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Effect of nonlocal-nonsingular Fractional Moore-Gibson-Thompson theory in semiconductor cylinder

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Abstract. This study is aimed to investigate the electrically conductive properties of epoxy nanocomposites exposed to an acidic environment under various mechanical loads. For simultaneous assessment of the acidic environment and mechanical load on the electrical conductivity of the samples, the samples with and without carbon nanotubes were exposed to the acidic environment under three different loading conditions for 20 days. Then, the aged samples' strength and flexural stiffness degradation under crude oil and bending stress were measured using a three-point flexural test. The aged samples in the acidic environment and under 80 percent of their intact ultimate strength revealed a 9% and 26% reduction of their electrical conductivity for samples with and without CNTs, respectively. The presence of nanoparticles declined flexural stiffness by about 16.39%. Scanning electron microscopy (SEM) images of the specimen were used to evaluate the dispersion quality of CNTs. The results of this study can be exploited in constructing conductive composite electrodes to be used in petroleum environments such as crude oil electrostatic tanks.

Keywords: carbon nanotubes (CNT); bending load; electrical conductivity; epoxy nanocomposites; stress corrosion; threshold

1. Introduction

A significant amount of attention has been paid to semiconductor materials in recent years owing to their physical properties. The conductivity of semiconductor materials at room temperature is not as good as that of copper or aluminum. These also don't make good insulators at room temperature, like glass. A semiconductor material's resistance gradually decreases as the temperature increases. As a result, they become electrically conductive. As a result, semiconductor materials play an increasingly important role in modern VLSI, electronic, and electrical engineering. The internal properties of semiconductor materials change when they are subjected to high temperatures. Due to thermal effects, two significant changes are caused, one is thermoelastic deformation, and the other is electronic deformation. There are many uses of these materials in renewable energy, especially in the solar cell industry.

Classical uncoupled thermoelasticity was introduced by Duhamel (1938). A couple of limitations are associated with this theory. Firstly, the state of elastic materials does not depend on their temperature. Similarly, the parabolic heat equation predicts that the temperature will travel at an infinite speed, which is also contrary to physical experiments. This impediment was overcome by Biot's (1956) hypothesis of coupled thermoelasticity. In accordance with this theory, the heat conduction equations are linked to the equations of elasticity. This theory, however, has limitations since it only predicts heat waves with unlimited propagation speeds. A broader form of Fourier law has been suggested by Cattaneo (1958) and Vernotte (1958, 1961) in which a temperature gradient is abruptly imposed on a heat flux vector $\boldsymbol{q}$ to establish a steady state as

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\boldsymbol{q}=-K_{ij}\nabla\mathrm{T} \tag{1}$$

Lord and Shulman (1967) then developed the generalized thermoelasticity theory with one relaxation time. According to this theory, the temperature can only move at a finite rate since the heat equation is hyperbolic. The two-temperature theory of thermoelasticity, which Chen and Gurtin (1968) developed, comprises the conductive temperature (the result of thermodynamic operations) and the thermodynamic temperature (the consequence of mechanical processes), and it is a more exact form of thermoelasticity. The linear heat conduction tensor's symmetry was demonstrated by Green and Lindsay (1972) in their work. For an anisotropic media, Dhaliwal and Sherief's (1980) equations for generalized thermo-elasticity were provided. Green and Naghdi (1991, 1992, 1993), however, made additional contributions to the thermoelastic theories with and without energy dissipation and advanced the development of Fourier law as

$$q=-K_{ij}\nabla\mathrm{T}-K_{ij}^{*}\nabla\vartheta,\dot{\vartheta}=T,T=\varphi-a_{ij}\varphi_{,ij} \tag{2}$$

The Moore-Gibson-Thompson (MGT) equation has been analyzed and explained by numerous academic papers over the past few decades. The Lasiecka and Wang (2015) theory was based on a third-order differential equation crucial to a number of fluid dynamics. The MGT equation with 2T was developed by Quintanilla (2019, 2020). With

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the MGT equation, the modified Fourier law is

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\boldsymbol{q}=-K_{ij}\nabla\mathbf{T}-K_{ij}^{*}\nabla\boldsymbol{\vartheta}, where, \dot{\boldsymbol{\vartheta}}=\boldsymbol{T} \quad (3)$$

The classic fractional-order constitutive model employs the Riemann-Liouville fractional-order derivative and is defined as (Caputo and Fabrizio 2015)

$$D_t^{\alpha}f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_0^t\frac{f(\xi)}{(t-\xi)^{\alpha}}d\xi, 0<\alpha\le 1, \quad (4)$$

The basic theories are the Riemann-Liouville and Caputo conceptions, which are focused on the single kernel as

$$K_e(t,\xi)=\frac{(t-\xi)^{-\alpha}}{\Gamma(1-\alpha)}, \qquad 0<\alpha\le 1. \quad (5)$$

The new fractional derivative of Atangana–Baleanu based on the Caputo sense $D_t^{(\propto)}$ of order $\alpha\in(0,1)$ is given by (Abdon and Dumitru 2016)

$$D_t^{(\propto)}f(t)=\frac{1}{(1-\alpha)}\int_0^t\frac{\partial f(\xi)}{\partial\xi}E_{\alpha}\left[-\frac{\alpha(t-\xi)^{\alpha}}{(1-\alpha)}\right]d\xi, 0<\alpha\le 1 \quad (6)$$

where $E_{\alpha}(-t^{\alpha})=\sum_{k=1}^{\infty}\frac{(-t)^{\alpha k}}{\Gamma(1+k\alpha)}$ denotes the generalized Mittag-Leffer function.

The importance of using the Laplace transform method to investigate differential equations is widely admitted. It is also recognized for $0<\alpha\le 1$ for this new fractional definition

$$\mathcal{L}\left[D_t^{(\propto)}f(t)\right]=\frac{1}{(1-\alpha)}\frac{s^{\alpha}\mathcal{L}[f(t)]-s^{\alpha-1}f(0)}{s^{\alpha}+\frac{\alpha}{1-\alpha}}, s>0 \quad (7)$$

Thus modified Fourier law (3) while dealing with the Atangana-Baleanu fractional derivative can be written as

$$(1+\tau_0D_t^{\alpha})\boldsymbol{q}=-K_{ij}\nabla\mathbf{T}-K_{ij}^{*}\nabla\boldsymbol{\vartheta}, where, \dot{\boldsymbol{\vartheta}}=\boldsymbol{T} \quad (8)$$

MGT thermoelastic equation was used by Bazarra *et al* (2021) to study a thermoelastic problem. If the semiconductor elastic material is exposed to external laser beams, excited free electrons with semiconductor gap energy E_g will produce a carrier-free charge density. Elastic vibration and electronic deformation vary in reaction to optical energy absorbed. In this instance, thermal-elastic-plasma waves will have an impact on heat conductivity equations. A generalized version of the updated Fourier law for semiconductor materials with plasma impact is written as.

$$(1+\tau_0D_t^{\alpha})\boldsymbol{q}=-K_{ij}\nabla\mathbf{T}-K_{ij}^{*}\nabla\boldsymbol{\vartheta}-\int\frac{E_g\boldsymbol{N}}{\tau}d\boldsymbol{x}, where, \dot{\boldsymbol{\vartheta}}=\boldsymbol{T} \quad (9)$$

The last term in Eq. (4) stands in for the photo-excitation effect. The outcome of differentiating the aforementioned equation with regard to x is

$$(1+\tau_0D_t^{\alpha})\boldsymbol{\nabla}.\boldsymbol{q}=-\nabla.\left(K_{ij}\nabla\mathbf{T}+K_{ij}^{*}\nabla\boldsymbol{\vartheta}\right)-\frac{E_g\boldsymbol{N}}{\tau}, where, \dot{\boldsymbol{\vartheta}}=\boldsymbol{T} \quad (10)$$

Some other researchers also worked on similar research on Hall current effect and semiconductor medium as, Mahdy *et al*. (2020), Kaur and Singh (2021a, b, 2022, 2023a, b), Marin *et al*. (2013, 2020a, b), Kaur *et al*. (2022, 2022), Alberto Conejero *et al*. (2015), Marin *et al*. (2020), Craciun *et al*. (2004, 2005), Nasr *et al*. (2022), Abouelregal and Marin (2020a, 2020b), Zhou *et al*. (2022, 2022).

In the literature review, however, no work has been carried out on transient studies of semiconductor cylinders using fractional Moore-Gibson Thompson Heat Transfer Models and ultrashort pulsed laser heating with photogenerated plasma at two temperatures. This study is motivated by the importance of this issue to provide a detailed analysis of it. A Fractional Moore-Gibson-Thompson-Photo-Thermal (MGTPT) theory incorporating Nonlocal-Nonsingular Kernels in the presence of two temperatures is presented in this study. Numerical estimates for the carrier density, temperature, stress, and displacement components have been depicted graphically to analyze the novelty of the present work due to the effect of the fraction operator.

1.1 Basic equations

In accordance with Abouelregal and Atta (2022), the constitutive laws, equations of motion, plasma diffusion equations, MGTPT heat conduction equations for thermal-plasma-elastic interactions with two temperature for an isotropic semiconducting material is given by

i. *Constitutive relations*

$$\sigma_{ij}=\left(\lambda u_{k,k}-\beta T-\delta_n N\right)\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right), \quad (11)$$

$$\beta=(3\lambda+2\mu)\alpha_t, \delta_n=(3\lambda+2\mu)d_n, T=\varphi-a_{ij}\varphi_{,ij}.$$

ii. *Equation of motion*

$$\sigma_{ij,j}+F_i=\rho\ddot{u}_i, \quad (12)$$

iii. *Plasma diffusion equation*

$$\frac{\partial\boldsymbol{N}}{\partial t}=D_E\nabla^2\boldsymbol{N}-\frac{\boldsymbol{N}}{\tau}+\kappa(1-a\nabla^2)\varphi, \quad (13)$$

where, $\kappa=\frac{T}{\tau}\frac{\partial N_0}{\partial T}$.

Modified fractional MGT heat conduction equation with nonlocal and nonsingular kernels and with two temperature

$$\left(K_{ij}\dot{\varphi}_{,j}\right)_{,i}+\left(K_{ij}^{*}\varphi_{,j}\right)_{,i}+\frac{E_g\dot{N}}{\tau}=(1+\tau_0D_t^{\alpha})\left[\rho C_E(1-a\nabla^2)\ddot{\varphi}+\beta_{ij}T_0\ddot{e}_{ij}-\rho\dot{Q}\right], \quad (14)$$

Where, $K_{ij}=K_i\delta_{ij}, K_{ij}^{*}=K_i^{*}\delta_{ij}$, i is not summed.

also, the subscript followed by ‘,’ a comma denotes the partial derivative w.r.t. space coordinates and a superposed ‘.’ signifies differentiation w.r.t. time variable t.

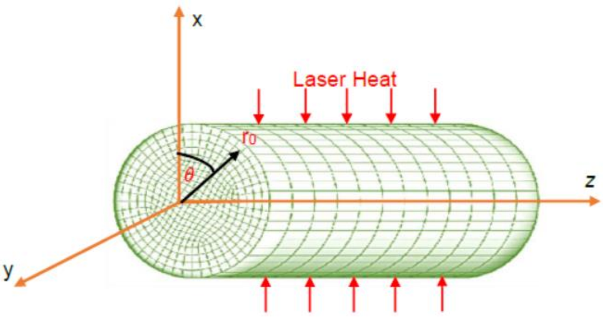

Fig. 1 Schematic of a three-layered beam including a core layer and two piezoelectric layers

1.2 Formulation and solution of the problem

We consider a semiconductor solid cylinder (Fig. 1) of radius r_0 that is symmetrical, thermally homogeneous, and one-dimensional (1D). Solid cylinders were illuminated externally with laser pulse heating. It was decided to use a cylindrical coordinate system (r,θ,z) and the z-axis was taken along the cylinder axis. The initial temperature of the cylinder is kept constant and uniform (T_0).

Moreover, all examined fields within the medium are assumed to be finite. A one-dimensional problem is symmetric, so all functions considered depend upon time t and radial distance r.

For the 1D problem, displacement components and the displacement–strain relations are given by

$$\boldsymbol{u} = (u(r,t),0,0), \tag{15}$$

$$e_{rr} = \frac{u_r}{r}, e_{\theta\theta} = \frac{\partial u}{\partial r}, e_{r\theta} = e_{rz} = e_{\theta z} = e_{zz} = 0. \tag{16}$$

The cubic dilation will become

$$e = \nabla \boldsymbol{u} = \frac{1}{r}\frac{\partial}{\partial r}(ru). \tag{17}$$

The operator ∇^2 is given by,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}. \tag{18}$$

The stress-strain–temperature–carrier relations Eqs. (11) using (15) will be the form

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} - (\beta(1-a\nabla^2)\varphi + \delta_n N), \tag{19}$$

$$\sigma_{\theta\theta} = 2\mu\frac{u}{r} + \lambda e - (\beta(1-a\nabla^2)\varphi + \delta_n N), \tag{20}$$

$$\sigma_{zz} = \lambda e - (\beta(1-a\nabla^2)\varphi + \delta_n N), \tag{21}$$

Hence, the equation of motion in cylindrical coordinates in the absence of external forces is given by

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho\frac{\partial^2 u}{\partial t^2}. \tag{22}$$

Using Eqs. (19)-(21), in Eqs. (22) and (15)-(16), the governing equations for the considered semi-conducting medium are:

$$\begin{aligned}&(\lambda+2\mu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru)}{\partial r}\right) - \beta\frac{\partial}{\partial r}\{(1-a\nabla^2)\varphi\}\\ &-\delta_n\frac{\partial N}{\partial r} = \rho\left(\frac{\partial^2 u}{\partial t^2}\right),\end{aligned} \tag{23}$$

$$\frac{\partial N}{\partial t} = D_E(\nabla^2 N) - \frac{N}{\tau} + \kappa(1-a\nabla^2)\varphi, \tag{24}$$

$$\begin{aligned}&K\frac{\partial}{\partial t}\nabla^2\varphi + K^*\nabla^2\varphi + \frac{E_g\dot{N}}{\tau}\\ &= (1+\tau_0 D_t^{\alpha})\left[\rho C_E\frac{\partial^2}{\partial t^2}\{(1-a\nabla^2)\varphi\} + \beta T_0\frac{\partial^2 e}{\partial t^2}\right],\end{aligned} \tag{25}$$

Pre-operating both sides of Eq. (23) by $\left(\frac{1}{r}+\frac{\partial}{\partial r}\right)$, we obtain

$$(\lambda+2\mu)\nabla^2 e - \beta\nabla^2(1-a\nabla^2)\varphi - \delta_n\nabla^2 N = \left(\frac{\partial^2 e}{\partial t^2}\right), \tag{26}$$

To obtain the above equations in dimensionless form, the dimensionless quantities are given by:

$$\begin{aligned}&(r',u') = v_0\eta(r,u), (T',N',\sigma'_{ij},\varphi')\\ &= \frac{1}{\rho v_0^2}(\beta T, \delta_n N, \sigma_{ij}, \varphi), (\tau'_0,\tau',t') = v_0^2\eta(\tau_0,\tau,t),\\ &\eta = \frac{\rho C_E}{K}, \rho v_0^2 = \lambda+2\mu, \qquad \gamma = \sqrt{\frac{\mu}{\lambda+2\mu}}\end{aligned} \tag{27}$$

Here, the magnetic parameter M (Also known as the Hartmann number) measures the strength of the magnetic field. Using Eq. (27) in Eqs. (24)-(26) and after suppressing the primes, yields

$$\nabla^2 e - \nabla^2(1-a\nabla^2)\varphi - \nabla^2 N = \left(\frac{\partial^2 e}{\partial t^2}\right), \tag{28}$$

$$\frac{\partial N}{\partial t} = \delta_1(\nabla^2 N) - \delta_2 N + \delta_3(1-a\nabla^2)\varphi, \tag{29}$$

$$\begin{aligned}&\frac{\partial}{\partial t}\nabla^2\varphi + \delta_4\nabla^2\varphi + \delta_5 N\\ &= (1+\tau_0 D_t^{\alpha})\left[\frac{\partial^2}{\partial t^2}(1-a\nabla^2)\varphi + \delta_6\frac{\partial^2 e}{\partial t^2}\right],\end{aligned} \tag{30}$$

where,

$$\begin{aligned}&\delta_1 = D_E\eta, \qquad \delta_2 = \frac{1}{\tau}, \qquad \delta_3 = \frac{\kappa\delta_n}{\beta},\\ &\delta_4 = \frac{K^*}{(\lambda+2\mu)C_E}, \qquad \delta_5 = \frac{E_g}{\delta_n C_E(\lambda+2\mu)\eta\tau},\\ &\delta_6 = \frac{\beta^2 T_0}{\rho C_E(\lambda+2\mu)}.\end{aligned} \tag{31}$$

Making use of dimensionless quantities defined by Eq. (27) in Eqs. (19)-(21) and after suppressing the primes, yields

$$\sigma_{rr} = 2\gamma^2\frac{\partial u}{\partial r} + (1-2\gamma^2)e - \left((1-a\nabla^2)\varphi + N\right), \tag{32}$$

$$\sigma_{\theta\theta} = 2\gamma^2 \frac{u}{r} + (1-2\gamma^2)e - \big((1-a\nabla^2)\varphi + N\big), \quad (33)$$

$$\sigma_{zz} = (1-2\gamma^2)e - \big((1-a\nabla^2)\varphi + N\big). \quad (34)$$

The initial conditions of the problem are taken as

$$u(r,0) = 0 = \frac{\partial u}{\partial r}(r,0) \quad (35)$$

$$\varphi(r,0) = 0 = \frac{\partial \varphi}{\partial r}(r,0) \quad (36)$$

$$N(r,0) = 0 = \frac{\partial N}{\partial r}(r,0) \quad (37)$$

The Laplace transform of a function f w.r.t. time variable t, with s as a Laplace Transform variable, is defined as

$$\mathcal{L}\big(f(t)\big) = \bar{f}(s) = \int_0^\infty f(t)e^{-st}dt \quad (38)$$

Using Laplace transforms on Eqs. (38) to Eqs. (28)-(30) we obtain

$$(\nabla^2 - s^2)\bar{e} - \nabla^2(1-a\nabla^2)\bar{\varphi} - \nabla^2\bar{N} = 0, \quad (39)$$

$$\big(\delta_1\nabla^2 - (\delta_2 + s)\big)\bar{N} + \delta_3(1-a\nabla^2)\bar{\varphi} = 0, \quad (40)$$

$$(1+\tau_0 P)\delta_6 s^2\bar{e} + \big(-(s+\delta_4)\nabla^2 + (1+\tau_0P)s^2(1-a\nabla^2)\big)\bar{\varphi} - \delta_5\bar{N} = 0,\ P = \frac{s^\alpha}{s^\alpha(1-\alpha)+\alpha} \quad (41)$$

Using Laplace transforms on Eqs. (30) to Eqs. (24)-(26), we obtain

$$\overline{\sigma_{rr}} = 2\gamma^2 \frac{\partial \bar{u}}{\partial r} + (1-2\gamma^2)\bar{e} - \big((1-a\nabla^2)\bar{\varphi} + \bar{N}\big), \quad (42)$$

$$\overline{\sigma_{\theta\theta}} = 2\gamma^2 \frac{\bar{u}}{r} + (1-2\gamma^2)\bar{e} - \big((1-a\nabla^2)\bar{\varphi} + \bar{N}\big), \quad (43)$$

$$\overline{\sigma_{zz}} = (1-2\gamma^2)\bar{e} - \big((1-a\nabla^2)\bar{\varphi} + \bar{N}\big). \quad (44)$$

When Eqs. (39) to (41) are decoupled, we get

$$(\nabla^6 - B\nabla^4 + C\nabla^2 - D)(\bar{e},\bar{\varphi},\bar{N}) = 0,$$

Where, $A = -\delta_1\delta_{11}$,

$$B = -\frac{\begin{pmatrix} a\delta_3\delta_5 s \pm A\delta_7 - \delta_1\delta_{10} - \delta_1\delta_9 \\ +\delta_8\delta_{11} - a\delta_8\delta_9 \end{pmatrix}}{A},$$

$$C = \frac{\begin{pmatrix} -\delta_3\delta_5 s + a\delta_3\delta_5\delta_7 + \delta_3\delta_9 - \delta_1\delta_7\delta_{10} \\ +\delta_8\delta_7\delta_{11} + \delta_8\delta_{10} + \delta_8\delta_9 \end{pmatrix}}{A}, \quad (45)$$

$$D = \frac{(\delta_3\delta_7\delta_5 s - \delta_8\delta_7\delta_{10})}{A},$$

$\delta_7 = (-s^2)$, $\delta_8 = \delta_2 + s$,
$\delta_9 = (1+\tau_0P)\delta_6 s^2$, $\delta_{10} = (1+\tau_0P)s^2$,
$\delta_{11} = -(s+\delta_4) - (1+\tau_0P)s^2 a$.

Presenting λ_i,i=1,2,3, in Eq. (45), we obtain

$$(\nabla^2 - \lambda_1^2)(\nabla^2 - \lambda_2^2)(\nabla^2 - \lambda_3^2)(\bar{e},\bar{\varphi},\bar{N}) = 0 \quad (46)$$

where, $\lambda_i^2, i = 1,2,3$, are the roots of the equation $(\lambda^6 - B\lambda^4 + C\lambda^2 - D) = 0$,

Which are given by

$$\lambda_1^2 = \frac{1}{3}(2\omega \sin\xi + B) \quad (47)$$

$$\lambda_2^2 = \frac{1}{3}\big(-\omega\sin\xi - \sqrt{3}\omega\cos\xi + B\big) \quad (48)$$

$$\lambda_3^2 = \frac{1}{3}\big(-\omega\sin\xi + \sqrt{3}\omega\cos\xi + B\big) \quad (49)$$

With $\omega = \sqrt{B^2 - 3C}$, $\xi = \frac{1}{3}\sin^{-1}\left(-\frac{2B^3 - 9BC + 27D}{2\omega^3}\right)$

The general solution of Eq. (45) can be written in the form

$$(\bar{e},\bar{\varphi},\bar{N}) = \sum_{i=1}^{3}(1,\zeta_i,\eta_i)\, g_i I_0(\lambda_i r) \quad (50)$$

where $I_n(\ )$ indicates the second type of modified Bessel functions of order n. We get the following relations by inserting Eq. (50) into Eqs. (39)-(41)

$$\zeta_i = \frac{-(\lambda_i^2 + \delta_7)(\delta_9\lambda_i^2 - \delta_5)}{\delta_3\delta_5 + (\delta_{11}\lambda_i^2 + \delta_{10})(\delta_1\lambda_i^2 - \delta_8)}, \quad (51)$$

$$\eta_i = \frac{-(\lambda_i^2 + \delta_7)(\delta_3)}{\delta_3\delta_5 + (\delta_{11}\lambda_i^2 + \delta_{10})(\delta_1\lambda_i^2 - \delta_8)}. \quad (52)$$

The displacement u may be represented as follows in the Laplace transform domain:

$$\bar{u} = \sum_{i=1}^{3}\frac{1}{\lambda_i} g_i I_1(\lambda_i r) \quad (53)$$

We obtained Eq. (53) with the help of the Bessel function relation

$$\int xI_0(x)dx = xI_1(x) \quad (54)$$

Differentiating Eq. (54) in terms of r gives

$$\frac{\partial\bar{u}}{\partial r} = \sum_{i=1}^{3} g_i\left[I_0(\lambda_i r) - \frac{1}{\lambda_i r}I_1(\lambda_i r)\right] \quad (55)$$

Thus, the final thermal stress solutions are generated in closed form as follows:

$$\overline{\sigma_{rr}} = \sum_{i=1}^{3} g_i\left\{l_i I_0(\lambda_i r) - \frac{2\gamma^2}{\lambda_i r}I_1(\lambda_i r)\right\}, \quad (56)$$

$$\overline{\sigma_{\theta\theta}} = \sum_{i=1}^{3} g_i\left\{\frac{2\gamma^2}{\lambda_i r}I_1(\lambda_i r) + l_i I_0(\lambda_i r)\right\}, \quad (57)$$

$$\overline{\sigma_{zz}} = \sum_{i=1}^{3} g_i\, l_i I_0(\lambda_i r),$$
$$l_i = 1 - 2\gamma^2 - (\zeta_i(1 - a\lambda_i^2) + \eta_i). \quad (58)$$

1.3 Boundary conditions

We presume that the cylinder's outside surface is compelled. Therefore, the mechanical boundary condition can be expressed as

$$u(r,t) = 0, \ \ at\ r = r_0 \quad (59)$$

Also, the boundary condition for variable heat flux (exponentially laser-pulsed heat) is applied to the boundary surface following Abouelregal and Atta (2022).

$$q_p = q_0 \frac{t^2}{16t_p^2} e^{-\frac{t}{t_p}}, \qquad at\ \ r = r_0 \quad (60)$$

Using dimensionless variables Eq. (27) on modified Fourier law i.e., Eq. (3) yields

$$(1 + \tau_0 D_t^{\alpha})\dot{q}_p = -\left(\frac{\partial}{\partial t} + \delta_4\right)\frac{\partial T}{\partial r} \quad (61)$$

Eqs. (60) and (61) give the following boundary condition

$$\frac{q_0}{16t_p^2}(1 + \tau_0 D_t^{\alpha})\frac{\partial}{\partial t}\left(t^2 e^{-\frac{t}{t_p}}\right) = -\left(\frac{\partial}{\partial t} + \delta_4\right)\frac{\partial T}{\partial r}, \quad at\ \ r = r_0 \quad (62)$$

"The carriers can reach the sample surface during the diffusion phase, with a finite probability of recombination". The boundary condition for the carrier density:

$$D_E \frac{\partial N}{\partial r} = s_v N, at\ \ r = r_0 \quad (63)$$

where s_v is the surface recombination velocity.

The boundary conditions Eq. (59), (62) and (63) have the following forms after performing the Laplace transform

$$\bar{u}(r_0, s) = 0, \quad (64)$$

$$\left.\frac{\partial(1 - a\nabla^2)\bar{\varphi}}{\partial r}\right|_{r=r_0} = -\frac{q_0(1 + \tau_0 P)s}{8(1 + st_p)^3(s + \delta_4)} = -\bar{G}(s), \quad (65)$$

$$\left.D_E \frac{\partial \bar{N}}{\partial r}\right|_{r=r_0} = s_v \bar{N}(r_0, s), \quad (66)$$

Eqs. (50) and (53) are substituted into Eq. (64)-(66), giving

$$\sum_{i=1}^{3} g_i \frac{1}{\lambda_i} I_1(\lambda_i r_0) = 0, \quad (67)$$

$$\sum_{i=1}^{3} g_i\, I_1(\lambda_i r_0)\zeta_i \lambda_i (1 - a\lambda_i^2) \quad (68)$$
$$= -\frac{q_0(1 + \tau_0 P)st_p}{8(1 + st_p)^3(s + \delta_4)} = -\bar{G}(s),$$

$$\sum_{i=1}^{3} \eta_i\, g_i\{D_E \lambda_i I_1(\lambda_i r_0) - s_v I_0(\lambda_i r_0)\} = 0, \quad (69)$$

The values of $g_i, i = 1,2,3$ can be obtained by solving Eqs. (59)-(61) by Cramer's rule

$$g_i(s) = \frac{\Delta_i}{\Delta},$$
$$\Delta = G_1[G_5G_9 - G_8G_6] - G_2[G_4G_9 - G_6G_7] + G_3[G_4G_8 - G_5G_7],$$
$$\Delta_1 = \bar{G}(s)[G_2G_9 - G_8G_3],$$
$$\Delta_2 = -\bar{G}(s)[G_1G_9 - G_7G_3],$$
$$\Delta_3 = \bar{G}(s)[G_1G_8 - G_2G_7],$$
$$G_i = \frac{1}{\lambda_i}\phi_i,$$
$$G_{i+3} = \phi_i \zeta_i \lambda_i (1 - a\lambda_i^2),$$
$$G_{i+6} = \eta_i\{D_E \lambda_i \phi_i - s_v \psi_i\},$$
$$I_1(\lambda_i r_0) = \phi_i, \quad I_0(\lambda_i r_0) = \psi_i, \quad i = 1,2,3.$$

and putting the values of $g_i(s)$ in Eqs. (50), (53), (56)-(58) the various components of displacement, temperature distribution, carrier density, and stresses are

$$\bar{u} = \frac{\bar{G}(s)}{\Delta}\left\{[G_2G_9 - G_8G_3]\frac{\theta_1}{\lambda_1} - [G_1G_9 - G_7G_3]\frac{\theta_2}{\lambda_2} + [G_1G_8 - G_2G_7]\frac{\theta_3}{\lambda_3}\right\}, \quad (70)$$

$$\bar{T} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3]\zeta_1\vartheta_1 - [G_1G_9 - G_7G_3]\zeta_2\vartheta_2 + [G_1G_8 - G_2G_7]\zeta_3\vartheta_3\}, \quad (71)$$

$$\bar{N} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3]\eta_1\vartheta_1 - [G_1G_9 - G_7G_3]\eta_2\vartheta_2 + [G_1G_8 - G_2G_7]\eta_3\vartheta_3\}, \quad (72)$$

$$\overline{\sigma_{rr}} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3](l_1\vartheta_1 - \mu_1) - [G_1G_9 - G_7G_3](l_2\vartheta_2 - \mu_2) + [G_1G_8 - G_2G_7](l_3\vartheta_3 - \mu_3)\}, \quad (73)$$

$$\overline{\sigma_{\theta\theta}} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3]\{\mu_1 + l_1\vartheta_1\} - [G_1G_9 - G_7G_3]\{\mu_2 + l_2\vartheta_2\} + [G_1G_8 - G_2G_7]\{\mu_3 + l_3\vartheta_3\}\}, \quad (74)$$

$$\overline{\sigma_{zz}} = \frac{\bar{G}(s)}{\Delta}\{[G_2G_9 - G_8G_3]l_1\vartheta_1 - [G_1G_9 - G_7G_3]l_2\vartheta_2 + [G_1G_8 - G_2G_7]l_3\vartheta_3\}, \quad (75)$$

where $\vartheta_i = I_0(\lambda_i r), \theta_i = I_1(\lambda_i r), \mu_i = \frac{2\gamma^2}{\lambda_i r} I_1(\lambda_i r), i = 1,2,3.$

2. Inversion of the transforms

To obtain the result of the problem in the physical domain transforms in Eqs. (70)-(75) are inverted using

$$f(x,t) = \frac{1}{2\pi i}\int_{e^{-i\infty}}^{e^{+i\infty}} \tilde{f}(x,s)e^{-st}ds \quad (76)$$

2.1 Numerical results and discussion

To demonstrate the theoretical results and to show the effect of the fractional order Moore-Gibson-Thompson-Photo-Thermal (MGTPT) theory graphically using MATLAB software, isotropic silicon (Si) material is used having its physical data (Mahdy *et al*. 2020), as given:

$\lambda = 3.64 \times 10^{10}\ Nm^{-2}$, $T_0 = 300$ K,

$\mu = 5.46 \times 10^{10}\ Nm^{-2}$, $H_0 = 1\ \mathrm{Jm^{-1}nb^{-1}}$,

$\beta = 7.04 \times 10^{6} Nm^{-2} deg^{-1}$, $\tau = 5 \times 10^{-5}$ s,

$\delta_n = -9 \times 10^{-31}\ m^{-3}$, $N_0 = 10^{20} m^{-3}$,

$\rho = 2.33 \times 10^{3} Kgm^{-3}$, $\varepsilon_0 = 8.838 \times 10^{-12}\mathrm{Fm^{-1}}$,

$C_E = 695\ JKg^{-1}K^{-1}$, $E_g = 1.11eV$,

$K = 150\ Wm^{-1}K^{-1}$ $\alpha_T = 3 \times 10^{-6} K^{-1}$,

$K^* = 1.54 \times 10^{2} Ws$, $s_v = 2ms^{-1}$.

$D_E = 2.5 \times 10^{-3} m^2 s^{-1}$, $H_0 = 10^{8} Col.cm^{-1}s^{-1}$,

$\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$ $\sigma_0 = 9.36 \times 10^{5} Col^2 C^{-1} m^{-1} s^{-1}$

$r_0 = 1$

Fig. 2 depicts the variation in the radial displacement u for MGTPT with two temperatures by varying the fractional parameter. It has been noticed that at $r = 0$ and $r = r_0$, the radial displacement is zero which satisfies the boundary conditions. However, as the value of the α parameters in MGTPT theory increases, the radial displacement increases in the center of the cylinder. Moreover, as radial distance increases, radial displacement decreases. Fig. 3 illustrates the variation in the temperature distribution for MGTPT with two temperature with variation in fractional parameter α. It has been noticed that temperature distribution is lower in the inner core of the cylinder as compared to the outer core of the cylinder.

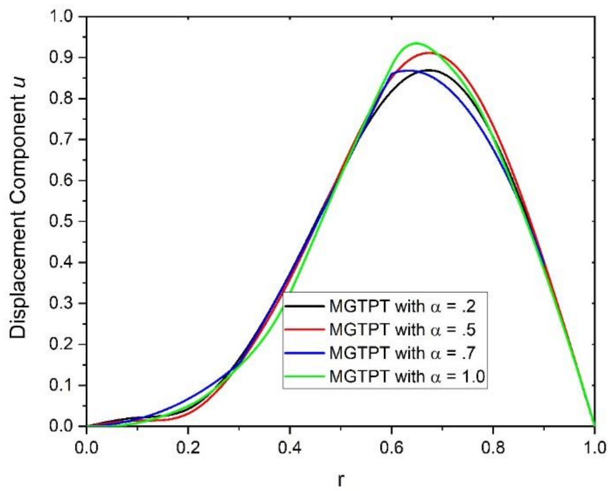

Fig. 2 The radial displacement variation for various models with two temperature

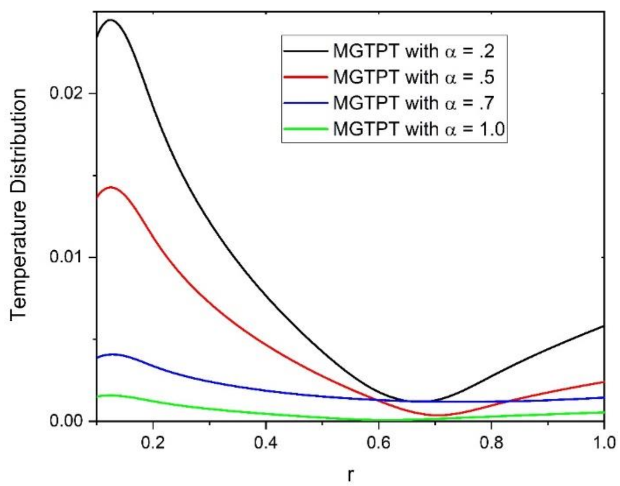

Fig. 3 The temperature variation for various models with two temperature

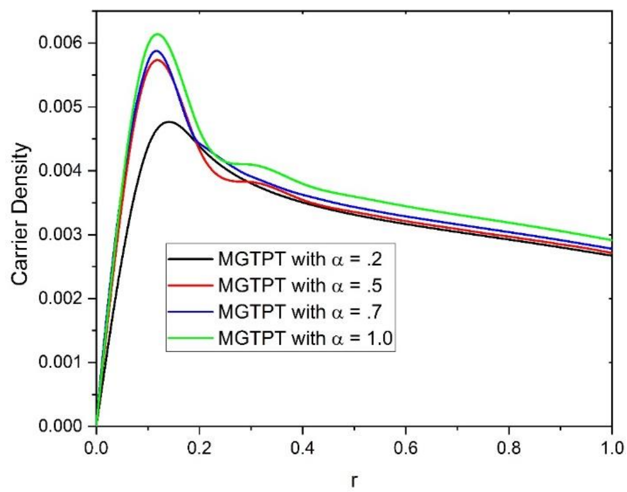

Fig. 4 The variation in carrier density for various models with two temperature

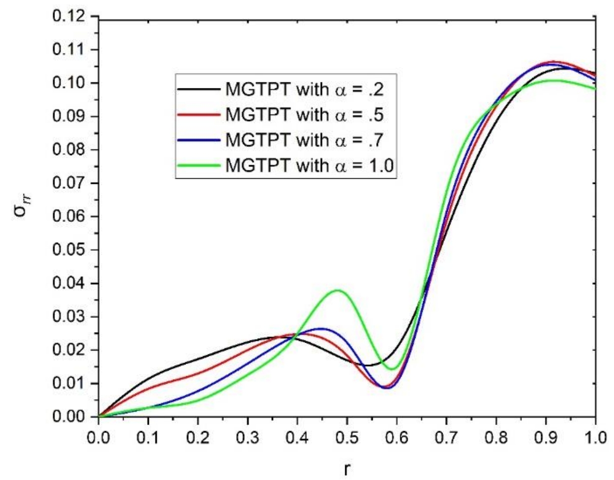

Fig. 5 The deviation in radial stress for various models with two temperature

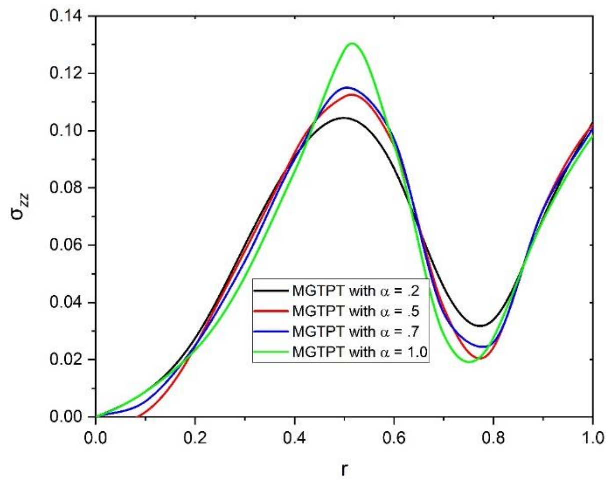

Fig. 7 The deviation in vertical stress for various models with two temperature

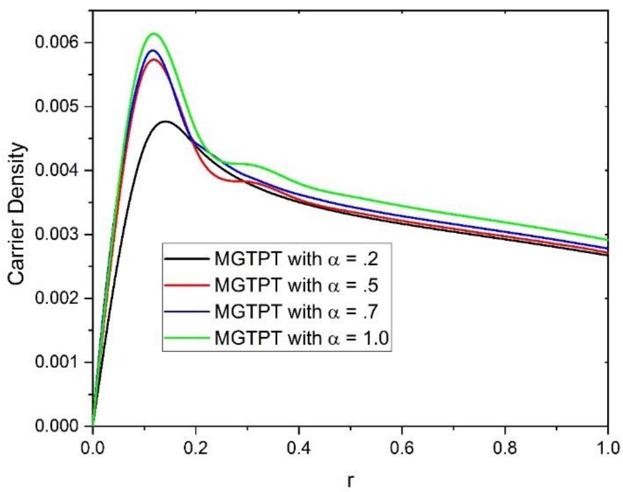

Fig. 6 The deviation in hoop stress for various models with two temperature

Fig. 4 shows the variation in the carrier density for MGTPT theories with two temperature with variations in fractional parameter α. It has been noticed that in as the radial distance increases, there is a sharp increase in the carrier density. However, as the value of α increases, the variation in carrier density also increases. Figs. 5-7 shows the variation in the stress components for MGTPT with two temperature. It has been noticed that radial stress is minimum in the initial range of the radial distance and then sharply increases with radial distance. Moreover, as the value of the fractional parameter increase, there are sharp increases in the stress components. It has been noticed that variation in the hoop and vertical stress components sharply varies as the radial distance r increases.

3. Conclusions

Nowadays, one of the most emerging areas in the field of thermoelasticity is the generalized theory of elasticity subject to the fractional order derivatives subject to the photothermal excitations. The dynamics of challenging real-world issues are closely connected to fractional calculus. Due to their non-local nature, fractional operators can more accurately and systematically represent a variety of natural phenomena. The findings for the fractional mathematical model are more precise than the classical mathematical modeling. Since the laser source is limited in width and duration, a nonlocal thermoelastic model has been used to predict these factors, which was proposed by Eringen. Only a few studies corresponding to this hypothesis have been found in the existing literature. In terms of theoretical developments and numerical computations, the following remarks are addressed from the present analysis.

• The current study presents photo-thermoelasticity models generalized to the fractional Moore-Gibson-Thompson photo-thermal (MGTPT) model with two temperatures. As part of this study, a type III Green-Naghdi photo-thermoelastic model was included. Some physical consequences of earlier models can be addressed with the generalized MGTPT model.

• Using an exponential laser pulse on the boundary surface of an infinite semiconducting solid cylinder, this study has examined the behavior of the solid cylinder.

• Using the fractional MGTPT with two temperatures, we can express the governing equations. In this paper, the fractional operator (Atangana–Baleanu) of order α is used to simulate the fractional MGT thermoelastic heat conduction model.

• Effects of the fractional operator (Atangana–Baleanu) of order α with two temperatures on the components of displacement, temperature field, carrier density, and thermal stresses are represented graphically. It is evident from the graphs that the fractional operator α significantly impacts every examined domain.

• Laser beams or sunlight beams will cause semi-conductors to vibrate due to the thermal effect when they are exposed to focused laser beams or sunlight beams. These materials have many uses in renewable energy, especially in the solar cells industry which depends heavily on semiconductor materials.

• Photothermal methods are not only simple and sensitive but they can also be used to gain some insight into the process of deexcitation in materials and optical absorption. The ideas presented in this paper will come in handy for physicists, material designers, thermal engineers, and geophysicists. A wide range of photo-thermoelasticity and thermodynamic problems can be solved using the technique used in the above study.

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Nomenclature

δ_{ij}	Kronecker delta,
e_{ij}	Strain tensors (mm^{-1}),
s_v	surface recombination velocity (ms^{-1})
D_E	Carrier diffusion coefficients(m^2s^{-1}),
N_0	Carrier concentration at equilibrium position (m^{-3})
λ, μ	Lame's elastic constants (Nm^{-2}),
β_{ij}	Thermal elastic coupling tensor ($Nm^{-2}deg^{-1}$)
κ	Coupling parameter for thermal activation,
T	Thermodynamic temperature(K),
C_E	Specific heat at constant strain ($JKg^{-1}K^{-1}$),
α_t	Linear thermal expansion coefficient (K^{-1}),
N	Carrier density (m^{-3}),
a_{ij}	Two temperature parameter (K),
δ_n	Electronic deformation coefficient (m^{-3}),
u_i	Components of displacement(m),
K^*_{ij}	Materialistic constant (Ws),
e_{kk}	Cubical dilatation,
ρ	Medium density (Kgm^{-3}),
φ	Conductive temperature (K),
t	Time(s),
σ_{ij}	Stress tensors (Nm^{-2}),
K_{ij}	Coefficient of Thermal conductivity,
T_0	Reference temperature s.t. $\lvert T/T_0\rvert << 1$,
ϵ_{ijk}	Permutation symbol,
E_g	The energy gap of the semiconductor parameter (eV),
τ	Photo-generated carrier lifetime (s)