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Thermoelastic deformation properties of non-localized and axially moving viscoelastic Zener nanobeams

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Abstract. This study aims to develop explicit models to investigate thermo-mechanical interactions in moving nanobeams. These models aim to capture the small-scale effects that arise in continuous mechanical systems. Assumptions are made based on the Euler-Bernoulli beam concept and the fractional Zener beam-matter model. The viscoelastic material law can be formulated using the fractional Caputo derivative. The non-local Eringen model and the two-phase delayed heat transfer theory are also taken into account. By comparing the numerical results to those obtained using conventional heat transfer models, it becomes evident that non-localization, fractional derivatives and dual-phase delays influence the magnitude of thermally induced physical fields. The results validate the significant role of the damping coefficient in the system's stability, which is further dependent on the values of relaxation stiffness and fractional order.
Keywords: fractional order; nonlocal theory; phase lags; viscoelastic beams; Zener model

1. Introduction

Over the past few decades, NEMS systems have made substantial progress. The mechanical behavior of nanoscale structural elements employed in NEMS has captivated the interest of scientists keen on comprehending their unique properties (Zhao *et al.* 2016 and Huang 2019). Nanomaterials are distinct from conventional materials, exhibiting novel characteristics that differentiate them from their larger counterparts.

Numerous experiments and simulations have provided compelling evidence supporting the significance of small-scale impact in design and optimization of MEMS and NEMS systems, including sensor systems, resonators, controllers, atomic force microscopes, and pumping systems (Aliasghary *et al*. 2022, Anjum *et al*. 2022, Faghidian *et al*. 2022). The conventional continuum concept, which lacks a length-scale parameter in its constitutive Eq.s, falls short of adequately explaining the mechanical behavior of micro- and nanostructures (Borjalilou *et al.* 2020). To overcome these limitations, researchers have developed non-classical continuum models within the framework of elasticity theory to capture the effects of reduced dimensions. Several novel approaches were devised to forecast mechanical behavior, taking into account the influence of size-dependent influences. At the nanoscale, size-dependent continuum mechanics models have been formulated, incorporating concepts such as surface elasticity, strain gradient elasticity, nonlocal elasticity, and couple stress elasticity (Toupin 1964, Mindlin *et al.* 1968, Lam *et al.* 2003, Yang *et al.* 2002, Eringen 1983). Among these models, Eringen (2002) introduced the concept of nonlocal elasticity as a size-dependent framework to tackle submicron structures. This model suggests that the strain in the surrounding region influences the stress tensor at a given location, deviating from the definition of the stress tensor in the conventional (local) continuum concept.

It is commonly acknowledged that the Euler-Bernoulli beam theory (EBT), which is extensively employed, neglects the influence of shear deformation. Long, narrow beams have the potential to yield dependable outcomes when utilized for forecasting the mechanical reaction. Nevertheless, in cases where the beam is thicker and shorter, the influence of shear deformation becomes significant. The Timoshenko beam model (TBT) is a pioneering beam theory that accounts for shear deformation. It postulates that the transverse shear stress and strain remain constant along the thickness direction. However, in real-world scenarios, it can be observed that the transverse shear stress and strain experienced by the top and bottom surfaces of the beam are negligible. Hence, the utilization of a transverse shear factor is implemented to account for the non-uniformity of the stress-strain distribution in the direction perpendicular to the thickness. Subsequently, a range of advanced beam theories have been developed to enhance the precision of predicting the mechanical characteristics of thick beams. One such theory is the sinusoidal shear deformation beam concept. Various versions of micro-polar elasticity, nonlocal elasticity, couple stress elasticity, and strain gradient elasticity, along with

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other related theories, have been extensively employed to investigate micro- and nanostructural phenomena (Koutsoumaris 2021, Xia *et al.* 2002, Askarian *et al.* 2020, Tung 2021, Pham *et al.* 2023, Zhang *et al.* 2022, Kumar *et al.* 2022, Abouelregal *et al.* 2022, Shan *et al.* 2022, Wu *et al.* 2023, Azandariani *et al.* 2022, Alazwari *et al.* 2022).

Many studies provide a comprehensive analysis of the impacts of many factors, including local volume fraction, nonlocal parameters, material length scale parameters, shear deformation, and surface energy, on stress- and strain-driven models. The results of the parametric analyses indicate that the two scale factors, namely the nonlocal parameters and material length characteristic parameters, have contrasting impacts on the stiffness of the nanobeams in the two driving models. On the other hand, the surface characteristics exert an equivalent influence on the stiffness of both driving models (Thai *et al*. 2022). The relationship between the slenderness ratio and the surface effect, as well as the scale effect, is characterized by an inverse correlation. Specifically, a rise in the slenderness ratio intensifies the influence of the surface effect while simultaneously diminishing the magnitude of the scale effect. Furthermore, disparities can be observed in the impact of higher-order modes on both products. The utilization of advanced methodologies yields more pronounced scale effects, whereas the impact of these methodologies on surface effects is contingent upon the elastic characteristics of the material's surface. Additionally, it is seen that the inclusion of surface elasticity leads to a wider disparity between the Euler-Bernoulli, Timoshenko, and other higher-order beam models. This suggests that the higher-order beam model yields more precise predictions when both surface and nonlocal effects are taken into account (Yang *et al.* 2023).

The theory of linear viscoelasticity is known for its effectiveness in determining constitutive relationships based on experimental data, as it provides a clear understanding of the interplay between stress, strain, and time. Mathematical models are frequently employed to gain deeper insights into the creep, relaxation, and oscillatory behavior exhibited by viscoelastic materials. To unravel the underlying molecular or microstructural deformation mechanisms, it is crucial to extract model parameters and establish their inter-relationships. These principles find application not only in sensory perception but also in various sectors, such as the food industry, to name just one example. However, before comparing the behavior of different materials, organizational states, or environmental conditions, it is imperative to select an appropriate model that allows for meaningful comparisons. By utilizing these mathematical techniques, one can predict a material's stress and strain response under a wide range of loading conditions. These parameters, also referred to as material characteristics, define the behavior of a material and enable accurate predictions of its mechanical response.

Materials possessing both elasticity and viscosity, such as polymers and rubbers, exhibit Viscoelastic characteristics. These materials have a molecular structure composed of chain-like molecules. The works (Mead 1999, Jones 2001) provided examples of various polymeric materials utilized in dampers, along with practical and engineering challenges associated with viscoelastic dampers. Mathematical models have been developed to simulate the behavior of linear viscoelastic materials. These models, known as viscoelastic traditional models (Vincent 1990), employ a combination of springs and dashpots arranged in series and parallel configurations due to the similarities between the responses of viscoelastic materials and those of elastic and viscous materials. These models depict the stress-strain relationship by applying first-order differential Eq.s to the spring and dashpot components. Extensive research has been conducted on the dynamic properties of such components, given their widespread use in pipeline networks (Vincent 1990). Although traditional viscoelastic models can capture some of the elastic and viscous behavior of materials, they may lack the precision required to predict material response across a broad range of frequencies. Enhancing the accuracy of these models can be achieved by incorporating additional springs and dashpots in series and parallel configurations. However, it is crucial to acknowledge that in such cases, a significant number of experimental constants would be necessary to accommodate the observed experimental results (Eldred *et al.* 2005).

Fractional derivatives provide a more precise and realistic representation of the behavior of complex materials, such as sludge, in viscoelastic models. The concept of fractional derivatives is an alternative approach to predicting the response of viscoelastic materials. Utilizing fractional derivatives in viscoelastic models enhances their performance in the linear viscoelastic domain and extends their applicability to higher frequencies. The power-law relaxation modulus is commonly observed in polymers, and these formulations have shown promise in capturing their behavior (Bagley *et al.* 1983). Fractional viscoelastic formulations excel due to the inclusion of fractional elements that bridge the gap between purely viscoelastic and completely elastic components (Farno *et al.* 2018). These formulations have the potential to accurately predict viscoelastic behavior across a wide frequency range without relying on numerous empirical constants. Researchers have investigated the molecular response of viscoelastic materials (Vincent 1990), demonstrating that employing a molecular model to describe viscoelastic materials leads to a fractional stress-strain relationship. Bagley and Torvik (1983) conducted experiments and compared the results with the fractional derivative model, finding that the fractional derivative model significantly improved predictions of viscoelastic material behavior.

According to the conventional concept of thermo elasticity, which is based on the Fourier model of thermal conductivity, a thermoplastic wave propagates at infinite speed under extreme temperature scenarios such as laser pulse heating and high-frequency operation. However, conclusive evidence against this traditional concept has been obtained from physical observations and experimental results (Zhou and Li 2017). Several non-Fourier theories of thermal conductivity, including Cattaneo-Vernotte heat wave modulation (Cattaneo 2011, Vernotte 1958), the Tzou (2014, 2015) double-phase lag (DPL) model, and the Roy Choudhuri three-phase (DPL) model (Roy Choudhuri 2017), have been proposed to address this problem. Among the most popular ideas and models in thermoelastic are the theory developed by Green and Naghdi (GN) (1992, 1993) and the Lord and Shulman (LS) model (1967), both of which utilize the energy Eq.

In their study, Arhami *et al*. (2022) conducted a coupled thermoelastic analysis on a nanobeam subjected to clamped-clamped and pinned-pinned boundary conditions. The analysis was performed using the Green-Naghdi theory, which incorporates energy dissipation. Abouelregal *et al*. (2023) present a novel approach to analyzing and exploring the transverse vibrational characteristics of rotating thermo-elastic nanobeams. This is achieved by utilizing nonlocal elasticity theory, taking into account the size-dependent nature of the system. The study employed a thermal conductivity model using two-phase delays (DPL) in its formulation. Borjalilou *et al*. (2020) introduce a comprehensive formulation that describes the thermoelastic damping phenomenon in nanobeams. This formulation takes into account the influence of small-scale influences on both the continuum mechanics and heat transfer aspects. In order to account for small-scale impacts, the coupled Eq.s governing motion and heat transfer are derived using the nonlocal elasticity model and the dual-phase-lag heat transfer theory. Jalil *et al*. (2023) employ a combined utilization of the nonlocal strain gradient concept (NSGT) and nonlocal dual-phase-lag (NDPL) heat transport theories to provide a mathematical framework and analytical solution for thermal elastic distortion (TED) in nanobeams. This approach allows for incorporating size impacts in structural and heat transmission domains. Grover and Seth (2018) conducted research on visco-thermoelastic micro-scale beam resonators by employing a dual phase-lagging model in their work. In (Hamidi *et al*. 2020), the shaking of a silver nanobeam resonator was looked at, taking into account both the surface impact and the temperature influence. The Green–Naghdi thermoelastic, nonlocal elasticity concept, and surface impact for the Euler–Bernoulli beam framework are used to get the governing Eq.s for shaking.

Based on the previous literature review, studies examining the behavior of moving rubber nanobeams using viscoelastic fractional models are scarce. Furthermore, it is evident that further research is necessary to gain a deeper understanding of how changes in volume and temperature impact the softening of toughness and strengthening of elasticity in nano- and microstructures. Fractional models are frequently employed due to their ability to accurately depict the dynamic behavior of a damper across a broad range of temperatures and vibrations using only a few parameters.

The Zener model, often referred to as the standard linear solid, is a theoretical framework employed for characterizing the mechanical response of viscoelastic materials. It involves using a linear combination of springs and dashpots to represent the elastic and viscous constituents. Frequently, the Maxwell model and the Kelvin-Voigt model are employed. However, it is essential to note that these models often have limitations. For instance, the Maxwell model fails to adequately characterize phenomena such as creep or recovery, while the Kelvin-Voigt model does not well capture stress relaxation (Oparnica and Süli 2020). The Zener model integrates elements from the Maxwell and Kelvin-Voigt models to accurately represent the overall response of a system subjected to certain loading circumstances. The new model possesses the capability to accurately depict a diverse range of linear viscoelastic materials. The linear Zener model has demonstrated its efficacy in characterizing the viscoelastic response of different materials under minor deformations, as evidenced by previous studies. The model incorporates commonly observed attenuation phenomena, such as quality factors, using a frequency power law (Blanc *et al*. 2016). The identification of the model parameters was conducted through laboratory experiments conducted at various temperatures and vibration frequencies.

This paper aims to study thermo-viscoelastic vibrations in nanobeam resonators by presenting a new mathematical framework for non-Fourier phase-delay thermal conductivity and the non-local Eringen theory. The idea of nonlocal elasticity is employed as a means of comprehending the influence of small-scale phenomena. Considering the effect of thermal effects, this article also contributes to the analysis of the viscoelastic vibrations of a moving nanobeam using the Zener model with fractional derivatives. The viscoelastic nanobeam undergoes axial movement at a constant speed under the influence of a sinusoidal pulse with varying temperatures. This study also aims to compare the results obtained from computing the dynamics of structural members using the aforementioned two models in order to assess the suitability of the fractional Kelvin-Voigt model. A set of solvable ordinary differential Equations is derived by transforming them into a Laplace field under the given initial conditions. Through numerical simulations, we examine how various viscoelastic properties, including phase delay, size-dependent effects, axial velocity, and fractional order parameters, impact the dynamic characteristics of a moving nanobeam. The findings of this investigation have potential applications in modeling Stockbridge dampers, energy harvesting devices, viscoelastic nanostructures, and military airplane wings. Fractional viscoelasticity can also find utility in environmental and building materials such as asphalt, concrete, rock mass, viscous petroleum products, polymer materials, gel forms, and food.

2. Formulation of the problem and fundamental Equations

NEMS resonators are commonly fabricated using nanobeams featuring square or rectangular cross-sections. In this study, we propose a viscoelastic moving nanobeam, as depicted in Fig. 1, which will be subjected to time-dependent thermal loading. The nanobeam possesses specific dimensions: length L, fixed cross-section area A, thickness h, and width b. Our hypothesis assumes that the nanobeam remains in a state of non-deformation and stress when in equilibrium at a temperature T_0 representing the surrounding environment's temperature. Additionally, the nanobeam exhibits an axial velocity denoted by υ and a

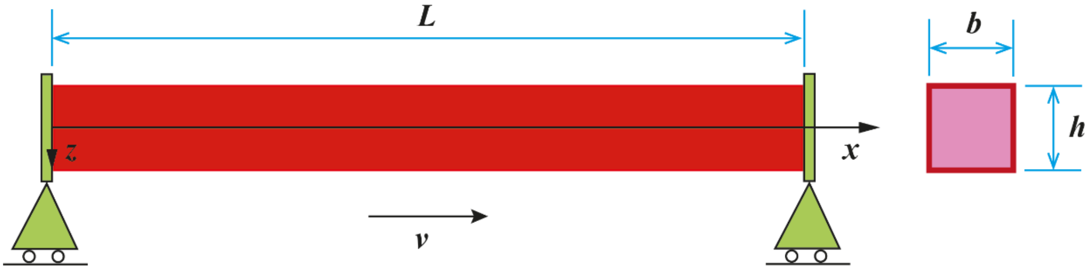

Fig. 1 The configuration of moving nanobeam resonator composed of viscoelastic material

flexural stiffness represented by EI, where I is equal to $bh^3/12$. The material properties are characterized by E, μ_P, and ρ, corresponding to Young's modulus, Poisson's ratio, and density, respectively.

Given that the transverse dimensions of the nanobeam are significantly smaller in comparison to its beam length, neglect of shear deformation is warranted. Consequently, this investigation employs the standard Euler-Bernoulli (EB) assumption. Under these circumstances, the displacement components can be expressed as follows:

$$\vec{u}(x,y,z,t) = \left(-z\frac{\partial w}{\partial x},\ 0,\ \ w(x,t)\right) \tag{1}$$

The variable $w(x,t)$ represents the beam's deflection, often known as its transverse displacement. To understand why the beam is oscillating in a transverse direction, we can utilize the following Eq. based on Newton's second law of motion (Liu *et al.* 2014, Chen 2005):

$$\frac{\partial^2 M}{\partial x^2} + \rho A\left(\frac{\partial^2 w}{\partial t^2} + 2v\frac{\partial^2 w}{\partial x \partial t} + v^2\frac{\partial^2 w}{\partial x^2}\right) = 0 \tag{2}$$

In this Eq., M is the bending moment and A is the cross-sectional area. Also, using the Hamiltonian idea, the motion Eq. (2) was established.

The nonlocal theory is the most widely adopted approach in nanostructured elasticity. Academics predominantly employ the differential form of this theory due to its inherent convenience in simplifying relationships. Nonlocal elasticity posits that the stress at a specific location along a nanobeam relies on the strain at that particular point and the themes present throughout the beam length (Eringen 1972). Eringen (2002) has derived a differential form of the constitutive Eq. that establishes a connection between nonlocal and local stresses. Consequently, it becomes feasible to express this Eq. for the nonlocal beam as (Eringen, 1972b):

$$\sigma_x - \xi\frac{\partial^2 \sigma_x}{\partial x^2} = -E\left(z\frac{\partial^2 w}{\partial x^2} + \alpha_T \theta\right) \tag{3}$$

In Eq. (3), the parameter $\xi = (e_0 a)^2$ is the size scale indicator, a denotes a material constant, and e_0 represents the internal characteristic length. Also, $\theta = T - T_0$ indicates the temperature increment, $\alpha_T = \alpha_t/(1-2\mu_P)$, α_t is the thermal expansion, and σ_x denotes the axial nonlocal thermal stress.

The influence of viscoelastic materials on the dynamic behavior of a system is widely recognized in the scientific community. The damping properties of a system play a crucial role in its dynamic behavior, and these properties are directly influenced by the characteristics of the viscoelastic material. Consequently, accurately determining the energy dissipation properties of a structure poses a significant challenge when predicting its dynamics. Therefore, employing an appropriate model of viscoelastic matter in the study of dynamics is imperative. In recent decades, fractional differential Equations have gained popularity to describe various physical processes (Podlubny 1999, Das 2011). These Equations allow for the modeling of spatially and temporally dependent physical events through the use of fractional derivatives (Sumelka 2015, Rahimi 2017). Fractional derivatives are particularly useful in modeling the damping characteristics of viscoelastic materials, especially those that exhibit a moderate frequency dependency (Di Pa *et al*. 2011).

Numerous studies have focused on investigating the beam dynamics of viscoelastic beams using the fractional Kelvin-Voigt Equations. One notable advantage of the fractional Kelvin-Voigt model is its versatility in examining the dynamics of different components within a building. By employing the Kelvin-Vo fractional viscoelastic material model to represent the dynamics of structural elements, we can formulate Equations that align with well-established approaches for addressing elastic issues.

Unfortunately, this paradigm has a few drawbacks as well. Specifically, it tends to overestimate material damping at higher frequencies. Through a comparison between the fractional Zener solid model and the fractional Kelvin-Voigt model, Caputo and Mainardi (1971) discovered that the former exhibited a superior fit to the experimental data. Therefore, when calculating the dynamics of beams and other structural components, it is crucial to assess the potentially significant differences between the fractional Zener model and the fractional Kelvin-Voigt model. Several academic articles have also been published on the subject of forced transient vibrations of viscoelastic beams utilizing the fractional Zener model. For instance, there is a scarcity of research on transiently induced vibrations of cantilever beams with a tip mass element modeled using a fractional Zener material.

The primary objective of this study is to ascertain how the complexity of a model influences the accuracy with which fractional-order models can match experimental data. This investigation concentrates on the fractional Zener model for describing the dynamic transient response of a nanobeam composed of a viscoelastic material. It is anticipated that the viscoelastic characteristics of the beam material can be described using the fractional Zener framework, which is formulated as follows (Mainardi *et al*. 2011, Lewandowski *et al*. 2017, 2017b):

$$(1+\tau_\sigma^\alpha D_t^\alpha)\sigma_x = -E_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(z\frac{\partial^2 w}{\partial x^2}+\alpha_T\theta\right) \tag{4}$$

where the parameters τ_σ and τ_ε are relaxation times satisfying $\tau_\varepsilon > \tau_\sigma > 0$ and $E_0 > 0$ is the relaxed stiffness modulus. Moreover, D_t^α is the e Caputo fractional derivative operator of the non-integer order $(0<\alpha<1)$ concerning the time t. The expression for the Caputo fractional derivative, $D_t^\alpha f(t)$, may be written as:

$$D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{1}{(t-\chi)^\alpha}\frac{\partial f(\chi)}{\partial t}d\,\chi, \quad \alpha\in(0,1), t>0 \tag{5}$$

$\Gamma(1-\alpha)$ is the Euler gamma function. Upon the use of fractional derivatives, conventional rheological models undergo a transformation into their corresponding fractional counterparts. The new model possesses the capability to accurately depict a diverse range of linear materials with viscoelastic properties.

The flexural moment of the nanobeam can be calculated using the following formula:

$$M(x,t) = b\int_{-h/2}^{h/2}\sigma_x z\,d\,z \tag{6}$$

The result of applying Eq. (4) to Eq. (6) is

$$\begin{aligned}&(1+\tau_\sigma^\alpha D_t^\alpha)\left(1-\xi\frac{\partial^2}{\partial x^2}\right)M\\ &= -IE_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^2 w}{\partial x^2}+\alpha_T M_T\right)\end{aligned} \tag{7}$$

where M_T stands for the thermal moment that is calculated by

$$M_T = \frac{12}{h^3}\int_{-h/2}^{h/2} z\theta\,d\,z \tag{8}$$

By making a double differentiation of Eq. (10) with respect to the variable x, we get

$$\begin{aligned}&(1+\tau_\sigma D_t^\alpha)\left(1-\xi\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 M}{\partial x^2}\\ &= -IE_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^4 w}{\partial x^4}+\alpha_T\frac{\partial^2 M_T}{\partial x^2}\right)\end{aligned} \tag{9}$$

When Eq. (2) is put into Eq. (9), it yields:

$$\begin{aligned}&\rho A(1+\tau_\sigma^\alpha D_t^\alpha)\left(1-\xi\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial t^2}+2\upsilon\frac{\partial^2 w}{\partial x\partial t}+\upsilon^2\frac{\partial^2 w}{\partial x^2}\right)\\ &= IE_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^4 w}{\partial x^4}+\alpha_T\frac{\partial^2 M_T}{\partial x^2}\right)\end{aligned} \tag{10}$$

The mathematical model for the transverse excitations of moving nonlocal viscoelastic nanobeams is represented by Eq. (10). When $\xi = 0$ is substituted into the Equations above, the classical Eq. of motion for an oscillating Euler-Bernoulli viscoelastic beam is recovered.

When Eq. (2) is replaced with Eq. (7), the bending moment Eq. may be written as follows:

$$\begin{aligned}&(1+\tau_\sigma^\alpha D_t^\alpha)M(x,t)\\ &= -\xi\rho A(1+\tau_\sigma^\alpha D_t^\alpha)\left(\frac{\partial^2 w}{\partial t^2}+2\upsilon\frac{\partial^2 w}{\partial x\partial t}+\upsilon^2\frac{\partial^2 w}{\partial x^2}\right)\\ &-IE_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^2 w}{\partial x^2}+\alpha_T M_T\right)\end{aligned} \tag{11}$$

To analyze the behavior of a moving thermoelastic nanobeam resonator, dual-phase lag (DPL) theory will be taken into account. According to the generalized DPL thermoelasticity model, the approximate modified heat transfer Eq. takes the following formula:

$$\vec{q}+\tau_q\frac{\partial\vec{q}}{\partial t} = -K\left(\nabla\theta+\tau_\theta\frac{\partial\nabla\theta}{\partial t}\right) \tag{12}$$

where τ_q and τ_θ are positive delay times, which are linked to the microstructure of the substance being studied, and $K\ (>0)$ indicates the thermal conductivity. The stability of the system is determined by the relationship between the time constant τ_θ and τ_q. Specifically, if τ_θ is greater than $2\tau_q$, the system is considered stable. Conversely, if τ_θ is less than $2\tau_q$, the system is deemed unstable.

The following formula can be used to represent the energy Equation for a thermoviscoelastic material:

$$C_E\frac{\partial\theta}{\partial t}+E_0\alpha_T T_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\frac{\partial e}{\partial t}-Q = -\nabla\cdot\vec{q} \tag{13}$$

where $C_E>0$ represents the specific heat, $e=u_{k,k}$ represents the cubical dilatation, and Q represents the internal supply of energy. The DPL heat transfer Equation for isotropic thermoviscoelastic material can be derived from Eqs. (12) and (13), as shown below:

$$\begin{aligned}&\left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\rho C_E\frac{\partial\theta}{\partial t}+E_0\alpha_T T_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\frac{\partial e}{\partial t}\right)\\ &-\left(Q+\tau_q\frac{\partial Q}{\partial t}\right) = K\left(\nabla^2\theta+\tau_\theta\frac{\partial}{\partial t}(\nabla^2\theta)\right)\end{aligned} \tag{14}$$

For $\tau_q = 0 = \tau_\theta$, the DPL thermoelastic model is reduced to the standard Fourier's law. For both and $\tau_\theta,\tau_q >0$, one gets the Lord and Shulman (LS) heat transfer theory.

The DPL heat transfer Equation can be derived in the case of neglecting the heat source $(Q=0)$ by substituting Eq. (1) into Eq. (14) to get:

$$\begin{aligned}&\left(1+\tau_q\frac{\partial}{\partial t}\right)\left[\rho C_E\frac{\partial\theta}{\partial t}-E_0\alpha_T T_0 z(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^3 w}{\partial t\partial x^2}\right)\right]\\ &= K\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\left[\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}\right]\end{aligned} \tag{15}$$

3. Problem solution

Furthermore, we make the assumption that the upper and lower surfaces of the beam exhibit adiabatic behavior, resulting in a condition where the temperature gradient $\frac{\partial\theta}{\partial z}$

is zero at $z = \pm \frac{h}{2}$. To address this problem, we will consider the sinusoidal growth of temperature increases along the thickness direction of the nanobeam in motion. Specifically, the temperature increase $\theta(x,z,t)$ can be expressed as follows:

$$\theta(x,z,t) = \Theta(x,t)\sin(\pi z/h) \tag{16}$$

By inserting Eq. (16) into Eqs. (10) and (11), the z variable is eliminated, and the resulting Eqs. are as follows:

$$\begin{aligned}\rho A(1+\tau_\sigma^\alpha D_t^\alpha)\left(1-\xi\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial t^2}+2v\frac{\partial^2 w}{\partial x\partial t}+v^2\frac{\partial^2 w}{\partial x^2}\right)\\ = IE_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^4 w}{\partial x^4}+\frac{24\alpha_T}{h\pi^2}\Theta\right)\end{aligned} \tag{17}$$

$$\begin{aligned}&(1+\tau_\sigma^\alpha D_t^\alpha)M(x,t)\\ &= -\xi\rho A(1+\tau_\sigma^\alpha D_t^\alpha)\left(\frac{\partial^2 w}{\partial t^2}+2v\frac{\partial^2 w}{\partial x\partial t}+v^2\frac{\partial^2 w}{\partial x^2}\right)\\ &-IE_0(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^2 w}{\partial x^2}+\frac{24\alpha_T}{h\pi^2}\Theta\right).\end{aligned} \tag{18}$$

Once again, integrating Eq. (15) with respect to z in the range $z \in [-h/2, h/2]$ after multiplying by $12z/h^3$, yields

$$\begin{aligned}&\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\left[\frac{\partial^2\Theta}{\partial x^2}-\frac{\pi^2}{h^2}\Theta\right]\\ &= \left(1+\tau_q\frac{\partial}{\partial t}\right)\left[\eta\frac{\partial\Theta}{\partial t}-\frac{E_0\alpha_T T_0\pi^2 h}{24K}(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^3 w}{\partial t\partial x^2}\right)\right]\end{aligned} \tag{19}$$

We utilize the following non-dimensional quantities for the sake of simplicity:

$$\begin{aligned}&(x',L',u',w',z') = \mathscr{g}\mathscr{v}(x,L,u,w,z),\Theta' = \frac{\Theta}{T_0},\sigma_x' = \frac{\sigma_x}{E_0},\\ &M' = -\frac{M}{\mathscr{g}\mathscr{v}IE_0},(t',t_0',\tau_q',\tau_\theta') = \mathscr{g}\mathscr{v}^2(t,t_0,\tau_q,\tau_\theta),v' = \frac{v}{c},\end{aligned} \tag{20}$$

where $\mathscr{g} = \frac{\rho C_E}{K}$ and $\mathscr{v} = \sqrt{\frac{E}{\rho}}$.

The dimensionless version of the coupled system of Equations may be constructed by substituting Eq. (20) into Eqs. (17-19) as

$$\begin{aligned}&\frac{12}{h^2}(1+\tau_\sigma^\alpha D_t^\alpha)\left(1-\xi\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial t^2}+2v\frac{\partial^2 w}{\partial x\partial t}+v^2\frac{\partial^2 w}{\partial x^2}\right)\\ &= (1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^4 w}{\partial x^4}+\mathscr{b}_1\frac{\partial^2\Theta}{\partial x^2}\right)\end{aligned} \tag{21}$$

$$\begin{aligned}&\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\left[\frac{\partial^2\Theta}{\partial x^2}-\frac{\pi^2}{h^2}\Theta\right]\\ &= \left(1+\tau_q\frac{\partial}{\partial t}\right)\left[\frac{\partial\Theta}{\partial t}-\mathscr{b}_2(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^3 w}{\partial t\partial x^2}\right)\right]\end{aligned} \tag{22}$$

$$\begin{aligned}&(1+\tau_\sigma^\alpha D_t^\alpha)M(x,t)\\ &= \xi(1+\tau_\sigma^\alpha D_t^\alpha)\left(\frac{\partial^2 w}{\partial t^2}+2v\frac{\partial^2 w}{\partial x\partial t}+v^2\frac{\partial^2 w}{\partial x^2}\right)\\ &+(1+\tau_\varepsilon^\alpha D_t^\alpha)\left(\frac{\partial^2 w}{\partial x^2}+\mathscr{b}_1\Theta\right)\end{aligned} \tag{23}$$

where $\mathscr{b}_1 = \frac{24T_0\alpha_T}{h\pi^2}$ and $\mathscr{b}_2 = \frac{\alpha_T E_0\pi^2 h}{24K\eta}$.

4. Transformed solution of the problem

Assuming homogeneous initial conditions, the starting conditions will be considered as follows:

$$\begin{aligned}w(x,t)|_{t=0} = 0 = \left.\frac{\partial w(x,t)}{\partial t}\right|_{t=0},\\ \Theta(x,t)|_{t=0} = 0 = \left.\frac{\partial\Theta(x,t)}{\partial t}\right|_{t=0}\end{aligned} \tag{24}$$

Moreover, the scenario wherein both ends of the beam are clamped is duly considered. Consequently, the boundary conditions exhibit the subsequent structure:

$$w(x,t)|_{x=0,L} = 0, \left.\frac{\partial w(x,t)}{\partial x}\right|_{x=0,L} = 0 \tag{25}$$

The scenario where the initial end (x=0) of the nanobeam experiences thermal loading will be considered. Consequently, the subsequent thermal boundary condition will be obtained:

$$\theta = \theta(z,t) = \theta_0\sin(pz)f(x,t) \quad \text{at } x = 0 \tag{26}$$

where θ_0 is a constant. We also assume that the function $f(x,t)$ is a time-dependent sinusoidal function with a pulse width t_0 and takes the following form:

$$f(x,t) = f(t) = \sin(\pi t/t_0),\ 0 \le t \le t_0 \tag{27}$$

In addition, it is supposed that the other end ($x = L$) of the nanobeam is thermally insulated and fulfills the following thermal condition:

$$\frac{\partial\Theta}{\partial x} = 0, \text{ at } x = L \tag{28}$$

Under initial conditions Eq. (24), applying the Laplace transform to Eqs. (21)-(23) yields the following:

$$\left(\frac{\mathrm{d}^4}{\mathrm{d}x^4}+A_3\frac{\mathrm{d}^3}{\mathrm{d}x^3}+A_2\frac{\mathrm{d}^2}{\mathrm{d}x^2}+A_1\frac{d}{dx}+A_0\right)\bar{w} = -B_1\frac{\mathrm{d}^2\bar{\Theta}}{\mathrm{d}x^2} \tag{29}$$

$$-A_4\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2} = \left(\frac{\mathrm{d}^2}{\mathrm{d}x^2}-B_2\right)\bar{\Theta} \tag{30}$$

$$\begin{aligned}\bar{M}(x,t) = (v^2\xi+\omega_0)\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2}+\left(2v\xi s\frac{d}{dx}+\xi s^2\right)\bar{w}\\ +\omega_0\mathscr{b}_1\bar{\Theta}\end{aligned} \tag{31}$$

where

$$\begin{aligned}&\omega_0 = \frac{(1+\tau_\sigma^\alpha s^\alpha)}{(1+\tau_\varepsilon^\alpha s^\alpha)}, \qquad \omega_1 = \frac{12\omega_0}{h^2},\\ &\omega_2 = \frac{s(1+\tau_q s)}{1+\tau_\theta s}, \qquad A_0 = \frac{\omega_1 s^2}{(1+\xi\omega_1 v^2)},\\ &A_1 = \frac{2v\omega_1 s}{(1+\xi\omega_1 v^2)}, \qquad A_2 = \frac{\omega_1(s^2\xi-v^2)}{(1+\xi\omega_1 v^2)},\\ &A_3 = \frac{2sv\xi\omega_1}{(1+\xi\omega_1 v^2)}, \qquad B_1 = \frac{\mathscr{b}_1}{(1+\xi\omega v^2)},\\ &B_2 = \frac{\pi^2}{h^2}+\omega_2, \qquad A_4 = \mathscr{b}_2\,\omega_2(1+\tau_\varepsilon^\alpha s^\alpha)\end{aligned} \tag{32}$$

Through the process of multiplying Eq. (29) by two and combining it with Eq. (30), followed by rearrangement, we arrive at the subsequent differential Eq.:

$$\begin{bmatrix} a_6 \frac{d^6}{dx^6} + a_5 \frac{d^5}{dx^5} + a_3 \frac{d^4}{dx^4} + a_3 \frac{d^3}{dx^3} \\ +a_2 \frac{d^2}{dx^2} + a_1 \frac{d}{dx} + a_0 \end{bmatrix} \{\overline{w}, \overline{\Theta}\} \tag{33}$$

where

$$a_5 = A_3, a_4 = A_2 - B_1 A_4,\ a_3 = A_1 - B_2 A_3, \\ a_2 = A_0 - B_2 A_2, \quad a_1 = -B_2 A_1, a_0 = -B_2 A_0. \tag{34}$$

The following formula can mathematically represent the solution to the differential Eq. (31):

$$\{\overline{w}, \overline{\Theta}\} = \sum_{i=1}^{6} \{C_i, C_i'\}\, e^{m_i x}, \tag{35}$$

where C_i and C_i' are constants that need to be calculated once the boundary conditions have been applied. Furthermore, the m_i parameters, where i ranges from 1 to 6, each represent a root of the following polynomial:

$$m^6 + a_5 m^5 + a_4 m^4 + a_3 m^3 + a_2 m^2 + a_2 m + a_0 = 0 \tag{36}$$

Kulkarni (2008) devised a technique for decomposing a six-degree polynomial issue into two cubic polynomials as constituent elements. The cubic polynomial Eq. is set to zero, and upon solving it, the solution yields six distinct roots for the Eq. (36). The salient feature of the Eq. solved in this manner is that the aggregate of its three roots is equivalent to the sum of its remaining three roots. He derived the requirement that the coefficients of a solved Eq. of this kind must fulfill.

Take into account the sixth-degree polynomial Eq. given below:

$$(m^3 + b_2 m^2 + b_1 m + b_0)^2 - (c_2 m^2 + c_1 m + c_0)^2 = 0, \tag{37}$$

The Eq. (37) involves unknown coefficients denoted as b_0, b_1, b_2, c_0, c_1, and c_2. These coefficients correspond to the component cubic and quadratic polynomials, respectively. Suppose the Eq. with a degree of six, denoted as Eq. (36), can be represented in the format of Eq. (37). In that case, it is possible to decompose it into two cubic polynomials, which will lead to the determination of its solution. The cubic Eqs. presented above are derived by setting the polynomial components equal to zero:

$$m^3 + (b_2 - c_2) m^2 + (b_1 - c_1) m + b_0 - c_0 = 0, \\ m^3 + (b_2 + c_2) m^2 + (b_1 + c_1) m + b_0 + c_0 = 0. \tag{38}$$

The process of solving these cubic Eqs. yields six distinct solutions that correspond to the roots of the given six-degree problem. Therefore, in order to express Eq. (36) in the format of Eq. (37), it is necessary for the parameters of both Eqs. to be identical. Consequently, this leads to

$$b_0 = \frac{a_3}{2} + \frac{a_5^3}{16} - \frac{a_5 a_4}{4}, \qquad b_1 = \frac{a_4}{2} - \frac{a_5^2}{8}, \\ b_2 = \frac{a_5}{2}, \qquad c_2 = 0, \\ c_1^2 = \frac{5a_5^4}{64} + \frac{a_4^4}{4} + \frac{a_5 a_3}{2} - \frac{3a_4 a_5^2}{8} - a_2, \qquad c_0^2 = \frac{a_7^2}{c_1^2}, \\ a_7 = \frac{a_3 a_4}{4} + \frac{3a_4 a_5^3}{16} - \frac{a_5 a_4^2}{8} - \frac{a_3 a_5^2}{16} - \frac{a_5^5}{128} - \frac{a_1}{2} \tag{39}$$

After solving for the unknown variables and substituting their values into the cubic Eqs. (38), we have all six solutions for Eq. (36) as

$$\begin{aligned} m_1 &= \frac{2}{3} p_1 \sin(q_1), \\ m_2 &= -\frac{1}{3} p_1 [\sqrt{3}\cos(q_1) + \sin(q_1)], \\ m_3 &= \frac{1}{3} p_1 [\sqrt{3}\cos(q_1) - \sin(q_1)], \\ m_4 &= \frac{2}{3} p_2 \sin(q_2), \\ m_5 &= -\frac{1}{3} p_2 [\sqrt{3}\cos(q_2) + \sin(q_2)], \\ m_6 &= \frac{1}{3} p_2 [\sqrt{3}\cos(q_2) - \sin(q_2)] \end{aligned} \tag{40}$$

where

$$p_1 = \sqrt{3c_1 - 3b_1}, \quad q_1 = \frac{1}{3}\sin^{-1}\left(\frac{27b_0 - 27a_7}{2p_1^3}\right), \\ p_1 = \sqrt{3c_1 + 3b_1}, \quad q_2 = \frac{1}{3}\sin^{-1}\left(\frac{27b_0 + 27a_7}{2p_2^3}\right). \tag{41}$$

Due to the fact that two Eqs. (30) and (35) are compatible with each other, the following relations can be obtained:

$$C_i' = \beta_i C_i, \quad \beta_i = -\frac{A_4 m_i^2}{m_i^2 - B_2} \tag{42}$$

With the help of the function $\overline{\Theta}$ and Eq. (16), the global solution for the temperature change in the transformed Laplace field can be found as follows:

$$\bar{\theta} = \sin\left(\frac{\pi z}{h}\right) \sum_{i=1}^{6} \beta_i C_i\, e^{m_i x} \tag{43}$$

After performing the Laplace transform, we obtain the axial displacement $\bar{u}$ by substituting Eq. (35) into Eq. (1), as follows:

$$\bar{u} = \sum_{i=1}^{6} m_i C_i\, e^{m_i x}, \tag{44}$$

The final expression for the bending moment M given in Eq. (31) can be rewritten with the help of Eq. (35) as:

$$\overline{M}(x,t) = \sum_{i=1}^{6} (v^2 \xi m_i^2 + \omega_0 m_i^2 + 2 v s \xi m_i + \xi s^2 \\ + \omega_0 \mathcal{b}_1 \beta_i) C_i e^{m_i x} \tag{45}$$

After applying the Laplace transform to the Eqs. (25)-(28), the boundary conditions of the problem are converted to:

$$\overline{w}(x,s)|_{x=0,L} = 0, \qquad \left.\frac{d\overline{w}(x,s)}{dx}\right|_{x=0,L} = 0, \\ \overline{\Theta}(x,s)|_{x=0} = \frac{\pi t_0}{\pi^2 + t_0^2 s^2} = \bar{G}(s), \quad \left.\frac{d\overline{\Theta}(x,s)}{dx}\right|_{x=L} = 0 \tag{46}$$

When Eq. (35) is substituted into the boundary conditions, the following linear Eqs. can be found as:

$$\sum_{i=1}^{6} e^{m_i L} C_i = 0, \quad \sum_{i=1}^{6} m_i C_i = 0, \quad \sum_{i=1}^{6} m_i e^{m_i L} C_i = 0,$$
$$\sum_{i=1}^{6} C_i = 0, \quad \sum_{i=1}^{6} \beta_i C_i = \bar{G}(s), \quad \sum_{i=1}^{6} \beta_i m_i e^{m_i L} C_i = 0 \tag{47}$$

The above set of linear Eqs. can be solved to calculate the unknown coefficients C_i, ranging from 1 to 6. Here, the complete solution to the problem has been found in the domain of the Laplace transform.

5. Laplace transform inversion

In order to compute the numerical calculations of the thermophysical fields examined in the nanobeam, including bending moment modes, temperature, displacement, and pressure, it is necessary to invert the Laplace transforms of these fields into the time domain. Due to the complexity of the expressions in different domains, direct inversion becomes challenging. To address this issue, the Riemann sum approximation algorithm will be employed as a reliable and validated approximation method. Following this approach, any function denoted by $\bar{F}(x,s)$ in the Laplace domain can be reversed into the time domain using the relation (Honig *et al*. 1984):

$$f(x,t) = \frac{e^{\beta t}}{t}\left[\frac{1}{2} Re[\bar{F}(x,\beta)] + Re\sum_{\varphi=0}^{N}\left(\bar{F}\left(x,\beta+\frac{i\varphi\pi}{t}\right)(-1)^n\right)\right] \tag{48}$$

where β and N represent constants that are consistently present, the precision of the Reimann sum approximation relies on the magnitude of the constant β in addition to the truncation error associated with the number N. It is feasible to develop a Mathematica program capable of numerically solving the Laplace inversion of Eq. (43).

6. Benchmark analysis

In this section, we demonstrate the alterations in thermal vibration of viscoelastic nanobeams when incorporating nonlocal elasticity theory and the DPL heat transfer framework. Additionally, numerical data is employed to examine the impacts of nonlocality, viscosity, and the axial transport velocity of the nanobeam on the dimensions of the thermophysical fields under investigation. We present numerical results for a silicon nanobeam with a fixed aspect ratio of $L = 10h$. The material parameters of the silicon nanobeam can be described as (Duwel *et al*. 2003):

$$E = 169\text{GPa}, L = 300\text{nm}, C_E = 713\text{J/(kgK)}, b = 15\text{nm},$$
$$\mu_P = 0.22, \alpha_T = 0.259\times10^{-5}K^{-1}, K = 1.56\times10^{2}\text{W/(mK)},$$
$$\rho = 2330(\text{kg/m}^3), h = 30\text{nm}, T_0 = 293\text{K}.$$

For only a limited number of specific materials, it is possible to determine the phase delays of heat flow τ_q and temperature gradient τ_θ. The values of these phase delay parameters can be deduced by considering the mechanical and thermal properties of the materials, as demonstrated in reference (Tzou 1995). By incorporating the physical non-dimensional quantities mentioned in Eq. (20), the numerical values of the non-dimensional fields under investigation can be computed, and corresponding graphs can be generated. In the present study, we consider the non-dimensional values $= 0.12,\ \xi = 0.001\ \ \tau_q = 0.2,\ \tau_\theta = 0.1,\ t_0 = 0.1,$ $L = 1,\ \ b = h/2$, unless they have been previously mentioned in the discussion. Graphical representations illustrating the variations of system variables, such as temperature change θ, deflection w, and deformation u in the x-axis direction, are presented at $z = h/6$. Including these variables reveals significant effects on the dynamic behavior of an axially traveling nanobeam. Three numerical cases will be thoroughly examined and discussed.

6.1 The small-scale effect

Experimental studies have demonstrated the significant role of the small-scale effect in the optimal design of micro- and nanoelectromechanical devices. Consequently, the first scenario aims to investigate the impact of changes in the non-local dimensionless parameter on the bending moment, thermodynamic temperature, displacement, and transverse vibration. Figures 2, 3, 4, and 5 depict the predicted variation of the nanobeam along the axial distance. The classical continuity theory (local) is compared with the nonlocal elasticity theory for different values $\xi = 0, 0.001, 0.002, 0.003$. In this context, we adopt an updated version of the two-phase delay theory (DPL) of heat transfer. It is assumed that the phase delay coefficients (τ_q and τ_θ), which are dimensionless, and the transport speed (v) remain constant.

As the nanobeams increase in size and shape, their non-local characteristics decrease, resulting in a further reduction of their influence with larger beam sizes. Consequently, the softening effect of a non-local parameter ξ becomes evident across all physical fields, accompanied by the thermal vibration of nanobeams when one of their sides is exposed to a sinusoidally changing temperature.

When considering the length scale coefficient ξ and the impact of sinusoidal temperature in comparison to the classical theory of continuum mechanics (as depicted in Fig. 2), varying degrees of deviation w are observed. Fig. 2 illustrates that incorporating the length scale parameter leads to a reduction in both the largest and smallest lateral deviations w. Notably, despite differences in quantities, the deflection behavior aligns with the findings of Borjalilou *et al*. (2020), confirming the accuracy of the current study's results.

Fig. 2 illustrates the geometric distribution of the lateral deflection, denoted as w. This deflection initiates from zero (i.e., it starts at zero) and adheres to the boundary conditions at $x = 0$ and $x = L$. It is noteworthy that the lateral deflection w attains its maximum value at a specific distance x from the edge of the nanobeam, after which it

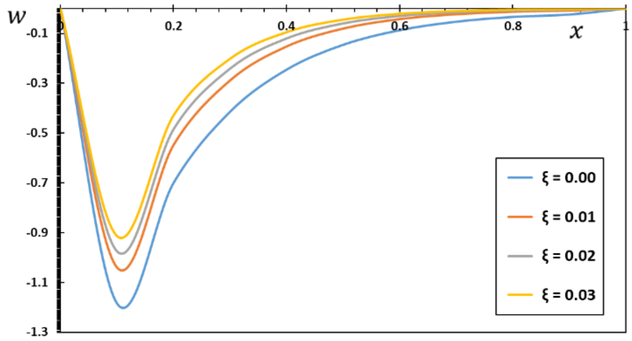

Fig. 2 The variation of deflection w wrt non-local coefficient ξ

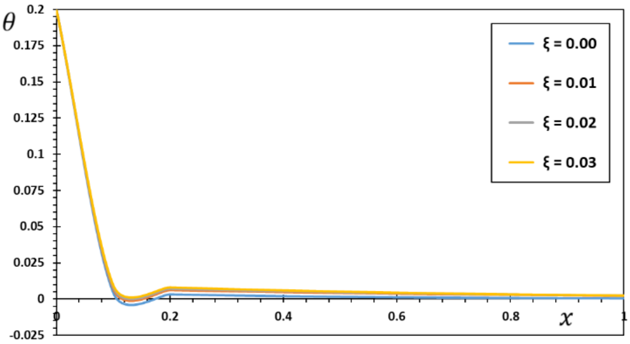

Fig. 3 The variation of temperature θ wrt non-local coefficient ξ

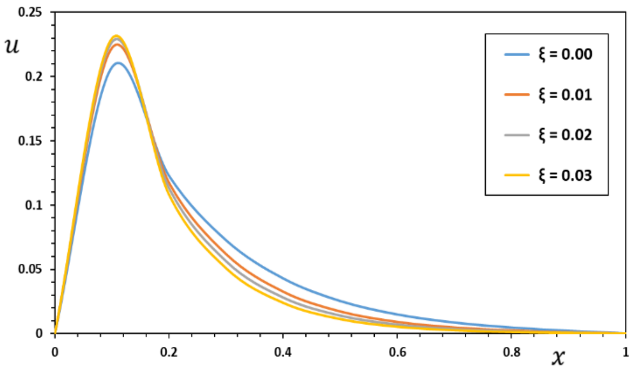

Fig. 4 The variation of displacement u wrt non-local coefficient ξ

diminishes with increasing separation. By augmenting the non-local factor, the lateral deflection propagates slower and disappears more rapidly from the graph. In contrast, the conventional vibration concept generates smaller signals, while the nonlocal effect produces significantly larger ones. In other words, when comparing outcomes derived from the traditional theory of vibration to those influenced by non-local effects, the latter often yields lower vibrational modes. The nonlocal results obtained in this study exhibit notable distinctions compared to Lim's findings (Lim *et al*. 2010).

Fig. 3 illustrates that the variations of non-local parameter ξ have negligible influence on the temperature profile θ. It is evident that the impact of non-local stress on the temperature θ of the nanobeam is trivial. Furthermore, as depicted in the figure, as one moves away from the heat source at the initial edge and increases the distance x, the temperature θ decreases. This observation indicates that thermal signals propagate through the nanobeam like a wave traveling at a specific velocity. This behavior can be attributed to the inclusion of phase delay parameters in the proposed thermoelastic model.

Fig. 4 is presented to depict the range of variations in the displacement u along the axis of the nanobeam within a short time ($t = 0.12$) while considering different values of the non-local coefficient ξ. The illustration demonstrates that the displacement u initiates with minute magnitudes and subsequently undergoes rapid growth until it reaches its maximum value, followed by a gradual decrease until it eventually diminishes. It is evident that the displacement magnitude diminishes as the non-local parameter ξ increases within the specified range. Consequently, understanding the disparity between the outcomes of local thermoplastic models and non-local theories becomes crucial. Since nanotechnology typically involves micro-scopic length scales, it is imperative to investigate further whether classical continuity models can be applied to such dimensions.

Fig. 5 illustrates the fluctuation in the bending moment M caused by variations in the non-dimensional, non-local parameter ξ. It is important to note that the beam-bending moment M reaches a constant maximum value at a specific distance from the tip of the first beam. Subsequently, it decreases until it reaches a constant minimum value at a certain distance from the ends of the beam. This trend is further supported by the data presented in Fig. 5, indicating the high sensitivity of the bending moment M to changes in the non-local parameter ξ. Consequently, nonlocal nano-materials models can potentially consider size effects at the nanoscale, which are not accounted for by traditional continuum models. This study demonstrates that the size effect, as quantified by the nonlocal nanoscale factor, is more pronounced in smaller nanostructured materials.

Indeed, empirical observations have demonstrated that size effects exert a significant influence on the physical properties and mechanical responses of nanoelectron-mechanical systems (NEMS) at the nanoscale (Abouelregal *et al*. 2023, Ece 2007), contradicting the assumptions made by most continuum models. In the realm of lattice dynamics, the impact of small scale is contingent upon the crystal structure and the specific physics under investigation (Abouelregal and Marin 2020). When nanobeams are subjected to sinusoidal thermal loads and phase delays, their bending deformation exhibits nonlocal behavior. The predictions of nonlocal theory regarding bending deflection, as per thermal load and thermoelastic modeling, may either surpass or fall short of those derived from the classical approach. In essence, the interplay between the various imposed thermal loads determines whether the bending stiffness of the nano-nonlocal beam is enhanced or not (Li *et al*. 2015).

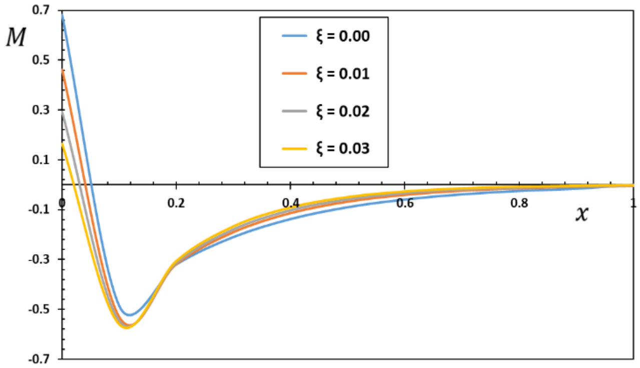

Fig. 5 The variation of bending moment M wrt non-local coefficient ξ

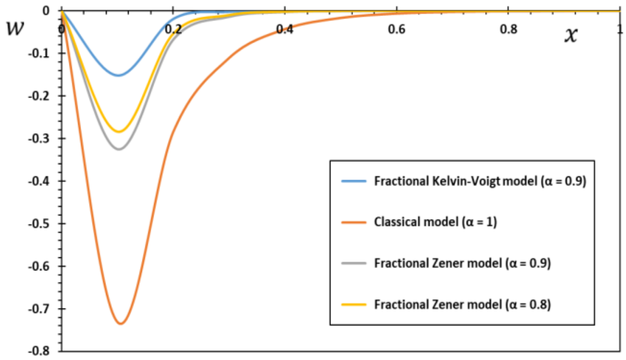

Fig. 6 The variation of deflection w for different viscoelastic models

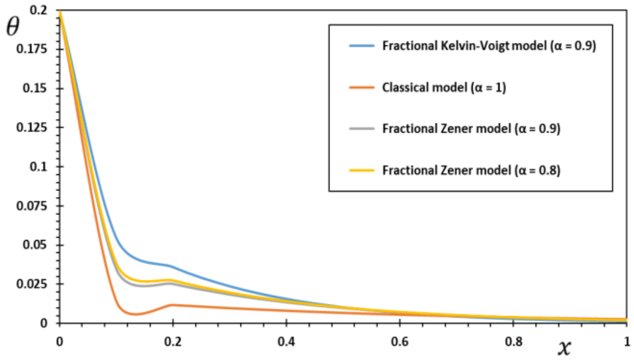

Fig. 7 The variation of temperature θ for different viscoelastic models

6.2 Comparison of fractional thermo-viscoelastic models

Numerous constitutive laws exist for viscoelastic materials, which can be employed to elucidate the mechanism by which the internal structure of nanobeams attenuates vibrations. In conventional models of visco-elasticity utilizing integer-order derivatives, it has been demonstrated that a substantial number of parameters are required to accurately match experimental curves. To address this issue, several researchers have proposed augmenting generalized models of viscoelasticity with fractional-order derivatives, aiming to provide an explanation for this phenomenon. The utilization of enhanced viscoelastic models based on fractional derivatives, such as the fractional Kelvin-Voigt (FKV) or Zener (FZ) models, assumes paramount importance due to the crucial role played by internal damping in nano-structured materials, both in theoretical investigations and practical applications of nanoscale devices. Consequently, the primary focus of this study revolves around incorporating the fractional-order Zener model into the constitutive Equations of a nonlocal Euler-Bernoulli nanobeam.

Section 2 demonstrates the distinct physical significance of the fractional order in the fractional Zener model, representing the width of the relaxation spectrum. When the relaxation behaviour occurs over a broader time range with a lower fractional order, it directly correlates with the microstructure of the materials. This work enhances our understanding of the physical importance of fractional viscoelastic models. The research and analysis in this example focus on studying the material significance of the fractional arrangement of fractional operators. Additionally, it explores the influence of fractional viscosity models on the behaviour of different physical domains within the moving nanobeam. Three standard models, namely relaxation, creep, and oscillatory tests, are used to measure viscoelastic. Selecting the appropriate test for a specific application or research issue can be challenging due to various criteria, including the capabilities of available experimental equipment and the types of physical deformation interest.

Figs. 6-9 illustrate that the propagation curves of thermomechanical waves inside viscoelastic materials differ significantly from those inside completely elastic materials. The findings highlight the greater significance of the viscoelastic model with derivatives and fractional order compared to conventional models that incorporate differential derivatives of integer order. This is because fractional models can better match the experimental data while requiring fewer parameters.

As time progresses, the bending deflection w exhibits a decrease in its maximum value across all values of the fractional derivative of order α, as depicted in Fig. 6. This phenomenon can be attributed to the dissipative nature of the material model. The decrease in bending deflection is primarily due to the increasing elasticity of the medium described by the fractional type constitutive Eq. (4) as α approaches zero. Conversely, as the fractional parameter α in Eq. (4) approaches a value of 1, the medium becomes more viscous. Additionally, when the fractional parameter α is small, the material demonstrates a gentler response, resulting in higher friction and energy dissipation. In contrast, Atanackovic and Pilipovic (2022) arrive at contrasting conclusions for a similar problem. However, our findings possess stronger physical justification. Upon examining the curves presented in Fig. 7, it becomes evident that thermal waves are diminished due to viscosity, particularly in the case of viscous fractional models. Figure 7 illustrates that an increase in the x parameter leads to a corresponding rise in temperature over time. This observation suggests that sludge viscosity escalates with higher temperatures and prolonged heat treatments. These

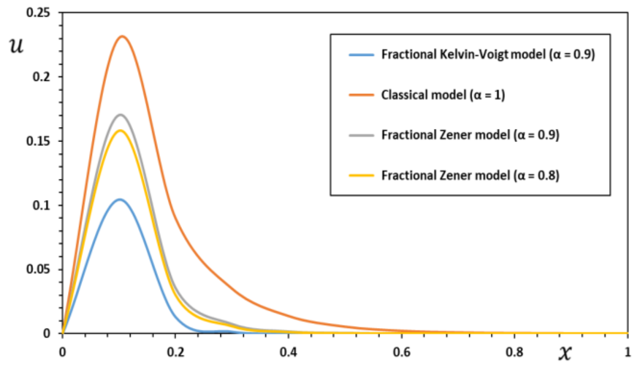

Fig. 8 The variation of displacement u for different viscoelastic models

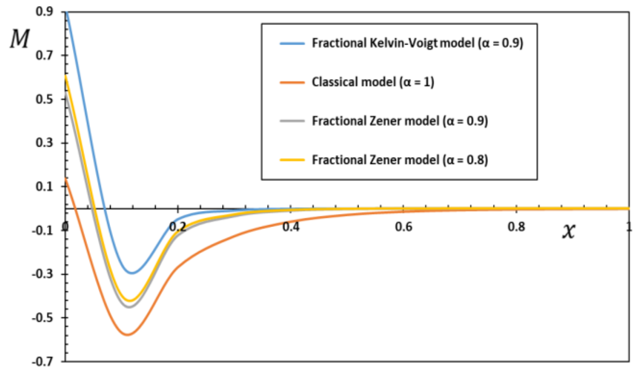

Fig. 9 The variation of bending moment M for different viscoelastic models

results align with Farno's experimental discoveries (Xiao *et al*. 2016).

Fig. 8 illustrates the influence of changes in the viscous fractional model and the order of the fractional differential on deformation. In contrast, viscous fractional models exhibit non-local properties, necessitating the consideration of all preceding deformation histories. Figure 9 depicts the representation of thermal stress for various fractional viscosity models. The results indicate that lower fractional order values lead to a slower response to thermal stress and relaxation, suggesting a wider range of relaxation time for relaxation to occur. Overall, the model predictions generally align well with the experimental results across different deformations, temperatures, and strain rates. As anticipated, both the fractional viscoelastic model and temperature change significantly affect the stress response of moving nanobeams. Specimens subjected to higher temperature deformation display less compliant stress responses compared to those experiencing lower temperature deformation. Decreasing the fractional order similarly reduces both temperature and deformation.

As viscoelastic materials, polymers typically exhibit broad relaxation spectra and require considerable time to reach equilibrium. The conventional rheological systems, such as the Kelvin concept and the Maxwell type, fail to describe long-term relaxation behavior adequately (Farno *et al*. 2014).

In order to accurately simulate nanostructures, it is necessary to incorporate the nonlocal concept and fractional-order viscoelasticity. By manipulating the system properties and the fractional order parameters, we can demonstrate a wider range of dynamic behaviors compared to traditional viscoelastic models. Additionally, these models may exhibit periodic behavior based on the parameter outcomes. Since generalized models require fewer factors than integer-order ones, they provide a natural framework for explaining damping characteristics, which is crucial from a practical perspective. Identifying numerous criteria to characterize the behavior of natural substances precisely could be a time-consuming process. However, fractional models have the potential to overcome these limitations by requiring fewer parameters to fully describe experimental results. This study illustrates that the three-parameter fractional Zener model could serve as an alternative to the generalized Maxwell model with multiple parameters (Xiao *et al*. 2016).

7. Conclusions

The thermoelastic behavior of moving viscoelastic nanobeams was investigated in this study using the non-local elasticity theory and the DPL thermal conductivity model for the first time. The constitutive relations and thermal conductivity were influenced by length scale parameters in this model, which also incorporated the generalized fractional Zener model. One of the primary objectives of this article was to examine the effectiveness of fractional thermo-viscoelasticity models in the context of moving nanobeams and highlight their superiority over traditional and commonly used viscoelasticity modeling methods.

The deductions that can be drawn based on the findings of the current investigation are as follows:

• By comparing data from various investigations, it is anticipated that a better understanding of the underlying physical significance of the behaviors of moving nanobeams can be achieved.

• A fractional thermoelasticity model was also developed to facilitate comprehension for researchers unfamiliar with fractional calculus. The results indicate that fractional viscoelastic models offer systematic techniques for capturing comparable material properties across experiments.

• At the nonlocal nanoscale, the stiffness of the nanobeam is significantly higher, resulting in more pronounced thermal and mechanical vibrations. Moreover, the nonlocal stiffness of the nanostructure loads can either be maintained or altered. The observed gradual increase in flexural stiffness supports a novel prediction made by the nonlocal effective stress model regarding the role of stiffness in supporting stiffness.

• Numerous real-world studies have indicated that their size influences these systems' mechanical and thermal behavior. Therefore, it is essential to employ size-dependent continuum concepts and heat transfer models to obtain more accurate measurements of viscoelastic behaviors in small-sized devices.

• This work enhances our understanding of the physical importance of fractional viscoelastic models. Theoretical and experimental data demonstrate that viscoelasticity offers a promising approach to harnessing energy from small materials.

References

Abouelregal, A.E., Marin, M. and Askar, S.S. (2023), "Analysis of the magneto-thermoelastic vibrations of rotating Euler–Bernoulli nanobeams using the nonlocal elasticity model", *Bound. Value Probl*., 21. https://doi.org/10.1186/s13661-023-01706-5

Abouelregal, A.E. and Marin, M. (2020), "The response of nanobeams with temperature-dependent properties using state-space method via modified couple stress theory", *Symmetry*, **12**, 1276. https://doi.org/10.3390/sym12081276

Abouelregal, A.E. and Sedighi, H.M. (2022), "Thermoelastic characteristics of moving viscoelastic nanams based on the nonlocal couple stress theory and dual-phase lag model", *Phys. Scr.*, **97**(11), 3. https://doi.org/10.1088/1402-4896/ac97cc

Alazwari, M.A., Esen I, Abdelrahman, A.A., Abdraboh, A.M. and Eltaher, M.A. (2022), "Dynamic analysis of functionally graded (FG) nonlocal strain gradient nanobeams under thermo-magnetic fields and moving load", *Adv. Nano Res.*, **12**(3), 231-51. https://doi.org/10.12989/anr.2022.12.3.231

Aliasghary, M., Mobki, H., Ouakad, H.M. (2022), "Pull-in phenomenon in the electrostatically micro-switch suspended between two conductive plates using the artificial neural network", *J. Appl. Comput. Mech.*, **8**(4), 1222-1235. https://doi.org/10.22055/jacm.2021.38569.3248

Anjum, N., He, J., He, C., Ashiq, A. (2022), "A brief review on the asymptotic methods for the periodic behaviour of microelectron-mechanical systems", *J. Appl. Comput. Mech.*, **8**(3), 1120-1140. https://doi.org/10.22055/jacm.2022.39404.3401

Arhami, M., Koochi, A. and Gharib, M.R. (2022), "Nonlocal coupled thermoelastic analysis of nanobeam under Casimir force", *Arch. Appl. Mech*., **92**, 3729-3746. https://doi.org/10.1007/s00419-022-02258-3

Askarian, A.R., Permoon, M.R. and Shakouri, M. (2020), "Vibration analysis of pipes conveying fluid resting on a fractional Kelvin-Voigt viscoelastic foundation with general boundary conditions", *Int. J. Mech. Sci.*, **179**, 105702. https://doi.org/10.1016/j.ijmecsci.2020.105702

Atanackovic, T.M. and Pilipovic, S. (2022), "Zener model with general fractional calculus: thermodynamical restrictions", *Fractals*, **6**, 617. https://doi.org/10.3390/fractalfract6100617

Azandariani, M.G., Gholami, M. and Nikzad, A. (2022), "Eringen's nonlocal theory for non-linear bending analysis of BGF Timoshenko nanobeams", *Adv. Nano Res.*, **12**(1), 37-47. https://doi.org/10.12989/anr.2022.12.1.037

Bagley, R.L. and Torvik, P.J. (1983), "Fractional calculus-a different approach to the analysis of viscoelastically damped structures", *AIAA J.*, **21**(5), 741-748. https://doi.org/10.2514/3.8142

Bagley, R.L. and Torvik, P.J. (1983), "A theoretical basis for the application of fractional calculus to viscoelasticity", *J. Rheol.*, **27**(3), 201-210. https://doi.org/10.1122/1.549724

E. Blanc, D. Komatitsch, E. Chaljub, B. Lombard, Z. Xie, (2016), "Highly-accurate stability preserving optimization of the Zener viscoelastic model, with application to wave propagation in the presence of attenuation," *Geophys. J. Int*., **205**, 427-439.

Borjalilou, V. and Asghari, M.E. (2020), "Thermoelastic damping nonlocal nanobeams considering dual-phase-lagging effect", *J. Vib. Control*, **26**(11-12), 1042-1053. https://doi.org/10.1177/1077546319891334

Caputo, M. and Mainardi, F. (1971), "A new dissipation model based on memory mechanism", *Pure Appl. Geophys.*, **91**, 147. https://doi.org/10.1007/BF00879562

Cattaneo, C. (2011), *Sulla Conduzione Del Calore*, Springer, Berlin Heidelberg.

Chen, L.Q. (2005), "Analysis and control of transverse vibrations of axially moving strings", *Appl. Mech. Rev.*, **58**(2), 91. https://doi.org/10.1115/1.1849169

Das, S. (2011), *Functional fractional calculus*, Springer, Berlinm Hidelberg.

Di Pa, M., Pirrotta, A., and Valenza, A. (2011), "Visco-elastic behavior through fractional calculus: easier method for best fitting experimental results," *Mech. Mat.*, **43**, 799-806. https://doi.org/10.1016/j.mechmat.2011.08.016

Duwel, A., Gorman, J., Weinstein, M., Borenstein, J. and Ward, P. (2003), "Experimental study of thermoelastic damping in MEMS gyros", *Sensors Actuat. A*, **20**, 70-75. https://doi.org/10.1016/S0924-4247(02)00318-7

Ece, A.E., Marin, M. and Askar, S.S. (2023), "Generalized MGT heat transfer model for an electro-thermal microbeam lying on a viscous-Pasternak foundation with a laser excitation heat source", *Symmetry*, **15**, 814. https://doi.org/10.3390/sym15040814

Ece, M.C. and Aydogdu, M. (2007), "Nonlocal elasticity effect on vibration in-plane loaded double-walled carbon nano-tubes", *Acta Mech.*, **190**(1-4), 185-195. https://doi.org/10.1007/s00707-006-0417-5

Eldred, L.B., Baker, W.P. and Palazotto, A.N. (2005), "Kelvin-Voigt versus fractional derivative model as constitutive relations for viscoelastic materials", *AIAA J.*, **33**, 547. https://doi.org/10.2514/3.12471

Eringen, A.C. (1972), "Nonlocal polar elastic continua", *Int. J. Eng. Sci.*, **10**, 1-16. https://doi.org/10.1016/0020-7225(72)90070-5

Eringen, A.C. (1972), "Linear theory nonlocal elasticity and dispersion of plane waves", *Int. J. Eng. Sci.*, **10**, 425-435. https://doi.org/10.1016/0020-7225(72)90050-X

Eringen, A.C. (1983), "On differential Eqs. of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710. https://doi.org/10.1063/1.332803

Eringen, A.C. (2002), "*Nonlocal Continuum Field Theories*", Springer, New York, NY, USA.

Farno, E., Baudez, J.C. and Eshtiaghi, N. (2018), "Comparison between classical Kelvin-Voigt and fractional derivative Kelvin-Voigt models in prediction of linear viscoelastic behaviour of waste activated sludge", *Sci. Total Environ.*, **613-614**, 1031-1036. https://doi.org/10.1016/j.scitotenv.2017.09.206

Faghidian SA, Tounsi A. (2022), "Dynamic characteristics of mixture unified gradient elastic nanobeams", *Facta Universitatis, Series: Mechanical Engineering*, **20**(3), 539-52.

Farno, E., Baudez, J.C., Parthasarathy, R. and Eshtiaghi, N. (2014), "Rheological characterisation of thermally-treated anaerobic digested sludge: impact of temperature and thermal history", *Water Res.*, **56**, 161. https://doi.org/10.1016/j.watres.2014.02.048

Jalil, A. T., Saleh, Z. M., Imran, A. F., Yasin, Y., Ruhaima, A. A. K., Gatea, M. A., and Esmaeili, S. (2023). A size-dependent generalized thermoelasticity theory for thermoelastic damping in vibrations of nanobeam resonators. *Int. J. Struct. Stab. Dyn.*, **23**(12), 2350133.

Green, A.E., and Naghdi, P.M. (1992), "On undamped heat waves in an elastic solid", *J. Therm. Stresses*, **15**, 253-264. https://doi.org/10.1080/01495739208946136

Green, A.E. and Naghdi, P.M. (1993), "Thermoelasticity without energy dissipation", *J. Elasticity*, **31**, 189-208. https://doi.org/10.1007/BF00044969

Grover, D., and Seth, R.K. (2018), "Viscothermoelastic micro-scale beam resonators based on dual-phase lagging model",

Microsyst. Technol., **24**, 1667-1672.
Hamidi, B.A., Hosseini, S.A., Hassannejad, R., and Khosravi, F. (2020), "Theoretical analysis of thermoelastic damping of silver nanobeam resonators based on Green-Naghdi via nonlocal elasticity with surface energy effects", *Eur. Phys. J. Plus*, **135**, 1-20.
Honig, G. and Hirdes, U. (1984), "A method for the numerical inversion of the Laplace transform", *J. Comput. Appl. Math.*, **10**, 113-132. https://doi.org/10.1016/0377-0427(84)90075-X
Huang, Y., Fu, J. and Liu, A. (2019), "Dynamic instability of Euler-Bernoulli nanobeams subject to parametric excitation", *Compos. B*, **164**, 234. https://doi.org/10.1016/j.compositesb.2018.11.088
Jones, D. (2001), *Handbook of Viscoelastic Vibration Damping*, John Wiley and Sons.
Kulkarni, R.G. (2008), "Solving sextic equations," *Atlantic Electr. J. Math.*, **3**(1), 56-60.
Koutsoumaris, C.C. and Eptaimeros, K.G. (2021), "Nonlocal integral static problems of nanobeams resting on elastic foundation", *Eur. J. Mech. A Solids*, **89**, 104295. https://doi.org/10.1016/j.euromechsol.2021.104295
Kumar, R., Kaushal, S. and Vikram, D. (2022), "Dynamic mathematical model of modified couple stressermoelastic diffusion with phase-lag", *Int. Appl. Mech.*, **58**, 348. https://doi.org/10.1007/s10778-022-01160-3
Lam, D.C.C., Yang, F., Chong, A.C.M., Wang, J. and Tong, P. (2003), "Experiments and theory in strain gradient elasticity", *J. Mech. Phys. Solids.*, **51**(8), 1477-1508. https://doi.org/10.1016/S0022-5096(03)00053-X
Lewandowski, R., Słowik, M. and Przychodzki, M. (2017), "Parameters identification of fractional models of viscoelastic dampers and fluids", *Struct. Eng. Mech.*, **63**(2), 181-193. https://doi.org/10.12989/sem.2017.63.2.181
Li, C., Yao, L. and Chen, W.S. (2015), "Comments on nonlocal effects in nano-cantver beams", *Int. J. Eng. Sci.*, **87**, 47-57. https://doi.org/10.1016/j.ijengsci.2014.11.006
Lim, C.W., Li, C. and Yu, J.L. (2010), "Dynamic behaviour of axially moving nanobeams based on nonlocal elasticity approach", *Acta Mech. Sinica*, **26**(5), 755-765. https://doi.org/10.1007/s10409-010-0374-z
Liu, N., Yang, G. and Chen, B. (2014), "Transverse vibration analysis of an axially moving beam with lumped mass", *J. Vibroeng.*, **16**(7), 3209-3217.
Lord, H.W. and Shulman, Y. (1967), "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solids*, **15**, 299-309. https://doi.org/10.1016/0022-5096(67)90024-5
Mainardi, F. and Sp, G. (2011), "Creep, relaxation and viscosity properties for basic fractional models in rheology", *Eur. Phys. J. Spec. Top.*, **193**, 133-160. https://doi.org/10.1140/epjst/e2011-01387-1
Mead, D. (1999), *Passive Vibration Control*, John Wiley and Sons Inc.
Mindlin, R.D. and Eshel, N.N. (1968), "On first strain-gradient theories in linear elasticity", *Int. J. Solids Struct.*, **4**(1), 109-124. https://doi.org/10.1016/0020-7683(68)90036-X
Oparnica, L., and Süli, E. (2020), "Well-posedness of the fractional zener wave equation for heterogeneous viscoelastic materials", *Fract. Calculus Appl. Anal.*, **23**(1), 126-166. https://doi.org/10.1515/fca-2020-0005
Pham, Q.H., Nguyen, P.C., Tran, V.K., Lieu, Q.X. and Tran, T.T. (2023), "Modified nonlocal couple stress isogeometric approach for bending and free vibration analysis of functionally graded nanoplates", *Eng. Comput.*, **39**, 993-1018. https://doi.org/10.1007/s00366-022-01726-2
Podlubny, I. (1999), *Fractional Differential Equations*, Academic Press, San Diego.
Rahimi, Z., Sumelka, W. and Yang, X.J. (2017), "Linear and non-linear free vibration of nano beams based on a new fractional non-local theory", *Eng. Comput*, **34**(5), 1754-1770. https://doi.org/10.1108/EC-07-2016-0262
Roy Choudhuri, S.K. (2007), "On a thermoelastic three-phase-lag model", *J. Therm. Stresses*, **30**, 231-238. https://doi.org/10.1080/01495730601130919
Shan, X. and Huang, A. (2022), "Intelligent simulation of the thermal buckling characteristics of a tapered functionally graded porosity-dependent rectangular small-scale beam", *Adv. Nano Res.*, **12**(3), 281-90. https://doi.org/10.12989/anr.2022.12.3.281
Sumelka, W., Blaszszyk, T. and Liebold, C. (2015), "Fractional Euler-Bernoulli beams: theory, numerical study and experimental validation", *Eur. J. Mech. A Solids*, **54**, 243-251. https://doi.org/10.1016/j.euromechsol.2015.07.002
Thai, C.H., Nguyen-Xuan, H. and Phung-Van, P. (2022), "A size-dependent isogeometric analysis of laminated composite plates based on the nonlocal strain gradient theory," *Eng. Comp.*, **39**, 331-345.
Toupin, R.A. (1964), "Theories of elasticity with couple-stress", *Arch. Ration. Mech. Anal.*, **17**(2), 85. https://doi.org/10.1007/BF00253050
Tung, D.X. (2021), "Wave propagation in nonlocal orthotropic micropolar elastic solids", *Arch. Mech.*, **73**(3), 237-251. https://doi.org/10.24423/aom.3764
Tzou, D.Y. (1995), "The generalized lagging response in small-scale and high-rate heating", *Int. J. Heat Mass Transfer*, **38**(17), 3231-3240. https://doi.org/10.1016/0017-9310(95)00052-B
Tzou, D.Y. (2015), "A unified field approach for heat conduction from macro- to micro-scales", *J. Heat Trans.*, **117**, 8-16. https://doi.org/10.1115/1.2822329
Tzou, D.Y. (2014), *Macro-to Microscale Heat Transfer: the Lagging Behavior*, John Wiley & Sons, West Sussex.
Vernotte, P. (1958), "Les paradoxes de la theorie continue de l'Eq. de la chaleur", *Comput. Rendus*, **246**, 3154. https://doi.org/10.1016/j.ijengsci.2017.06.006
Vincent, J.F.V. (1990), *Structural Biomaterials*, Princeton University Press.
Vajjaramatti, A., Balavalad, K.B. and Ashokkumar M. (2020), "Design, simulation and analysis of nems based piezoresistive pressure sensor", *Int. J. Eng. Res. Technol.*, **9**(7), 808-811. https://doi.org/10.17577/IJERTV9IS070342
Wu, F. and She, G.L. (2023), "Wave propagation in double nano-beams in thermal environments using the Reddy's high-order shear deformation theory", *Adv. Nano Res.*, **14**(6), 495-506. https://doi.org/10.12989/anr.2023.14.6.495
Xia, Z.X., Zhang, G.Y., Cong, Y. and Gu, S.T. (2002), "A non-classical couple stress based Mindlin plate finite element framework for tuning band gaps of periodic composite micro plates", *J. Sound Vib.*, **529**, 116889. https://doi.org/10.1016/j.jsv.2022.116889
Xiao, R., Yakacki, C.M., Guo, J., Frick, C.P. and Nguyen, T.D. (2016), "A predictive parameter for the shape memory behavior of thermoplastic polymers", *J. Polym. Sci. B Polym. Phys.*, **54**(14), 1405-1414. https://doi.org/10.1002/polb.23981
Xiao, R., Sun, H., Chen, W. (2016), "An equivalence between generalized Maxwell model and fractional Zener model", *Mech. Mater.*, **100**, 148-153. https://doi.org/10.1016/j.mechmat.2016.06.016
Yang, F., A.C.M., Lam, D.C.C. and Tong, P. (2002), "Couple stress based strain gradient theory for elasticity", *Int. J. Solids Struct.*, **39**(10), 2731-2743. https://doi.org/10.1016/S0020-7683(02)00152-X
W. Yang, S. Wang, W. Kang, T. Yu, and Y. Li, (2023), "A unified high-order model for size-dependent vibration of nanobeam based on nonlocal strain/stress gradient elasticity with surface effect," *Int. J. Eng. Sci.*, **182**, 103785
Zhang, Y., Sahmani, S., Yang, Z. and Safaei, B. (2022), "Nonlocal

and couple stress tensors three-dimensional nonlinear dynamical stability behavior of microshells manufactured by smart materials", *Acta Mech.*, **233**, 5377-5401. https://doi.org/10.1007/s00707-022-03394-1

Zhao, D., Liu, J. and Wang, L. (2016), "Linear free vibration of a cantilever nanobeam with surface effects: Semi-analytical solutions", *Int. J. Mech. Sci.*, **113**, 184-195. https://doi.org/10.1016/j.ijmecsci.2016.05.001

Zhou, H. and Li, P. (2017), "Thermoelastic damping in micro- and nanobeam resonators with non-fourier heat conduction", *IEEE Sens. J.*, **17**, 6966-6977. https://doi.org/10.1016/j.ijmecsci.2019.105211

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