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Static analysis of nonlinear FG-CNT reinforced nano-composite beam resting on Winkler/Pasternak foundation

Mostefa Sekkak[1], Rachid Zerrouki[2], Mohamed Zidour[2], Abdelouahed Tounsi*[3,4]
Mohamed Bourada[1], Mahmoud M Selim[5] and Hosam A. Saad[6]

[1]*Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[2]*Laboratory of Geomatics and Sustainable Development, University of Tiaret, Algeria*
[3]*Department of Civil and Environmental Engineering, Lebanese American University, 309 Bassil Building, Byblos, Lebanon*
[4]*Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia*
[5]*Department of Mathematics, College of Science and Humanities, Prince Sattam bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia*
[6]*Department of Chemistry, College of Science, Taif University, P.O. Box 11099, Taif, 21944, Saudi Arabia*

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Abstract. In this study, the static analysis of carbon nanotube-reinforced composites (CNTRC) beams resting on a Winkler-Pasternak elastic foundation is presented. The developed theories account for higher-order variation of transverse shear strain through the depth of the beam and satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam. To study the effect of carbon nanotubes distribution in functionally graded (FG-CNT), we introduce in the equation of CNT volume fraction a new exponent equation. The SWCNTs are assumed to be aligned and distributed in the polymeric matrix with different patterns of reinforcement. The rule of mixture is used to describe the material properties of the CNTRC beams. The governing equations were derived by employing Hamilton's principle. The models presented in this work are numerically provided to verify the accuracy of the present theory. The analytical solutions are presented, and the obtained results are compared with the existing solutions to verify the validity of the developed theories. Many parameters are investigated, such as the Pasternak shear modulus parameter, the Winkler modulus parameter, the volume fraction, and the order of the exponent in the volume fraction equation. New results obtained from bending and stresses are presented and discussed in detail. From the obtained results, it became clear the influence of the exponential CNTs distribution and Winkler-Pasternak model improved the mechanical properties of the CNTRC beams.
Keywords: beam; critical buckling; FG-CNTRC; nanotube; shear deformation; volume fraction

1. Introduction

The discovery of carbon nanotubes by Iijima (1991), through his article titled "Helical microtubules of graphitic carbon," this new research leads the world to a new industrial revolution that offers many benefits to humanity. Several studies investigating CNTs have indicated that there are numerous carbon nanotube properties that make him an excellent candidate for the reinforcement of polymer composites, including mechanical strength (Coleman *et al*. 2006, Cui *et al*. 2017), electrical properties, and thermal conductivity (Koppad *et al*. 2013, Wu *et al*. 2017). The mechanical behavior of CNT is one of the basic problems on the nano-composite. To describe this problems, molecular dynamics (MD) Simulations methods have been adopted to study the mechanical behavior of nanowires, for instance, their bending, flexion, buckling, and compression behavior (Vakili-Nezhaad *et al*. 2017, Singh *et al*. 2022, Avcar *et al*. 2023). On the other hand, continuum mechanics has been applied to predict the mechanical properties of composites and nano-composites (Yaylacı 2016, Karimzadeh *et al*. 2007, Talebitooti and Zarastvand 2018, Yaylacı *et al*. 2022c, Huang and Rodrigue 2013, Zarastvand *et al*. 2022a). AsadiJafari *et al*. (2023) presented doubly curved truss core composite shell system for broadband diffuse acoustic insulation. Sound propagation of a three-dimensional sandwich panels: influence of three-dimensional re-entrant auxetic core has been studied by Ghafouri *et al*. (2022). Zarastvand *et al*. (2022b) investigated the acoustic wave transmission features of the multilayered plate constructions. A review approach for sound propagation prediction of plate constructions has been studied by Zarastvand *et al*. (2021b). Ghassabi *et al*. (2020) investigated the state vector computational solution on modeling of wave propagation through functionally graded nanocomposite doubly curved thick structures.

The CNT/polymer nanocomposites are a promising new class of composite materials finding their excellent mechanical, electrical, and thermal properties (Sankar *et al*. 2016, Heidari *et al*. 2020).

The novel generation of composite materials introduced in nanotechnology, which are called "Functionally Graded Materials (FGM)", has provided solutions to many technical problems by producing new materials with properties that cannot be obtained in conventional materials. (Yaylacı 2022d, e, Giunta *et al*. 2010, Yaylaci *et al*. 2021, Garg *et al*.

*Corresponding author, Ph.D., Professor,
E-mail: tou_abdel@yahoo.com

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2021). Yaylacı *et al*. (2023) investigated the vibration and buckling of FGM beam with an edge crack using finite element and multilayer perception methods. Adıyaman *et al*. (2023) studied the contact problem of a layer consisting of functionally graded material (FGM) in the presence of body force. Using computational, finite element and artificial neural network methods, the plane receding contact between two functionally graded layers has been developed by Öner *et al*. (2022). Turan *et al*. (2023) presented the free vibration and buckling of FG porous beams using analytical, finite element, and artificial neural network methods. By using analytical and numerical methods, the contact problem of FG layers resting on HP and rigid foundation pressed has been solved, respectively, by Yaylacı *et al*. (2022a, b).

In current applications, reinforcement based carbon nanotubes (CNTs) have been widely adopted in place of conventional fibre bulk due to their exceptional properties to enhance the mechanical, electrical, and thermal properties of composite structures (Mayandi and Jeyaraj 2015, Kumar and Srinivas 2017, Ansari and Kumar 2019, Dat *et al*. 2020, Khazaei and Mohammadimehr 2020, AlSaid-Alwan and Avcar 2020, Akhavan *et al*. 2021, Nguyen *et al*. 2022). To develop their use in current applications, it is necessary to observe the overall response of the nanocomposite structural element. Notwithstanding, a number of studies have been carried out on the mechanical behavior of macroscopical structures made of functionally graded carbon nanotubes (FG-CNTRC). All research used a linear distribution. Zhu *et al*. (2012) investigate the bending, buckling, and vibration analyses of CNTRC plates resting on the Pasternak elastic foundation, including the shear layer and Winkler springs. Wang *et al*. (2019) studied the bending and elastic vibration of a novel functionally graded polymer nanocomposite beam reinforced by grapheme nanoplatelets. Sayyad and Ghugal (2020) presented a theoretical unification of twenty-one nonlocal beam theories for the bending, buckling, and vibration analysis of FG nanobeams. Tayeb *et al*. (2020) studied the effect of CNT distribution on the critical buckling load of CNTRC plates using the shear deformation theory. According to their results, parabolic distributions have higher critical buckling loads than linear distributions. In a study involving deep cylindrical panels reinforced by functionally graded carbon nanotubes under thermo-mechanical loading, Salami *et al*. (2021) evaluated the effects of distributions and volume fractions of carbon nanotubes (CNTs) on nonlinear bending behavior.

Consequently, the main aim of this study is to examine the static behavior of FG-CNTRC beams under an elastic foundation. The novelty of the manuscript is the influence of the CNT nonlinearity configuration. The authors propose, for the first time, a novel exponential equation for the CNT volume fraction. This equation captures the impact of nonlinear distribution on both mechanical and economic aspects. On the mechanical side, the nonlinear distribution of CNTs has demonstrated a notable increase in the beam's rigidity compared to the linear distribution. It is crucial to note that this enhancement in rigidity is achieved with the same quantity of CNTs used in both linear and nonlinear distributions. This underscores the significance of nonlinear distribution from an economic standpoint. Additionally, economic benefits can be realized, for instance by improving the mechanical properties of the beam by increasing the exponent (n) rather than raising the volume fraction of CNTs.

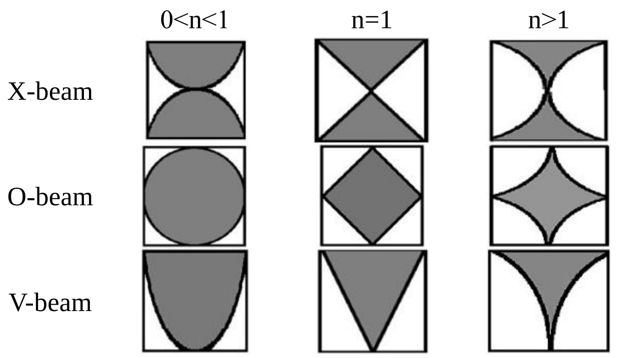

Fig. 1 Cross sections of different patterns CNTRC beams

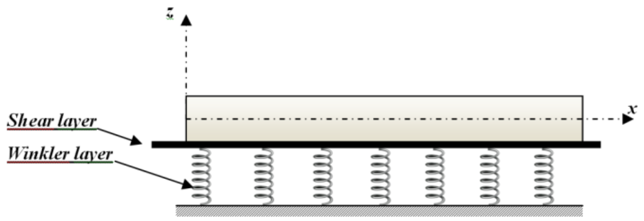

Fig. 2 Geometry of carbon nanotube-reinforced composite CNTRC beam

2. Geometrical and properties of FG-CNTRC beams

In this work, the CNT are considered to have uniform (UD) or FG distributions (FG-X, FG-O, FG-V) in the beam thickness (Fig.1). The CNT volume fraction is assumed to obey an exponential function used for describing the uniform and symmetrical FG distribution of aligned CNTs along the thickness direction of the beams represented in Fig. 1.

In terms of foundation, the Pasternak model used in the present work studied the shear interaction between springs by connecting the ends of the springs to a layer consisting of incompressible vertical elements that deform only by transverse shear. The Pasternak model is a more realistic representation of interaction than the Winkler model. Bao and Liu (2020) indicated that the Pasternak model yields more realistic results than the Winkler model because of its consideration of the continuity of foundation media via the shear interaction. The research results using the Pasternak model have attracted the attention of many researchers (Asadijafari *et al*. 2021, Madani *et al*. 2016, Zarastvand *et al*. 2021a, Zhang *et al*. 2015, Lei *et al*. 2016, Heidari-Rarani *et al*. 2015).

It is considered that the FG-CNTRC beam is made of an isotropic matrix reinforced by aligned single-walled carbon nanotubes (SWCNTs). The effective material properties (Young's modulus and shear modulus) of the FG-CNTRC material are computed using the extended rule of mixture as: (Wattanasakulpong and Ungbhakorn 2013):

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E^p \tag{1a}$$

$$E_{22} = \eta_2 \frac{E_{22}^{cnt} E^p}{V_p E_{22}^{cnt} + V_{cnt} E^p} \tag{1b}$$

$$G_{12} = \eta_3 \frac{G_{12}^{cnt} G^p}{V_{cnt} G^p + V_p G_{12}^{cnt}} \tag{1c}$$

where E_{11}^{cnt}, E_{22}^{cnt}, E^p and G_{12}^{cnt}, G^p indicate the Young's modulus and shear modulus of SWCNTs and polymer matrix, respectively. The CNT efficiency parameters (η) related to the volume fraction (V_{cnt}^*) which is shown in Table 1, are given by Yas and Samadi (2012):

The Poisson's ratio (υ) and mass density (ρ) of beams are written as (Yas and Samadi 2012):

$$\rho = V_{cnt} \rho^{cnt} + V_p \rho^p \tag{2a}$$

$$\nu_{12} = V_{cnt} \nu_{12}^{cnt} + V_p \nu^p \tag{2b}$$

$$V_{cnt} + V_p = 1 \tag{2c}$$

where V_{cnt} and V_p are the volume fractions of the CNT and polymer, respectively.

For the different functionally graded distributions of the CNTs, the volume fraction CNT alongthe beam thickness direction becomes an exponential function (Zerrouki *et al.* 2020):

$$V_{cnt} = \begin{cases} V_{cnt}^* & \text{for UD-beam} \\ (n+1)(1-2\frac{|z|}{h})^n V_{cnt}^* & \text{for O-beam} \\ (n+1)(2\frac{|z|}{h})^n V_{cnt}^* & \text{for X-beam} \\ (n+1)(\frac{1}{2}+\frac{z}{h})^n V_{cnt}^* & \text{for V-beam} \end{cases} \tag{3}$$

where n is the exponent degree of V_{cnt} equation and $V_{cnt}^*$is the volume fraction of CNTs:

$$V_{cnt}^* = \frac{W_{cnt}}{W_{cnt} + (\rho^{cnt}/\rho^p)(1 - W_{cnt})} \tag{4}$$

Here W_{cnt} is the mass fraction of the CNTs.

3. Equations of motion

The displacement fields of various shear deformation beam theories are chosen based on the following assumptions: (1) the axial and transverse displacements are partitioned into bending and shear components; (2) the bending component of axial displacement is similar to that given by the CBT; and (3) the shear component of axial displacement gives rise to the higher-order variation of shear strain and hence to shear stress through the depth of the beam in such a way that shear stress vanishes on the top and bottom surfaces.

Assumptions involved concerning beam displacement components are that the beam axial displacement u(x, z, t)

Table 1 The CNT efficiency parameters (η)

case	η_1	$\eta_2 = \eta_3$
$V_{cnt}^* = 0.12$	1.2833	1.0566
$V_{cnt}^* = 0.17$	1.3414	1.7101
$V_{cnt}^* = 0.28$	1.3238	1.7380

and transverse displacement w(x, t) along the x and z directions, respectively, are made of bending and shearing components.

$$u_b = -z\frac{\partial w_b}{\partial x}, u_s = -f(z)\frac{\partial w_s}{\partial x}, \ w = w_b + w_s \tag{5a}$$

The bending component axial u_b and transverse w_b displacement of the beam do not take part in the beam transverse shear strain. But the shearing component, axial u_s and transverse w_s displacement are the only beam displacement components that take part in the transverse shear strain. These displacement, components, in conjunction contribute towards the transverse shear strain in such a way that it remains zero at beam surfaces z=±h/2. This implies that the transverse shear stress developed also remains zero at beam surfaces.

The mathematical model of the FG-CNTRCs beam is developed based on the refined HSDT. The axial displacement, u, and the transverse displacement of any point of the beam, w, are given as (Tagrara *et al.* 2015):

$$\begin{cases} u(x,z,t) = u_0 - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ w(x,z,t) = w_b(x,t) + w_s(x,t) \end{cases} \tag{5b}$$

where u_0 is the axial displacement, w_b and w_s are the bending and shear components of transverse displacement along the mid-plane of the beam.

The shape function f(z) is chosen based on a third order shear deformation theory (TSDT) as described by Reissner (1945):

$$f(z) = z - \frac{5}{4}z(1 - \frac{4z^2}{3h^2}) \tag{6}$$

The normal and shear strain components associated with the displacement field in Eq. (5) are as follows:

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \\ \gamma_{xz} = g(z)\frac{\partial w_s}{\partial x} \end{cases} \tag{7a}$$

where

$$g(z) = 1 - \frac{df(z)}{dz} \tag{7b}$$

The normal stress σ_x and shear stress γ_{xz} are given by linear elastic constitutive law as:

$$\begin{cases} \sigma_x = Q_{11}(z)\varepsilon_x \\ \tau_{xz} = Q_{55}(z)\gamma_{xz} \end{cases} \tag{8a}$$

where

$$\begin{cases} Q_{11}(z) = \frac{E_{11}(z)}{1 - v^2} \\ Q_{55}(z) = G_{12}(z) \end{cases} \tag{8b}$$

The governing equations of motion are obtained by the principle of Hamilton's (Medani *et al*. 2019):

$$\delta U + \delta V = 0 \tag{9}$$

where δ represents the virtual variational symbol, U is the strain energy, and V is the potential energy of the beam.

The virtual strain energy of the beam is (Gafour *et al*. 2020):

$$\delta U = \int_0^L \int_{-h/2}^{h/2} (\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz}) dz dx = \int_0^L (N\frac{d\delta u_0}{dx} - M_b\frac{d^2\delta u_b}{dx^2} - M_s\frac{d^2\delta u_s}{dx^2} + Q\frac{d\delta w_s}{dx}) \tag{10}$$

The stress resultants N, Mb, Ms and Q are defined as

$$(N, M_b, M_s) = \int_0^L (1, z, f)\, \sigma_x dz,\ \ Q = \int_{-h/2}^{h/2} g\tau_{xz}\, dz \tag{11}$$

The virtual work done (δV), by the transverse load (q), and the density of the reaction force of foundation $f_e$can be expressed as follows: (Tagrara *et al*. 2015)

$$\delta V = -\int_0^L [(q - f_e)(\delta w_b + \delta w_s)]\, dx \tag{12}$$

$$f_e = K_w w - K_s\frac{\partial^2 w}{\partial x^2} \tag{13}$$

where K_w and K_s are the winkler and shearing layer spring constants can be expressed as:

$$K_w = \beta_w\frac{A_{110}}{L^2} \tag{14a}$$

$$K_s = \beta_s A_{110} \tag{14b}$$

where β_w and β_s are the corresponding spring constant factors. A_{110} is the extension stiffness or the value of A_{10} of a homogeneous beam made of pure matrix material.

Substituting the expressions of δU and δV from Eqs. (10) and (12), into Eq. (9) and integrating by parts versus both space and time variables, and collecting the coefficients of $\delta u_0, \delta w_b$, and δw_s, the following equations of motion of the CNTRC beam are obtained

$$\begin{cases} \delta u_0: \dfrac{dN}{dx} = 0 \\ \delta w_b: \dfrac{d^2M_b}{dx^2} + q - f_e = 0 \\ \delta w_s: \dfrac{d^2M_s}{dx^2} + \dfrac{dQ}{dx} + q - f_e = 0 \end{cases} \tag{15}$$

From the above relations, all stress resultants can be written in the form of material stiffness components and displacements as follows:

$$\begin{cases} N = A_{11}\dfrac{du_0}{dx} - B_{11}\dfrac{d^2w_b}{dx^2} - B_{11}^s\dfrac{d^2w_s}{dx^2} \\ M_b = B_{11}\dfrac{du_0}{dx} - D_{11}\dfrac{d^2w_b}{dx^2} - D_{11}^s\dfrac{d^2w_s}{dx^2} \\ M_s = B_{11}^s\dfrac{du_0}{dx} - D_{11}^s\dfrac{d^2w_b}{dx^2} - H_{11}^s\dfrac{d^2w_s}{dx^2} \\ Q = A_{55}^s\dfrac{dw_s}{dx} \end{cases} \tag{16}$$

where A_{11}, D_{11} etc. are the beam stiffness, defined by

Table 2 The material properties of PMMA

v^p	ρ^p	E^p
0.3	1190Kg/m^3	2.5GPa

Table 3 The material properties of CNT (armchair (10,10) SWCNTs)

V_{cnt}	ρ^{cnt}	E_{11}^{cnt}	E_{22}^{cnt}	G_{12}^{cnt}
0.19	1400Kg/m^3	600GPa	10GPa	17.2GPa

Table 4 The dimensionless deflection and stresses

$\overline{w}$	$\overline{\sigma_x}$	$\overline{\tau_{xz}}$
$100\dfrac{E_p h^3}{q_0 L^4} w$	$\dfrac{h}{q_0 L}\sigma_x\left(\dfrac{L}{2}, \dfrac{h}{2}\right)$	$\dfrac{h}{q_0 L}\tau_{xz}(0,0)$

$$\begin{cases} (A_{11}, B_{11}, D_{11}) = \displaystyle\int_{-h/2}^{h/2} (1, z, z^2) dz \\ (B_{11}^s, D_{11}^s, H_{11}^s) = \displaystyle\int_{-h/2}^{h/2} (f(z), zf(z); f^2(z)) dz \\ A_{55}^s = \displaystyle\int_{-h/2}^{h/2} Q_{55}[g(z)] dz \end{cases} \tag{16}$$

By substituting the expression from Eq. (16) of stress resultants into Eq. (17), it is obtained

$$\begin{cases} A_{11}\dfrac{d^2u_0}{dx^2} - B_{11}\dfrac{d^3w_b}{dx^3} - B_{11}^s\dfrac{d^3w_s}{dx^3} = 0 \\ B_{11}\dfrac{d^3u_0}{dx^3} - D_{11}\dfrac{d^4w_b}{dx^4} - D_{11}^s\dfrac{d^4w_s}{dx^4} = 0 \\ B_{11}^s\dfrac{d^3u_0}{dx^3} - D_{11}^s\dfrac{d^4w_b}{dx^4} - D_{11}^s\dfrac{d^4w_s}{dx^4} + \dfrac{d^2w_s}{dx^2} = 0 \end{cases} \tag{18}$$

4. Analytical solution

For the numerical solution, the Navier solution method for a simply supported CNTRC beam resting under a Pasternak elastic foundation is adopted. Thus, the following expansions of displacements (u_0, wb, ws) are assumed as (Draoui *et al*. 2019):

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m\, cos(\lambda x)e^{iwt} \\ W_{bm}\, sin(\lambda x)e^{iwt} \\ W_{sm}\, sin(\lambda x)e^{iwt} \end{Bmatrix} \tag{19}$$

where U_m, W_{bm} and W_{sm} are arbitrary parameters to be determined, ω is the eigenfrequency associated with the mth eigenmode, and $\lambda = m\pi/L$. The transverse load q is also expanded in the Fourier series as:

The coefficient Q_m are given below for some typical loads

$$\begin{cases} q(x) = \displaystyle\sum_{n=1}^{\infty} Q_m \sin(\lambda x) \\ Q_m = \dfrac{2}{L}\displaystyle\int_0^L q(x)\sin(\lambda x)dx \quad \text{load amplitude} \end{cases} \tag{20a}$$

Table 5 Dimensionless displacements and stresses of UD-Beams under uniform and sinusoidal loads

Vcnt	L/h	Theory	uniform loads			sinusoidal loads		
			$\bar{w}$	$\bar{\sigma}$	$\bar{\tau}$	$\bar{w}$	$\bar{\sigma}$	$\bar{\tau}$
0.12	10	Present	0.594	7.054	0.602	0.474	5.889	0.399
		Wattanasakulpong *et al*. (2013)	0.594	7.053	0.602	0.475	5.890	0.399
		Error (%)	0	-0.014	0	0.211	0.017	0
		Tagrara *et al*. (2015)	0.593	7.103	0.617	0.474	5.937	0.411
		Error (%)	-0.169	0.690	2.431	0	0.808	2.920
	20	Present	0.311	10.318	0.521	0.247	8.535	0.322
		Wattanasakulpong *et al*. (2013)	0.311	10.316	0.520	0.247	8.535	0.322
		Error (%)	0	-0.019	-0.192	0	0	0
		Tagrara *et al*. (2015)	0.311	10.336	0.536	0.247	8.555	0.322
		Error (%)	0	0.174	2.799	0	0.234	0
0.17	10	Present	0.402	7.375	0.638	0.320	6.126	0.424
		Wattanasakulpong *et al*. (2013)	0.403	7.374	0.638	0.321	6.126	0.424
		Error (%)	0.248	-0.014	0	0.312	0	0
		Tagrara *et al*. (2015)	0.401	7.419	0.654	0.320	6.169	0.436
		Error (%)	-0.249	0.593	2.446	0	0.697	2.752
	20	Present	0.232	11.569	0.576	0.184	9.520	0.362
		Wattanasakulpong *et al*. (2013)	0.232	11.568	0.575	0.184	9.520	0.362
		Error (%)	0	-0.009	-0.174	0	0	0
		Tagrara *et al*. (2015)	0.232	11.587	0.592	0.184	9.539	0.373
		Error (%)	0	0.155	2.703	0	0.199	2.949
0.28	10	Present	0.299	7.870	0.647	0.239	6.573	0.435
		Wattanasakulpong *et al*. (2013)	0.299	7.869	0.647	0.239	6.573	0.435
		Error (%)	0	-0.013	0	0	0	0
		Tagrara *et al*. (2015)	0.299	7.933	0.662	0.239	6.632	0.447
		Error (%)	0	0.794	2.266	0	0.890	2.685
	20	Present	0.167	12.753	0.614	0.133	10.485	0.393
		Wattanasakulpong *et al*. (2013)	0.167	12.751	0.613	0.133	10.485	0.393
		Error (%)	0	-0.016	-0.163	0	0	0
		Tagrara *et al*. (2015)	0.167	12.781	0.631	0.133	10.514	0.406
		Error (%)	0	0.219	2.694	0	0.276	3.202

$$\begin{cases} Q_m = q_0, m=1 \quad \text{for sinusoidal load} \\ Q_m = \dfrac{4q_0}{m\pi}, m=1,3,5 \quad \text{for uniform load} \end{cases} \tag{20b}$$

Substituting the expansions of u_0, w_b, w_s and q from Eqs. (19) and (20b) into the equations of motion Eq.(18), the analytical solutions can be determined from the following equations:

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{bmatrix} \begin{bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{bmatrix} = \begin{bmatrix} 0 \\ Q_m \\ Q_m \end{bmatrix} \tag{21a}$$

where

$$S_{11} = A_{11}\lambda^2;\ S_{12} = -B_{11}\lambda^3;\ S_{13} = -B_{11}^s\lambda^3$$
$$S_{22} = D_{11}\lambda^4 + K_w + K_s\lambda^2; \tag{21b}$$

$$S_{23} = D^s{}_{11}\lambda^4 + K_w + K_s\lambda^2;$$
$$S_{33} = H^s{}_{11}\lambda^4 + A_{55}^s\lambda^2 + K_w + K_s\lambda^2; \tag{21b}$$

5. Results and discussion for static analysis

The analytical solutions of the static analysis of CNTRC beams resting on the Pasternak elastic foundation are presented. Assuming the beams (CNTRC) is made from CNTs and polymethyl methacrylate (PMMA). The following parameters are employed at ambient temperature (Yas and Samadi 2012).

Before starting to study the static analysis of carbon nanotube-reinforced composites (CNTRC) beams resting on a winkler-Pasternak elastic foundation, a comparison of the dimensionless deflection and stresses is made between the present results and those from the open literature (Tagrara

Table 6 Effect of Winkler modulus parameter on the dimensionless transverse displacements of CNTRC beams under uniform load (βs=0 , Vcnt=0.12, L/h=10)

	$\bar{w}$						
	UD-beam	X-beam		O-beam		V-beam	
βw	n=0	n=1	n=3	n=1	n=3	n=1	n=3
0	0.704	0.577	0.504	1.098	2.091	0.878	1.315
0.1	0.662	0.548	0.482	1.000	1.766	0.815	1.179
0.2	0.625	0.523	0.462	0.919	1.528	0.760	1.068
0.3	0.592	0.499	0.444	0.849	1.346	0.712	0.976
0.4	0.562	0.478	0.427	0.790	1.202	0.669	0.898
0.5	0.535	0.458	0.411	0.738	1.086	0.632	0.832
0.6	0.511	0.440	0.396	0.692	0.990	0.598	0.775
0.7	0.488	0.423	0.383	0.651	0.910	0.567	0.725
0.8	0.468	0.408	0.370	0.615	0.841	0.540	0.681
0.9	0.449	0.393	0.358	0.583	0.782	0.515	0.642
1	0.431	0.380	0.346	0.554	0.730	0.492	0.607

Table 7 Effect of Pasternak shear modulus parameter on the dimensionless transverse displacements of CNTRC beams under uniform load (Bw=0.4, Vcnt=0.12, L/h=10)

	$\bar{w}$						
	UD-beam	X-beam		O-beam		V-beam	
Bs	n=0	n=1	n=3	n=1	n=3	n=1	n=3
0	0.562	0.478	0.427	0.790	1.202	0.669	0.898
0.01	0.536	0.459	0.411	0.739	1.088	0.632	0.833
0.02	0.512	0.441	0.397	0.694	0.994	0.599	0.777
0.03	0.490	0.425	0.384	0.654	0.915	0.569	0.728
0.04	0.470	0.409	0.371	0.619	0.847	0.542	0.684
0.05	0.451	0.395	0.360	0.587	0.789	0.518	0.646
0.06	0.434	0.382	0.349	0.558	0.738	0.495	0.611
0.07	0.418	0.370	0.338	0.532	0.693	0.475	0.580
0.08	0.403	0.358	0.328	0.509	0.653	0.456	0.552
0.09	0.390	0.347	0.319	0.487	0.618	0.438	0.527
0.1	0.377	0.337	0.311	0.467	0.586	0.422	0.504

et al. 2015, Wattanasakulpong and Ungbhakorn 2013) in order to validate the present formulation.

For bending analysis of CNTRC beams resting on the Winkler-Pasternak elastic foundation, Table 5 shows the comparison of dimensionless displacements and stresses of UD-Beams under uniform and sinusoidal loads between the present results and the results obtained by (Tagrara *et al*. 2015, Wattanasakulpong and Ungbhakorn2013). it is very clear that the present results show good agreement with other available results. For all aspect ratios and volume fraction of CNT, The maximum difference in dimensionless displacements and normal stresses between the obtained results of this study and those used inthe high order shear deformation beam theory (HSDT) is 0.89 %. But the maximum difference in shear stresses observed between the results is 2.94%. It can be explained by the different transverse shear strain shape functions f (z) used in this study in equation 6 and the function used by (Tagrara *et al*. 2015).

Tables 6 and 7 show the effects of the Winkler modulus parameter and the Pasternak shear modulus parameter, respectively, on the dimensionless central deflection of CNTRC beams under uniform load. In this case, the degree of exponent (n) takes three values (n=0 for uniform distribution, n=1 for linear distribution, and n=3 for nonlinear distribution) for different patterns (UD-beam, X-beam, O-beam and V-beam). It can be seen that increasing the Winkler modulus parameter and/or the Pasternak shear modulus parameter decreases the dimensionless deflection. In addition, minimum dimensionless deflection is achieved in the X-beam pattern, especially in the case of n=3 (nonlinear distribution).

The FG-CNTRC beams based on the polymer PMMA as a matrix and SWCNT (10, 10) as a nano-reinforcement have been studied by several researchers, such as Yas and Samadi (2012), Wattanasakulpong and Ungbhakorn (2013) and Tagrara *et al*. (2015). These researchers used three

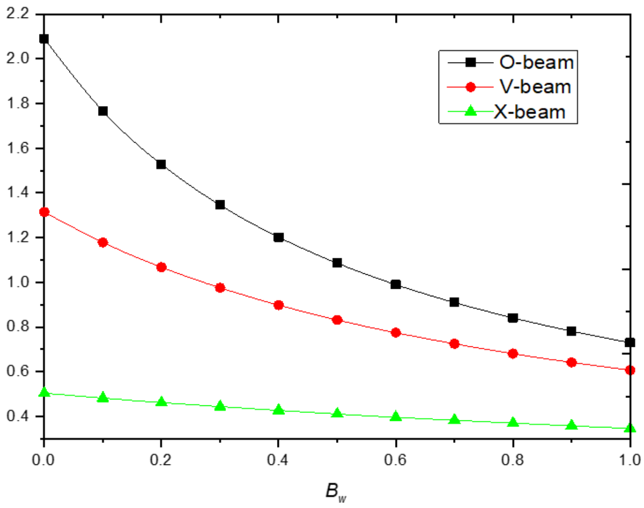

Fig. 3 Effect of Winkler modulus parameter on the dimensionless transverse displacements of CNTRC beams under uniform load (L/h=10; βs=0; Vcnt*=0.12 ; n=3)

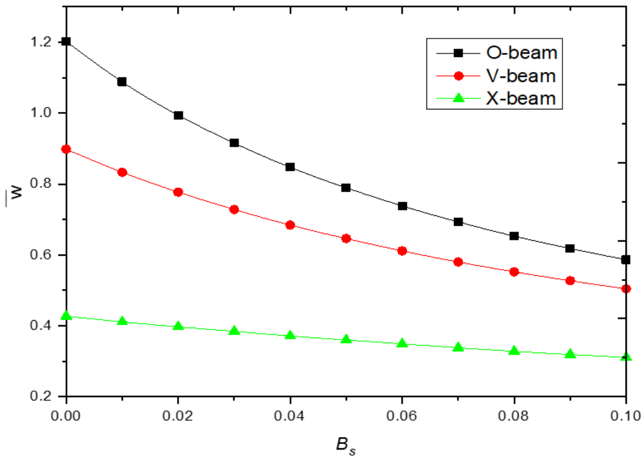

Fig. 4 Effect of Pasternak shear modulus on the dimensionless transverse displacements of CNTRC beams under uniform load (L/h=10; βw=0.4; Vcnt*=0.12, n=3)

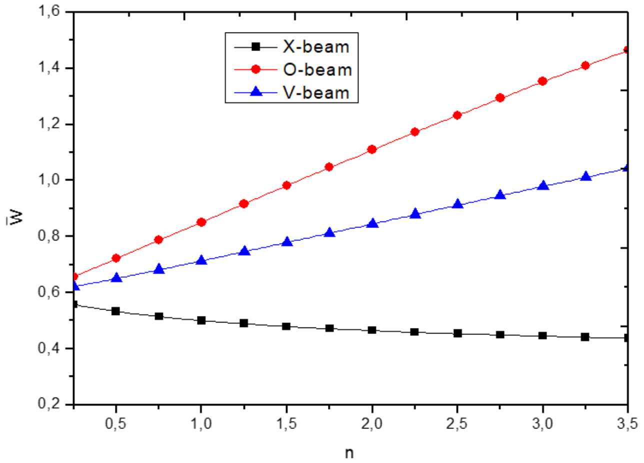

Fig. 5 Effect of the degree of exponent (n) on the dimensionless transverse displacements of CNTRC beams resting on elastic foundation under uniform load (βw=0.1, βs=0.02, Vcnt=0.12, L/h=10,)

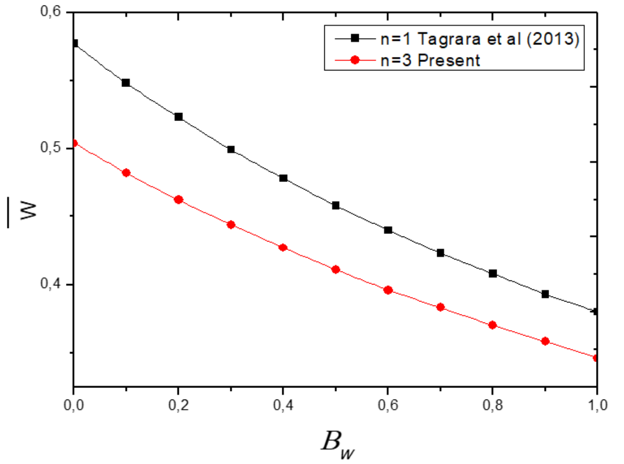

Fig. 6 Effect of Winkler modulus parameter on the dimensionless transverse displacements of CNTRC beams under uniform load (L/h=10; βs=0; $V^*_{cnt} = 0.12$, X-beam)

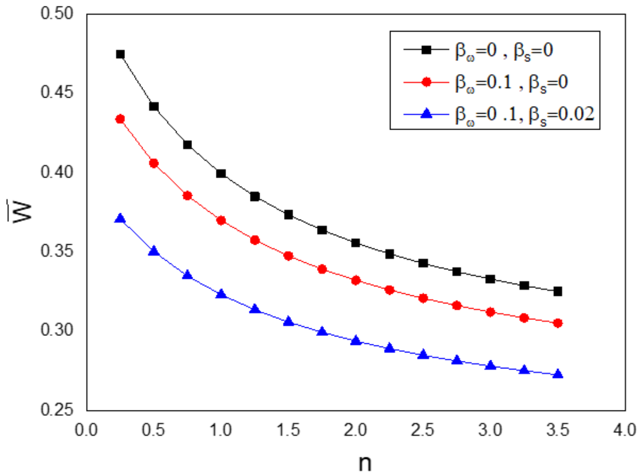

Fig. 7 Effect of elastic foundation and the degree of the exponent (n) on the dimensionless transverse displacements of CNTRC beams under uniform load (L/h=15, X-beam)

CNT volume fractions (0.12, 0.17 and 0.28). In the linear distribution, the maximum of CNT volume fractions reaches to $2Vcnt*$ (0.24, 0.34 and 0.58). In this work, the maximum of CNT volume fractions in the nonlinear distribution reaches to $(n+1)nt*$. For this reason, we have chosen the degree of exponent (n) in which the CNT volume fraction does not exceed the value 0.56 which is the maximum value in the linear distribution.

These results obtained from the previous tables are very obvious from the presentation of Figs. 3 and 4. The influence of the Winkler modulus parameter and the Pasternak shear modulus on the dimensionless results of transverse displacements is shown in Fig. 3 and 4, respectively. In this case, the distribution of CNTs exhibits a non-linear pattern (n=3).

It is clear that the increase of Winkler modulus and Pasternak shear modulus in the (Figs. 3 and 4) respectively, increases the dimensionless transverse displacements. So, the non-linearity of the distribution of the CNT volume fraction plays an important role in improving the strength and stiffness of FG-CNTRC beams. It concluded that the Pasternak model with a nonlinear distribution is more accurate than the Winkler model. In addition, the X-Beam configuration is the strongest of all CNTRC beams in supporting transverse displacements. This is also due to the increase in the concentration of CNTs at both ends of the beam.

Fig. 5 presents the effect of the degree of exponent (n) on the dimensionless central deflection of FG-CNTRC beams with three distribution patterns of reinforcements resting on a Winkler-Pasternak elastic foundation. In this figure, increasing the degree of exponent (n) increases the dimensionless central deflection in the O-beam and V-beam configurations. On the other hand, the dimensionless central deflection decreases in the X-beam configuration.

Fig. 6 shows the influence of the Winkler modulus parameter on the dimensionless transverse displacements in the X-beam configuration. The present results (nonlinear distribution n=3) are compared with the results predicted by Tagrara *et al*. (2013) (linear distribution n=1). The results in Fig. 6 indicate that the dimensionless central deflection is at its minimum in a nonlinear distribution (n=3). It can be noted that the effect of non-linear distribution has a clear impact on the displacements, and the increase of the degree in the exponent of the exponential equation makes the beam stiffer and strength.

Fig. 7 displays the effect of the elastic foundation and the degree of exponent (n) on the dimensionless transverse displacements. It is observed that the presence of the elastic foundation obviously reduced the dimensionless transverse displacement. In addition, it is observed that the dimensionless transverse displacement decrease with the increases of the degree of exponent (n).

6. Conclusions

In this paper, we present the bending analysis of a nonlinear FG-CNT reinforced nano-composite beam resting on Winkler/Pasternak. The (FG-CNTRC) beam is made of an isotropic matrix reinforced by aligned single-walled carbon nanotubes (SWCNTs). Both dimensionless displacements and stresses for different patterns (UD-beam, X-beam, O-beam, and V-beam) under uniform and sinusoidal loads are included in the derived equations of motion. The CNT volume fraction is assumed to obey an exponential function used for describing the uniform and functionally graded distribution of aligned CNTs along the thickness direction of beams.

This paper investigates a new exponential equation of CNT volume fraction based upon high Order Shear Deformation Theory (HSDT) to comprehensively study the bending problems of FG-CNTRC beams in different pattern, of SWCNT distributions in the polymeric matrix resting on a Winkler-Pasternak elastic foundation. The results of this study have been compared to other studies that are currently accessible; the comparison indicates that there is good agreement across all CNT distributions. The findings of this investigation and those derived using the high order shear deformation beam theory (HSDT) differs by a maximum of 0.89% in dimensionless displacements and normal stresses.

Based on the results of this work, parametric analysis is performed to investigate the effect of degree of exponent, elastic foundation parameter, CNT volume fraction, and type of configuration on the bending and stresses. This study shows that:

• The results of all the proposed beam theories are almost identical to each other and agree well with the existing solutions.

• The mechanical properties such as strength and stiffness of FG-CNTRC beams in the X-CNT configuration can be improved by increasing the degree of the exponent (n) and elastic foundation parameter (β_w, and β_s).

• Increasing the volume fraction of CNTs decreased the dimensionless displacements. While this has a significant effect on the stiffness of the beam. Furthermore, this finding was independent of the CNT distributions.

• The X-Beam configuration is the strongest of all CNTRC beams for supporting dimensionless displacements. This is also due to the increase in the concentration of CNTs (as the degree of the exponent (n) increases) at both ends of the beam thickness.

• The influence of the elastic foundation also has a clear effect on CNTRC beams by decreasing the dimensionless displacements. The elastic foundation plays the role of a support that prevents buckling and increases the stability of the beam.

Finally, it should be noted that the novel exponential equation of CNTs distribution has an impact on mechanical and economic aspects. Mechanically, the nonlinear graduated distribution of CNTs has proven its effect by increasing the rigidity of the beam compared to the linear distribution. In the economic aspect, the mechanical properties of the beam are improved by changing the distribution rather than increasing the volume fraction of CNTs. FG-CNT structure has numerous practical applications, including aerospace structures, civil engineering, nuclear reactors, the biomedical industry, optical semiconductors, automotive industry, and smart structures. These applications highlight the versatility and potential impact of functionally graded CNT beams across various industries. It is interesting to develop new concepts for the application of the new formulation in future research. In addition, an experimental strategy must be considered in order to manufacture the new functionally graded structure sought by such applications.

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