \tag{11}$$

$$M = \int_A \sigma_{xx} z dA \tag{12}$$

By substituting Eqs. (11) and (12) into Eq. (10), S is derived as follows (Jena *et al.* 2019):

$$S = -\frac{1}{2} \int_0^L M \partial_{xx} \mathrm{w}(\mathrm{x}, \mathrm{t})\, dx \tag{13}$$

The kinetic energy of the nanobeam (*K*) is given as (Jena *et al.* 2019):

$$K = \frac{1}{2} \int_0^L \int_A \rho_c (\partial_t \mathrm{w}(\mathrm{x}, \mathrm{t}))^2 \, dA dx \tag{14}$$

By neglecting the rotary inertia, the work done by the external forces (V_{ext}) can be calculated as follows (Demir and Civalek 2017):

$$V_{ext} = \frac{1}{2} \int_0^L (P(T)(\partial_x \mathrm{w}(\mathrm{x}, \mathrm{t}))^2 - k_w \, \mathrm{w}(\mathrm{x}, \mathrm{t})^2) dx \tag{15}$$

where ρ_c and k_w represent the mass density of the composite material and the Winkler stiffness, respectively. $P(T)$ represents the axial force and can be shown as follows (Usmani *et al.* 2001):

$$P(T) = -E_c A \alpha_x \Delta T \tag{16}$$

Here, α_x and ΔT represent the thermal expansion coefficient in linear form and change in temperature, respectively. The Hamilton's principle is described by (Demir and Civalek 2017):

$$\delta \int_0^t (K - (S - V_{ext})) dt = 0 \tag{17}$$

Here, δ is the variation operator. The first variations of Eqs. (13)-(15) are substituted in Eq. (17). Then, a new equation of motion is obtained by considering the free harmonic motion $[w(x,t) = \mu(x) e^{-i\omega t}]$. Here, the displacement function is denoted by $\mu(x) = \mu$, while the circular frequency is denoted by ω. The equation of motion of the embedded SFR nano-beam influenced by the resulting thermal load is derived as follows (Demir and Civalek 2017):

$$d_{xx} M = -\rho_c A \omega^2 \mu + k_w \mu + P(T) d_{xx} \mu \tag{18}$$

4. Derivation of vibration equations for non-local strain gradient elasticity

In specifying the total stress statement (σ_{xx}), the scaling effect of both stress and strain gradients in NSGT is added to the strain energy equation. Then the new total stress expression is obtained as follows (Lim *et al.* 2015):

$$\sigma_{xx} = \sigma_{xx}^{c} - \nabla\sigma_{xx}^{ho} \tag{19}$$

Here, ∇ is the Laplacian operator. Classical non-local (σ_{xx}^{c}) and higher-order non-local stress tensors (σ_{xx}^{ho}) are formulated as below (Lim *et al.* 2015):

$$\sigma_{xx}^{c} = \int_0^L E_c\mu_0(x,x',\tau_0 a)\epsilon'_{xx}(x')\mathrm{d}x' \tag{20}$$

$$\sigma_{xx}^{ho} = \iota^2\int_0^L E_c(x,x',\tau_1 a)\nabla\epsilon'_{xx}(x')\mathrm{d}x' \tag{21}$$

The terms $\mu_0(x,x',\tau_0 a)$ and $\mu_1(x,x',\tau_1 a)$ are used to denote Eringen's NET. Here $\tau_0 a$ and $\tau_1 a$ refer to the non-local parameters, while ι is the material length scale parameter (MLSP). The differential operator in the NET is shown as below (Karmakar and Chakraverty 2022):

$$l_i = 1-(\tau_i a)^2\nabla^2, i = 0{,}1 \tag{22}$$

Eqs. (20)-(22) are used together to create a NSGT model for a one-dimensional material and is shown as follows (Karmakar and Chakraverty 2022):

$$\begin{aligned}&(1-(\tau_0 a)^2\partial_{xx})(1-(\tau_1 a)^2\partial_{xx})\sigma_{xx}\\&= E_c\epsilon_{xx}[(1-(\tau_1 a)^2\partial_{xx})-\iota^2(\partial_{xxxx}-(\tau_0 a)^2\partial_{xx})]\end{aligned} \tag{23}$$

Assuming $\tau_0 = \tau_1 = \tau$, Eq.(23) can be modified as follows (Karmakar and Chakraverty 2022):

$$(1-\tau^2\partial_{xx})\sigma_{xx} = E_c\epsilon_{xx}(1-\iota^2\partial_{xx}) \tag{24}$$

Multiplying Eq. (24) by z and then integrating over the cross-sectional area gives M^{nl} as follows (Karmakar and Chakraverty 2022):

$$M^{nl} = \tau^2 d_{xx}M - E_c I d_{xx}\mu + \iota^2 E_c I d_{xxxx}\mu \tag{25}$$

I denotes the moment of inertia in the above equation. By inserting Eq. (18) into Eq. (25), the non-local bending moment M^{nl} and the corresponding shear force Q^{nl} can be written as below (Karmakar and Chakraverty 2022):

$$M^{nl} = -E_c I d_{xx}\mu + \iota^2 E_c I d_{xxxx}\mu - \tau^2\rho_c A\omega^2\mu + \tau^2 P(T)d_{xx}\mu + \tau^2 k_w\mu \tag{26a}$$

$$Q^{nl} = -E_c I d_{xxx}\mu + \iota^2 E_c I d_{xxxxx}\mu - \tau^2\rho_c A\omega^2 d_x\mu + \tau^2 P(T)d_{xxx}\mu + \tau^2 k_w d_x\mu \tag{26b}$$

Finally, using Eqs. (18) and (25), the governing equation of SFR nano-beam for thermo-mechanical vibration based on NSGT derived as below (Karmakar and Chakraverty 2022):

$$\begin{aligned}&d_{xx}(\iota^2 E_c I d_{xxxx}\mu + \left(-E_c I + \tau^2 P(T)\right)d_{xx}\mu - \tau^2\rho_c A\omega^2\mu\\&+\tau^2 k_w\mu) = (-\rho_c A\omega^2 + k_w)\mu + P(T)d_{xx}\mu\end{aligned} \tag{27}$$

5. Solution method for vibration and derivation of eigenvalue problem in thermo-mechanical SFR nanobeam

This chapter presents the solution methodology for thermo-mechanical vibration of embedded constrained SFR nanobeam based on non-local strain gradient effects. In this solution method, an eigenvalue problem will be derived that can reveal the effects of Winkler stiffness, non-local parameter, MLSP, temperature variation and transverse springs on the frequencies of Euler-Bernoulli thermo-mechanical nanobeam. As shown in Fig. 1a, transverse springs are employed to support the thermo-mechanical SFR nanobeam at both ends, which helps to study different support conditions in the model. The displacement function $[\mu(x)]$ is shown below (Yaylı *et al.* 2022):

$$\mu(x) = \mu_0 \qquad x = 0 \tag{28}$$

$$\mu(x) = \mu_L \qquad x = L \tag{29}$$

$$\mu(x) = \sum_{m=1}^{\infty} D_m sin\left(\frac{m\pi}{L}x\right) \qquad 0 < x < L \tag{30}$$

Here, D_m is the unknown Fourier coefficient. As the derivation of the D_m and the calculation of the displacement function's derivatives have been covered in recent publications (Uzun *et al.* 2023, Yaylı *et al.* 2022), it is not necessary to repeat these computations in this discussion. With the help of Fourier series, a general eigenvalue problem is formulated and the natural frequencies of a thermomechanically SFR nano-beam embedded in Winkler foundation are derived. Stokes transformation is used in combination with Fourier series in combination with Fourier series to derive the eigenvalue problem of the nano-beam. The first six derivatives of $\mu(x)$ are obtained as follows (Yaylı *et al.* 2022):

$$\begin{aligned}\mu'(x) &= \frac{\mu_L-\mu_0}{L}\\&+\sum_{m=1}^{\infty} cos\left(\frac{m\pi}{L}x\right)\left[\frac{2[(-1)^m\mu_L-\mu_0]}{L}+\frac{m\pi}{L}D_m\right]\end{aligned} \tag{31a}$$

$$\mu''(x) = -\sum_{m=1}^{\infty}\frac{m\pi}{L}sin\left(\frac{m\pi}{L}x\right)\left[\frac{2[(-1)^m\mu_L-\mu_0]}{L}+\frac{m\pi}{L}D_m\right] \tag{31b}$$

$$\begin{aligned}\mu'''(x) = \frac{\mu''_L-\mu''_0}{L}+\sum_{m=1}^{\infty} cos\left(\frac{m\pi}{L}x\right)&\left[\frac{2[(-1)^m\mu''_L-\mu''_0]}{L}\right.\\&\left.-\left(\frac{m\pi}{L}\right)^2\left(\frac{2[(-1)^m\mu_L-\mu_0]}{L}+\frac{m\pi}{L}D_m\right)\right]\end{aligned} \tag{31c}$$

$$\begin{aligned}\mu^{(4)}(x) = -\sum_{m=1}^{\infty}\frac{m\pi}{L}sin\left(\frac{m\pi}{L}x\right)&\left[\frac{2[(-1)^m\mu''_L-\mu''_0]}{L}\right.\\&\left.-\left(\frac{m\pi}{L}\right)^2\left(\frac{2[(-1)^m\mu_L-\mu_0]}{L}+\frac{m\pi}{L}D_m\right)\right]\end{aligned} \tag{31d}$$

$$\begin{aligned}\mu^{(5)}(x) = \frac{\mu_L^{(4)}-\mu_0^{(4)}}{L}+\sum_{m=1}^{\infty} cos\left(\frac{m\pi}{L}x\right)&\left[\frac{2\left[(-1)^m\mu_L^{(4)}-\mu_0^{(4)}\right]}{L}\right.\\&\left.-\left(\frac{m\pi}{L}\right)^2\left(\frac{2[(-1)^m\mu''_L-\mu''_0]}{L}+\frac{m\pi}{L}D_m\right)\right]\end{aligned} \tag{31e}$$

$$\begin{aligned}\mu^{(6)}(x) = -\sum_{m=1}^{\infty}\frac{m\pi}{L}sin\left(\frac{m\pi}{L}x\right)&\left[\frac{2\left[(-1)^m\mu_L^{(4)}-\mu_0^{(4)}\right]}{L}\right.\\&-\left(\frac{m\pi}{L}\right)^2\left(\frac{2[(-1)^m\mu''_L-\mu''_0]}{L}\right)\\&\left.+\left(\frac{m\pi}{L}\right)^4\left(\frac{2[(-1)^m\mu_L-\mu_0]}{L}+\frac{m\pi}{L}D_m\right)\right]\end{aligned} \tag{31f}$$

Via these derivatives and the conditions $d_{xx}\mu_L = d_{xx}\mu_0 = d_{xxxx}\mu_L = d_{xxxx}\mu_0 = 0$, D_m can be obtained as follows:

$$D_m = \frac{\begin{array}{c} 2\mathit{\Pi}_m(-P(T) - \tau^2\rho_c \mathrm{A}\omega^2 + \tau^2 k_w \\ +(E_c I - P(T)\tau^2)\mathit{\Pi}_m^2 + E_c I\iota^2\mathit{\Pi}_m^4)(\mu_0 - (-1)^m\mu_L) \end{array}}{\begin{array}{c} L(-\rho_c \mathrm{A}\omega^2 - (P(T) + \tau^2\rho_c \mathrm{A}\omega^2)\mathit{\Pi}_m^2 \\ +(E_c I - P(T)\tau^2)\mathit{\Pi}_m^4 + E_c I\iota^2\mathit{\Pi}_m^6 + k_w(1 + \tau^2\mathit{\Pi}_m^2)) \end{array}} \tag{32}$$

Here $\mathit{\Pi}_m = \frac{M\mathit{\Pi}}{L}$, once D_m is obtained, taking into account the expressions in Eq. (32), $\mu(x)$ can be expressed with respect to D_m as follows:

$$\mu(x) = \sum_{n=1}^{\infty} \frac{\begin{array}{c} 2\mathit{\Pi}_m(-P(T) - \tau^2\rho_c \mathrm{A}\omega^2 + \tau^2 k_w \\ +(E_c I - P(T)\tau^2)\mathit{\Pi}_m^2 + E_c I\iota^2\mathit{\Pi}_m^4)(\mu_0 - (-1)^m\mu_L) \end{array}}{\begin{array}{c} L(-\rho_c \mathrm{A}\omega^2 - (P(T) + \tau^2\rho_c \mathrm{A}\omega^2)\mathit{\Pi}_m^2 \\ +(E_c I - P(T)\tau^2)\mathit{\Pi}_m^4 + E_c I\iota^2\mathit{\Pi}_m^6 + k_w(1 + \tau^2\mathit{\Pi}_m^2)) \end{array}} sin(\mathit{\Pi}_m x) \tag{33}$$

Here, the equation $\mu(x)$ becomes an analytical static solution for the suggested model, which depends on the boundary displace-ments and consists of an infinite series. A Fourier sine series solution for thermo-mechanical vibration analysis of a simply supported SFR nano-beam has been published previously (Karmakar and Chakraverty 2022). The following force boundary conditions are considered for the restricted thermo-mechanical SFR nano-beam seen in Fig. 1b:

$$Q^{nl} = S_0\mu_0 \qquad x = 0 \tag{34}$$

$$Q^{nl} = -S_L\mu_L \qquad x = L \tag{35}$$

Here S_0 and S_L are the stiffnesses of the transversely deformable springs. The nano-beam is supported by deformable springs at the initial and final ends. By varying the stiffness of the transversely deformable springs at both edges of the nano-beam, the support conditions can be varied. The force boundary conditions for the initial and final ends of the thermomechanically loaded and embedded SFR nano-beam are shown below, respectively:

$$\begin{aligned} -E_c I d_{xxx}\mu + \iota^2 E_c I d_{xxxxx}\mu - \tau^2\rho_c A\omega^2 d_x\mu \\ + \tau^2 P(T) d_{xxx}\mu + \tau^2 k_w d_x\mu = S_0\mu_0 \end{aligned} \tag{36}$$

$$\begin{aligned} -E_c I d_{xxx}\mu + \iota^2 E_c I d_{xxxxx}\mu - \tau^2\rho_c A\omega^2 d_x\mu \\ + \tau^2 P(T) d_{xxx}\mu + \tau^2 k_w d_x\mu = -S_L\mu_L \end{aligned} \tag{37}$$

In this study, the Stokes' transform technique and Fourier sine series are used to present an eigenvalue problem for the free vibrational frequencies of embedded thermo-mechanical SFR nano-beam. Two homogeneous equations for the free thermo-mechanical vibration analysis of SFR nanobeam can be derived as follows:

$$\begin{aligned} &\left(-S_0 + \frac{\tau^2\lambda_1}{L} + \sum_{m=1}^{\infty} \frac{\begin{array}{c} 2L\lambda_1(-E_c I m^2\pi^2\phi_1 \\ +L^2\tau^2\phi_2 - L^4\tau^2 k_w) \end{array}}{\begin{array}{c} \lambda_2 + m^2\pi^2\phi_3(-E_c I + P(T)\tau^2) \\ +L^2\phi_3\phi_4 - \phi_5 k_w \end{array}}\right)\mu_0 \\ &+ \left(-\frac{\tau^2\lambda_1}{L} - \sum_{m=1}^{\infty} \frac{\begin{array}{c} 2(-1)^m L\lambda_1(-E_c I m^2\pi^2\phi_1 \\ +L^2\tau^2\phi_2 - L^4\tau^2 k_w) \end{array}}{\begin{array}{c} \lambda_2 + m^2\pi^2\phi_3(-E_c I + P(T)\tau^2) \\ +L^2\phi_3\phi_4 - \phi_5 k_w \end{array}}\right)\mu_L \\ &= 0 \end{aligned} \tag{38}$$

$$\begin{aligned} &\left(-\frac{\tau^2\lambda_1}{L} - \sum_{m=1}^{\infty} \frac{2(-1)^m L\lambda_1\begin{pmatrix} -E_c I m^2\pi^2\phi_1 \\ +L^2\tau^2\phi_2 - L^4\tau^2 k_w \end{pmatrix}}{\begin{array}{c} \lambda_2 + m^2\pi^2\phi_3(-E_c I + P(T)\tau^2) \\ +L^2\phi_3\phi_4 - \phi_5 k_w \end{array}}\right)\mu_0 \\ &+ \left(-S_L + \frac{\tau^2\lambda_1}{L} + \sum_{m=1}^{\infty} \frac{2L\lambda_1\begin{pmatrix} -E_c I m^2\pi^2\phi_1 \\ +L^2\tau^2\phi_2 - L^4\tau^2 k_w \end{pmatrix}}{\begin{array}{c} \lambda_2 + m^2\pi^2\phi_3\begin{pmatrix} -E_c I \\ +P(T)\tau^2 \end{pmatrix} \\ +L^2\phi_3\phi_4 - \phi_5 k_w \end{array}}\right)\mu_L \\ &= 0 \end{aligned} \tag{39}$$

where,

$$L^2 + m^2\pi^2\iota^2 = \phi_1 \tag{40}$$

$$m^2 P(T)\pi^2 + L^2\rho_c \mathrm{A}\omega^2 = \phi_2 \tag{41}$$

$$L^2 m^2\pi^2 = \phi_3 \tag{32}$$

$$P(T) + \tau^2\rho_c \mathrm{A}\omega^2 = \phi_4 \tag{43}$$

$$L^6 + L^4\tau^2\phi_3 = \phi_5 \tag{44}$$

$$\rho_c \mathrm{A}\omega^2 - k_w = \lambda_1 \tag{45}$$

$$-E_c I m^6\pi^6\iota^2 + L^6\rho_c \mathrm{A}\omega^2 = \lambda_2 \tag{46}$$

The eigenvalue problem can be used to identify the lateral vibration frequencies of the constrained nano-beam under the effects of non-local parameter, MLPS, thermal load and foundation. And this eigenvalue problem can be expressed as follows using the Eqs. (38) and (39):

$$\begin{bmatrix} \text{И}_{11} & \text{И}_{12} \\ \text{И}_{21} & \text{И}_{22} \end{bmatrix} \begin{bmatrix} \mu_0 \\ \mu_L \end{bmatrix} = 0 \tag{47}$$

The solution based on the eigenvalue problem is applied to analyze various cases between simply-supported and free boundary conditions. Here $\text{И}_{i,j}$ represents the coefficient matrix and its determinant is set to zero:

$$|\text{И}_{i,j}| = 0 \qquad (i, j = 1,2) \tag{48}$$

where,

$$\begin{aligned} \text{И}_{11} &= -S_0 + \frac{\tau^2\lambda_1}{L} \\ &+ \sum_{m=1}^{\infty} \frac{2L\lambda_1(-E_c I m^2\pi^2\phi_1 + L^2\tau^2\phi_2 - L^4\tau^2 k_w)}{\lambda_2 + m^2\pi^2\phi_3(-E_c I + P(T)\tau^2) + L^2\phi_3\phi_4 - \phi_5 k_w} \end{aligned} \tag{49}$$

$$\begin{aligned} \text{И}_{12} &= -\frac{\tau^2\lambda_1}{L} \\ &- \sum_{m=1}^{\infty} \frac{2(-1)^m L\lambda_1(-E_c I m^2\pi^2\phi_1 + L^2\tau^2\phi_2 - L^4\tau^2 k_w)}{\lambda_2 + m^2\pi^2\phi_3(-E_c I + P(T)\tau^2) + L^2\phi_3\phi_4 - \phi_5 k_w} \end{aligned} \tag{50}$$

$$\begin{aligned} \text{И}_{21} &= -\frac{\tau^2\lambda_1}{L} \\ &- \sum_{m=1}^{\infty} \frac{2(-1)^m L\lambda_1(-E_c I m^2\pi^2\phi_1 + L^2\tau^2\phi_2 - L^4\tau^2 k_w)}{\lambda_2 + m^2\pi^2\phi_3(-E_c I + P(T)\tau^2) + L^2\phi_3\phi_4 - \phi_5 k_w} \end{aligned} \tag{51}$$

Table 1 Comparison of the first and second modes' dimensionless frequencies of homogeneous simply-supported nano-beams

Mode number	τ/L = 1							
	P_{temp}	ι/L	Karmakar and Chakraverty (2022)	This study	P_{temp}	ι/L	Karmakar and Chakraverty (2022)	This study
1	-0.4	0.5	11.6198	11.6198	-0.2	0.5	11.5346	11.5346
		1	14.1900	14.1900		1	14.1203	14.1203
	0.4	0.5	11.2750	11.2750	0.2	0.5	11.3621	11.3621
		1	13.9090	13.9090		1	13.9798	13.9798
2	-0.4	0.5	23.1150	23.1150	-0.2	0.5	22.9436	22.9436
		1	40.9186	40.9186		1	40.8220	40.8220
	0.4	0.5	22.4214	22.4214	0.2	0.5	22.5968	22.5968
		1	40.5309	40.5309		1	40.6281	40.6281

$$\text{И}_{22} = -S_L + \frac{\tau^2 \lambda_1}{L} + \sum_{m=1}^{\infty} \frac{2L\lambda_1(-E_c I m^2 \pi^2 \phi_1 + L^2 \tau^2 \phi_2 - L^4 \tau^2 k_w)}{\lambda_2 + m^2 \pi^2 \phi_3(-E_c I + P(T)\tau^2) + L^2 \phi_3 \phi_4 - \phi_5 k_w} \tag{52}$$

In this theoretical work, the thermo-mechanical vibration of embedded restrained SFR nano-beams is shown. With the help of Eq. (48), the thermo-mechanical vibration frequencies of SFR nano-beam can be calculated in terms of Winkler stiffness, elastic spring stiffness, MLSP, non-local parameter, thermal load and fiber and matrix properties. It should also be noted that in this study, the vibrations of SFR nanofibers are investigated with respect to size, Winkler foundation and thermal load effects. Deformable boundary conditions are considered in the analysis. The reason is that under rigid boundary conditions, collapse and/or rotation are assumed to be fully constrained, but in practice, it is obvious that rigidities cannot be fully achieved when the elements are supported. By using the eigenvalue problem obtained in this study, which includes spring parameters representing deformable boundary conditions, the effect of deformable boundaries can be easily analyzed. Furthermore, assigning higher stiffnesses to the spring parameters makes the boundaries simple supported, while assigning smaller stiffnesses to the spring parameters makes the boundaries free. Therefore, the presented solution is effective for investigating both rigid and non-rigid end conditions.

6. Numerical results

In this part of the study, comparison studies and numerical examples for SFR nano-beams demonstrated. Via a number of figures, the thermo-mechanical vibration frequencies of SFR nano-beams are examined in terms of various parameters such as fiber and matrix mass densities ratio, Winkler foundation, temperature variation, fiber and matrix Young's moduli ratio, MLSP and non-local parameter. While calculating the effects of these parameters, temperature and Winkler foundation effects are considered in dimensionless forms as follows:

$$K_W = \frac{k_w L^4}{E_m I} \tag{53}$$

$$P_{temp} = \frac{PL^2}{E_m I} \tag{54}$$

Also, non-dimensional frequencies are obtained as follows:

$$\varpi = \omega L^2 \sqrt{\frac{\rho_m \mathrm{A}}{E_m I}} \tag{56}$$

In our study, rectangular nano-beam is considered and for this reason moment of inertia is defined by: $I = bh^3/12$. The work is based on some fixed values of parameters. Unless otherwise stated for the examples, the following parameters are constant throughout the work: $\rho_f/\rho_m = 2.5$, $E_f/E_m = 5$, $S_0 = S_L = 10^{18}\, nN/nm$, $b = 2\, nm$, $h = 3\, nm$, $L = 15h\, nm$, $l/d = 4$, $m = 35$, $V_f = 0.5$, $\tau = 1\, nm, \iota = 2nm$.

Before proceeding with the examination of the SFR nano-beam, a validation study is included. This comparison study for homogeneous nano-beams is shown in Table 1. As can be understood, the eigenvalue solution presented in this study is effective and gives accurate results. Karmakar and Chakraverty (2022) presented the frequencies of thermal vibration of nano-beams with fixed and changing cross-section according to the NSGT. Some dimensionless frequency values found for nano-beams with fixed cross-section and $K_W = k_w L^4/EI = 100$ in the study of Karmakar and Chakraverty (2022) are considered for comparison. Karmakar and Chakraverty (2022) calculated the dimensionless frequencies via the following expression:

$$\varpi^2 = \frac{\rho \mathrm{A} \omega^2 L^4}{EI} \tag{55}$$

Figs. 2-7 show the effect of varying *l/d* ratios on the composite nano-beam's frequencies. *l/d* values are chosen as 2, 4, 6, 8, 10, 12. It is clear from the graphs that as the *l/d* ratio increases, the dimensionless frequencies also increase. Different parameters are used in the graphs for the first four modes. In these Figs., some parameters are taken differently in order to make comparative studies. In the analysis for Fig. 2, both foundation and temperature effects are neglected. That is, $K_W = 0$ & $P_{temp} = 0$. In Fig. 3, the temperature effect is neglected ($P_{temp} = 0$) and the foundation effect is included in the analysis by taking $K_W = 30$. As can be understood, the frequencies at which the foundation

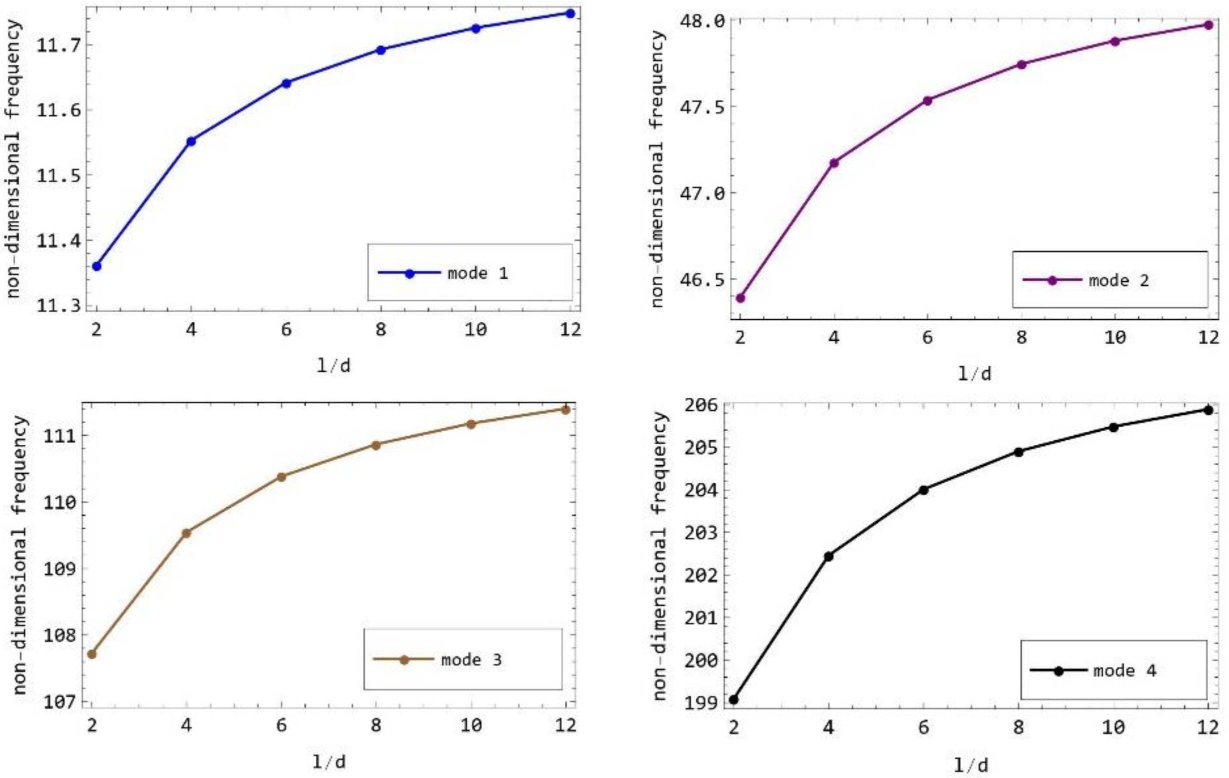


Fig. 2 $\bar{\omega}$ of reinforced nano-beams for various *l*/*d* ratios ($K_W = 0$ & $P_{temp} = 0$)

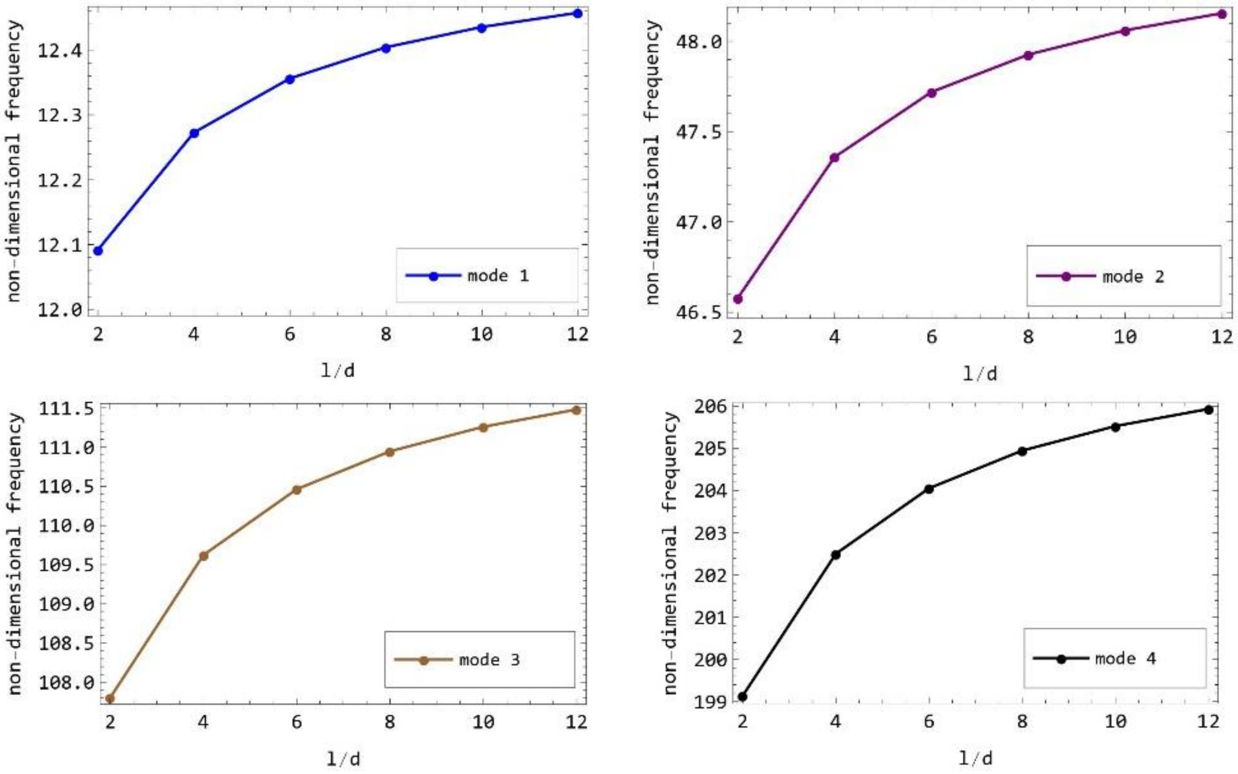


Fig. 3 $\bar{\omega}$ of reinforced nano-beams for various *l*/*d* ratios ($K_W = 30$ & $P_{temp} = 0$)

effect is included in the analysis are higher. It can be easily concluded that the Winkler foundation strengthens the composite nano-beam and increases the frequency.

In Figs. 4 and 5, P_{temp} is +1.5 and -1.5 respectively, while both figures are independent of the foundation effect. Since Fig. 2 is independent of both the foundation effect

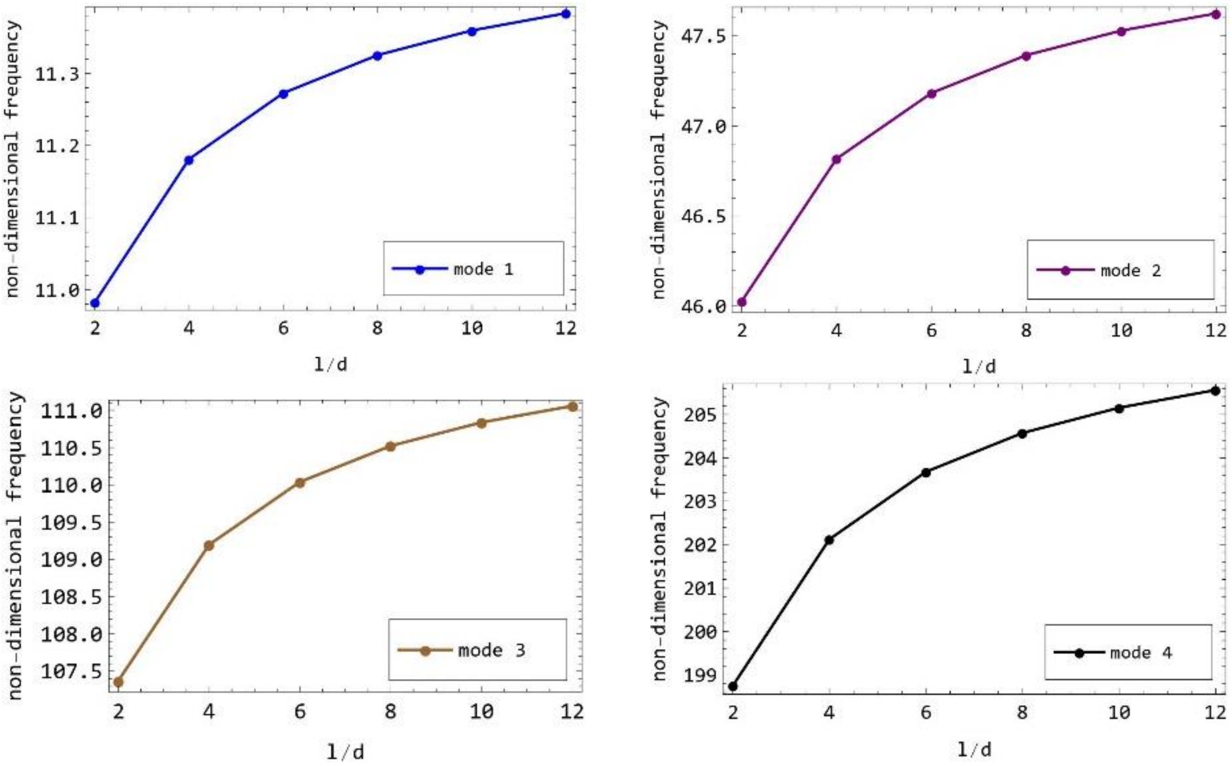


Fig. 4 $\overline{\omega}$ of reinforced nano-beams for various *l/d* ratios ($K_W = 0$ & $P_{temp} = +1.5$)

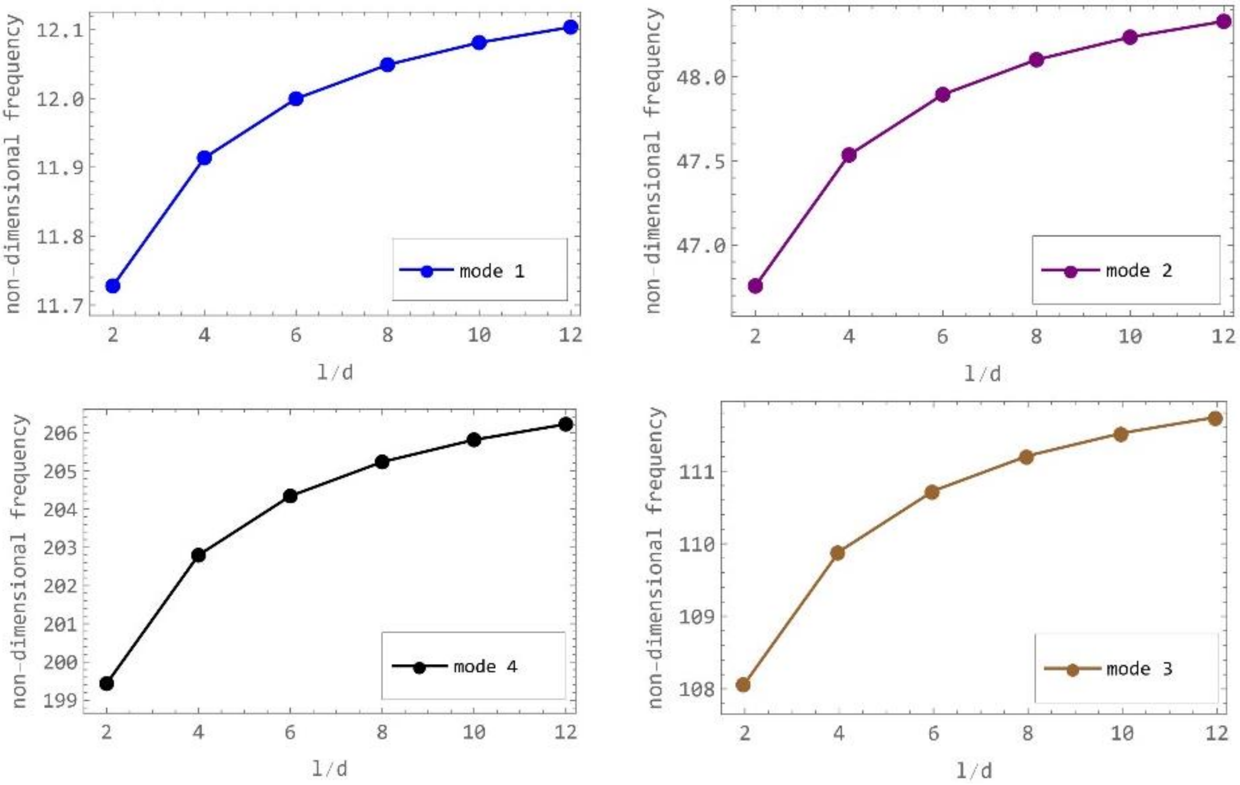


Fig. 5 $\overline{\omega}$ of reinforced nano-beams for various *l/d* ratios ($K_W = 0$ & $P_{temp} = -1.5$)

and the temperature effect, it can be compared with Figs. 2, 4, 5 how the temperature effect drives the frequencies. Looking at these three figures, it is clear that in the case where $P_{temp} = 0$, the frequency values are between the

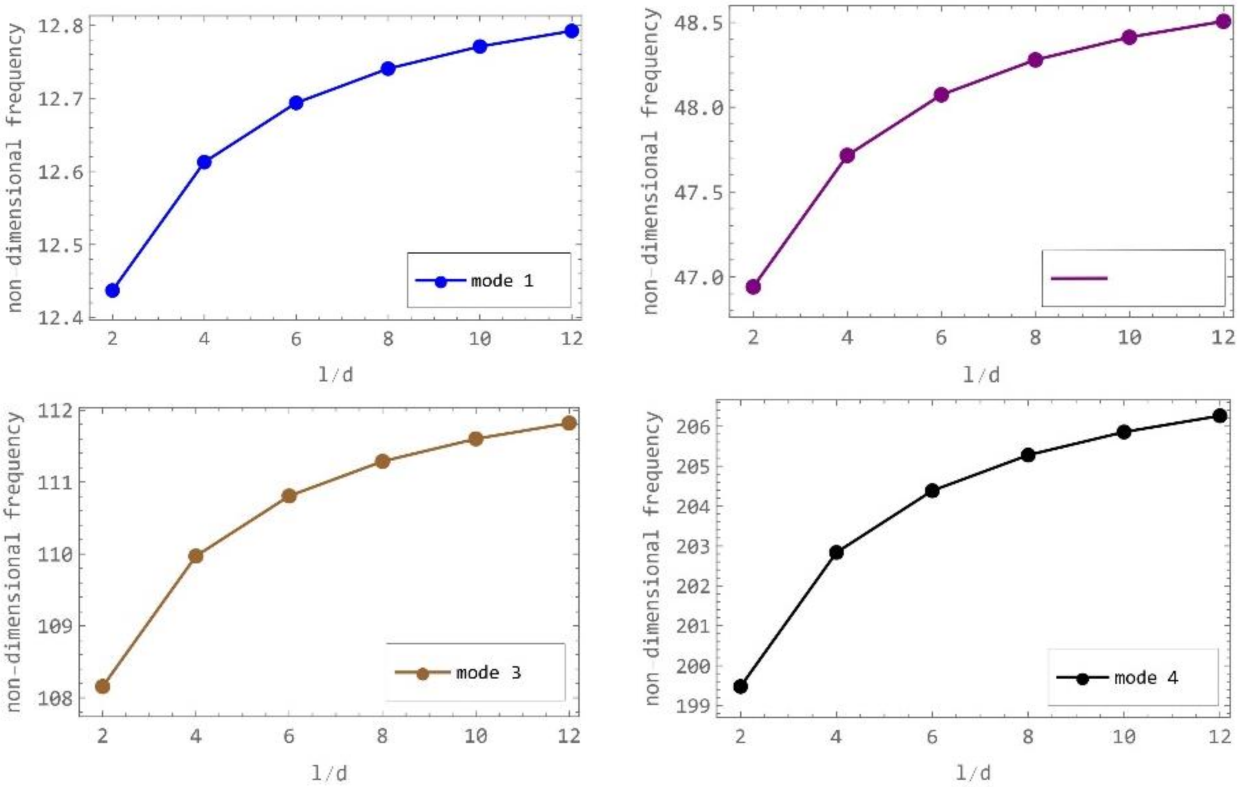


Fig. 6 $\bar{\omega}$ of reinforced nano-beams for various *l/d* ratios ($K_W = 30$ & $P_{temp} = -1.5$)

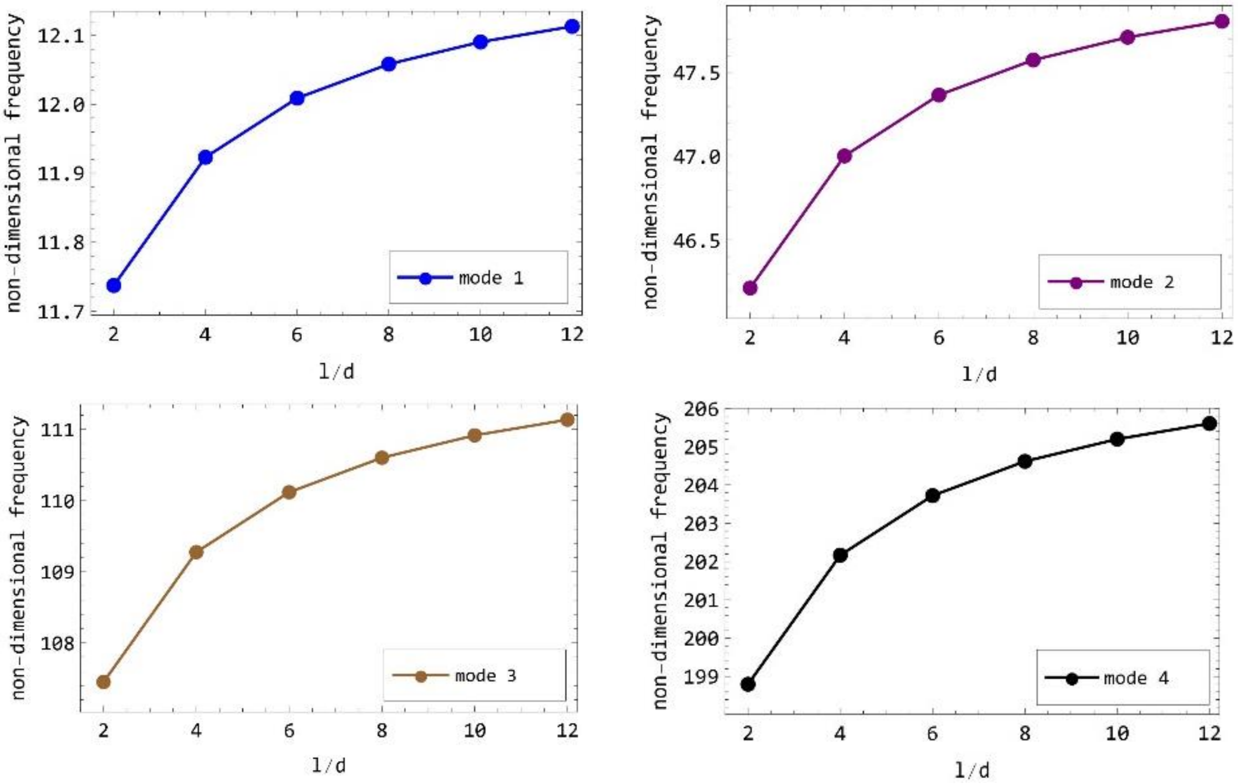


Fig. 7 $\bar{\omega}$ of reinforced nano-beams for various *l/d* ratios ($K_W = 30$ & $P_{temp} = +1.5$)

cases where P_{temp} is +1.5 and -1.5. In Fig. 4, where P_{temp} =+1.5, the frequencies are the lowest. It is understood from this that the $\bar{\omega}$ increases when the temperature change is negative, while the $\bar{\omega}$ decreases

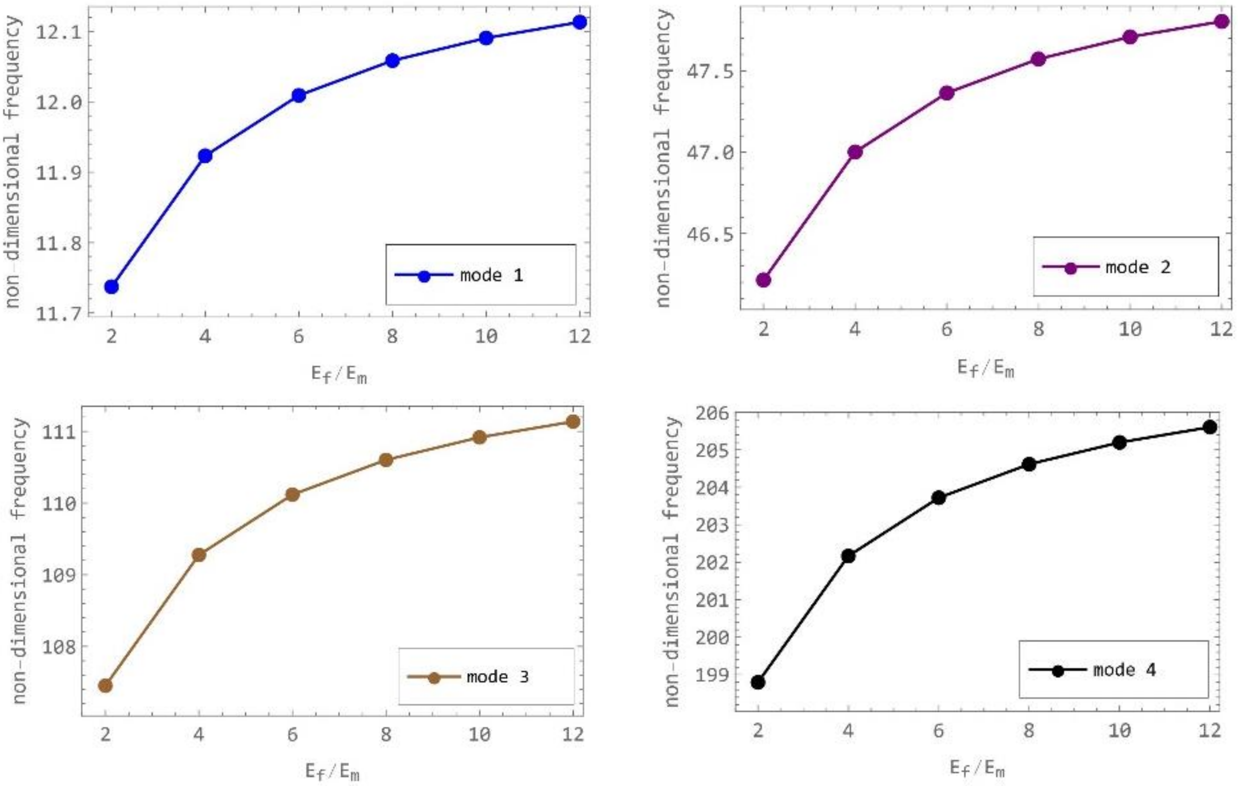


Fig. 8 $\bar{\omega}$ of reinforced nano-beams for various E_f/E_m ratios ($K_W = 30$ & $P_{temp} = -1.5$)

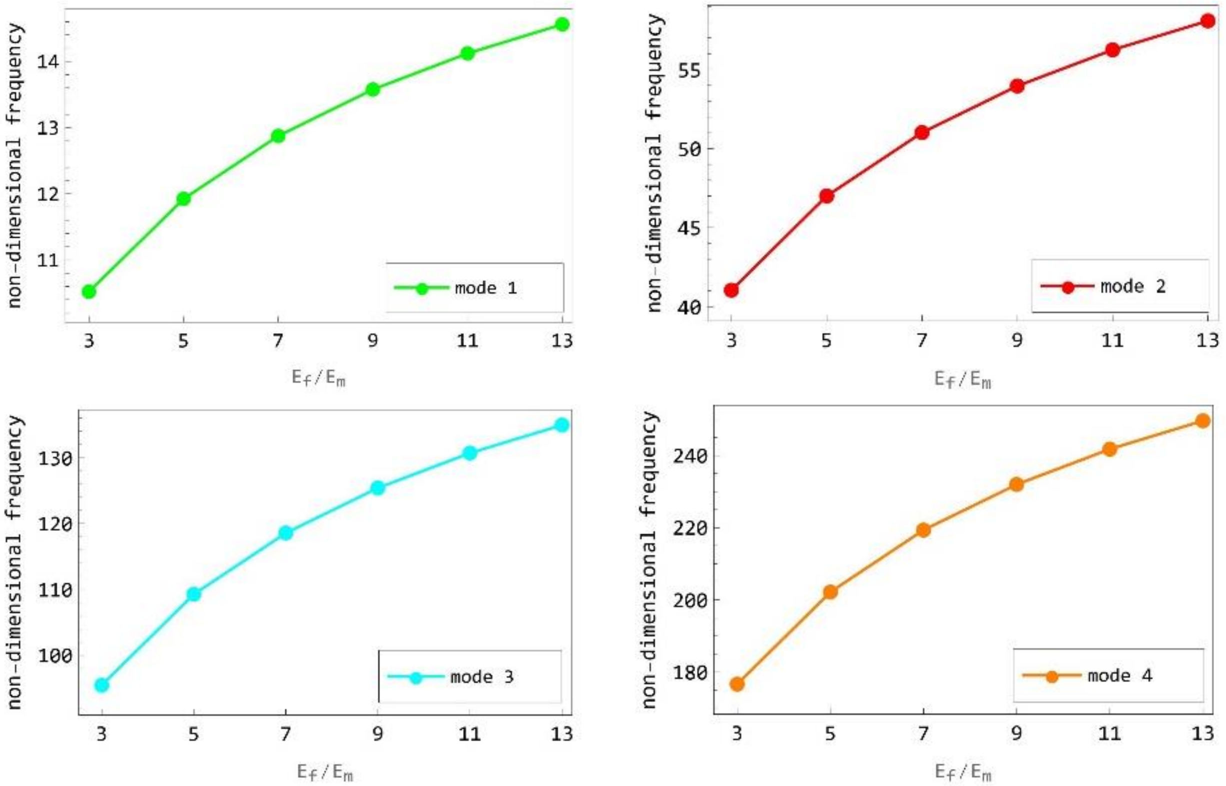


Fig. 9 $\bar{\omega}$ of reinforced nano-beams for various E_f/E_m ratios ($K_W = 30$ & $P_{temp} = +1.5$)

when the temperature change is positive.

In Figs. 6 and 7, both the foundation effect and the temperature effect are considered. In Fig. 6, P_{temp} is -1.5, while in Fig. 7, P_{temp} is +1.5. Fig. 3, where the foundation

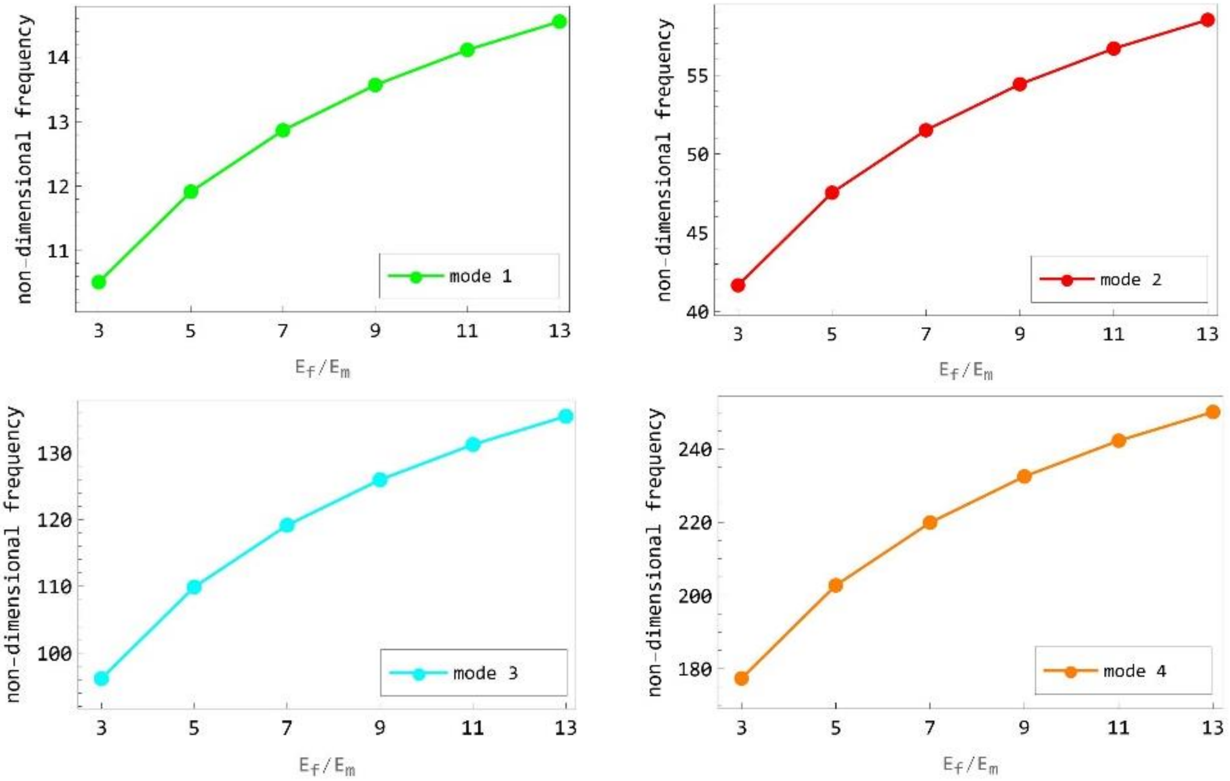


Fig. 10 ϖ of reinforced nano-beams for various E_f/E_m ratios ($K_W = 0$ & $P_{temp} = -1.5$)

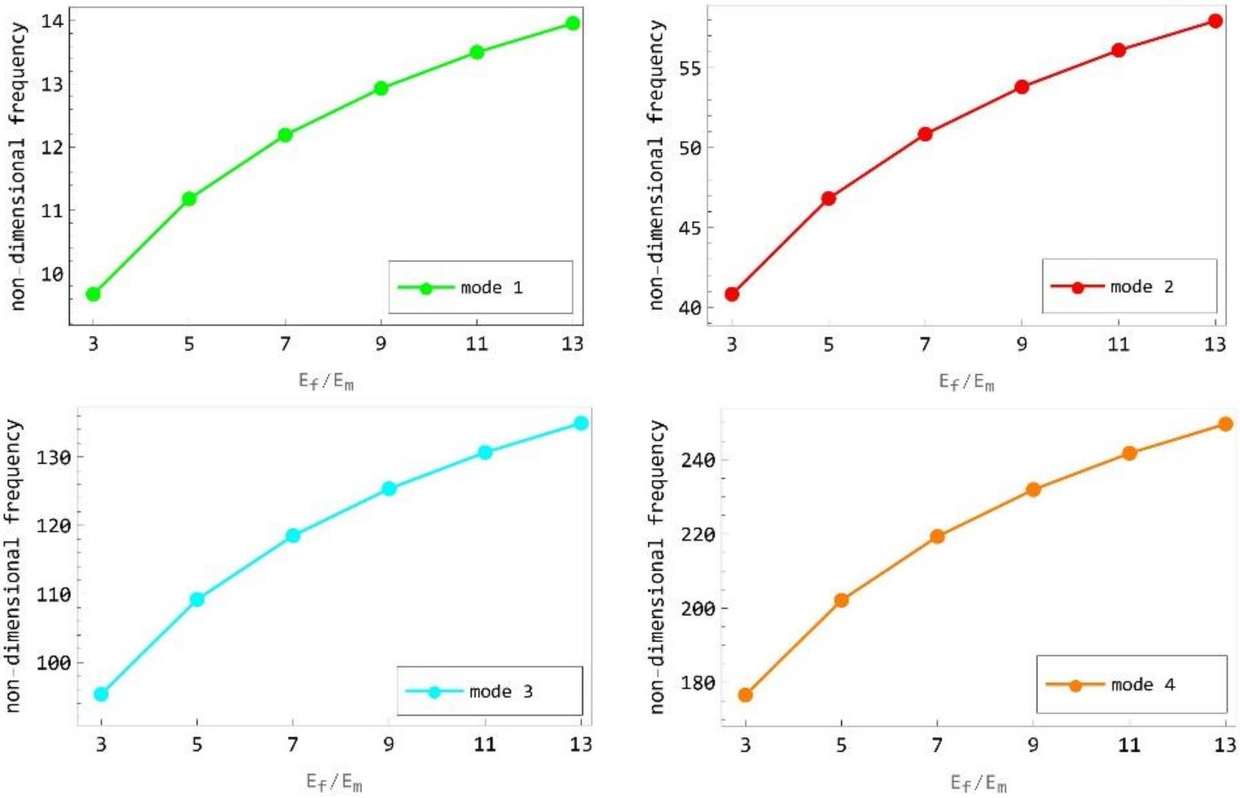


Fig. 11 ϖ of reinforced nano-beams for various E_f/E_m ratios ($K_W = 0$ & $P_{temp} = +1.5$)

effect is taken into account but the temperature effect is not taken into account, can be compared with these two graphs. When the changes in frequencies in the *l/d* range are compared, it becomes clear that the effect of *l/d* is at least

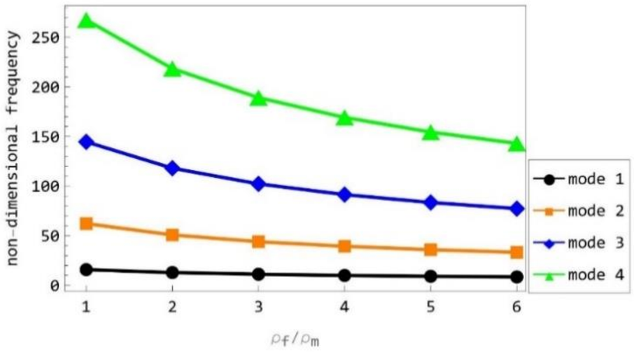


Fig. 12 ϖ of reinforced nano-beams for various ρ_f/ρ_m ratios ($K_W = 30$ & $P_{temp} = +1.5$)

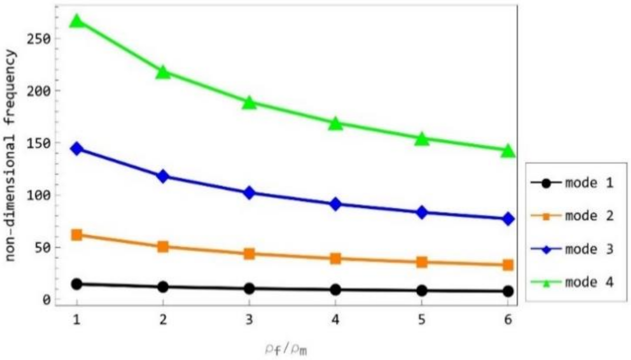


Fig. 13 ϖ of reinforced nano-beams for various ρ_f/ρ_m ratios ($K_W = 0$ & $P_{temp} = +1.5$)

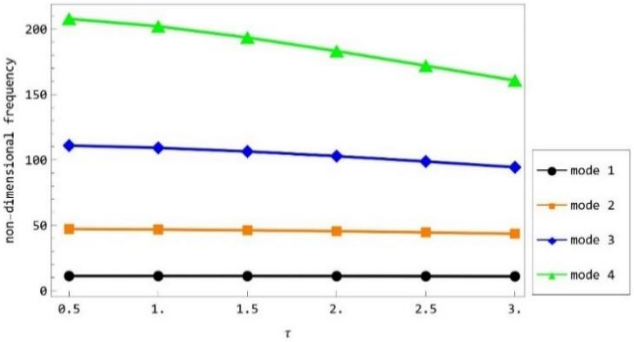


Fig. 14 Non-local parameter effect on ϖ ($K_W = 0$ & $P_{temp} = +1.5$)

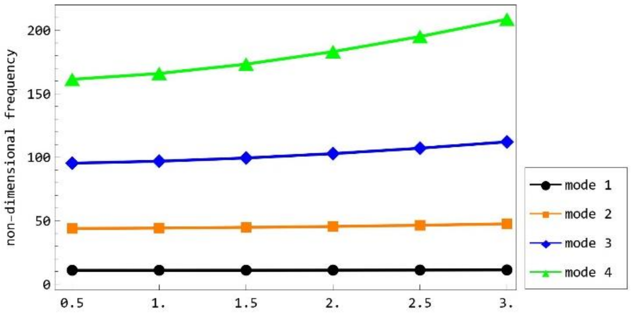


Fig. 15 MLSP effect on ϖ ($K_W = 0$ & $P_{temp} = +1.5$)

negative P_{temp} values are calculated. The conclusion to be drawn here is that as P_{temp} increases, the effect of *l/d* on frequencies increases.

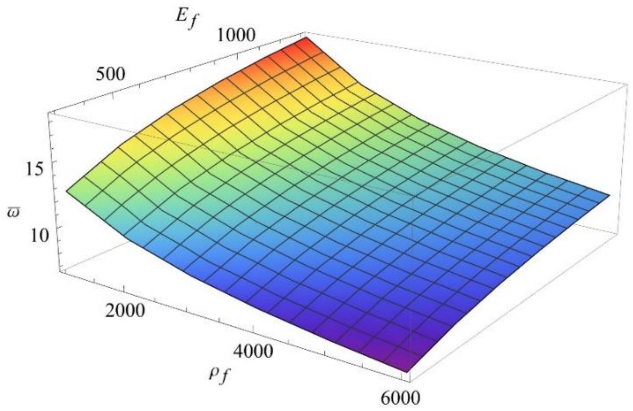


Fig. 16 Effect of ρ_f and E_f on non-dimensional frequencies ($K_W = 0$ & $P_{temp} = +1.5$)

Figs. 8-11 show the effect of the E_f/E_m ratio. For these graphs, the first four modes are considered. When determining the E_f/E_m ratio, the modulus of elasticity of the matrix is kept constant and the modulus of elasticity of the fiber is increased. It is clear from the graphs that increasing the E_f/E_m ratio, i.e. increasing E_c, increases the dimensionless frequencies. Because the increase in the modulus of elasticity of the fibers strengthening the element also strengthens the composite and the frequencies increase.

In Figs. 8 and 9, the effect of E_f/E_m is examined for $K_W = 30$ also P_{temp} = -1.5 and $P_{temp} =$ +1.5 cases, respectively. When the increases in frequencies are examined, it is understood that the effect of E_f/E_m is higher in the analysis made for P_{temp} = +1.5. This effect can also be confirmed by Figs. 10 and 11, which are considered independently of the foundation effect. In addition, if Figs. 8 and 10 or Figs. 9 and 11 are compared, the differences in the effect of E_f/E_m on the frequencies in the cases of foundation-dependent and foundation-independent can be examined. As a result of this analysis, it is revealed that, independent of the foundation effect, the frequencies increase more with the increase of E_f/E_m.

Figs. 12 and 13 are drawn to show the effect of the ρ_f/ρ_m ratio on ϖ. For this purpose, the mass density of the matrix is kept constant and the mass density of the fiber is increased. In the graphs where the ratio is considered as 1, 2, 3, 4, 5, 6, it is clear that increasing this ratio results in a decrease in non-dimensional frequencies. This can be explained by the increased mass density of the element. The increase in the mass density of the fiber leads to an increase in the mass density of the composite and causes a decrease in frequencies.

As is well known, the theory considered in this paper includes the effects of both τ and the ι. To illustrate the effect of the non-local parameter, the vibration frequencies of the first four modes are plotted in Fig. 14. It is clear from the graph that the third and fourth mode frequencies decrease as τ increases. For the first two modes, the frequencies remain the same or the change is so small that it is not easily noticeable. It is understood from this figure that there is a decrease in each mode. The rate of this decrease is not the same in each mode. It is clear from Fig. 14 that as the mode number increases, the reduction due to the increase of the non-local parameter is much greater.

In Fig. 15, the effect of ι is analyzed. Here the variation of the frequencies of the first four modes is considered for varying values of ι. While the increases in the third and fourth modes are clearly observed, it is necessary to look more carefully for the first two modes. Increasing ι leads to an increment in the frequency of each mode. The effect of this parameter, which has a similar but opposite effect to τ, is obviously more pronounced for the higher modes.

As can be seen, there are too many parameters and too many variables in elastic boundary conditions. These include: the effect of elastic springs on thermo-mechanical vibration frequencies, elastic foundation effect, non-local and strain gradient effects, reinforcing effect of short-fibers used. As it is known, if all these effects are analyzed, the article would be quite long. Here, some effects not found in the literature review are plotted and graphed. In the figure above (Fig. 16), the effect of increasing the elasticity modulus and mass density of the fibers on the frequencies is shown together. To repeat, as the elasticity modulus of the fibers increases, the frequencies increase and as the mass of the fibers increases, the frequencies decrease.

7. Conclusions

In this work, a general solution method for thermo-mechanical vibration of a SFR nano-beam is presented. The SFR nano-beam is modeled with transverse springs at both ends and also, it is rested on Winkler foundation. The shear force expression at the end points of the SFR nano-beam is written via a non-local strain gradient elasticity found in the literature, which includes both non-local and material length scale parameters. Then, a set of linear equations including of infinite series is derived. By excluding the deformable transverse springs, a matrix of coefficients is obtained consisting temperature load, foundation parameter, composite material properties and size parameters is gained. The determinant of this coefficients matrix gives the thermo-mechanical vibration frequencies of the embedded SFR nano-beam.

References

Abdelrahman, A.A., Esen, I., Daikh, A.A. and Eltaher, M.A. (2023), “Dynamic analysis of FG nanobeam reinforced by carbon nanotubes and resting on elastic foundation under moving load”, *Mech. Based Des. Struct. Mach.*, **51**(10), 5383-5406. https://doi.org/10.1080/15397734.2021.1999263.

Abdullah, S.S., Hosseini-Hashemi, S., Hussein, N.A. and Nazemnezhad, R. (2020), “Thermal stress and magnetic effects on nonlinear vibration of nanobeams embedded in nonlinear elastic medium”, *J. Therm. Stress.*, **43**(10), 1316-1332. https://doi.org/10.1080/01495739.2020.1780175.

Abrate, S. (1986), “The mechanics of short fiber-reinforced composites: A review”, *Rubber Chem. Technol.*, **59**(3), 384-404. https://doi.org/10.5254/1.3538207.

Ahmadi, I., Sladek, J. and Sladek, V. (2024), “Size dependent free vibration analysis of 2D-functionally graded curved nanobeam by meshless method”, *Mech. Adv. Mater. Struct.*, **31**(18), 4352-4373. https://doi.org/10.1080/15376494.2023.2195400.

Alfred, P.B., Ossia, C.V. and Big-Alabo, A. (2024), “Free nonlinear vibration analysis of a functionally graded microbeam resting on a three-layer elastic foundation using the continuous piecewise linearization method”, *Arch. Appl. Mech.*, **94**(1), 57-80. https://doi.org/10.1007/s00419-023-02505-1.

Attia, M.A., Matbuly, M.S., Osman, T. and AbdElkhalek, M. (2024), “Dynamic analysis of double cracked bi-directional functionally graded nanobeam using the differential quadrature method”, *Acta Mech.*, **235**(4), 1961-2012. https://doi.org/10.1007/s00707-023-03797-8.

Barati, M.R. and Zenkour, A. (2018), “Forced vibration of sinusoidal FG nanobeams resting on hybrid Kerr foundation in hygro-thermal environments”, *Mech. Adv. Mater. Struct.*, **25**(8), 669-680. https://doi.org/10.1080/15376494.2017.1308603.

Beni, Y.T. (2023), “Size-dependent electro-thermal buckling analysis of flexoelectric microbeams”, *Int. J. Struct. Stabil. Dyn.*, **24**(8). https://doi.org/10.1142/S0219455424500937.

Chen, J., Qu, W., Ye, C., Zhao, Z. and Wang, H. (2024), “Nonlinear vibration and dynamic stability of dielectric sandwich micro-beams”, *Int. J. Mech. Sci.*, https://doi.org/10.1016/j.ijmecsci.2023.108738.

Civalek, Ö., Uzun, B. and Yaylı, M.Ö. (2022), “Nonlocal free vibration of embedded short-fiber-reinforced nano-/micro-rods with deformable boundary conditions”, *Materials*, **15**(19), 6803. https://doi.org/10.3390/ma15196803.

Civalek, Ö., Uzun, B. and Yaylı, M.Ö. (2023), “Torsional static and free vibration analysis of noncircular short-fiber-reinforced microwires with arbitrary boundary conditions”, *Polym. Compos.*, Early View. https://doi.org/10.1002/PC.27321.

Dean, D., Obore, A.M., Richmond, S. and Nyairo, E. (2006), “Multiscale fiber-reinforced nanocomposites: Synthesis, processing and properties”, *Compos. Sci. Technol.*, **66**(13), 2135-2142. https://doi.org/10.1016/J.COMPSCITECH.2005.12.015.

Dehkordi, H.R.B. and Beni, Y.T. (2023), “Size-dependent coupled bending-torsional vibration of functionally graded carbon nanotube reinforced composite Timoshenko microbeams”, *Arch. Civ. Mech. Eng.*, **23**(3), 1-12. https://doi.org/10.1007/s43452-023-00725-4.

Demir, Ç. and Civalek, Ö. (2017), “A new nonlocal FEM via Hermitian cubic shape functions for thermal vibration of nano beams surrounded by an elastic matrix”, *Compos. Struct.*, **168**, 872-884. https://doi.org/10.1016/J.COMPSTRUCT.2017.02.091.

Ebnali Samani, M.S. and Beni, Y.T. (2018), “Size dependent thermo-mechanical buckling of the flexoelectric nanobeam”, *Mater. Res. Express*, **5**(8), 085018. https://doi.org/10.1088/2053-1591/AAD2CA.

Ebrahimi, F., Daman, M. and Mahesh, V. (2019), “Thermo-mechanical vibration analysis of curved imperfect nano-beams based on nonlocal strain gradient theory”, *Adv. Nano Res.*, **7**(4), 249-263. https://doi.org/10.12989/anr.2019.7.4.249.

Ebrahimi, F. and Jafari, A. (2016), “Thermo-mechanical vibration analysis of temperature-dependent porous FG beams based on Timoshenko beam theory”, *Struct. Eng. Mech.*, **59**(2), 343-371. https://doi.org/10.12989/SEM.2016.59.2.343.

El-Shahrany, H.D. and Zenkour, A.M. (2023), “A nonlocal vibration suppression for a multilayered magneto-viscoelastic nanobeam on a three-parameter-type medium”, *Mech. Adv. Mater. Struct.*. https://doi.org/10.1080/15376494.2023.2278763.

Feo, L., Lovisi, G. and Penna, R. (2023), “Free vibration analysis of functionally graded nanobeams based on surface stress-driven nonlocal model”, *Mech. Adv. Mater. Struct.*, 1-9. https://doi.org/10.1080/15376494.2023.2289079.

Frigione, M. and Lettieri, M. (2018), “Durability issues and challenges for material advancements in FRP employed in the construction industry”, *Polymers*, **10**(3), 247. https://doi.org/10.3390/polym10030247.

Ghobadi, A., Beni, Y.T. and Golestanian, H. (2020), "Nonlinear thermo-electromechanical vibration analysis of size-dependent functionally graded flexoelectric nano-plate exposed magnetic field", *Arch. Appl. Mech.*, **90**(9), 2025-2070. https://doi.org/10.1007/S00419-020-01708-0/FIGURES/11.

Ghobadi, A., Golestanian, H., Beni, Y.T. and Żur, K.K. (2021), "On the size-dependent nonlinear thermo-electro-mechanical free vibration analysis of functionally graded flexoelectric nano-plate", *Commun. Nonlinear Sci. Numer. Simul.*, **95**, 105585. https://doi.org/10.1016/J.CNSNS.2020.105585.

Ghorbani Shenas, A., Ziaee, S. and Malekzadeh, P. (2019), "A unified higher-order beam theory for free vibration and buckling of FGCNT-reinforced microbeams embedded in elastic medium based on unifying stress-strain gradient framework", *Iran. J. Sci. Technol. Trans. Mech. Eng.*, **43**(1), 469-492. https://doi.org/10.1007/s40997-018-0171-z.

Gul, U. (2024), "Dynamic analysis of short-fiber reinforced composite nanobeams based on nonlocal strain gradient theory", *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, **238**(7), 2641-2676. https://doi.org/10.1177/09544062241227086.

Gul, U. and Aydogdu, M. (2023), "On the axial vibration of viscously damped short-fiber-reinforced nano/micro-composite rods", *J. Vib. Eng. Technol.*, **11**(3), 1327-1341. https://doi.org/10.1007/s42417-022-00643-4.

Guzmán de Villoria, R. and Miravete, A. (2007), "Mechanical model to evaluate the effect of the dispersion in nano-composites", *Acta Mater.*, **55**(9), 3025-3031. https://doi.org/10.1016/j.actamat.2007.01.007.

Herisanu, N., Marinca, B. and Marinca, V. (2023), "Longitudinal-transverse vibration of a functionally graded nanobeam subjected to mechanical impact and electromagnetic actuation", *Symmetry*, **15**(7), 1376. https://doi.org/10.3390/sym15071376.

Hosseini, S.M.H. and Beni, Y.T. (2023), "Free vibration analysis of rotating piezoelectric/flexoelectric microbeams", *Appl. Phys. A Mater. Sci. Process.*, **129**(5), 1-13. https://doi.org/10.1007/s00339-023-06615-z.

Jena, S.K., Chakraverty, S. and Jena, R.M. (2019), "Propagation of uncertainty in free vibration of Euler-Bernoulli nanobeam", *J. Brazilian Soc. Mech. Sci. Eng.*, **41**(10), 1-18. https://doi.org/10.1007/s40430-019-1947-9.

Junaedi, H., Baig, M., Dawood, A., Albahkali, E. and Almajid, A. (2020), "Mechanical and physical properties of short carbon fiber and nanofiller-reinforced polypropylene hybrid nano-composites", *Polym*, **12**(12), 2851. https://doi.org/10.3390/POLYM12122851.

Karmakar, S. and Chakraverty, S. (2022), "Thermal vibration of nonhomogeneous Euler nanobeam resting on Winkler foundation", *Eng. Anal. Bound. Elem.*, **140**, 581-591. https://doi.org/10.1016/j.enganabound.2022.04.020.

Khan, S.U., Li, C.Y., Siddiqui, N.A. and Kim, J.K. (2011), "Vibration damping characteristics of carbon fiber-reinforced composites containing multi-walled carbon nanotubes", *Compos. Sci. Technol.*, **71**(12), 1486-1494. https://doi.org/10.1016/j.compscitech.2011.03.022.

Ladmek, M., Belkacem, A., Daikh, A.A., Bessaim, A., Garg, A., Houari, M.S.A., Belarbi, M.O. and Ouldyerou, A. (2023a), "Free vibration of functionally graded carbon nanotubes reinforced composite nanobeams", *Adv. Mater. Res.*, **12**(2), 161-177. https://doi.org/10.12989/amr.2023.12.2.161.

Ladmek, M., Belkacem, A., Houari, M.S.A., Daikh, A.A., Bessaim, A., Belarbi, M.O., Tounsi, A., Khdair, A.I. and Eltaher, M.A. (2023b), "On vibration responses of advanced functionally graded carbon nanotubes reinforced composite nanobeams", *J. Nano Res.*, **80**, 49-63. https://doi.org/10.4028/p-u9eXPt.

Lal, R. and Dangi, C. (2020), "Comprehensive effect of in-plane load and nonlinear thermal field on dynamic response of embedded bi-directional functionally graded tapered thick nanobeams", *J. Therm. Stress.*, **43**(12), 1577-1600. https://doi.org/10.1080/01495739.2020.1831416.

Lei, H.F., Zhang, Z.Q. and Liu, B. (2012), "Effect of fiber arrangement on mechanical properties of short fiber reinforced composites", *Compos. Sci. Technol.*, **72**(4), 506-514. https://doi.org/10.1016/j.compscitech.2011.12.011.

Li, C. and Qing, H. (2024), "Theoretical thermal damping vibration analysis of functionally graded viscoelastic Timoshenko microbeam with integral nonlocal strain gradient model", *Mech. Based Des. Struct. Mach.*, **52**(7), 4337-4360. https://doi.org/10.1080/15397734.2023.2227702.

Lim, C.W., Zhang, G. and Reddy, J.N. (2015), "A higher-order nonlocal elasticity and strain gradient theory and its applications in wave propagation", *J. Mech. Phys. Solids*, **78**, 298-313. https://doi.org/10.1016/j.jmps.2015.02.001.

Lin, Y., Min, J., Teng, H., Lin, J., Hu, J. and Xu, N. (2020), "Flexural performance of Steel-FRP composites for automotive applications", *Automot. Innov.*, **3**(3), 280-295. https://doi.org/10.1007/s42154-020-00109-x.

Lovisi, G., Feo, L., Lambiase, A. and Penna, R. (2024), "Application of surface stress-driven model for higher vibration modes of functionally graded nanobeams", *Nanomaterials*, **14**(4), 350. https://doi.org/10.3390/nano14040350.

Marinca, B., Herisanu, N. and Marinca, V. (2023), "Investigating nonlinear forced vibration of functionally graded nanobeam based on the nonlocal strain gradient theory considering mechanical impact, electromagnetic actuator, thickness effect and nonlinear foundation", *Eur. J. Mech. A/Solids*, **102**, 105119. https://doi.org/10.1016/j.euromechsol.2023.105119.

Modanloo, V., Mashayekhi, A., Taghipour Lahijani, Y. and Akhoundi, B. (2023), "Prediction of large deflection of chromium nanobeams using a hybrid meta-heuristic algorithm", *J. Eng. Res.*, In Press. https://doi.org/10.1016/J.JER.2023.12.004.

Mohammadi, M., Farajpour, A. and Rastgoo, A. (2023), "Coriolis effects on the thermo-mechanical vibration analysis of the rotating multilayer piezoelectric nanobeam", *Acta Mech.*, **234**(2), 751-774. https://doi.org/10.1007/s00707-022-03430-0.

Mojahedi, M. (2024), "Analytical and Numerical Investigation of a Nonlinear Nanobeam Model", *J. Vib. Eng. Technol.*, **12**(3), 3471-3485. https://doi.org/10.1007/s42417-023-01058-5.

Moradi, Z., Ebrahimi, F. and Davoudi, M. (2023), "On the vibration and energy harvesting of the piezoelectric MEMS/NEMS via nonlocal strain gradient theory", *Adv. Nano Res.*, **15**(3), 203-213. https://doi.org/10.12989/anr.2023.15.3.203.

Murín, J., Aminbaghai, M. and Kutiš, V. (2010), "Exact solution of the bending vibration problem of FGM beams with variation of material properties", *Eng. Struct.*, **32**(6), 1631-1640. https://doi.org/10.1016/j.engstruct.2010.02.010.

Nalbant, M.O., Bağdatli, S.M. and Tekin, A. (2023), "Investigation of nonlinear vibration behavior of the stepped nanobeam", *Adv. Nano Res.*, **15**(3), 215-224. https://doi.org/10.12989/anr.2023.15.3.215.

Pan, N. (1993), "Theoretical determination of the optimal fiber volume fraction and fiber-matrix property compatibility of short fiber composites", *Polym. Compos.*, **14**(2), 85-93. https://doi.org/10.1002/PC.750140202.

Penna, R., Lambiase, A., Lovisi, G. and Feo, L. (2023), "Investigating hygrothermal bending behavior of FG nano-beams via local/nonlocal stress gradient theory of elasticity with general boundary conditions", *Mech. Adv. Mater. Struct.*, 1-10. https://doi.org/10.1080/15376494.2023.2269938.

Qing, H. (2023), "Size-dependent nonlinear forced vibration of microbeam using two-phase local/nonlocal viscoelastic integral model with thermal effects", *J. Vib. Control.*, 10775463231210030.

https://doi.org/10.1177/10775463231210030.

Semmah, A., Heireche, H., Bousahla, A.A. and Tounsi, A. (2019), "Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT", *Adv. Nano Res.*, **7**(2), 89-98. https://doi.org/10.12989/anr.2019.7.2.089.

Shafiei, N., Ghadiri, M. and Mahinzare, M. (2019), "Flapwise bending vibration analysis of rotary tapered functionally graded nanobeam in thermal environment", *Mech. Adv. Mater. Struct.*, **26**(2), 139-155. https://doi.org/10.1080/15376494.2017.1365982.

Sobhy, M. (2017), "Hygro-thermo-mechanical vibration and buckling of exponentially graded nanoplates resting on elastic foundations via nonlocal elasticity theory", *Struct. Eng. Mech.*, **63**(3), 401-415. https://doi.org/10.12989/sem.2017.63.3.401.

Sreejith, M. and Rajeev, R.S. (2021), "Fiber reinforced composites for aerospace and sports applications", *Fiber Reinf. Compos. Const. Compat. Perspect. Appl.*, 821-859. https://doi.org/10.1016/B978-0-12-821090-1.00023-5.

Tounsi, A., Benguediab, S., Adda Bedia, E.A., Semmah, A. and Zidour, M. (2013), "Nonlocal effects on thermal buckling properties of double-walled carbon nanotubes", *Adv. nano Res.*, **1**(1), 1-11. https://doi.org/10.12989/anr.2013.1.1.001.

Tuyen, B. Van, and Du, N.D. (2023), "Analytic solutions for static bending and free vibration analysis of FG nanobeams in thermal environment", *J. Therm. Stress.*, **46**(9), 871-894. https://doi.org/10.1080/01495739.2023.2211642.

Usmani, A.S., Rotter, J.M., Lamont, S., Sanad, A.M. and Gillie, M. (2001), "Fundamental principles of structural behaviour under thermal effects", *Fire Saf. J.*, **36**(8), 721-744. https://doi.org/10.1016/S0379-7112(01)00037-6.

Uzun, B., Civalek, Ö. and Yaylı, M.Ö. (2023), "A hardening nonlocal approach for vibration of axially loaded nanobeam with deformable boundaries", *Acta Mech.*, **234**(5), 2205-2222. https://doi.org/10.1007/s00707-023-03490-w.

Uzun, B. and Yayli, M.Ö. (2024), "Rotary inertia effect on dynamic analysis of embedded FG porous nanobeams under deformable boundary conditions with the effect of neutral axis", *J. Brazilian Soc. Mech. Sci. Eng.*, **46**(2), 1-20. https://doi.org/10.1007/s40430-023-04605-z.

Vahidi-Moghaddam, A., Rajaei, A., Azadi Yazdi, E., Eghtesad, M. and Necsulescu, D.S. (2023), "Nonlinear forced vibrations of nonlocal strain gradient microbeams", *Mech. Based Des. Struct. Mach.*, **51**(2), 1035-1053. https://doi.org/10.1080/15397734.2020.1860773.

Wattanasakulpong, N. and Ungbhakorn, V. (2014), "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", *Aerosp. Sci. Technol.*, **32**(1), 111-120. https://doi.org/10.1016/j.ast.2013.12.002.

Wei, D., Zhang, N., Jiao, Y., Fan, Y., Yu, H. and Koochakianfard, O. (2024), "Magneto-hygro-thermo-mechanical vibration analysis of spinning nanobeams with axisymmetric cross-sections incorporating surface, rotary inertia, and thickness effects", *Eng. Struct.*, **305**, 117702. https://doi.org/10.1016/j.engstruct.2024.117702.

Yang, W., Wang, S., Kang, W., Yu, T. and Li, Y. (2023), "A unified high-order model for size-dependent vibration of nanobeam based on nonlocal strain/stress gradient elasticity with surface effect", *Int. J. Eng. Sci.*, **182**, 103785. https://doi.org/10.1016/j.ijengsci.2022.103785.

Yaylı, M.Ö., Uzun, B. and Deliktaş, B. (2022), "Buckling analysis of restrained nanobeams using strain gradient elasticity", *Wave Random Complex Med.*, **32**(6), 2960-2979. https://doi.org/10.1080/17455030.2020.1871112.

Yuan, Y., Niu, Z. and Smitt, J. (2023), "Magneto-hygro-thermal vibration analysis of the viscoelastic nanobeams reinforcedwith carbon nanotubes resting on Kerr's elastic foundation based on NSGT", *Adv. Compos. Mater.*, **32**(4), 568-590. https://doi.org/10.1080/09243046.2022.2122766.

Zaman, A., Gutub, S.A. and Wafa, M.A. (2013), "A review on FRP composites applications and durability concerns in the construction sector", *J. Reinf. Plast. Compos.*, **32**(24), 1966-1988. https://doi.org/10.1177/0731684413492868.

Zhao, X., Zheng, S. and Li, Z. (2022), "Bending, free vibration and buckling analyses of AFG flexoelectric nanobeams based on the strain gradient theory", *Mech. Adv. Mater. Struct.*, **29**(4), 548-563. https://doi.org/10.1080/15376494.2020.1779880.

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