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On dynamic flight response of golf ball containing nanoparticles for improving quality

Yuwei Du[1], Guowen Ai[*2], M. Kaffash[3]

[1]*College of Leisure and Digital Sports, Guangzhou Sport University, Guangzhou 510000 Guangdong, China*
[2]*School of Physical Education, Hainan Normal University, Haikou 570100, Hainan, China*
[3]*School of Mechanical Engineering, Malaya University, Malaysia*

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Abstract. This research delves into the intricate dynamics of the flight response exhibited by a golf ball that incorporates nanoparticles with the goal of enhancing its overall quality. The golf ball is meticulously modeled utilizing beam elements, and the impact of nanoparticles is intricately captured through the application of the Halpin-Tsai theory. Employing a numerical solution, the study thoroughly explores the flight response of the golf ball, taking into account the nuanced effects of the embedded nanoparticles. By scrutinizing the aerodynamic characteristics through advanced simulations, this investigation aims to provide valuable insights that could potentially revolutionize the design and performance of golf equipment, offering a pathway towards superior quality and enhanced functionality in the realm of golf ball technology. Results show that increase in the volume percent of nanoparticles, improves the flight response of the golf ball.

Keywords: dynamic flight; golf ball; model; nanoparticles; numerical solution

1. Introduction

In the pursuit of elevating the performance and quality of golf equipment, this study focuses on the dynamic flight response of a golf ball engineered with the incorporation of nanoparticles. The flight dynamics of the golf ball are intricately modeled using beam elements, offering a detailed representation of its structural behavior. The impact of nanoparticles on the ball’s properties is systematically addressed by applying the Halpin-Tsai theory, providing a theoretical framework to understand the enhanced characteristics induced by these nanoscale additives. Through the utilization of a numerical solution, this research aims to unravel the complexities associated with the flight response of the golf ball. The investigation explores how the presence of nanoparticles influences the aerodynamic performance, stability, and overall behavior of the golf ball during its trajectory. By delving into the dynamic aspects of the golf ball’s flight, this study seeks to contribute valuable insights that may pave the way for advancements in golf ball design, ultimately leading to an improved quality and enhanced performance on the golf course.

A sandwich nanocomposite porous smart micro-beam has several potential practical applications in engineering, especially in aerospace, mechanical, and civil engineering. For example, in aerospace structures, these smart micro-beams can be used in the construction of lightweight and high-strength components for aircraft and spacecraft. They can improve the structural integrity of wings, fuselages, and other critical parts while reducing overall weight, contributing to fuel efficiency and better performance. In energy harvesting, smart materials embedded within the micro-beam can convert mechanical vibrations or deformations into electrical energy. Also, in vibration dampening, the porous structure in the micro-beam can be designed to absorb and dampen vibrations. In addition, in MEMS (Micro-Electro-Mechanical Systems) devices, such as accelerometers or gyroscopes, porous smart micro-beams can be employed to enhance sensitivity, reduce noise, and improve overall performance. Furthermore, smart micro-beams can be embedded within sport like ball. Any deviations or damage can be detected early, allowing for timely maintenance or repairs. For modelling in this field, (Hadji *et al.* 2021) delved into the intricacies of hyperbolic shear deformation plate theory, exploring its applicability in the bending and free vibration analysis of functionally graded plates. Notably, their investigation accounted for porosities that may arise within functionally graded materials (FGMs) during the fabrication process. (Zerrouki *et al.* 2021) contributed a novel numerical tool for analyzing the bending responses of carbon nanotube-reinforced composite (CNTRC) beams, providing valuable insights into the structural behavior of these advanced materials. (Mudhaffar *et al.* 2021) delved into the bending behavior of an advanced functionally graded ceramic–metal plate subjected to hygro-thermo-mechanical loads. Their study, utilizing a simple higher-order integral shear deformation theory, incorporated the influence of a viscoelastic foundation. (Benaberrahmane *et al.* 2021) presented a comprehensive exploration of the free vibration characteristics of bidirectional functionally graded (FG) beams, particularly focusing on their interaction with variable elastic foundations. (Hirane *et al.* 2021) introduced

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[*] Corresponding author, Ph.D.,
E-mail: 202109110135@hainnu.edu.cn

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a novel C0 higher-order layerwise finite element model, offering a versatile tool for static and free vibration analysis of functionally graded materials (FGM) sandwich plates. (Nam *et al*. 2020) delved into the nonlinear large deflection torsional buckling of functionally graded carbon nanotube (CNT) orthogonally reinforced composite cylindrical shells. Their study considered the thermal effects and the influence of Pasternak's elastic foundations. (Viet Hoang *et al*. 2020) explored the nonlinear vibration characteristics of functionally graded graphene nanoplatelet-reinforced composite doubly curved shallow shells, investigating their behavior on elastic foundations. (Ninh *et al*. 2021) presented a study on the nonlinear dynamic behavior of functionally graded graphene nanoplatelets reinforced (FG-GPLRC) shells. Their analysis incorporated high-order functions for the shell radius, employing the classical thin shell theory (CTST) and Von Karman-Donnell geometrical nonlinearity assumption. (Yaylaci *et al*. 2020) investigated both continuous and discontinuous contact problems involving functionally graded (FG) layers resting on rigid foundations. (Belarbi *et al*. 2020) contributed to the understanding of the bending behavior of functionally graded single-layered, symmetric, and non-symmetric sandwich beams. Their study employed a new higher-order shear deformation theory, offering insights into the structural response of these complex configurations.

Numerous endeavors have been dedicated to scrutinizing the mechanical characteristics of sandwich structures. In contemporary research, there is a discernible focus on the exploration of smart sandwich structures. For instance, (Thi Phuong *et al*. 2018) delved into the nonlinear vibration axisymmetric analysis of functionally graded sandwich shallow spherical caps under external pressure, considering an elastic foundation in a thermal environment. (Dong *et al*. 2020) contributed insights into the nonlinear thermomechanical buckling behaviors of sandwich functionally graded plates subjected to axial compression and external pressure, with a meticulous analytical analysis resting on nonlinear elastic foundations. (Nguyen *et al*. 2020) explored a semi-analytical approach for the nonlinear free and forced asymmetric vibration of corrugated sandwich functionally graded cylindrical shells containing fluid under harmonic radial loads. (Tho Hung *et al*. 2020) presented a semi-analytical investigation into the nonlinear buckling and post-buckling of spiral corrugated sandwich functionally graded (FGM) cylindrical shells under external pressure and surrounded by a two-parameter elastic foundation based on the Donnell shell theory. (Ninh *et al*. 2020) conducted an analytical study on the nonlinear vibration of W-Cu sandwich shells containing heavy water surrounded by an elastic foundation under thermo-mechanical loads. (Solmaz and Topkaya 2020) proposed an examination of the flexural fatigue behaviors of honeycomb sandwich composites subjected to low-velocity impact damage, considering parameters such as the type and thickness of the face sheet material, cell size, and core height. (Topkaya and Solmaz 2018) empirically studied the low-velocity impact loading behaviors of honeycomb sandwich composites. (Topkaya and Solmaz 2016) presented an analysis of the fatigue behaviors of honeycomb-sandwich composites under buckling and three-point bending loads. (Hirane *et al*. 2021) introduced a novel C0 higher-order layerwise finite element model for static and free vibration analysis of functionally graded materials (FGM) sandwich plates. (Daikh *et al*. 2021b) investigated the static stability buckling of cross-ply single-walled (SW) carbon nanotube-reinforced composite (CNTRC) curved sandwich nanobeams in a thermal environment, based on a novel quasi-3D higher-order shear deformation theory. (Daikh *et al*. 2021a) explored the free vibration response of rectangular functionally graded material sandwich nanoplates with simply supported boundary conditions. (Belarbi *et al*.) delved into the bending behavior of functionally graded single-layered, symmetric, and non-symmetric sandwich beams, employing a new higher-order shear deformation theory.

This study presents a novel exploration into the dynamic flight response of a golf ball enriched with nanoparticles, aiming to enhance its overall quality. Unlike conventional analyses, the ball's intricate behavior is meticulously modeled using beam elements, incorporating the influence of nanoparticles through the application of the Halpin-Tsai theory. Leveraging advanced numerical solutions, this research offers a comprehensive investigation into the flight response of the golf ball, unraveling the nuanced effects and interactions of nanoparticles during its trajectory. This innovative approach not only deepens our understanding of the aerodynamic dynamics of nanoparticle-infused golf balls but also holds the potential to revolutionize the design and performance optimization of sports equipment, ushering in a new era of precision and efficiency in golf ball technology.

2. Formulation

Fig. 1 reveals a sandwich golf ball which is modeled by beam element with length of L and thickness of h.

In this paper, for simulating the structure mathematically, Euler theroy has been exerted where the displacement vectors are written (Timoshenko and Gere 1961) as:

$$U(x,z,t) = u_0(x,t) - z\frac{\partial w_0(x,t)}{\partial x}, \tag{1}$$

$$V(x,z,t) = 0, \tag{2}$$

$$W(x,z,t) = w_0(x,t), \tag{3}$$

where respectively, u_0 and w_0 are the displacements of the middle plate in the x and z axis, respectively, The strain components can be described as follows (Pandey *et al*. 2019, Timoshenko and Gere 1961):

$$\varepsilon_{xx} = \frac{\partial}{\partial x}u_0 - z\frac{\partial^2}{\partial x^2}w_0, \tag{4}$$

$$\varepsilon_{xz} = 0, \tag{5}$$

If the volume fraction of the GPLs be lower than 1%, the Halpin-Tsai model is suitable to determine the Young's

Fig. 1 A golf ball modeled by beam element

modulus of the CNTs reinforced for top layer. The effective Young modulus of the piezoelectric layer is written by: (Fakhar and Kolahchi 2018)

$$E_t = \frac{3}{8}\frac{1+\left(2\frac{l}{t}\right)\left(\frac{\left(\frac{E_{GPL}}{E_M}-1\right)}{(E_{GPL}/E_M)+2l/t}\right)V_{GPL}}{1-\left(\frac{\left(\frac{E_{GPL}}{E_M}-1\right)}{(E_{GPL}/E_M)+2l/t}\right)V_{GPL}}E_M + \frac{5}{8}\frac{1+(2\frac{w}{t})\left(\frac{\left(\frac{E_{GPL}}{E_M}-1\right)}{(E_{GPL}/E_M)+2l/t}\right)V_{GPL}}{1-\left(\frac{(\frac{E_{GPL}}{E_M}-1)}{(E_{GPL}/E_M)+2w/t}\right)V_{GPL},}E_M \tag{6}$$

$$\upsilon_t = V_{GPL} \times \upsilon_{GPL} + (1 - V_{GPL})\upsilon_M, \tag{7}$$

$$\rho_t = V_{GPL} \times \rho_{GPL} + (1 - V_{GPL})\rho_M, \tag{8}$$

Respectively, *t*, *W*, *l* are the thickness, width and average length of the GPLs. Respectively, E_{GPL} and E_M indicate the moduli of GPL and polymer matrix, The Poisson's ratio of matrix and GPL are υ_M and υ_{GPL}, respectively, V_{GPL} is the volume percent of the GPL as follows (Berghouti *et al*. 2019):

$$V_{GPL} = \frac{W_{GPL}}{W_{GPL} + (\rho_{GPL}/\rho_M) - (\rho_{GPL}/\rho_M)W_{GPL}} \tag{9}$$

respectively, ρ_M, ρ_{GPL} and W_{GPL} denote the mass densities of matrix and GPLs, the weight fraction of GPLs.

The relation for the stresses, σ, as well as flux density, D, of top layer based on piezo elasticity theory can be indicated as (Farrokhian 2020a, b):

$$\sigma_{ijt} = C_{ijkl}\varepsilon_{klt} - e_{ijk}E_k, \tag{10}$$

$$D_k = e_{kli}\varepsilon_{kl} + \in_{ij} E_j, \tag{11}$$

in this relations, respectively, σ_{ijt} and D_k are tensor stresses and electric displacement vector components, C_{ijkl}, e_{ijk} and $\in_{ij}$ are the matrix of elastic coefficients, piezo-electric constants and dielectric coefficients further, E_k named the electric field which can be defined as a function of electric potential (ϕ) as follows (Mehar and Panda 2019):

$$E = -\nabla\varphi \tag{12}$$

The elastic bending rigidity is expressed as follows (Farrokhian *et al*. 2020a, Nguyen *et al*. 2020, Vu *et al*. 2021):

$$C_{ij}^{(k)} = C_{ij}^{(k)}\left(1 + g\frac{\partial}{\partial t}\right) \tag{13}$$

in this relation g indicates the damping structural coefficient.

In elastomer layer of ball, the normal and the shear stresses in the *x*-*y* plane can be neglected. The magnetic field in the *z* direction is applied and the Young's modulus is very small as well as, the shear strains can be given as (Fakhar and Kolahchi 2018):

$$\varepsilon_{xzc} = \frac{H(\frac{\partial}{\partial x}w_0 + \frac{u_{0t}-u_{0b}}{H})}{h_c} \tag{14}$$

According to the Hook's law, the stress relations of core layer are written by:

$$\sigma_{xzc} = \frac{G \times H(\frac{\partial}{\partial x}w_0 + \frac{u_{0t}-u_{0b}}{H})}{h_c}, \tag{15}$$

where G is related to the complex shear modulus of LPRE which is (Keshtegar *et al*. 2020b):

$$G = G' + iG'' \tag{16}$$

in which respectively G'' and G' are loss modulus and storage modulus of the LPRE layer. Respectively, the loss modulus and storage modulus of the core can be defined $2600B + 1700$ and $50000B^2$ in which B is the electric field in kV/mm.

Based on the energy model, the potential energy *U* for a beam can be obtained as (Keshtegar *et al*. 2020a):

$$U = (\frac{1}{2}\int(\sigma_{ij}\varepsilon_{ij} - D_{ii}E_{ii})\,dv \tag{17}$$

In above relation

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{18}$$

The kinetic energy is:

$$K = \frac{\rho}{2}\int \int_{-h/2}^{(h/2)} [(\frac{\partial w}{\partial t})^2]dzdA \tag{19}$$

The force of impact on the ball is (Fakhar and Kolahchi 2018):

$$w = \int qdA \tag{20}$$

Utilizing Hamilton's principle, we have:

$$\delta u_0\text{:}\left(C_{11}h\frac{\partial^2}{\partial x^2}u_0 + C_{11}h^3/3\frac{\partial^3}{\partial x^3}w_0 - e_{31}\frac{\partial}{\partial x}\varphi\right) + \frac{G(\frac{\partial}{\partial x}w_0 + u_0)}{h^2} - I_{0t}\frac{\partial^2}{\partial t^2}u_{0t} = 0 \tag{21}$$

$$\begin{aligned}\delta w_0 :+ \frac{G}{(h)^2}\frac{\partial}{\partial x} + (\frac{\partial}{\partial x} w_0 + u_0)h \\ + \rho h^3/12\left(\frac{\partial^4}{\partial x^2 \partial t^2} w_0\right) \\ - \frac{\partial^2}{\partial x^2}\left(C_{11} h \frac{\partial}{\partial x} u_0 + C_{11} h^3/3 \frac{\partial^2}{\partial x^2} w_0 - e_{31}\varphi\right) \\ - \rho h\left(\frac{\partial^2}{\partial t^2} w_0\right) = 0,\end{aligned} \tag{22}$$

$$\delta\varphi : -e_{31}\frac{\partial^2}{\partial x^2}\varphi + C_{11} h \frac{\partial}{\partial x} u_0 - C_{11} h^3/3 \frac{\partial^2}{\partial x^2} w_0 = 0, \tag{23}$$

3. Solving method

In the analytical procedure, to derive the function can be used as (Keshtegar 2020b):

$$d(x, y, t) = d_0\, e^{(k_x x - \omega t)} \tag{24}$$

In which d_0, ω, k_x respectively, are the amplitudes of the wave motion, frequency and the wave number. In addition, the flight speed is defined as:

$$C = \frac{\omega}{k_x} \tag{25}$$

By substituting Eq. (70) into motion equations yields as:

$$([K] - \omega[D] + \omega^2[M])[d_0] = [0] \tag{26}$$

In which [K], [D] and [M], respectively, represent stiffness, damper and mass matrix.

4. Results and discussion

In this part, respect to study the numerical results of this paper concentrating on particular components. The elastic constant and density are assumed as $E = 65Mpa$, $\rho_M = 1100kg/m^3$.

In most of the figure of this paper, illustrate the variations of the flight velocity of golf ball (C) versus various wave numbers (K_x). Fig. 2 illustrates the effect of diameter on the flight speed of the golf ball. It's vivid the flight speed will be decreased when the diameter and the porosity are increased. It is because porosity introduces empty spaces or voids within the material's structure. These voids reduce the effective stiffness of the material. Also, the presence of voids increases the compliance or deformability of the material. In simple terms, it becomes easier for the material to deform under the influence of an external force, which includes wave propagation. This increased compliance results in a lower flight speed. In addition, porous materials often have lower densities due to the voids. The flight speed in a material is inversely proportional to the square root of its density. As the density decreases, the flight speed decreases as well.

Fig. 3 reveals the influence of structure damping on the flight velocity. It is shown that when the damping is considered, the flight velocity is lower. From the physical side, when a wave propagates through a damped material, part of its energy is absorbed and converted into heat due to

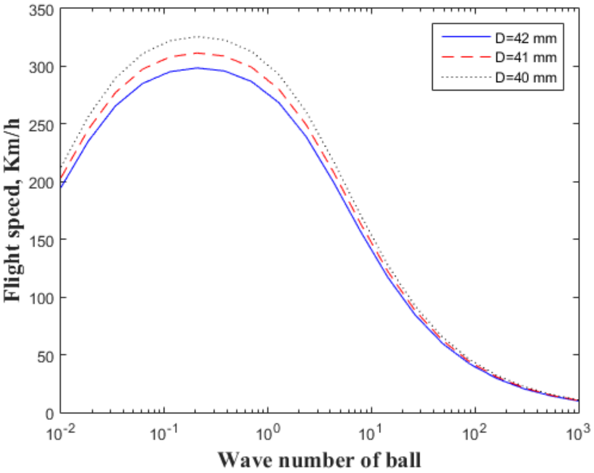

Fig. 2 Influence of diameter on the flight speed of the golf ball

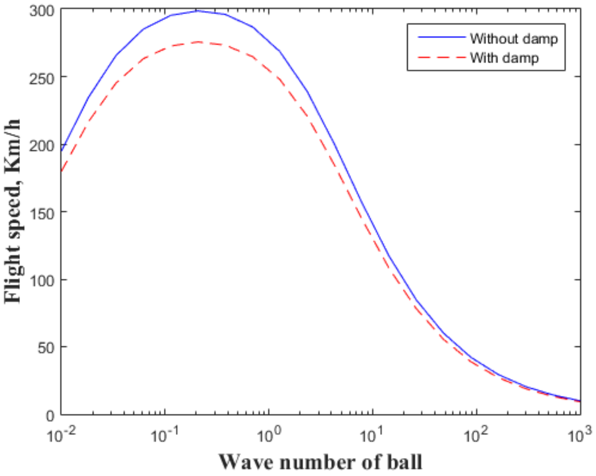

Fig. 3 Influence of structural damping on the flight speed of the golf ball

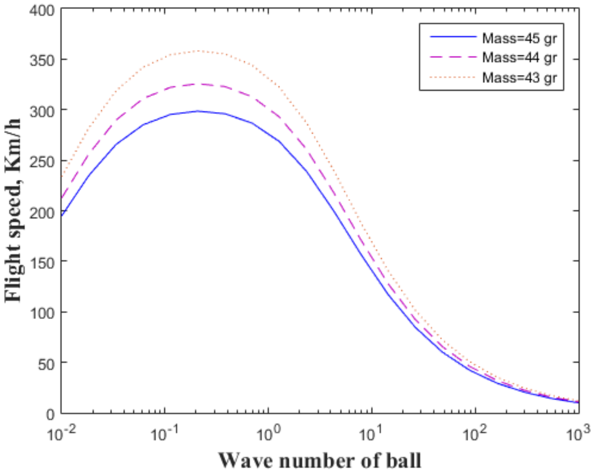

Fig. 4a The effect golf ball mass on the flight speed of the golf ball

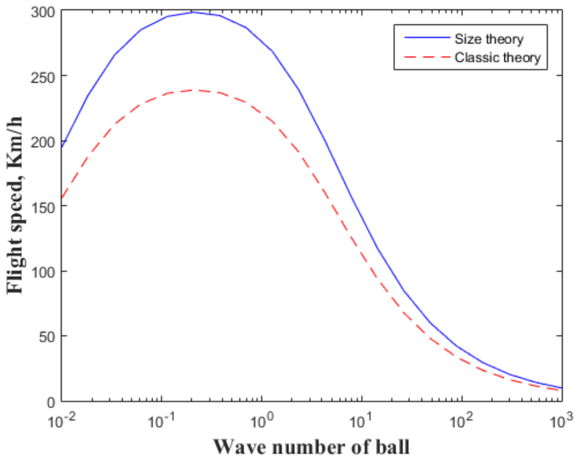

Fig. 4b Influence of theory type on the flight speed of the golf ball

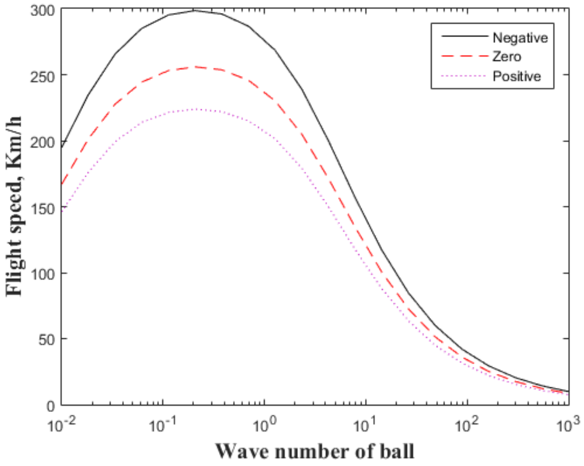

Fig. 5 The effect of the external voltage on the flight speed of the golf ball

internal friction and other dissipative mechanisms within the material. This energy loss effectively reduces the overall kinetic energy of the wave, resulting in a decrease in flight speed. In essence, damping acts as a resistance to motion, slowing down the wave's progress. This reduction in flight speed is especially pronounced in viscoelastic materials, where the energy dissipation due to internal friction is significant, leading to a more pronounced decrease in flight speed compared to purely elastic materials. According to the real fact of the sandwich structure as a viscoelastic material, results can be determined more accurately than without considering damping effects.

Figs. 4a and 4b the effects of golf ball mass upon the flight speed. Increasing the golf ball mass can reduce the flight speed due to the material properties. Rheological elastomers are known for their viscoelastic behavior, which combines characteristics of both elasticity and viscosity. When the mass of the elastomer core is increased, it typically results in a softer and less stiff core material. This reduced stiffness affects the overall stiffness of the sandwich structure, making it more compliant. As a result, waves traveling through the softer elastomer core experience lower wave velocities because the core material is less resistant to deformation.

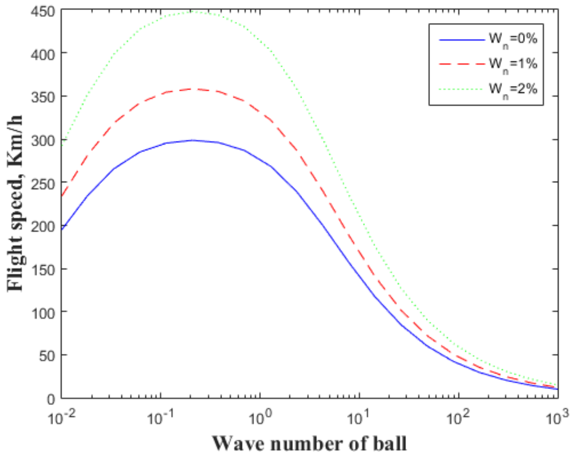

Fig. 6 Effect of GPL weight percent on the flight speed of the golf ball

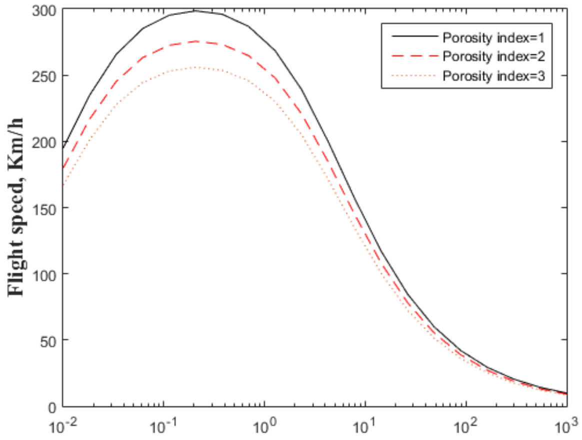

Fig. 7 Effect of porosity index on the flight speed of the golf ball

The influence of theory type on the flight velocity as function of wave numbers is indicated in Fig. 5. Considering size theory yields increase of flight velocity. This is primarily due to the additional microstructural effects introduced by couple stress theory. In addition, the microstructure contributes to the overall stiffness and deformation behavior of the material, leading to differences in wave propagation characteristics.

Fig. 5 illustrates effect of electric voltage on the flight velocity. It is inferred that using positive voltage decrease flight velocity of the golf ball. When a negative external voltage is applied to a piezoelectric material, it induces an electric field that aligns the polarized domains of the material in a way that increases its stiffness. This increased stiffness results in higher phase velocities for mechanical waves traveling through the material. Essentially, the material becomes more rigid under the influence of the negative voltage, leading to faster wave propagation. Conversely, when a positive external voltage is applied, it induces an electric field that aligns the polarized domains in the opposite direction, which can reduce the material's stiffness. This decreased stiffness leads to a decrease in phase velocity for mechanical waves.

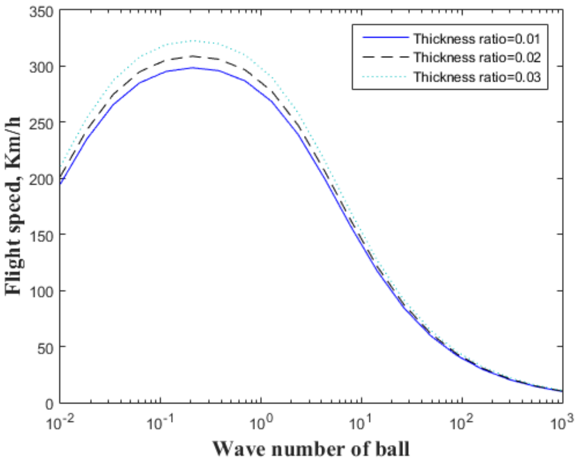

Fig. 8 The effect piezoelectric to elastomer thickness ratio on the flight speed of the golf ball

Fig. 6 presents the variations of the flight speed versus GPLs volume fraction. As it is expected the weight percent of GPLs raises, the flight velocity enhances. GPL is an incredibly stiff and strong material. When incorporated into a polymer matrix, it enhances the overall stiffness of the composite. This increased stiffness can lead to higher phase velocities for mechanical waves. Essentially, the composite becomes less deformable and transmits waves more quickly. Also, GPL exceptional mechanical properties can also enhance the structural integrity of the composite. This can lead to more efficient transmission of mechanical waves, contributing to increased phase velocities.

Fig. 7 shows the flight speed of the sandwich beam according to porosity index. As it is seen raising the porosity index, the flight speed reduces. It is vivid, enhance of porosity index lead to decrement of the stiffness and the elasticity modulus.

Fig. 8 reveals the flight speed of the piezoelectric to elastomer thickness ratio. Increasing the piezoelectric to elastomer thickness ratio, enhance in the flight velocity, since the bending rigidity of the golf ball get raised.

5. Conclusions

In conclusion, this study delves into the dynamic flight response of a golf ball enhanced with nanoparticles, employing a beam element model and integrating the Halpin-Tsai theory to capture the impact of nanoparticles on its performance. Through a rigorous numerical analysis, we have unveiled critical insights into the flight behavior of the golf ball, shedding light on the intricate interplay between its structural elements and the introduced nanoparticles. The findings underscore the potential for substantial advancements in golf ball technology, with implications for improved quality, aerodynamic efficiency, and overall performance. As we navigate the complex dynamics of nanoparticle-infused sports equipment, this research contributes not only to our comprehension of golf ball flight mechanics but also paves the way for future innovations in the realm of sports science and engineering. It's vivid the flight speed will be decreased when the diameter and the porosity are increased. As it is expected the weight percent of GPLs raises, the flight velocity enhances. Increasing the piezoelectric to elastomer thickness ratio, enhance in the flight velocity Increasing the piezoelectric to elastomer thickness ratio, enhance in the flight velocity.

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