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Application of numerical methods for dynamic response induced by moving load on concrete shells containing nanoparticles with economic study

Taoqian Xie[1], Wei Han[2], Haoqi Chang*[3,4] and M.R. Motaghedfer[5]

[1]School of Public Administration, Central China Normal University, Wuhan 430000, Hubei, China
[2]Wanpu (Wuhan) Institute of Education, Wuhan 430000, Hubei, China
[3]College of Cities and environmental sciences, Central China Normal University, Wuhan 430000, Hubei, China
[4]Warner College of Natural Resources, Colorado State University, Fort Collins, CO 80523, USA
[5]College of Engineering, Azad University, Iran

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Abstract. This paper conducts a thorough economic evaluation of integrating nanoparticles into concrete structures within the construction industry, aiming to elevate the material properties of concrete. Employing the Halpin-Tsai micromechanics theory for deriving the effective material properties of the nanocomposite concrete structure, the research investigates the nuanced impact of nanoparticles on various mechanical properties, including the modulus of elasticity, compressive strength, and their indirect effects on the percentage of reinforcement. Implementing the Euler theory to formulate the governing equation based on Hamilton's principle, the study delves into the pricing dynamics of nanoparticles and their influence on the overall cost structure of concrete structures. Notably, the findings reveal that a measured increase in the volume percentage of nanoparticles, up to 1%, results in a remarkable 78% improvement in elastic modulus and a substantial 142% reduction in armature percentage. Remarkably, from an economic perspective, the incremental cost associated with the integration of nanoparticles is relatively modest (around $1 per ton of concrete), considering the substantial enhancements in mechanical properties achieved.
Keywords: concrete; dynamics; economic study; nanoparticles

1. Introduction

This exploration into nanotechnology unveils diverse strategies for elevating concrete by integrating nano-sized building blocks or surface modifications. These techniques provide control over material behavior and introduce novel properties. Nanoparticles, nanotubes, and grafted molecules serve to manipulate surface functionality, fostering specific interfacial interactions. Operating as elementary building blocks, nanoparticles form clusters within the 1-100 nm size range, characterized by a high surface area-to-volume ratio. Noteworthy alterations in their properties, such as surface energy, chemistry, and morphology, manifest with size reduction (Scrivener 2009, Zhou 2016).

In the realm of mathematical modeling for civil engineering and related fields (Yang and Yu 2017, Padhy and Panda 2017, Zhao *et al*. 2017, Rishikeshan and Ramesh 2017, Wen *et al*. 2017, Torres-Jimenez and Rodriguez-Cristerna 2017, Liu *et al*. 2018), computer programs play a pivotal role. Researchers have undertaken mechanical analyses of nanostructures, leading to significant enhancements in the mechanical performance of various materials, spanning metals, polymers, ceramics, and concrete composites (Amoli *et al*. 2018, Arbabi *et al*. 2017, Azmi *et al*. 2019, Bakhshande Amnieh *et al*. 2018, Faramoushjan *et al*. 2021, Jo *et al*. 2007). For instance, nanosilica (nano-SiO_2) emerges as a key player, enhancing workability and strength in high-performance and self-compacting concrete (Sanchez and Sobolev 2010).

Advancements in instrumentation have empowered the nanoscale characterization of concrete and the measurement of local mechanical properties in micro- and nanoscopic phases (Trtik and Bartos 2001). Techniques like nuclear magnetic resonance, atomic force microscopy, micro- and nano-indentation, neutron scattering, ultrasonic force microscopy, and focus-ion beam (FIB) nanotomography offer insights into nano-scale processes in cementitious materials. For example, atomic force microscopy (AFM) has unveiled a highly ordered structure in nanoscale C-S-H, challenging conventional assumptions. This improved understanding of concrete's nano-level structure facilitates better performance control and the customization of desired properties, potentially influencing concrete production and application methods (Farokhian and Kolahchi 2020, 2018, Golabchi *et al*. 2018, Hajmohammad *et al*. 2017, 2018a, b, c, 2019a, b, 2021, Heidarzadeh *et al*. 2018, Jafarian Arani 2016, Jassas *et al*. 2019, Jamali *et al*. 2016, 2019, Jafari Natanzi 2018, Keshtegar *et al*. 2018, 2020a, b, c, Kolahdouzan *et al*. 2020, Motezaker *et al*. 2017a, b, 2021, Naseri Taheri *et al*. 2020, Taherifar *et al*. 2020, 2021).

Another avenue of nanotechnology application in concrete lies in the "bottom-up" potential of nano-chemistry, giving rise to innovative superplasticizers and advanced coating materials (Babazadeh *et al*. 2016). These coatings can boast self-cleaning properties, resistance to discoloration and graffiti, and high scratch-and-wear resistance. Moreover, self-cleaning materials based on photocatalyst technology, utilizing substances like titanium

*Corresponding author, Professor,
E-mail: changhaoqi66@163.com

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dioxide (TiO2), have been explored. TiO2 acts as a photocatalyst under UV light, enabling self-cleaning and disinfecting properties, with surface hydrophilicity preventing the attachment of dust and dirt. While prior concrete technology advancements employed super-fine particles like fly ash and silica fume, recent progress in nano-chemistry and nanoparticle synthesis methods is anticipated to offer new possibilities for enhancing concrete performance (Flores *et al*. 2010).

Integrating nanoparticles into traditional construction materials opens avenues for introducing advanced or smart properties, particularly relevant to high-rise, long-span, and intelligent infrastructure systems (Flores *et al*. 2010). Researchers have delved into nonlinear vibration analyses of embedded nanocomposite concrete (Shokravi *et al*. 2017), wave propagation analyses in concrete models reinforced with silicon dioxide (SiO2) nanoparticles (Bakhshandeh Amnieh and Zamzam, 2017), and stability analyses of concrete pipes mixed with nanoparticles conveying fluid (Zamani Nouri 2016).

To the best of the author's knowledge, no prior reports have explored the technical and economic evaluation of utilizing silica nanoparticles for constructing concrete structures. Driven by this gap, this study aims to enhance the optimal design of concrete structures by investigating the effects of nanoparticles on mechanical properties such as modulus of elasticity, compressive strength, and their indirect influence on the percentage of reinforcement.

2. Halpin-Tsai formulation

2.1 Materials

A concrete beam with length L and thickness h is assumed. Based on Euler theory of beam, we have (Zamanian *et al*. 2017, Zarei *et al*. 2017):

$$U(x,z,t) = u_0(x,t) - z\frac{\partial}{\partial x}w_0(x,t), \quad (1)$$

$$V(x,z,t) = 0, \quad (2)$$

$$W(x,z,t) = w_0(x,t), \quad (3)$$

where respectively, u_0, w_0 are the mid-plane displacements in the x and z axis. The normal and shear strains are:

$$\varepsilon_{xx} = \frac{\partial}{\partial x}u_{0t} - z\frac{\partial^2}{\partial x^2}w_0, \quad (4)$$

$$\varepsilon_{xz} = 0, \quad (5)$$

The stress-strain relation is:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad (6)$$

where C_{ijkl} is the matrix of elastic. The effective Young's modulus, Poisson's ratio and density for the micro concrete beam using the Halpin-Tsai model are:

$$E = \frac{3}{8}\frac{1+\left(2\frac{l}{t}\right)\left(\left(\frac{E_{NP}}{E_M}-1\right)/\left(\frac{E_{NP}}{E_M}\right)+2\frac{l}{t}\right)V_{NP}}{1-\left(\left(\frac{E_{NP}}{E_M}-1\right)/\left(\frac{E_{NP}}{E_M}+2\frac{l}{t}\right)\right)V_{NP}}E_M + \quad (7)$$

$$\frac{5}{8}\frac{1+\left(2\frac{w}{t}\right)\left(\left(\frac{E_{NP}}{E_M}-1\right)/\left(\frac{E_{NP}}{E_M}\right)+2\frac{w}{t}\right)V_{NP}}{1-\left(\left(\frac{E_{NP}}{E_M}-1\right)/\left(\frac{E_{NP}}{E_M}\right)+2\frac{w}{t}\right)V_{NP},}E_M,$$

$$\upsilon = V_{NP}\upsilon_{NP} + (1-V_{NP})\upsilon_M, \quad (8)$$

$$\rho = V_{NP}\rho_{NP} + (1-V_{NP})\rho_M, \quad (9)$$

where t, W, l are the thickness, width and average length of nanoparticles, respectively, E_{NP} and E_M are the moduli of nanoparticles and matrix, respectively, υ_M and υ_{NP} are the Poisson's ratio of matrix and nanoparticles, ρ_M and ρ_{GPL} respectively are densities of matrix and nanoparticles, V_{NP} is:

$$V_{NP} = \frac{W_{NP}}{W_{NP} + (\rho_{NP}/\rho_M) - (\rho_{NP}/\rho_M)W_{NP}}, \quad (10)$$

where W_{NP} is the weight fraction of nanoparticles. The potential and kinetic energies of the structure are:

$$U = \frac{1}{2}\int(\sigma_{ij}\varepsilon_{ij})\,dv \quad (11)$$

$$K = \int\rho\left[\left(\frac{\partial U}{\partial t}\right)^2 + \left(\frac{\partial W}{\partial t}\right)^2\right]dxdt, \quad (12)$$

Utilizing Hamilton's principle can be obtained the motion final equations as:

$$\delta\int_0^T[U \quad - K]dt = 0, \quad (13)$$

Based on ACI, the elastic modulus of the concrete without nanoparticles can be calculated by

$$E_c = 4700\sqrt{f_c^{'}} = 4700\sqrt{21} = 21.5GPa \quad (14)$$

Noted that the final relations are presented in Appendix A and B.

3. Results and discussion

The elastic modulus of concrete, as predicted by the Halpin-Tsai model, is depicted in Fig. 1 for various beam length. The graph clearly illustrates that an increase in the volume percentage of nanoparticles corresponds to a significant rise in the elastic modulus. Specifically, reinforcing the concrete with 1% nanoparticles results in an impressive 78% enhancement of the elastic modulus. This improvement can be attributed to the increased stiffness of the structure as the volume percentage of silica nano-particles rises.

However, it is important to note that the length of beam can lead to a reduction in the elastic modulus of up to 24%. To mitigate this effect, proper dispersion of nanoparticles in the water during the construction process becomes crucial. Ensuring effective dispersion will help minimize the agglomeration of nanoparticles and maintain the desired enhancement in the elastic modulus of the concrete.

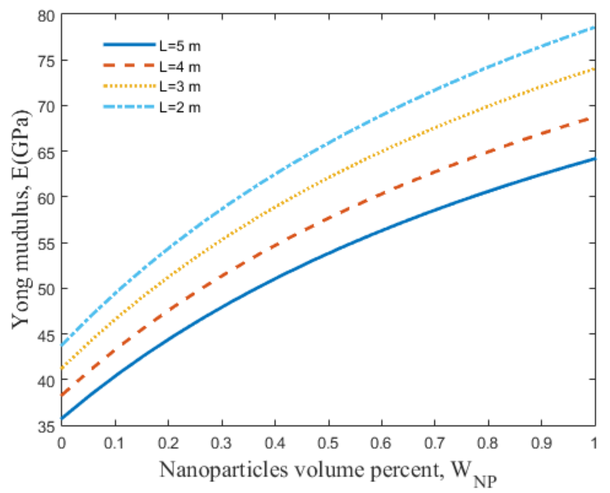

Fig. 1 The effect of nanoparticles and length on the elastic modulus

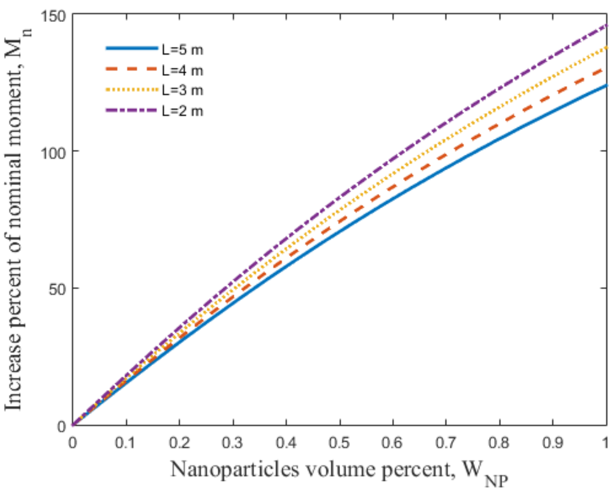

Fig. 2 The effect of nanoparticles and length on the armature percentage

Fig. 2 illustrates the impact of nanoparticle volume percentage and length on the armature percentage in concrete. The graph shows that as the volume percentage of nanoparticles increases, the armature percentage decreases. For instance, reinforcing the concrete with 1% nanoparticles results in a significant reduction of approximately 142% in the armature percentage. This implies that the addition of nanoparticles allows for a reduction in the amount of reinforcement required in the concrete structure.

On the other hand, it should be noted that the length of beam has the opposite effect, leading to an increase in the armature percentage. This means that when nanoparticles, it becomes necessary to use a higher percentage of reinforcement in the concrete to compensate for this effect. Therefore, it is crucial to ensure proper dispersion of nanoparticles during the construction process to minimize length and achieve the desired reduction in the armature percentage.

Fig. 3 demonstrates the influence of nanoparticle volume percentage and length on the compressive strength of the concrete. The graph clearly shows that as the volume

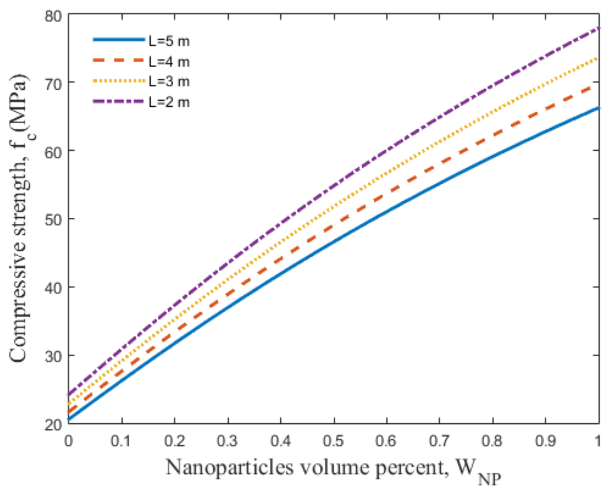

Fig. 3 The effect of nanoparticles and length on the compressive strength

Table 1 Price analysis of using silica nanoparticle in the concrete

Concrete with 40 GPa	Cement price	Silica nanoparticle price	Total price
Sample 1	(512/1000)*25=12.8$	0	12.8$
Sample 2	(288/1000)*25=7.3$	(45/1000)*150=6.75$	14.05$

percentage of nanoparticles increases, the compressive strength of the concrete also increases. This is attributed to the enhanced stiffness of the structure resulting from the presence of nanoparticles.

For instance, when no nanoparticles are present, the compressive strength of the concrete is 20 MPa. However, reinforcing the concrete with 1% nanoparticles leads to a substantial increase in compressive strength, reaching 72.69 MPa. In other words, the addition of 1% nanoparticles results in the concrete exhibiting a compressive strength approximately 2.5 times higher than that of the concrete without nanoparticles.

Furthermore, it should be noted that the length of beam has a detrimental effect on the compressive strength of the concrete. Increasing length leads to a decrease in compressive strength compared to the dispersed state of nanoparticles. Hence, it is important to ensure proper dispersion of nanoparticles during the concrete preparation process in order to achieve the desired increase in compressive strength.

To perform a pricing analysis of using silica nanoparticles, let’s consider information from the “Alibaba” website. The price of 1 ton of silica nanoparticles is $150, while the price of cement for 1 ton is $25.

Table 1 indicates that to achieve a compressive strength of 40 GPa, two samples are considered: one without nanoparticles (sample 1) and the other with nanoparticles (sample 2).

For sample 1 (without nanoparticles), 512 kg of cement is required to obtain the desired compressive strength.

For sample 2 (with nanoparticles), considering the enhanced properties discussed earlier, 288 kg of cement and 45 kg of silica nanoparticles are needed to achieve the same compressive strength.

Now, let’s calculate the cost implications:

For sample 1:

Cement: 512 kg x $25/ton = $12.8

For sample 2:

Cement: 288 kg x $25/ton = $7.2

Silica nanoparticles: 45 kg x $150/ton = $6.75

Therefore, the total cost increase in 1 ton of concrete, when reinforcing it with silica nanoparticles (sample 2), is $7.2.

Considering the cost difference and the significant improvements in compressive strength (2.5 times higher) and reduced armature percentage (up to 142% reduction), it can be concluded that the use of silica nanoparticles is a cost-effective option. The increase in price is relatively small compared to the enhanced performance achieved by incorporating nanoparticles into the concrete.

4. Conclusions

The study delves into the technical and economic evaluation of employing silica nanoparticles in the construction of concrete structures. The Mori-Tanaka model is employed to ascertain the equivalent material properties of the structure while considering nanoparticle agglomeration. The investigation explores the impact of these nanoparticles on concrete’s mechanical properties, including modulus of elasticity, compressive strength, and their indirect influence on armature percentage. Furthermore, the study scrutinizes the pricing aspect of silica nanoparticles and their influence on the overall cost escalation of concrete structures. The integration of nanoparticles substantially enhances the modulus of elasticity, with a noteworthy 78% increase observed by elevating the volume percentage of silica nanoparticles up to 1%. Compressive strength experiences a significant boost through nanoparticle addition, resulting in a 2.5 times increase with 1% nanoparticle reinforcement compared to nanoparticle-free concrete. The armature percentage in concrete decreases with escalating volume percentages of silica nanoparticles, exemplified by a remarkable 142% reduction when reinforcing with 1% silica nanoparticles. The length of the beam negatively affects both elastic modulus and compressive strength in concrete. Ensuring proper nanoparticle dispersion during the construction process is crucial to minimizing length-related impacts and maximizing desired material properties. A comprehensive price analysis, incorporating the costs of silica nanoparticles and cement, reveals a relatively modest increase in price (approximately $1 per ton of concrete) when utilizing silica nanoparticles, underscoring the cost-effectiveness of achieving significant enhancements in mechanical properties. In conclusion, the research underscores that the incorporation of nanoparticles in concrete yields notable improvements in mechanical properties without compromising cost-effectiveness. This study contributes valuable insights for integrating nanotechnology into the construction industry and underscores the potential advantages of using silica nanoparticles in concrete structures.

References

Arbabi, A., Kolahchi, R. and Rabani Bidgoli, M. (2017), “Concrete columns reinforced with Zinc Oxide nanoparticles subjected to electric field: buckling analysis”, *Wind Struct.*, **24**(5), 431-446. https://doi.org/10.12989/was.2017.24.5.431

Amoli, A., Kolahchi, R. and Rabani Bidgoli, M. (2018), “Seismic analysis of AL2O3 nanoparticles-reinforced concrete plates based on sinusoidal shear deformation theory”, *Earthq. Struct.*, **15**(3), 285-294. https://doi.org/10.12989/eas.2018.15.3.285.

Azmi, M., Kolahchi, R. and Rabani Bidgoli, M. (2019), “Dynamic analysis of concrete column reinforced with Sio2 nanoparticles subjected to blast load”, *Adv. Concr. Constr.*, **7**(1), 51-63. https://doi.org/10.12989/acc.2019.7.1.051.

Bakhshandeh Amnieh, H. and Zamzam, M.S. (2017), “Theoretical and experimental analysis of wave propagation in concrete blocks subjected to impact load considering the effect of nanoparticles”, *Comput. Concr.*, **20**(6), 711-718. https://doi.org/10.12989/cac.2017.20.6.711

Babazadeh, A., Burgueño, R. and Silva, P.F. (2016), “Evaluation of the critical plastic region length in slender reinforced concrete bridge columns”, *Eng. Struct.*, **125**, 280-293.

Fakhar, A. and Kolahchi, R. (2018), “Dynamic buckling of magnetorheological fluid integrated by visco-piezo-GPL reinforced plates”, *Int. J. Mech. Sci.*, **144**, 788-799. https://doi.org/10.1016/j.ijmecsci.2018.06.036.

Farokhian, A. and Kolahchi, R. (2020), “Frequency and instability responses in nanocomposite plate assuming different distribution of CNTs”, *Struct. Eng. Mech.*, **73**(5), 555-563. https://doi.org/10.12989/sem.2020.73.5.555.

Flores, I., Sobolev, K., Torres, L.M., Valdez, P.L., Zarazua, E. and Cuellar, E.L. (2010), “Performance of cement systems with nano- sio2 particles produced using sol-gel method”, *TRB First International Conference in North America on Nanotechnology in Cement and Concrete*, Irvine, California, USA, May 5-7.

Faramoushjan, S.G., Jalalifar, H. and Kolahchi, R. (2021), “Mathematical modelling and numerical study for buckling study in concrete beams containing carbon nanotubes”, *Adv. Concr. Constr.*, **11**(6), 521-529. https://doi.org/10.12989/acc.2021.11.6.521.

Golabchi, H., Kolahchi, R. and Rabani Bidgoli, M. (2018), “Vibration and instability analysis of pipes reinforced by SiO2 nanoparticles considering agglomeration effects”, *Comput. Concrete*, **21**, 431-440. https://doi.org/10.12989/cac.2018.21.4.431

Hajmohammad, M.H., Azizkhani, M.B. and Kolahchi, R. (2018a), “Multiphase nanocomposite viscoelastic laminated conical shells subjected to magneto-hygrothermal loads: Dynamic buckling analysis”, *Int. J. Mech. Sci.*, **137**, 205-213. https://doi.org/10.1016/j.ijmecsci.2018.01.026.

Hajmohammad, M.H., Sharif Zarei, M., Nouri, A. and Kolahchi, R. (2017), “Dynamic buckling of sensor/functionally graded-carbon nanotube-reinforced laminated plates/actuator based on sinusoidal-visco-piezoelasticity theories”, *J. Sandw. Struct. Mater.*, 1099636217720373. https://doi.org/10.1177/1099636217720373.

Hajmohammad, M.H., Zarei, M.S., Farrokhian, A. and Kolahchi, R. (2018b), “A layerwise theory for buckling analysis of truncated conical shells reinforced by CNTs and carbon fibers integrated with piezoelectric layers in hygrothermal environment”, *Adv. Nano Res.*, **6**(4), 299-321. https://doi.org/10.12989/anr.2018.6.4.299.

Hajmohammad, M.H., Nouri, A.H., Zarei, M.S. and Kolahchi, R. (2019a), “A new numerical approach and visco-refined zigzag theory for blast analysis of auxetic honeycomb plates integrated by multiphase nanocomposite facesheets in hygrothermal”, *Eng. Comput.*, **35**(4), 1141-1157.

https://doi.org/10.1007/s00366-018-0655-x.

Hajmohammad, M.H., Maleki, M. and Kolahchi, R. (2018c), "Seismic response of underwater concrete pipes conveying fluid covered with nano-fiber reinforced polymer layer", *Soil Dyn. Earthq. Eng.*, **110**, 18-27. https://doi.org/10.1016/j.soildyn.2018.04.002

Hajmohammad, M.H., Zarei, M.S., Kolahchi, R. and Karami, H. (2019b), "Visco-piezoelasticity-zigzag theories for blast response of porous beams covered by graphene platelet-reinforced piezo-electric layers", *J. Sandw. Struct. Mater.*, 1099636219839175. https://doi.org/10.1177/1099636219839175.

Hajmohammad, M.H., Farrokhian, A. and Kolahchi, R. (2021), "Dynamic analysis in beam element of wave-piercing Catamarans undergoing slamming load based on mathematical modelling", *Ocean Eng.*, **234**, 109269. https://doi.org/10.1016/j.oceaneng.2021.109269.

Jafarian Arani, A. and Kolahchi, R. (2016), "Buckling analysis of embedded laminated porous concrete beams armed with carbon nanotubes", *Comput. Concr.*, **17**(5), 567-578. https://doi.org/10.12989/cac.2016.17.5.567

Jo, B.W., Kim, C.H. and Lim, J.H. (2007), "Investigations on the development of powder concrete with nano-SiO2 particles", *KSCE J. Civil Eng.*, **11**, 37-42. https://doi.org/10.1007/BF02823370

Jamali, M., Shojaee, T., Kolahchi, R. and Mohammadi, B. (2016), "Buckling analysis of nanocomposite cut out plate using domain decomposition method and orthogonal polynomials", *Steel Compos. Struct.*, **22**(3), 691-712. https://doi.org/10.12989/scs.2016.22.3.691.

Jassas, M.R., Rabani Bidgoli, M. and Kolahchi, R. (2019), "Forced vibration analysis of concrete plates reinforced by agglomerated SiO_2 nanoparticles based on numerical methods", *Constr. Build. Mater.*, **211**, 796-806. https://doi.org/10.1016/j.conbuildmat.2019.03.263.

Javani, R., Rabani Bidgoli, M. and Kolahchi, R. (2019), "Buckling analysis of plates reinforced by Graphene platelet based on Halpin-Tsai and Reddy theories", *Steel Compos. Struct.*, **31**(4), 419-426. https://doi.org/10.12989/scs.2019.31.4.419.

Jafari Natanzi, A., Soleimani Jafari, G. and Kolahchi, R. (2018), "Vibration and instability of nanocomposite pipes conveying fluid mixed by nanoparticles resting on viscoelastic foundation", *Comput. Concr.*, **21**(5), 569-582. https://doi.org/10.12989/cac.2018.21.5.569.

Jamali, M., Shojaee, T., Mohammadi, B. and Kolahchi, R. (2019), "Cut out effect on nonlinear post-buckling behavior of FG-CNTRC micro plate subjected to magnetic field via FSDT", *Adv. Nano Res.*, **7**(6), 405-417. https://doi.org/10.12989/anr.2019.7.6.405.

Keshtegar, B., Motezaker, M., Kolahchi,R. and Trung, N.T. (2020a), "Wave propagation and vibration responses in porous smart nanocomposite sandwich beam resting on Kerr foundation considering structural damping", *Thin Wall Struct.*, **154**, 106820. https://doi.org/10.1016/j.tws.2020.106820

Keshtegar, B. and Kolahchi, R. (2018), "Reliability analysis-based conjugate map of beams reinforced by ZnO nanoparticles using sinusoidal shear deformation theory", *Steel Compos. Struct.*, **28**(2), 195-207. https://doi.org/10.12989/scs.2018.28.2.195.

Keshtegar, B., Tabatabaei, J., Kolahchi, R. and Trung, N.T. (2020c), "Dynamic stress response in the nanocomposite concrete pipes with internal fluid under the ground motion load", *Adv. Concr. Constr.*, **9**(3), 327-335. https://doi.org/10.12989/acc.2020.9.3.327.

Keshtegar, B., Farrokhian, A., Kolahchi, R. and Trung, N.T. (2020b), "Dynamic stability response of truncated nano-composite conical shell with magnetostrictive face sheets utilizing higher order theory of sandwich panels", *Eur. J. Mech. A Solids*, **82**, 104010. https://doi.org/10.1016/j.euromechsol.2020.104010

Kolahchi, R. (2017a), "A comparative study on the bending, vibration and buckling of viscoelastic sandwich nano-plates based on different nonlocal theories using DC, HDQ and DQ methods", *Aerosp. Sci. Technol.*, **66**, 235-248. https://doi.org/10.1016/j.ast.2017.03.016.

Kolahdouzan, F., Mosayyebi, M., Ghasemi, F.A., Kolahchi, R. and Mousavi Panah, S.M. (2020), "Free vibration and buckling analysis of elastically restrained FG-CNTRC sandwich annular nanoplates", *Adv. Nano Res.*, **9**(4), 237-250. https://doi.org/10.12989/anr.2020.9.4.237.

Liu, H., Ma, J., Huang, W. (2018), "Sensor-based complete coverage path planning in dynamic environment for cleaning robot", *CAAI Trans. Intell. Technol.*, **3**, 65-72.

Motezaker, M. and Kolahchi, R. (2017a), "Seismic response of concrete columns with nanofiber reinforced polymer layer", *Comput. Concr.*, **20**(3), 361-368. https://doi.org/10.12989/cac.2017.20.3.361.

Motezaker, M. and Kolahchi, R. (2017b), "Seismic response of SiO_2 nanoparticles-reinforced concrete pipes based on DQ and newmark methods", *Comput. Concr.*, **19**(6), 745-753. https://doi.org/10.12989/cac.2017.19.6.745.

Motezaker, M., Kolahchi, R., Kumar Rajak, D. and Mahmoud, S. R. (2021), "Influences of fiber reinforced polymer layer on the dynamic deflection of concrete pipes containing nanoparticle subjected to earthquake load", *Polym. Compos.*, **42**(8), 4073-4081. https://doi.org/10.1002/pc.26118.

Naseri Taheri, M., Sabet, S.A. and Kolahchi, R. (2020), "Experimental investigation of self-healing concrete after crack using nano-capsules including polymeric shell and nanoparticles core", *Smart Struct. Syst.*, **25**(3), 337-343. https://doi.org/10.12989/sss.2020.25.3.337.

Sanchez, F. and Sobolev, K. (2010), "Nanotechnology in concrete - A review". *Constr. Build. Mater.*, **24**, 2060-2071.

Taherifar, R., Zareei, S.A., Rabani Bidgoli, M. and Kolahchi, R. (2021), "Application of differential quadrature and Newmark methods for dynamic response in pad concrete foundation covered by piezoelectric layer", *J. Computat. Appl. Math.*, **382**, 113075. https://doi.org/10.1016/j.cam.2020.113075.

Rishikeshan, C.A. and Ramesh, H. (2017), "A novel mathematical morphology based algorithm for shoreline extraction from satellite images", *Geo-spatial Inform. Sci.*, **20**, 345-352.

Padhy, S. and Panda, S. (2017), "A hybrid stochastic fractal search and pattern search technique based cascade PI-PD controller for automatic generation control of multi-source power systems in presence of plug in electric vehicles", *CAAI Trans. Intell. Technol.*, **2**, 12-25 .

Scrivener, K.L. (2009), "Nanotechnology and cementitious materials", *Proceedings of the NICOM3 3rd Int Symp Nanotech Const*, Prague, Czech Republic.

Shokravi, M. (2017), "Vibration analysis of silica nanoparticles-reinforced concrete beams considering agglomeration effects", *Comput. Concr.*, **19**(3), 333-338. https://doi.org/10.12989/cac.2017.19.3.333

Trtik, P. and Bartos, P.J.M. (2001), "Nanotechnology and concrete: What can we utilise from the upcoming technologies?", *Proceedings pf the 2nd Annamaria Workshop: Cement Concrete: Trends & Challenges*.

Torres-Jimenez, J. and Rodriguez-Cristerna, A. (2017), "Metaheuristic post-optimization of the NIST repository of covering arrays", *CAAI Trans. Intell. Technol.*, **2**, 31-38 .

Yang, H. and Yu, L. (2017), "Feature extraction of wood-hole defects using wavelet-based ultrasonic testing", *J. Forest. Res.*, **28**, 395-402.

Wen, Q., He, J., Guan, Sh., Chen, T., Hu, Y., Wu, W., Liu, F., Qiao, Y. (2017), "The TripleSat constellation: A new geospatial data service model", *Geo-spatial Inform. Sci.*, **20**, 163-173.

Zamanian, M., Kolahchi, R. and Rabani Bidgoli, M. (2017), "Agglomeration effects on the buckling behaviour of embedded concrete columns reinforced with SiO_2 nanoparticles", *Wind. Struct.*, **24**(1), 43-57. https://doi.org/10.12989/was.2017.24.1.043.
Zarei, M.S., Azizkhani, M.B., Hajmohammad, M.H. and Kolahchi, R. (2017), "Dynamic buckling of polymer–carbon nanotube–fiber multiphase nanocomposite viscoelastic laminated conical shells in hygrothermal environments", *J. Sandw. Struct. Mater.*, 1099636217743288. https://doi.org/10.1177/1099636217743288.
Zamani, A., Kolahchi, R. and Rabani Bidgoli, M. (2017), "Seismic response of smart nanocomposite cylindrical shell conveying fluid flow using HDQ-Newmark methods", *Comput. Concr.*, **20**(6), 671-682. https://doi.org/10.12989/cac.2017.20.6.671.
Zhou, X., Liu, J., Wang, X. and Frank Chen, Y. (2016), "Behavior and design of slender circular tubed-reinforced-concrete columns subjected to eccentric compression", *Eng. Struct.*, **124**, 17-28. https://doi.org/10.1016/j.engstruct.2016.05.036
Zhao, B., Gao, L., Liao, W. and Zhang, B. (2017), "A new kernel method for hyperspectral image feature extraction", *Geo-spatial Inform. Sci.*, **20**, 309-318. https://doi.org/10.1080/10095020.2017.1403088

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Appendix A

$$\gamma_x^t = \frac{\partial^2 u_{0t}}{\partial x^2} - z\frac{\partial^3 w_0}{\partial x^3} + f(z)\frac{\partial^2 \alpha}{\partial x^2} \tag{A1}$$

$$\gamma_x^b = \frac{\partial^2 u_{0b}}{\partial x^2} - z\frac{\partial^3 w_0}{\partial x^3} + f(z)\frac{\partial^2 \alpha}{\partial x^2} \tag{A2}$$

$$\gamma_z = -\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} \tag{A3}$$

$$p_x^t = 2\mu I_{00}^2\left(\frac{\partial^2 u_{0t}}{\partial x^2} - z\frac{\partial^3 w_0}{\partial x^3} + f(z)\frac{\partial^2 \alpha}{\partial x^2}\right) \tag{A4}$$

$$p_x^b = 2\mu I_{00}^2\left(\frac{\partial^2 u_{0b}}{\partial x^2} - z\frac{\partial^3 w_0}{\partial x^3} + f(z)\frac{\partial^2 \alpha}{\partial x^2}\right) \tag{A5}$$

$$p_z = 2\mu I_{00}^2\left(-\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial \ f(z)}{\partial z}\frac{\partial \ \alpha}{\partial x}\right) \tag{A6}$$

$$\eta_{xxx}^t = \frac{2}{5}\frac{\partial^2 u_{0t}}{\partial x^2} - \frac{2}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A7}$$

$$\eta_{xxx}^b = \frac{2}{5}\frac{\partial^2 u_{0b}}{\partial x^2} - \frac{2}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A8}$$

$$\eta_{xxz}^t = \frac{4}{5}\frac{\partial \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2} \tag{A9}$$

$$\eta_{xxz}^b = \frac{4}{5}\frac{\partial \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2} \tag{A10}$$

$$\eta_{xyy}^t = -\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A11}$$

$$\eta_{xyy}^b = -\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A12}$$

$$\eta_{xzx} = \frac{4}{5}\frac{\partial f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2} \tag{A13}$$

$$\eta_{xzz}^t = -\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{1}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A14}$$

$$\eta_{xzz}^b = -\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{1}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A15}$$

$$\eta_{yxy}^t = -\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A16}$$

$$\eta_{yxy}^b = -\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A17}$$

$$\eta_{yyx}^t = \frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} - \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A18}$$

$$\eta_{yyx}^b = \frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} - \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A19}$$

$$\eta_{yyz} = \frac{4}{5}\frac{\partial \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{1}{15}\frac{\partial^2 w_0}{\partial x^2} \tag{A20}$$

$$\eta_{yzy} = -\frac{4}{5}\frac{\partial \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} + \frac{1}{15}\frac{\partial^2 w_0}{\partial x^2} \tag{A21}$$

$$\eta_{zxx} = +\frac{4}{5}\frac{\partial \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2} \tag{A22}$$

$$\eta_{zxz}^t = -\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A23}$$

$$\eta_{zxz}^b = -\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A24}$$

$$\eta_{zyy} = -\frac{1}{5}\frac{\partial\ \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} + \frac{1}{15}\frac{\partial^2 w_0}{\partial x^2} \tag{A25}$$

$$\eta_{zzx}^t = -\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A26}$$

$$\eta_{zzx}^b = -\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha \tag{A27}$$

$$\eta_{zzz} = -\frac{3}{5}\frac{\partial\ \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} + \frac{1}{5}\frac{\partial^2 w_0}{\partial x^2} \tag{A28}$$

$$\eta_{xxz}^c = \frac{8}{15}\frac{H}{h_c}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\frac{\partial u_{0t}}{\partial x} - \frac{\partial u_{0b}}{\partial x}}{H}\right) \tag{A29}$$

$$\eta_{xzx}^c = \frac{8}{15}\frac{H}{h_c}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\frac{\partial}{\partial x}u_{0t} - \frac{\partial}{\partial x}u_{0b}}{H}\right) \tag{A30}$$

$$\eta_{yyz}^c = -\frac{2}{15}\frac{H}{h_c}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\frac{\partial}{\partial x}u_{0t} - \frac{\partial}{\partial x}u_{0b}}{H}\right) \tag{A31}$$

$$\eta_{yzyc} = -\frac{2 * H(\frac{\partial^2}{\partial x^2}w_0 + \frac{\frac{\partial}{\partial x}u_{0t} - \frac{\partial}{\partial x}u_{0b}}{H})}{15hc} \tag{A32}$$

$$\eta_{zxx}^c = -\frac{8}{15}\frac{H}{hc}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{1}{H}\left(\frac{\partial u_{0t}}{\partial x} - \frac{\partial u_{0b}}{\partial x}\right)\right) \tag{A33}$$

$$\eta_{zyy}^c = -\frac{2}{15}\frac{H}{hc}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{1}{H}\left(\frac{\partial u_{0t}}{\partial x} - \frac{\partial u_{0b}}{\partial x}\right)\right) \tag{A34}$$

$$\eta_{zzz}^c = -\frac{2}{5}\frac{H}{hc}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{1}{H}\left(\frac{\partial u_{0t}}{\partial x} - \frac{\partial u_{0b}}{\partial x}\right)\right) \tag{A35}$$

$$\tau_{xxx}^t = 2\mu I_{11}^2\left(\frac{2}{5}\frac{\partial^2 u_{0t}}{\partial x^2} - \frac{2}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A36}$$

$$\tau_{xxx}^b = 2\mu I_{11}^2\left(\frac{2}{5}\frac{\partial^2 u_{0b}}{\partial x^2} - \frac{2}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A37}$$

$$\tau_{xxz}^t = 2\mu I_{11}^2\left(\frac{4}{5}\frac{\partial\ \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A38}$$

$$\tau_{xxz}^b = 2\mu I_{11}^2\left(\frac{4}{5}\frac{\partial\ \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A39}$$

$$\tau_{xyy}^t = 2\mu I_{11}^2\left(-\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A40}$$

$$\tau_{xyy}^b = 2\mu I_{11}^2\left(-\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A41}$$

$$\tau_{xzx} = 2\mu I_{11}^2\left(\frac{4}{5}\frac{\partial f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A42}$$

$$\tau_{xzz}^t = 2\mu I_{11}^2\left(-\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{1}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A43}$$

$$\tau_{xzz}^b = 2\mu I_{11}^2\left(-\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{2}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{1}{5}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A44}$$

$$\tau_{yxy}^t = 2\mu I_{11}^2\left(-\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A45}$$

$$\tau_{yxy}^b = 2\mu I_{11}^2\left(-\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} + \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} - \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} - \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A46}$$

$$\tau_{yyx}^t = 2\mu I_{11}^2\left(\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2} - \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A47}$$

$$\tau_{yyx}^b = 2\mu I_{11}^2\left(\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2} - \frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3} + \frac{1}{5}f(z)\frac{\partial^2 \alpha}{\partial x^2} + \frac{2}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\right) \tag{A48}$$

$$\tau_{yyz} = 2\mu I_{11}^2\left(\frac{4}{5}\frac{\partial\ \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{1}{15}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A49}$$

$$\tau_{yzy} = 2\mu I_{11}^2\left(-\frac{4}{5}\frac{\partial\ \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} + \frac{1}{15}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A50}$$

$$\tau_{zxx} = 2\mu I_{11}^2\left(+\frac{4}{5}\frac{\partial\ \ f(z)}{\partial z}\frac{\partial \alpha}{\partial x} - \frac{4}{15}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A51}$$

$$\tau_{zxz}^{t}=2\mu I_{11}^{2}\begin{pmatrix}-\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2}+\frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3}\\ -\frac{1}{5}f(z)\frac{\partial^2\alpha}{\partial x^2}+\frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\end{pmatrix} \tag{A52}$$

$$\tau_{zxz}^{b}=2\mu I_{11}^{2}\begin{pmatrix}-\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2}+\frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3}\\ -\frac{1}{5}f(z)\frac{\partial^2\alpha}{\partial x^2}+\frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\end{pmatrix} \tag{A53}$$

$$\tau_{zyy}=2\mu I_{11}^{2}\left(-\frac{1}{5}\frac{\partial f(z)}{\partial z}\frac{\partial\alpha}{\partial x}+\frac{1}{15}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A54}$$

$$\tau_{zzx}^{t}=2\mu I_{11}^{2}\begin{pmatrix}-\frac{1}{5}\frac{\partial^2 u_{0t}}{\partial x^2}+\frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3}\\ -\frac{1}{5}f(z)\frac{\partial^2\alpha}{\partial x^2}+\frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\end{pmatrix} \tag{A55}$$

$$\tau_{zzx}^{b}=2\mu I_{11}^{2}\begin{pmatrix}-\frac{1}{5}\frac{\partial^2 u_{0b}}{\partial x^2}+\frac{1}{5}z\frac{\partial^3 w_0}{\partial x^3}\\ -\frac{1}{5}f(z)\frac{\partial^2\alpha}{\partial x^2}+\frac{8}{15}\frac{\partial^2 f(z)}{\partial z^2}\alpha\end{pmatrix} \tag{A56}$$

$$\tau_{zzz}=2\mu I_{11}^{2}\left(-\frac{3}{5}\frac{\partial f(z)}{\partial z}\frac{\partial\alpha}{\partial x}+\frac{1}{5}\frac{\partial^2 w_0}{\partial x^2}\right) \tag{A57}$$

$$\tau_{xxz}^{c}=2\mu I_{11}^{2}\left(\frac{8}{15}\frac{H}{h_c}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\frac{\partial u_{0t}}{\partial x}-\frac{\partial u_{0b}}{\partial x}}{H}\right)\right) \tag{A58}$$

$$\tau_{xzx}^{c}=2\mu I_{11}^{2}\left(\frac{8}{15}\frac{H}{h_c}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\frac{\partial}{\partial x}u_{0t}-\frac{\partial}{\partial x}u_{0b}}{H}\right)\right) \tag{A59}$$

$$\tau_{yyz}^{c}=2\mu I_{11}^{2}\left(-\frac{2}{15}\frac{H}{h_c}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\frac{\partial}{\partial x}u_{0t}-\frac{\partial}{\partial x}u_{0b}}{H}\right)\right) \tag{A60}$$

$$\chi_{xy}=\frac{\frac{\partial}{\partial x}(-(1/2)\frac{\partial}{\partial x}w_0+(1/2)(\frac{d}{dz}f(z))\alpha-(1/2)\frac{\partial}{\partial x}w_0)}{2} \tag{A61}$$

$$\chi_{yz}=\frac{(\frac{d^2}{dz^2}f(z))\alpha}{4} \tag{A62}$$

Appendix B

$$(A_{11},B_{11},H_{11},P_{11},S_{11},T_{11})=\int_{h_c/2}^{(h_c/2)+h_t}(1,z,z^2,f(z),zf(z),f^2(z))c_{11t}dz \tag{B1}$$

$$(Q_{11},R_{11},Y_{11},K_{11},F_{11},Z_{11})=\int_{-(h_c/2)-h_b}^{-(h_c/2)}(1,z,z^2,zf(z),f(z),f^2(z))c_{11b}dz \tag{B2}$$

$$(M_{31},J_{31},Z_{31})=\int_{h_c/2}^{(h_c/2)+h}(1,z,f(z))e_{31t}dz \tag{B3}$$

$$(X_{31},M_{22},Z_{22})=\int_{h_c/2}^{(h_c/2)+h}(1,z,f(z))\frac{\pi}{h_t}cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_t}\right)e_{31t}dz \tag{B4}$$

$$O_{31}=\int_{h_c/2}^{(h_c/2)+h_t}\left(\frac{\pi}{h_t}cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_t}\right)\right)^2\in_{33t}dz \tag{B5}$$

$$(U_{11},N_{11})=\int_{h_c/2}^{(h_c/2)+h_t}(c_{55t},c_{55b})\left(\frac{\partial}{\partial z}f(z)\right)^2dz \tag{B6}$$

$$V_{11}=\int_{h_c/2}^{(h_c/2)+h_t}e_{51t}\left(\frac{\partial f(z)}{\partial z}\right)sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_t}\right)dz \tag{B7}$$

$$U_{22}=\int_{h_c/2}^{(h_c/2)+h_t}\in_{11t}sin^2\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_t}\right)dz \tag{B8}$$

$$A_\gamma^t,B_\gamma^t,R_\gamma^t,P_\gamma^t,F_\gamma^t,D_\gamma^t=\int_{h_c/2}^{(h_c/2)+h_t}2(1,z,z^2,f(z),zf(z),f^2(z))\mu_t I^2{}_{00}dz \tag{B9}$$

$$Q_\gamma^t,H_\gamma^t=\int_{h_c/2}^{(h_c/2)+h_t}2\left(\frac{\partial f(z)}{\partial z},\left(\frac{\partial f(z)}{\partial z}\right)^2\right)\mu I^2{}_{00}dz \tag{B10}$$

$$A_\gamma^b,B_\gamma^b,P_\gamma^b,R_\gamma^b,F_\gamma^b,D_\gamma^b=\int_{-(h_c/2)-h_b}^{-(h_c/2)}2\begin{pmatrix}1,z,z^2,f(z),\\ zf(z),f^2(z)\end{pmatrix}\mu_b I^2{}_{00}dz \tag{B11}$$

$$Q_\gamma^b,H_\gamma^b=\int_{-(h_c/2)-h_b}^{-(h_c/2)}2\left(\frac{\partial f(z)}{\partial z},\left(\frac{\partial f(z)}{\partial z}\right)^2\right)\mu_b I^2{}_{00}dz \tag{B12}$$

$$A_\eta^t,B_\eta^t,P_\eta^t,C_\eta^t=\int_{h_c/2}^{(h_c/2)+h_t}2\left(1,z,f(z),\frac{\partial^2 f(z)}{\partial z^2}\right)\mu_t I^2{}_{11}dz \tag{B13}$$

$$R_\eta^t,F_\eta^t,G_\eta^t,D_\eta^t=\int_{h_c/2}^{(h_c/2)+h_t}2\left(z^2,zf(z),z\frac{\partial^2 f(z)}{\partial z^2},f^2(z)\right)\mu_t I^2{}_{11}dz \tag{B14}$$

$$L_\eta^t,Q_\eta^t,H_\eta^t,O_\eta^t=\int_{h_c/2}^{(h_c/2)+h_t}2\begin{pmatrix}f(z)\frac{\partial^2 f(z)}{\partial z^2},\left(\frac{\partial^2 f(z)}{\partial z^2}\right)^2,\\ \frac{\partial\ f(z)}{\partial z},\left(\frac{\partial\ f(z)}{\partial z}\right)^2\end{pmatrix}\mu_t I^2{}_{11}dz \tag{B15}$$

$$A_\eta^b,B_\eta^b,P_\eta^b,C_\eta^b=\int_{-(h_c/2)-h_b}^{-(h_c/2)}2\left(1,z,f(z),\frac{\partial^2 f(z)}{\partial z^2}\right)\mu_b I^2{}_{11}dz \tag{B16}$$

$$R_\eta^b, F_\eta^b, G_\eta^b, D_\eta^b = \int_{-(h_c/2)-h_b}^{-(h_c/2)} 2\left(z^2, zf(z), z\frac{\partial^2 f(z)}{\partial z^2}, f^2(z)\right)\mu_b I^2{}_{11} dz \quad \text{(B17)}$$

$$L_\eta^b, Q_\eta^b, H_\eta^b, O_\eta^b = \int_{-(h_c/2)-h_b}^{-(h_c/2)} 2\begin{pmatrix} f(z)\frac{\partial^2 f(z)}{\partial z^2}, \left(\frac{\partial^2 f(z)}{\partial z^2}\right)^2, \\ \frac{\partial\ f(z)}{\partial z}, \left(\frac{\partial\ f(z)}{\partial z}\right)^2 \end{pmatrix}\mu_b I^2{}_{11} dz \quad \text{(B18)}$$

$$A_\eta^c = \int_{-(h_c/2)}^{+(h_c/2)} 2\mu_c I^2{}_{11} dz \quad \text{(B19)}$$

$$A_\chi^t, C_\chi^t, H_\chi^t, O_\chi^t = \int_{h_c/2}^{(h_c/2)+h_t} 2\begin{pmatrix} 1, \frac{\partial^2 f(z)}{\partial z^2}, \frac{\partial\ f(z)}{\partial z}, \\ \left(\frac{\partial}{\partial z} f(z)\right)^2 \end{pmatrix}\mu_t I^2{}_{22} dz \quad \text{(B20)}$$

$$A_\chi^b, C_\chi^b, H_\chi^b, O_\chi^b = \int_{-(h_c/2)-h_b}^{-(h_c/2)} 2\begin{pmatrix} 1, \frac{\partial^2 f(z)}{\partial z^2}, \frac{\partial\ f(z)}{\partial z} \\ , \left(\frac{\partial\ f(z)}{\partial z}\right)^2 \end{pmatrix}\mu_b I^2{}_{22} dz \quad \text{(B21)}$$

$$C_\chi^c = \int_{-(h_c/2)}^{+(h_c/2)} 2\frac{\partial^2 f(z)}{\partial z^2}\mu_c I^2{}_{22} dz \quad \text{(B22)}$$