

Buckling analysis of perforated nano/microbeams with deformable boundary conditions via nonlocal strain gradient elasticity

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Abstract. This work aims to present a solution for the buckling behavior of perforated nano/microbeams with deformable boundary conditions using nonlocal strain gradient theory (NLSGT). For the first time, a solution that can provide buckling loads based on the non-local and strain gradient effects of perforated nanostructures on an elastic foundation, while taking into account both deformable and rigid boundary conditions. Stokes' transformation and Fourier series are used to realize this aim and determine the buckling loads under various boundary conditions. We employ the NLSGT to account for size-dependent effects and utilize the Winkler model to formulate the elastic foundation. The buckling behavior of the perforated nano/microbeams restrained with lateral springs at both ends is studied for various parameters such as the number of holes, the length and filling ratio of the perforated beam, the internal length, the nonlocal parameter and the dimensionless foundation parameter. Our results indicate that the number of holes and filling ratio significantly affect the buckling response of perforated nano/microbeams. Increasing the filling ratio increases buckling loads, while increasing the number of holes decreases buckling loads. The effects of the non-local and internal length parameters on the buckling behavior of the perforated nano/microbeams are also discussed. These material length parameters have opposite effects on the variation of buckling loads. This study presents an effective eigenvalue solution based on Stokes' transformation and Fourier series of the restrained nano/microbeams under the effects of elastic medium, perforation parameters, deformable boundaries and nonlocal strain gradient elasticity for the first time.

Keywords: buckling analysis; elastic foundation; nonlocal strain gradient theory; perforated nano/microbeams; Stokes' transformation

1. Introduction

Nanostructures are commonly modeled as nanobeams, nanorods, nanoshells, and nanoplates (Demir and Civalek 2017, Numanoğlu *et al.* 2018). Carbon nanotubes (CNTs) have garnered significant research interest due to their unique properties, which have been exploited in various applications, including nanoelectronics, nano-composites, nanoscale containers, and electro-mechanical systems (Altenbach and Öchsner 2020). CNTs are often modeled as nanobeams in formulations involving small deformations (Harik 2002). The development of technology and the growing number of applications for nanostructures have led to an increase in research on nanobeams. Nanobeam structures have a wide range of applications, including nanowires, nanosystems, biosensors, and especially nano/micro-electro-mechanical systems (NEMS-MEMS) (Eltaher *et al.* 2014).

As nanostructures are being used in a growing number of fields, it has become increasingly important to develop mathematical models to compute their static and dynamic

responses. The study of responses and mechanical properties of nanostructures can be performed experimentally, computationally (through simulations), and theoretically (through analytical approaches). Experimental methods require specialized equipment and expertise, making them costly. Computational methods, on the other hand, can be limited by the system's complexity, as the number of calculations grows exponentially with the number of degrees of freedom. Therefore, analytical modeling is often preferred for its efficiency and effectiveness (Uzun *et al.* 2022).

Classical beam theories (CBTs) such as Timoshenko, Euler-Bernoulli, and Reddy are widely used to study the buckling, bending, and vibration response of beams, but they neglect the small-size effects of these structures. It has been shown experimentally and theoretically that CBTs are unable to accurately predict the deformation of micro- and nano-sized structures due to their small size (Yaylı 2019).

To overcome the limitations of CBTs at the nano and micro scales, scientists have developed size-dependent nonlocal theories. Some of the theories commonly used by researchers include Eringen's nonlocal elasticity theory (ENET) (Eringen 1983, 1987, 2002), couple stress theory (CST) (Toupin 1962), modified couple stress theory (MCST) (Yang *et al.* 2002), strain gradient theory (SGT) (Mindlin 1964, 1965), higher-order shear deformation theory (HSDT) (Panda and Singh 2010, 2013a, b), nonlocal strain gradient theory (NLSGT) (Lim 2015a, Faghidian 2021a, b), the higher-order nonlocal gradient elasticity

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theory (HNLGT) (Faghidian 2020a, b, Faghidian *et al.* 2022a, b) and the mixture stress gradient theory (Faghidian *et al.* 2022c). Unlike CBTs, which assume that the stress at a point depends only on the strain at that point, nonlocal theories take into account the strains at points near the point of interest. In ENET, for example, the stress at one point is related to the strain at all other points. Based on nonlocal theories of elasticity, researchers have studied the static and dynamic behavior of micro- and nanostructures for bending (Arefi *et al.* 2019, Civalek *et al.* 2020, Güçlü and Artan 2020, Hamed *et al.* 2019, Pham *et al.* 2021, Reddy 2010), buckling and post-buckling (Murmu and Pradhan 2009, Katariya and Panda 2016, Kar and Panda 2017, Katariya *et al.* 2017, Mehar *et al.* 2019, Ebnali Samani and Beni 2018, Arefi and Amabili 2021, Daghigh *et al.* 2020, Li *et al.* 2018, Soltani *et al.* 2021, Tang and Qing 2023), vibration (Farajpour *et al.* 2012, Kolahchi *et al.* 2017, Lim *et al.* 2015b, Najafzadeh *et al.* 2020, Uzun *et al.* 2020b, Zeighampour and Beni 2021), dynamic stability (Beni and Beni 2022) and wave propagation (Eltaher *et al.* 2016, Wang *et al.* 2018, Wang and Liang 2019). In addition, in the context of size-dependent mechanical response of nanomaterials, there is also very recent research on hyperelastic continuum models of graphene and graphene nanocomposites (Pellicciari *et al.* 2022, Pellicciari and Tarantino 2021, 2022).

Recent research has focused on using SGT and NLSGT to study the behavior of micro- and nanostructures. In SGT, strain energy and strain are predicted as well as displacement. This method considers the displacement gradient and the size effect of the material. Fleck and Hutchinson (Fleck and Hutchinson 1993, 2001, Hutchinson and Fleck 1997) developed Mindlin's SGT and reformulated it. The new theory separates the quadratic strain gradient tensor into two different pieces: the stress gradient and the rotational gradient tensors. Traditional stability equations are used in the theory's formulations, and higher-order stability conditions are neglected. Lam *et al.* (2003) further developed the theory by introducing three material parameters and defining three gradient tensors: dilatation, deviation stress, and symmetric rotation. New higher-order deformation metrics were defined along with higher-order stress tensors and corresponding constitutive relations (B. Wang *et al.* 2010). It is worth noting that setting two of the three material parameters described by Lam *et al.* (2003) to zero in their SGT degenerates to the MCST of Yang *et al.* (2002).

Nanostructures' mechanical behavior has been studied and analyzed using SGT in many articles (Alazwari *et al.* 2022, Arda 2022, Barretta *et al.* 2021, Esen and Özmen 2022, Li and Hu 2016, Lim *et al.* 2015a, Lu *et al.* 2017, Nematollahi *et al.* 2017, Şimşek 2016, Tang *et al.* 2016, Xu *et al.* 2017, Yaylı *et al.* 2021, Zeighampour *et al.* 2018, Zhu and Li 2017). According to SGT and MCST, Akgöz and Civalek (2011) developed a model for the buckling analysis of microscale beams loaded axially. Lim *et al.* (2015a) used strain gradients and nonlocal stresses to create a model for studying nanostructures' wave propagation. Apuzzo *et al.* (2018) applied NLSGT to analyze Euler-Bernoulli nano-beams' axial and flexural free vibration, in line with the

elastodynamic problem. Mirjavadi *et al.* (2020) studied the forced vibration behavior of perforated and functionally graded (FG) nano-shells using classical shell theory and NLSGT. Noroozi *et al.* (2020) investigated the torsional vibration of bi-directional FG nanocones through NLSGT. Shariati *et al.* (2020) studied the vibrational behavior of graphene sheets placed on elastic media using NLSGT. By combining the higher-order strain gradient model and the higher-order nonlocal elasticity within the context of stationary variation, Faghidian *et al.* (2022b) established HNLGT and used it to investigate the tension issue of FG nanobars. Fakher and Hosseini-Hashemi (2022) used SGT and NLSGT to develop a model for investigating the transversal vibrations of a nanobeam. In order to create the mixture unified gradient theory of elasticity, Faghidian *et al.* (2022d, 2023a) combined the theories of stress gradient, strain gradient and classical elasticity. And then applied this theory to the electro-static and elasto-dynamic analysis of nanorods and the characterization of the nanoscopic features of nonlinear flexure of nanobeams (Faghidian *et al.* 2022d, 2023b). In the case of moving loading, Abdelrahman *et al.* (2021a) studied the free and forced vibration of nonlocal strain gradient (NSG) nanobeams. Esen *et al.* (2022a) developed an analytical model for studying the vibrations and buckling of functionally graded (FG) NSG nanobeams. Esen *et al.* (2021a, b) performed a detailed analysis of the dynamic behavior of sigmoid or symmetrical FG NSG nanobeams reinforced with CNTs, under the presence of moving loads. Civalek *et al.* (2022a) proposed using NLSGT to model the torsional vibration of restrained nano-sized rods with deformable boundaries.

Many researchers have studied the dynamic behavior of structural elements on elastic foundations due to their many engineering applications. Asghari *et al.* (2012) used SGT to derive the geometrically nonlinear differential equations of motion for Timoshenko microbeams with large deflections and the corresponding boundary conditions. Sari and Pakdemirli (2013) investigated the kinematics of a microbeam coupled to a nonlinear elastic foundation. Yaylı (2017) studied CNTs embedded in an elastic foundation with mounted springs. The nanotube interaction simulation with the elastic foundation was based on the Winkler-type elastic foundation (WEF). Bensaid *et al.* (2018) studied the free vibration characteristics of a nanobeam resting on an elastic Pasternak foundation using NLSGT. Ebrahimi and Barati (2017, 2018a, b) investigated the buckling behavior of different FG nanobeams resting on an elastic foundation through NLSGT. Barati (2017) also studied the forced vibration behavior of perforated nanoplate on an elastic foundation and used NLSGT to account for size-dependent effects. Yaylı (2019) used SGT to investigate the buckling behavior of microbeams, specifically rotationally restrained microbeams embedded in a WEF. Jena *et al.* (2019) studied the nonlocal vibration of nanobeams resting on various WEFs, including constant, linear, parabolic, and sinusoidal ones, using the differential quadrature method.

Researchers are also focusing on perforated, graded and FG nano/microbeams. Mehar *et al.* (2016) used HSDT to investigate the free vibration behavior of an FG-CNT-reinforced composite plate subjected to an elevated thermal

environment and examined the effects of different parameters on the behavior of the composite plate. Mehar and Panda (2019) investigated the thermal buckling behavior of the graded CNT-reinforced composite shell structure, using a combination of HSDT and multiscale modeling. The critical buckling temperatures under different thermal conditions are calculated using a finite element method, and the accuracy of the model is validated through comparison and convergence studies. The findings of the study are demonstrated through numerical examples involving various shell geometries and design parameters. Uzun *et al.* (2020a) proposed a finite element method to analyze the vibrations of FG nanobeams embedded in a WEF.

Perforated nano/microbeams are created using a variety of techniques. One such technique is the use of focused ion beam (FIB) milling. FIB milling is a process that uses a beam of ions to remove material from a sample. The beam can be focused to a very small spot size, allowing for the creation of very small features (Giannuzzi and Stevie 1999, Reyntjens and Puers 2001, Langford *et al.* 2007). Another technique is laser ablation, which uses a laser to remove material from a sample. Laser ablation can be used to create very precise patterns in materials (Tsuji *et al.* 2003, Zhang *et al.* 2002). Perforated nano/microbeams have a variety of applications (Almitani *et al.* 2020, Eltaher *et al.* 2023). They can be used to create filters and heat exchangers (Jeong and Amabili 2006) that are more efficient than traditional filters and heat exchangers. In addition, perforated nano/microbeams can be used in fuel cells to improve their performance and in NEMS as resonators and sensors. They are also used in sound absorbers (Park 2013, Temiz *et al.* 2016) and Radio-Frequency MEMS shunt switches (Guha *et al.* 2015). Due to the increase in usage areas research on the mechanical behavior of perforated nano/microbeams is increasing. Perforated nano/microbeams are one of the fundamental structural elements of MEMS and NEMS, requiring proper modeling of hole numbers, hole sizes, and scale effects (Abdelrahman and Eltaher 2022). Luschi and Pieri (2014) synthesized the micromechanical properties of microbeams and proposed a local form for vibration frequency. Bourouina *et al.* (2016) developed a nonlocal model using ENET to study a perforated nanobeam's dynamic vibration, but the model only considers the perforation parameters, not the adsorption process. In another research, Bourouina *et al.* (2020) investigated how perforation and adsorption affect the induced nonlocal frequency shift for aperiodic square hole networks. Their initial goal was to calculate the adsorption-induced energy. Ebrahimi *et al.* (2019) analyzed the frequency response of magneto flexo electric rotary porous nanobeams via NLSGT. Eltaher *et al.* (2018a, b) theoretically investigated nonlocal perforated nanobeams supported by a simple frame for bending, buckling, and vibration responses. In another study, Eltaher and Mohamed (2020) examined the effect of long-range atomic interactions, hole sizes, and hole numbers on the vibration response of a perforated nonlocal nanobeam under different boundary conditions. The combined effect of surface energy

and microstructure on the bending of a perforated nanobeam has been studied by Abdelrahman *et al.* (2022) using analytical techniques and based on nonlocal elasticity theory. Eltaher *et al.* (2020a, b) numerically analyzed the dynamic response of perforated piezoelectric nanobeams. Esen *et al.* (2022b) used a modified continuum model based on MCST to study the dynamic behavior and responses of perforated microbeams in the presence of moving loads. Abdelrahman *et al.* (2021a, b) investigated NSG nanobeams with moving loads under free and forced conditions, developing a nonclassical model for studying and analyzing the dynamic behavior of perforated higher-order NSG nanobeams under moving loads. Eltaher *et al.* (2023) developed a mathematical model and analysis to study the vibration response of perforated nonlocal visco-elastic thin and thick nanobeams using Kelvin-Voigt viscoelasticity constituent relations under a range of boundary conditions. Furthermore, Melaibari *et al.* (2022) investigated the dynamic response of perforated sandwich nanobeams with flexoelectricity, based on a mathematical size-dependent model via NLSGT.

When we look at the literature, it is seen that the boundary conditions of the beams analyzed at nanoscales are generally assumed to be rigid. However, it is not possible to achieve this rigidity in practice. Although it is desirable to apply the support rigidly, it will not be possible in practice, so deformation will definitely occur in the supports. With the solution approach presented in this study, it is possible to investigate deformable states. In addition, it has recently been observed that nano- and micro-scale structures are modeled as perforated. When we look at the literature, a solution that can give buckling loads based on both non-local and strain gradient effects of perforated nanostructures on an elastic foundation under both deformable and rigid boundary conditions is presented for the first time in this study. In the present work, nano/microbeams are considered with holes on sections. It is assumed that the axial point pressure load, which is the most important parameter for the buckling behavior, acts on the geometric center of the nanobeam. In addition, this perforated nanobeam is modeled on an elastic foundation with elastic springs attached at both ends. These springs are tuned to allow transverse deflection. During the analysis, the stiffness of the transverse springs can be considered at any value and their effects can be analyzed in a very comprehensive way. This study presents an eigenvalue-based approach to the size-dependent buckling problem of restrained, perforated nano/microbeams under different parameters. The boundary conditions of the perforated nano/microbeams are considered deformable and size dependency is considered using NLSGT. After the mathematical steps, an eigenvalue problem is obtained that accounts for the effects of perforation, deformable boundaries, and size on the buckling behavior of the beam. Finally, a number of numerical examples are provided to demonstrate the effectiveness of the proposed approach. These examples illustrate the effects of various parameters, such as the length of the perforated beam, the number of holes, the filling ratio, the small-scale parameters, and the foundation parameter, on the buckling behavior of the beam.

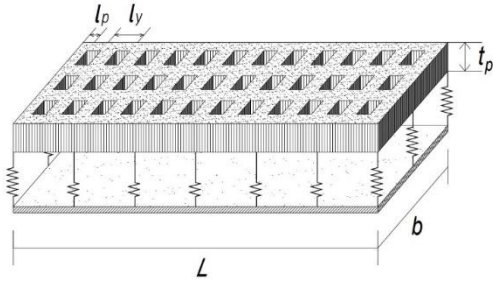


Fig. 1 The geometry of the perforated nano/microbeam

2. The perforated nano/microbeam

The geometry of the perforated nano/microbeam, which consists of holes in the cross-sectional area of dimensions L , b , and t_p , is illustrated in Figure 1. Next, we define filling ratio β as (Abdelrahman *et al.* 2021b):

$$\beta = \frac{l_p}{l_y} \quad 0 \leq \beta \leq 1 \quad (1)$$

where l_y is the spatial period, l_p is the period length. For instance, for the filling ratio, $\beta = 1$, the beam is fully filled and the beam is partially filled for $0 < \beta < 1$.

Luschi and Pieri (2014) developed the equivalent bending stiffness as;

$$(EI)_{eq} = EI \frac{\beta(H+1)(H^2+2H+\beta^2)}{\check{Y}_1 H^3 + 3\beta H^2 + \check{Y}_2 \beta^2 H + \beta^3} \quad (2)$$

where E is the elasticity modulus, I is the moment of inertia, H is the number of holes along the cross-section, and \check{Y}_1 , \check{Y}_2 is defined as;

$$\check{Y}_1 = (1 - \beta^2 + \beta^3) \quad (3)$$

$$\check{Y}_2 = (3 + 2\beta - 3\beta^2 + \beta^3) \quad (4)$$

2.1 Non-local strain gradient theory

Experimental and theoretical studies have shown that nanostructures exhibit both softening and hardening behavior. Lim *et al.* (2015a) conducted a study that blended SGT with ENET. They described the stress tensor in their work as the following (Lim *et al.* 2015a):

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma}^0 - \nabla \boldsymbol{\sigma}^1 \quad (5)$$

where $\boldsymbol{\sigma}^*$ is the NLSGT total stress tensor, ∇ is the symbol of gradient, the higher-order stress tensor $\boldsymbol{\sigma}^1$ and stress tensor $\boldsymbol{\sigma}^0$ are given by (Lim *et al.* 2015a)

$$\boldsymbol{\sigma}^0 = \int_{\Gamma} B_0(\mathbf{y}, \mathbf{x}, \alpha_0) \mathbf{C} : \boldsymbol{\varepsilon}^y(\mathbf{y}) d\Gamma \quad (6)$$

$$\boldsymbol{\sigma}^1 = \gamma^2 \int_{\Gamma} B_1(\mathbf{y}, \mathbf{x}, \alpha_1) \mathbf{C} : \nabla \boldsymbol{\varepsilon}^y(\mathbf{y}) d\Gamma \quad (7)$$

where C is the fourth-order elasticity tensor, $\boldsymbol{\varepsilon}$ is the strain tensor, γ is the internal length parameter, α_0 and α_1 are the nonlocal parameters, Γ is the volume of the beam and $B_i(\mathbf{y}, \mathbf{x}, \alpha_i)$, $i = 0, 1$ attenuation kernel function.

The aforementioned integral constitutive equations are difficult to solve theoretically; hence Lim *et al.* (2015a)

suggested a simple constitutive equation utilizing Eringen's method;

$$(1 - \alpha^2 \nabla^2) \boldsymbol{\sigma}^* = \mathbf{C} : \boldsymbol{\varepsilon} - \gamma^2 \mathbf{C} : \nabla^2 \boldsymbol{\varepsilon} \quad (8)$$

For one-dimensional problems, Eq. (8) is simplified to (Xu *et al.* 2017):

$$(1 - \alpha^2 \frac{d^2}{dx^2}) \sigma_{xx} = E \varepsilon_{xx} - \gamma^2 E \varepsilon_{xx,xx} \quad (9)$$

Eq. (9) contains two material length parameters, it should be noticed. The nonlocal effect is the first (α), and the size effect caused by the higher-order strain gradient is the second (γ). SGT and ENET can be obtained by accepting these as 0, respectively.

2.2 Nonlocal strain gradient beams' basic equations

Consider an elastic beam with dimensions L , b , and t_p . L is the length, b is the width and t_p is the thickness of the beam. Measurements of the beam's x - and z -axes are made along L and t_p , respectively. In Euler-Bernoulli beam theory, displacements (u_1, u_2, u_3) of a beam (x, z) can be expressed as follows (Xu *et al.* 2017):

$$u_1(x, z) = u - zw', \quad u_2(x, z) = 0, \quad u_3(x, z) = w \quad (10)$$

where u and w are the axial and transverse displacements relative to the beam midplane, respectively.

The strain displacement relation's only non-zero component, ε_{xx} , the longitudinal strain, can be written as follows:

$$\varepsilon_{xx} = u' + \frac{1}{2} w'^2 - zw'' \quad (11)$$

Proceeding to the derivation of the main equation and boundary conditions, the virtual work of the strain energy can be computed as follows (Xu *et al.* 2017):

$$\begin{aligned} \delta U &= \int_{\Gamma} (\sigma_{xx} \delta \varepsilon_{xx} + \sigma'_{xx} \delta \varepsilon_{xx,x}) d\Gamma \\ &= \int_{\Gamma} (\sigma_{xx} \delta \varepsilon_{xx} + \sigma'_{xx} \delta \varepsilon_{xx,x}) d\Gamma + \left[\int_A \sigma'_{xx} \delta \varepsilon_{xx} dA \right] \Big|_0^L \\ &= \int_0^L [N_c (\delta U' + w' \delta w') - M_c \delta w''] dx \\ &\quad + [N_n (\delta U' + w' \delta w') - M_n \delta w''] \Big|_0^L \end{aligned} \quad (12)$$

where A is the area of cross-sectional. Identified stress resultants are given by (Xu *et al.* 2017);

$$(N_c, M_c) = \int_A \sigma_{xx}(1, z) dA \quad (13a)$$

$$(N_n, M_n) = \int_A \sigma'_{xx}(1, z) dA \quad (13b)$$

The external forces conduct the following virtual work:

$$\delta V = \int_0^L (f \delta u + q \delta w) dx \quad (14)$$

where $q(x)$ and $f(x)$ are transverse and distributed axial loads, respectively.

According to the variational principle:

$$\delta U - \delta V = 0 \quad (15)$$

By substituting the expressions for δU and δV from Eqs. (12) and (14) into Eq. (15) and integrating by parts, we obtain the equations of motion of the beam and boundary conditions as follows (Xu *et al.* 2017):

$$\begin{aligned} \delta u : N_c' + f &= 0 \\ \delta w : M_c'' + q + (N_c w')' &= 0 \end{aligned} \quad (16a)$$

$$\begin{aligned} \delta u : N_c &= 0 \text{ either } u = 0 \\ \delta u' : N_n &= 0 \text{ either } u' = 0 \\ \delta w : M_c' + N_c w' &= 0 \text{ either } w = 0 \\ \delta w' : M_c + N_n w' &= 0 \text{ either } w' = 0 \\ \delta w'' : M_n &= 0 \text{ either } w'' = 0 \end{aligned} \quad (16b)$$

The equations of motion can be written in terms of displacements as follows (Xu *et al.* 2017):

$$\left[A_{xx} \left(u' + \frac{1}{2} w'^2 \right) \right]' - \left[A_{xx} \gamma^2 (u'''' + w' w'''' + w''^2) \right]' - \alpha^2 f'' + f = 0 \quad (17)$$

$$\begin{aligned} D_{xx} \gamma^2 w'''''' - D_{xx} w'''' + q - \alpha^2 q'' \\ + \left(1 - \alpha^2 \frac{d^2}{dx^2} \right) (N_c w')' = 0 \end{aligned} \quad (18)$$

where

$$(A_{xx}, D_{xx}) = \int_A [E(z) \times (1, z)] dA \quad (19)$$

A solid beam with rectangular cross section it is known that $E(z) = E$, which gives the following result:

$$(A_{xx}, D_{xx}) = \int_A [E(z) \times (1, z)] dA \quad (20)$$

2.3 Buckling solutions

The next part of the study will continue considering that the motion in the axial direction is neglected. According to Xu *et al.* (2017), once $q = 0$ and $N_c = -N_b$ are taken into consideration for the buckling case of a beam and applied to Eq. (18), the governing equations are as follows:

$$EI \gamma^2 w'''''' - EI w'''' - \left(1 - \alpha^2 \frac{d^2}{dx^2} \right) (N_b w'') = 0 \quad (21)$$

subjected to the following boundary conditions at the beam ends (Xu *et al.* 2017)

$$\begin{aligned} Q &= - \left(1 - \gamma^2 \frac{d^2}{dx^2} \right) EI w'''' - \left(1 - \alpha^2 \frac{d^2}{dx^2} \right) (N_b w') \\ &= 0 \text{ or } w = 0 \\ M &= \left(1 - \gamma^2 \frac{d^2}{dx^2} \right) EI w'' - \alpha^2 N_b w'' = 0 \text{ or } w' = 0 \end{aligned} \quad (22)$$

$$M_h = EI \gamma^2 w'''' = 0 \text{ or } w'' = 0$$

where Q , M and M_h are the non-classical stress resultants. Q is the shear force resultant, M is the bending moment resultant and M_h is the higher-order bending moment resultant.

For dimensionless boundary value problem, an additional non-dimensional parameter $\bar{N} = N_b L^2 / EI$ is introduced to rewrite the boundary value problem represented in the Eqs.

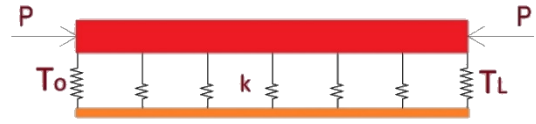


Fig. 2 Perforated nano/microbeam subjected to axial load

(21)-(22) as follows (Xu *et al.* 2017):

$$W'''''' + \bar{l}_2^2 (\bar{N} \tau_1^2 - 1) W'''' + (-\bar{N} \bar{l}_2^2) W'' = 0 \quad (23)$$

where the following dimensionless parameters are introduced:

$$W(x) = w(x)/L, \quad \bar{l}_2 = L/\gamma, \quad \tau_1 = \alpha/L \quad (24)$$

and the boundary conditions can be expressed as:

$$\begin{aligned} \bar{Q} = \frac{QL^2}{EI} &= - \left(1 - \bar{l}_2^{-2} \frac{d^2}{dx^2} \right) W'''' - \left(1 - \tau_1^2 \frac{d^2}{dx^2} \right) (\bar{N} W') \\ &= 0 \text{ or } W = 0 \\ \bar{M} = \frac{ML}{EI} &= \left(1 - \bar{l}_2^{-2} \frac{d^2}{dx^2} \right) W'' - \tau_1^2 \bar{N} W'' = 0 \text{ or } W' = 0 \end{aligned} \quad (25)$$

$$\bar{M}_h = \frac{M_h \bar{l}_2^{-2}}{EI} = W'''' = 0 \text{ or } W'' = 0$$

Xu *et al.* (2017) presented the analytical solution of a nonlocal strain gradient elastic beam for buckling load as follows:

$$\bar{N} = \frac{n^2 \pi^2 (\bar{l}_2^2 + n^2 \pi^2)}{\bar{l}_2^2 (1 + n^2 \pi^2 \tau_1^2)} \quad (26)$$

where n is the mode number of buckling.

3. Buckling analysis for embedded perforated nano/microbeam

Buckling formulations independent from perforation and elastic medium effects are given in the previous section of the study. The perforation and elastic medium effects are added to the formulations of this section. Only transverse loads and the distributed axial loads are considered in Eq. (14).

Buckling is one of the most important responses of engineering structures and components. Especially elements under axial compression are prone to buckling. Determination of the critical buckling load, which is the smallest buckling load, is very important for these elements. Buckling is a stability problem. In case of buckling, since the displacements will suddenly go to infinity, unforeseen damages occur and there is no time to take precautions. Therefore, the determination of buckling loads is necessary for a correct design. In this paper, the buckling response of restrained, perforated nano/microbeam is analyzed by means of the following governing equation.

If one is to investigate the elastic medium effect, the expression $-\int_0^L k \delta w dx$ work done by external forces has to be added. After the mathematical steps with this expression, the following governing equation is obtained for the embedded perforated nano/microbeam:

$$\alpha^2 k \frac{d^2 w(x)}{dx^2} + \alpha^2 P \frac{d^4 w(x)}{dx^4} - (EI)_{eq} \frac{d^2 w(x)}{dx^2} + \gamma^2 (EI)_{eq} \frac{d^6 w(x)}{dx^6} - P \frac{d^2 w(x)}{dx^2} - kw(x) = 0 \tag{27}$$

where, k represents the Winkler foundation parameter, P denotes the axial load effecting perforated nonlocal strain gradient nanobeam. Therefore, when the perforated beam is reduced to a solid beam, it means $P = N_b$.

It is noteworthy that there are several approaches to solving structural issues in the literature where the kinematic field variables (Uzun *et al.* 2020b, c, Akbaş *et al.* 2021) or kinetic field variables (Farrahi *et al.* 2009, 2010) or the autonomous series solution of both field variables (Faghidian *et al.* 2022c, Kamil Žur and Faghidian 2021) are presupposed to have a series solution form. The kinematic field variables are used to formulate the governing equation in this investigation. This study uses the Stokes transformation and Fourier series to analyze the buckling loads at deformable boundary conditions.

3.1 Stokes' transformation

This part of the work focuses on analyzing the mathematical steps of the buckling response according to the NLSGT for embedded nano-sized beams in the context of normal force action from both ends. The approach presented in this paper targets engineering elements with nano/micro-sized applications. In NEMS/MEMS applications, the elements must somehow be supported somewhere. These supports are assumed to be idealized when studied theoretically. However, this idealization can not be achieved in practice. So much so that, to give an example from nano/microbeams, it is not possible to completely eliminate rotation or displacement during bearing. At this point, it is also not possible to provide simply support or clamped conditions. This is why deformable boundary conditions are considered in the solution approach presented in this paper. Three distinct areas, two for boundary points and one for the locations in-between these points are used to define the transverse displacement function $w(x)$ (Civalek *et al.* 2022b, c, Uzun and Yaylı 2022):

$$w(x) = \begin{cases} w_0 & x = 0 \\ w_L & x = L \\ \sum_{t=1}^{\infty} A_t \sin\left(\frac{t\pi}{L}x\right) & 0 < x < L \end{cases} \tag{28}$$

In Eq. (29), an unknown Fourier coefficient A_t for the stability of the nanobeam is described as follows:

$$A_t = \frac{2}{L} \int_0^L w(x) \sin(\Psi_t x) dx \tag{29}$$

here, Ψ_t is described as:

$$\Psi_t = \frac{t\pi}{L} \tag{30}$$

If one calculates the first derivative of Eq. (29), the following relation can be derived:

$$w'(x) = \sum_{t=1}^{\infty} \Psi_t A_t \cos(\Psi_t x) \tag{31}$$

The first derivative of $w_0(x)$ can be re-written as follows:

$$w'(x) = \frac{a_0}{L} + \sum_{t=1}^{\infty} a_t \cos(\Psi_t x) \tag{32}$$

The explicit form of the Fourier constants a_0 and a_t are demonstrated as:

$$a_0 = \frac{2}{L} \int_0^L w'(x) dx = \frac{2}{L} (w_0(L) - w_0(0)) \tag{33}$$

$$a_t = \frac{2}{L} \int_0^L w'(x) \cos(\Psi_t x) dx \quad (t = 1, 2, \dots) \tag{34}$$

Using integration by part rule;

$$a_t = \frac{2}{L} [w(x) \cos(\Psi_t x)]_0^L + \frac{2}{L} \left(\alpha_t \int_0^L w(x) \sin(\Psi_t x) dx \right) \tag{35}$$

$$a_t = \frac{2}{L} ((-1)^t w(L) - w(0)) + \Psi_t A_t \tag{36}$$

In order to calculate the buckling of the nonlocal strain gradient beam embedded in the WEF, the definition of the first six derivatives of $w(x)$ is needed. The first six derivatives of $w(x)$ are defined as Eqs. (37)-(42) (Yaylı *et al.* 2021):

$$\frac{dw(x)}{dx} = \frac{w_L - w_0}{L} \sum_{t=1}^{\infty} \cos(\Psi_t x) \left(\frac{2((-1)^t w_L - w)}{L} + \Psi_t A_t \right) \tag{37}$$

$$\frac{d^2 w(x)}{dx^2} = - \sum_{t=1}^{\infty} \Psi_t \sin(\Psi_t x) \left(\frac{2((-1)^t w_L - w_0)}{L} + \Psi_t A_t \right) \tag{38}$$

$$\frac{d^3 w(x)}{dx^3} = \frac{w'_L - w'_0}{L} + \sum_{t=1}^{\infty} \cos(\Psi_t x) \left(\frac{2((-1)^t w'_L - w'_0)}{L} - \Psi_t^2 \left(\frac{2((-1)^t w_L - w_0)}{L} + \Psi_t A_t \right) \right) \tag{39}$$

$$\frac{d^4 w(x)}{dx^4} = - \sum_{t=1}^{\infty} \Psi_t \sin(\Psi_t x) \left(\frac{2((-1)^t w'_L - w'_0)}{L} - \Psi_t^2 \left(\frac{2((-1)^t w_L - w)}{L} + \Psi_t A_t \right) \right) \tag{40}$$

$$\frac{d^5 w(x)}{dx^5} = \frac{w''_L - w''_0}{L} + \sum_{t=1}^{\infty} \left(\frac{2((-1)^t w''_L - w''_0)}{L} - \Psi_t^2 \left(\frac{2((-1)^t w'_L - w'_0)}{L} + \Psi_t A_t \right) \right) \tag{41}$$

$$\frac{d^6 w(x)}{dx^6} = - \sum_{t=1}^{\infty} \Psi_t \sin(\Psi_t x) \left(\frac{2((-1)^t w''_L - w''_0)}{L} - \Psi_t^2 \left(\frac{2((-1)^t w'_L - w'_0)}{L} + \Psi_t^2 \left(\frac{2((-1)^t w_L - w_0)}{L} + \Psi_t A_t \right) \right) \right) \tag{42}$$

The Fourier coefficient A_t can be derived via the above six derivatives by the following:

$$A_t = \frac{(2\Psi_t(-P + k\alpha^2 + (B - P\alpha^2)\Psi_t^2 + B\gamma^2\Psi_t^4) (w_0 - (-1)^t w_L))}{(L(k + (-P + k\alpha^2)\Psi_t^2 + (B - P\alpha^2)\Psi_t^4 + B\gamma^2\Psi_t^6))} \quad (43)$$

Consistent with the Fourier coefficient, $w(x)$ is re-written as below:

$$w(x) = \sum_{t=1}^{\infty} \frac{\left[\frac{2\Psi_t \left(\begin{matrix} -P + k\alpha^2 \\ +((EI)_{eq} - P\alpha^2)\Psi_t^2 \\ + (EI)_{eq}\gamma^2\Psi_t^4 \end{matrix} \right)}{(w_0 - (-1)^t w_L)} \right]}{\left[\frac{L \left(\begin{matrix} k + (-P + k\alpha^2)\Psi_t^2 \\ +((EI)_{eq} - P\alpha^2)\Psi_t^4 \\ + (EI)_{eq}\gamma^2\Psi_t^6 \end{matrix} \right)}{L} \right]} \sin(\Psi_t x) \quad (44)$$

This article mainly plans to present an efficient general solution method that obtains the buckling loads of nonlocal strain gradient perforated beams embedded in a WEF. The force boundary conditions for constrained nonlocal strain gradient perforated nanobeams are expressed as the following:

$$\alpha^2 \left(P \frac{d^3 w_0}{dx^3} + k \frac{dw_0}{dx} \right) - (EI)_{eq} \frac{d^3 w_0}{dx^3} + \gamma^2 (EI)_{eq} \frac{d^5 w_0}{dx^5} = T_0 w_0 \quad (45)$$

$$\alpha^2 \left(P \frac{d^3 w_0}{dx^3} + k \frac{dw_0}{dx} \right) - (EI)_{eq} \frac{d^3 w_0}{dx^3} + \gamma^2 (EI)_{eq} \frac{d^5 w_0}{dx^5} = T_L w_L \quad (46)$$

$$\left. \frac{d^2 w_0(x)}{dx^2} \right|_{x=0} = 0 \quad (47)$$

$$\left. \frac{d^2 w_0(x)}{dx^2} \right|_{x=L} = 0 \quad (48)$$

where, T_0 and T_L given in Eqs. (45)-(46) represent the stiffnesses of the transversely deformable springs at both ends of the embedded nonlocal strain gradient beam.

A number of techniques are available to obtain the buckling loads of nanobeams. However, when we look at the studies on these techniques, we notice the lack of a solution that provides general elastic boundary conditions. This is due to the analysis on rigid boundary conditions and separate solutions for each boundary condition. The method presented in this paper is capable of handling both rigid and deformable boundary conditions in an effortless manner. The studies in the literature use the sine function for the simply supported nano/microbeams and the cosine function for clamped nano/microbeams in the analytical solution since they satisfy the boundary conditions. The results found are only for simply supported and clamped. In other words, a different function is needed for each boundary condition. In the present paper, Stokes transformation is applied to the sine function to force it to satisfy the boundary condition. Thus, the boundary condition is satisfied with a single function and a different function is not needed each time.

3.2 Eigenvalue problems

This method calculates the buckling loads of embedded nonlocal strain gradient nano-sized beams and is based on the eigenvalue problem. Two simultaneous homogeneous equations can be given for the stability problem of the nonlocal strain gradient beam embedded in the WEF. The Fourier coefficient A_t was found in the previous section of the study. After obtaining the Fourier coefficient, the force boundary conditions given above are used. By applying the force boundary conditions and making the necessary mathematical arrangements, Eqs. (49) - (50) are obtained as follows:

$$\begin{aligned} & \left(-T_0 - \frac{k\alpha^2}{L} - \sum_{t=1}^{\infty} \frac{2kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \right) w_0 \\ & + \left(\frac{k\alpha^2}{L} + \sum_{t=1}^{\infty} \frac{2(-1)^t kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \right) w_L \\ & = 0 \end{aligned} \quad (49)$$

$$\begin{aligned} & \left(\frac{k\alpha^2}{L} + \sum_{t=1}^{\infty} \frac{2(-1)^t kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \right) w_0 \\ & + \left(-T_L - \frac{k\alpha^2}{L} - \sum_{t=1}^{\infty} \frac{2kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \right) w_L \\ & = 0 \end{aligned} \quad (50)$$

where

$$\theta_1 = -L^4 + t^4 \pi^4 \gamma^2 \quad (51)$$

$$\theta_2 = -L^4 t^2 P \pi^2 \quad (52)$$

$$\theta_3 = L^2 t^4 \pi^4 ((EI)_{eq} - P\alpha^2) \quad (53)$$

$$\theta_4 = L^6 + L^4 t^2 \pi^2 \alpha^2 \quad (54)$$

$$\theta_5 = t^6 \pi^6 \gamma^2 \quad (55)$$

With the above two simultaneous homogeneous equations for the embedded nonlocal strain gradient beam, we can derive an eigenvalue problem involving transverse deformable springs, nonlocal parameter, WEF parameter and strain gradient parameter of nonlocal strain gradient beam as follows:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} w_0 \\ w_L \end{bmatrix} = 0 \quad (56)$$

The eigenvalue problem defined by the above equation allows us to calculate the buckling loads of embedded nonlocal strain gradient perforated beams under the effects of the nonlocal parameter, the WEF parameter, and the strain gradient parameter for random boundary conditions. The buckling loads of constrained nonlocal strain gradient perforated beams embedded in the WEF are calculated by setting the determinant of the coefficient matrix $(\Phi_{\kappa,\zeta})$ to zero after the transverse spring parameters T_0 and T_L are selected.

$$|\Phi_{\kappa,\zeta}| = 0 \quad (\kappa, \zeta = 1, 2) \quad (57)$$

in which,

$$\Phi_{11} = -T_0 - \frac{k\alpha^2}{L} - \sum_{t=1}^{\infty} \frac{2kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \quad (58)$$

$$\Phi_{12} = \frac{k\alpha^2}{L} + \sum_{t=1}^{\infty} \frac{2(-1)^t kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \quad (59)$$

$$\Phi_{21} = \frac{k\alpha^2}{L} + \sum_{t=1}^{\infty} \frac{2(-1)^t kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \quad (60)$$

$$\Phi_{22} = -T_L - \frac{k\alpha^2}{L} - \sum_{t=1}^{\infty} \frac{2kL(L^4(k+P)\alpha^2 + (EI)_{eq}\theta_1)}{\theta_2 + \theta_3 + k\theta_4 + (EI)_{eq}\theta_5} \quad (61)$$

Here it is necessary to emphasize a number of important points about the elements derived above. Thanks to these elements that form the coefficients matrix, it is possible to investigate some important parameters that play a role in the buckling behavior of a nano/microbeam. The first of these is the perforation effect. While formulating the problem, the number of holes and the filling ratio, which have important effects on the bending stiffness of the nano/microbeam, were considered. Also, the one-parameter elastic foundation effect is included and incorporated into the matrix. Another significant issue is the spring parameters included in the two elements of the matrix. These spring parameters can be adjusted to the desired value and buckling loads can be obtained. For example, when the spring stiffnesses are zero, the perforated nano/microbeam will behave in accordance with the free boundary condition at both ends. Choosing the spring stiffness large enough to be close to infinity corresponds to the simply supported boundary condition.

4. Numerical results and discussion

In this work, we investigate the buckling behavior of perforated nano/microbeams with deformable boundary conditions using NLSGT. NLSGT is employed to account for nonlocal effects on the buckling behavior of the perforated nano/microbeam with known geometrical properties. The type of elastic foundation used to model the deformable boundary conditions of the perforated nano/microbeam is based on the WEF. The critical buckling load (P_{cr}) of the perforated nano/microbeam was calculated by considering the effects of various parameters on its buckling behavior. These parameters include hole property-dependent parameters (e.g., H and β), size-dependent parameters (e.g., γ and α), the dimensionless foundation parameter (K), and the length of the beam.

The dimensionless foundation parameter can be calculated as follows:

$$K = \frac{kL^4}{EI} \quad (62)$$

In this study, buckling loads are obtained by solving the eigenvalue problem. In order to obtain the solution based on

Table 1 Comparison of critical buckling loads [nN] combining Eqs. (2) and (26) and this study

L (nm)	P_{cr} (nN)	
	Combining Eqs. (2) and (26)	This study
10	184.0530	184.0530
20	46.0133	46.0133
30	20.4503	20.4503
40	11.5033	11.5033
50	7.3621	7.3621

Table 2 Critical buckling loads [nN] depending on H and α ($\beta = 0.5, \gamma = 0.5$)

A	H					
	0	1	2	4	6	8
0.00	16.47	16.01	13.76	11.96	11.21	10.80
0.40	16.46	16.00	13.75	11.95	11.20	10.79
0.80	16.41	15.95	13.71	11.91	11.17	10.76
1.20	16.33	15.87	13.64	11.86	11.11	10.70
1.60	16.22	15.76	13.55	11.78	11.04	10.63
2.00	16.08	15.63	13.43	11.67	10.94	10.54

Eq. (57), it is necessary to truncate the infinite series in the problem at a certain point. In other words, it is necessary to determine the number of terms for the solution. In this study, the solutions were realized using 60 terms. Another point is the stiffness of the spring parameters. It is possible to realize the solution by giving values to the spring parameters ranging from zero to infinity. In this study, the solutions were realized by choosing $T_0 = T_L = 10^{20}$ nN/nm.

If we combine the equivalent bending stiffness in Eq. (2) given by Luschi and Pieri (2014) and the analytical solution of a nonlocal strain gradient elastic beam for buckling load given by Xu *et al.* (2017), shown in Eq. (26), we obtain a solution for the rigid boundary conditions of the critical buckling load (P_{cr}) of the perforated nano/microbeam. In this study, a method is developed that considers deformable and non-deformable boundary conditions. As a comparison of this method, the results obtained with the equations mentioned above are presented in a comparison table in Table 1, taking a very low value of k in the method that is the subject of this study. The material parameters of the nano/microbeam used in this study were taken as $E = 1$ TPa, $b = 4$ nm, and $t_p = 2$ nm. For Table 1 $\beta = 0.5, \alpha = 0.5, \gamma = 0.5, n = 1$ and $H = 5$ and the values of P_{cr} for different L values are compared.

For Tables 2-6, the length of the beam (L) was taken as 40 nm. Tables and figures were used to illustrate how each parameter affects the buckling behavior of the perforated nano/microbeam. Tables 2-4 show the effects of the change in α, γ , and β on P_{cr} for different values of H . Figs. 3-6 plot these effects.

Table 2 and Fig. 3 show the critical buckling loads depending on the number of holes and the nonlocal parameter. As can be seen, the critical buckling load decreases as the number of holes in the cross-sectional area increases, as expected. Additionally, the critical buckling load decreases as

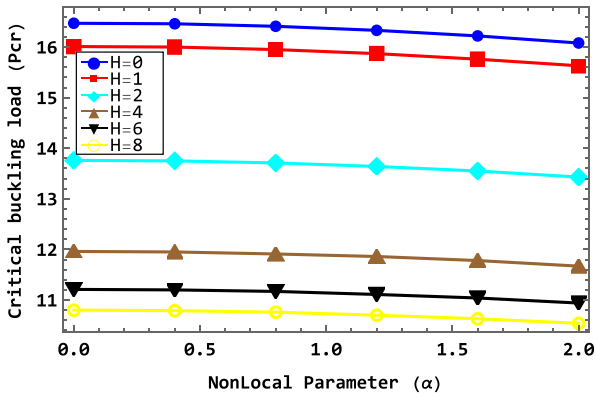


Fig. 3 Critical buckling loads [nN] depending on H and α ($\beta = 0.5, \gamma = 0.5$)

Table 3 Critical buckling loads [nN] depending on H and γ ($\beta = 0.5, \alpha = 0.5$)

A	H					
	0	1	2	4	6	8
0.00	16.42	15.96	13.72	11.92	11.18	10.77
0.40	16.44	15.98	13.73	11.94	11.19	10.78
0.80	16.49	16.03	13.78	11.97	11.22	10.81
1.20	16.57	16.11	13.84	12.03	11.27	10.86
1.60	16.68	16.22	13.94	12.11	11.35	10.94
2.00	16.83	16.36	14.06	12.22	11.45	11.03

the nonlocal parameter increases. When nonlocal effects are not considered, the critical buckling load will be the maximum for $\alpha = 0$. The reason for this decrease in buckling loads is the effect of the number of holes on the rigidity of the nanobeam. Because increasing the number of holes causes a negligible decrease in the bending stiffness of the nanobeam. This results in a decrease in buckling loads.

Table 3 and Fig. 4 demonstrate the dependence of the critical buckling load on the number of holes and the internal length parameter. As shown in Fig. 3, an increase in the number of holes leads to a decrease in the critical buckling load. In contrast, Table 3 and Fig. 4 indicate that the critical buckling load increases slightly as the internal length parameter increases. These results suggest that the nonlocal parameter has a weakening effect on the buckling strength of the beam, while the internal length parameter has a strengthening effect.

Figs. 3 and 4 show that the number of holes has a significant impact on the critical buckling load, especially when the filling ratio remains constant. Specifically, the average reduction in the critical buckling load is 2.80% between the $H = 0$ and $H = 1$ cases, and more than 14% when $H = 2$ instead of $H = 1$. This indicates that increasing the number of holes while maintaining the filling ratio significantly weakens the buckling strength of nano/microbeams. Additionally, as the number of holes increases further, the rate of change in the critical buckling load decreases.

Table 4 and Fig. 5 illustrate the variations in the critical buckling load as a function of the number of holes and the filling ratio. As the filling ratio increases, the critical

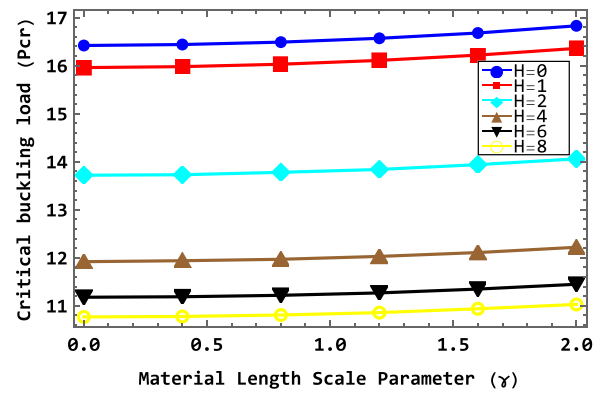


Fig. 4 Critical buckling loads [nN] depending on H and γ ($\beta = 0.5, \alpha = 0.5$)

Table 4 Critical buckling loads [nN] depending on H and β ($\alpha = 0.5, \gamma = 0.5$)

β	H					
	0	1	2	4	6	8
0.10	16.49	7.50	4.31	2.90	2.46	2.25
0.30	16.49	14.11	10.24	7.88	7.04	6.61
0.50	16.49	16.03	13.78	11.97	11.22	10.81
0.70	16.49	16.44	15.66	14.89	14.52	14.31
0.90	16.49	16.49	16.41	16.32	16.28	16.25

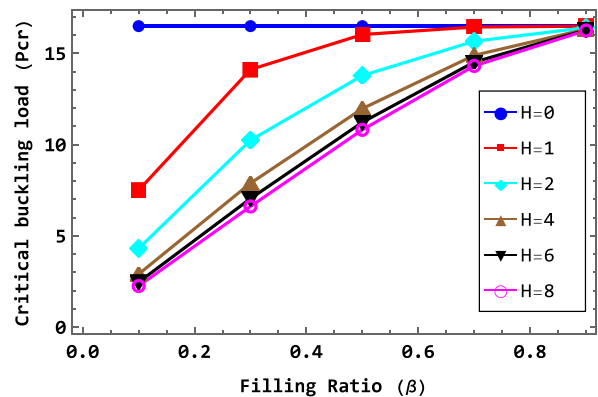


Fig. 5 Critical buckling loads [nN] depending on H and β ($\alpha = 0.5, \gamma = 0.5$)

buckling load increases. This increase in buckling loads is due to the effect of the occupancy rate on the stiffness of the perforated nanobeam. Because an increase in the hole filling ratio causes an increase in the bending stiffness of the perforated nanobeam. This results in an increase in buckling loads. In particular, when the filling ratio decreases, the rate of decrease in the critical buckling load increases significantly with the increase in the number of holes. For instance, when the number of holes increases from 1 to 2 with a filling ratio of 0.10, the critical buckling load decreases by nearly half, whereas with a filling ratio of 0.90, the decrease in the critical buckling load remains below 1%. Furthermore, the rate of decrease in the critical buckling load decreases significantly with the increase in the number of holes.

Table 5 Critical buckling loads [nN] depending on H and K

K	H				
	0	1	2	4	8
0	16.49	14.11	10.24	7.88	6.61
10	18.18	15.80	11.93	9.57	8.29
20	19.87	17.49	13.61	11.26	9.98
30	21.55	19.18	15.30	12.95	11.67
40	23.24	20.87	16.99	14.64	13.36

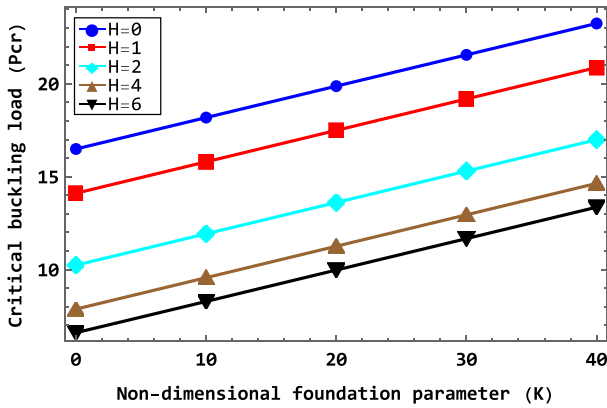


Fig. 6 Critical buckling loads [nN] depending on H and K

Table 6 Critical buckling loads [nN] depending on β and K

K	β				
	0.1	0.3	0.5	0.7	0.9
0	7.50	14.11	16.03	16.44	16.49
10	9.19	15.80	17.72	18.13	18.18
20	10.88	17.49	19.40	19.82	19.87
30	12.56	19.18	21.09	21.51	21.55
40	14.25	20.87	22.78	23.20	23.24

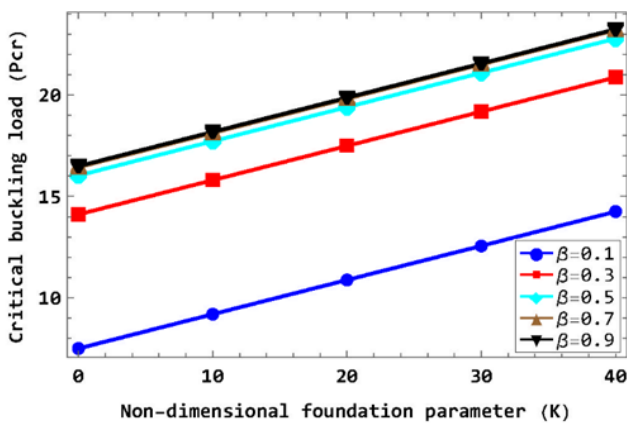


Fig. 7 Critical buckling loads [nN] depending on β and K

Table 5 examines the effect of the number of holes and the dimensionless foundation parameter on the critical buckling load, while the non-local parameter, internal length parameter, and filling ratio are held constant. Table 6 presents the effects of the filling ratio and the dimensionless foundation parameter on the critical buckling load. In both

Table 7 Critical buckling loads [nN] depending on β and L

β	L (nm)					
	40	44	48	52	56	60
0.10	7.50	6.19	5.20	4.43	3.82	3.33
0.30	14.11	11.66	9.79	8.34	7.19	6.26
0.50	16.03	13.24	11.12	9.47	8.17	7.11
0.70	16.44	13.58	11.41	9.72	8.38	7.30
0.90	16.49	13.62	11.44	9.75	8.40	7.32

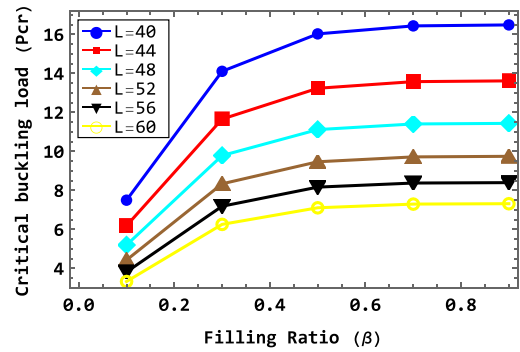


Fig. 8 Critical buckling loads [nN] depending on β and L

tables, $\gamma = 0.8$ and $\alpha = 0.5$. Additionally, Table 5 considers the case where $\beta = 0.3$, and Table 6 calculates the critical buckling load with the number of holes set to 1. The results are shown in Figs. 6 and 7.

Table 5 and Fig. 6 indicate that the dimensionless foundation parameter has a strengthening effect on the buckling strength of nano/microbeams. The increase in buckling loads when the perforated nanobeam is connected to an elastic foundation is due to the formation of a more rigid system. By choosing $K=0$, the effect of the elastic foundation is neglected and the buckling loads of the nanobeam not supported by the foundation are reduced. Additionally, it can be seen that increasing the number of holes while maintaining the filling ratio has a weakening effect on the buckling strength of the beams.

Table 6 and Fig. 7 provide further evidence of the strengthening effect of the dimensionless foundation parameter on the buckling behavior of nano/microbeams. Additionally, it is worth noting that the critical buckling load increases as the filling ratio increases. However, after the filling ratio reaches 0.5, the increase in the critical buckling load becomes limited. This suggests that the critical value of the filling ratio is 0.5. Overall, these results highlight the importance of the dimensionless foundation parameter and the filling ratio in determining the buckling behavior of nano/microbeams.

Table 7 and Fig. 8 investigate the effects of the filling ratio and the length of the nano/microbeam on the critical buckling load. In this analysis, $\gamma = 0.5$, $\alpha = 0.5$, and $H = 1$, and the number of holes is held constant. The results show that the critical buckling load decreases as the length of the nano/microbeam increases. This indicates that an increase in the length of the beam has a negative effect on its buckling behavior. Furthermore, the negative effect is independent of the filling ratio, as shown by the constant change rate in the critical buckling load for different filling

ratios. For example, when the void ratio is 0.10, the change rate in the critical buckling load is approximately 56% as the length increases from 40 nm to 60 nm. Similarly, when the void ratio is 0.90, the change rate is also around 56%.

5. Conclusions

This study presents a solution for the first time, that provides buckling loads of perforated nano/microbeams with non-local and strain gradient effects on an elastic foundation with deformable and rigid boundary conditions. A size-dependent continuum theory, non-local strain gradient theory (NLSGT), is used to present the buckling behavior of restrained perforated nano/microbeams embedded in an elastic medium. It is aimed to show an eigenvalue solution calculating buckling loads of perforated nano/microbeams based on various effects such as deformable boundary, material length parameters, filling ratio, number of holes and elastic medium. The NLSGT accounts for non-local and strain gradient effects, while the Winkler elastic foundation (WEF) model is used to consider the effects of the elastic foundation.

Some important results of our analysis are summarized as follows:

- The number of holes has a significant effect on the buckling strength of the beams. Buckling loads of perforated nano/microbeams start to decrease with increasing number of holes.
- An increase in the length of the perforated nano/microbeams leads to a decrease in the critical buckling load.
- The critical buckling load increases as the filling ratio increases. Also, 0.5 represents a critical threshold for the filling ratio of perforated nano/microbeams.
- The nonlocal parameter weakens the buckling strength of the beam. In contrast, the internal length parameter and the dimensionless foundation parameter have a strengthening effect on the buckling behavior of the beam.

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