

Bending behavior of microfilaments in living cell with nonlocal effects

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Abstract. Dynamics of protein filamentous has been an active area of research since the last few decades as the role of cytoskeletal components, microtubules, intermediate filaments and microfilaments is very important in cell functions. During cell functions, these components undergo the deformations like bending, buckling and vibrations. In the present paper, bending and buckling of microfilaments are studied by using Euler Bernoulli beam theory with nonlocal parametric effects in conjunction. The obtained results show that the nonlocal parametric effects are not ignorable and the applications of nonlocal parameters well agree with the experimental verifications.

Keywords: bending; buckling; euler bernoulli beam theory; microfilament; nonlocal effects

1. Introduction

One of the components of cytoskeleton is the microfilaments (MFs) also known as the actin filaments, are the polymers of protein actin within the cytoplasm of the cell. The cytoskeleton, in fact the network of protein fibers that extend within the whole cell by giving structure and shape to the cell, also keeping the organelles in their proper places (Wayne 2009). MFs are the smallest cytoskeletal components having the role of movements, contractions of muscles and the division of cell (Gunning *et al.* 2015). Structurally, MFs are composed of two constituents of protein actin subunits twisted in a coiled shape. Precisely, the protein actin subunits that are the basic units or building block of MFs are known as the globular actin or G-actin, once they joined together to form filamentous actin or F-actin (Pieper *et al.* 2013, Sadegh *et al.* 2017).

MFs are polar in nature like MTs with plus and minus ends. The plus end is barbed end while the negative end is pointed. Polarization, caused by the molecular binding pattern of the molecules that are the structural units of MFs. Similar to MTs, plus end grows faster than the minus end (Dickinson and Purich 2007). The diameter of MFs is the thinnest of all the cytoskeletal components, about 6 to 7 nanometers (Fuchs and Cleveland 1998). MFs are formed, when the three G-actin joined together to form a trimer.

After that, more actin are fixed to the plus end (Dickinson *et al.* 2002). This process is known as the self-assembly and is supported by the proteins known as the autoclampin proteins that act as motors for the process of making MFs (Gervasi *et al.* 2018, Gokhin and Fowler 2016). The two long strands of actin protein are arranged in a spiral to form MFs (Dickinson *et al.* 2004).

Cytoskeleton of almost all types of cells, a basic unit of living organisms, consists of three major components: microtubules (MTs), microfilaments (MFs) and intermediate filaments (IFs), biochemically composed of proteins subunits (Alberts *et al.* 2013). IFs are of rod shaped with diameter 10 nm which is intermediate of MTs and MFs diameters which are 12 nm and 7 nm respectively, and persistence lengths of IFs is varied in the range of a few hundred nm to few μm (Mofrad 2006, Franke *et al.* 1978, Herrmann *et al.* 2007). The rope like structure of IFs formed by extended 45 nm and thin 2 – 3 nm rod-like dimmers (Strelkov *et al.* 2003) provides the strength and shape to cytoskeleton along with other components (Chang *et al.* 2004). The most important function of MFs is to contract the muscles therefore the concentrations of MFs in the muscle's cell is high where they form the myofibrils which are the basic building block of muscle cell. Actin protein is a vital protein for the movement of muscle and the MFs are also called actin filaments as the actin proteins are the most prominent in the muscular system of the body. In the cells of muscle, actin units work with protein myosin and allow the muscle to contract and relax (Galland *et al.* 2013).

Neither actin nor myosin work properly without each other and they jointly form a complex called actomyosin

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(Dickinson and Purich 2006). This actomyosin are found in sarcomeres which is the basic unit of muscle tissue (Akintewe *et al.* 2017). The subunit of IFs is dimeric fiber protein which has both amino-terminal and carboxy-terminal globular on each monomer, and a central area is a α -helical coiled-coiled and rather stiff enough. Smallest unit of IFs observed in cytoplasm is, a tetramer of two such subunits, leaning head to tail (Wang *et al.* 2001). This position of tetramer keeps it indistinguishable on both ends and therefore non-polarized in nature (Qin *et al.* 2009), (Hanukoglu *et al.* 2014). IFs units line up with their long axis and width is determined by cross link of the dimmers and association and dissociation of these dimmers occurs all along their length (Crewther *et al.* 1983, Green *et al.* 1992). IFs are of different kinds known as vimentin, desmin, glial fibrillary acid protein (GFAP) and neurofilaments (Bornheim *et al.* 2008, Hanukoglu and Fuchs, 1983), some are specialized to particular cell types such as neurofilaments are found particularly in neurons and some are expressed in variety of cells as vimentin is found in almost all cell types. Under these conditions, IFs are just like to buckle, but still there are no theoretical studies on buckling of IFs. IFs organize themselves in such a way that can provide mechanical stability to plasma membrane where it comes into contact with other cells or with the extracellular matrix (Fletcher and Mullins 2010). They also provide support to mechanically stressed cells such as muscle cells, neurons and some epithelial cells and also provide integrity to the entire cytoskeleton by organizing in rope like structure (Potschka *et al.* 1990, Wang *et al.* 2001). They are often most developed in areas of the cell which are subjected to regular mechanical stress by the extracellular environment. They can also provide support for the (MFs) and (MTs) which are more fragile and are connected to both of the other types of cytoskeletal fibres by plakin proteins. IFs are necessary for stretching in epithelial cells (Gittes *et al.* 1993). Under such circumstances, the IFs are just like to be buckle. But to date, there are no studies on buckling of these filaments. (Ackbarow *et al.* 2007) found these properties at molecular level. Later, in (Qin *et al.* 2009) an atomistic model is used for vimentin intermediate filament to measure mechanical stress, under tensile loading at nanolevel and found that stretching causes the transition from alpha-helices to beta-sheets, a phenomenon known as alpha-beta transition. IFs are semi-flexible biopolymers as their persistence length is rather short as compared to MTs and MFs which varies from 7 μm to several mm (Block *et al.* 2015) while for IFs, it ranges from few hundred nm to few μm . Thus, they are one order of magnitude more flexible than MFs. Furthermore, (Ramm *et al.* 2014) both experimentally and theoretically analyzed vimentin IFs for the folding and stability properties and observed that their physical and mechanical properties have a distinct dependence on the diameter/width which shows that due to large ratio of surface area to volume at nanoscale, the surface effects may affect their physical and mechanical properties (Guzman *et al.* 2006). Experimentally, the surface effects on nanomaterials have been discussed by using atomic force microscopy AFM and found that these effects have profound effects on elastic properties of nanowires due to large diameter. Theoretically, the size

dependence of the stretching, bending and buckling behavior of such nanosized structural elements has been investigated. Akbaş (2018a) investigated the forced vibration responses of a cantilever nanobeam with crack for modified couple stress theory with damping effect. The crack is modeled with a rotational spring. The Kelvin–Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory in the time domain. Influences of the geometry, crack and material parameters on forced vibration responses of cracked nanobeams were examined and discussed.

Benmansour *et al.* (2019a) analyzed the dynamic and bending behaviors of isolated protein microtubules. Microtubules (MTs) can be considered as bio-composite structures that are elements of the cytoskeleton in eukaryotic cells and possess considerable roles in cellular activities. They have higher mechanical characteristics such as superior flexibility and stiffness. Besides the above types, the main type of IFs is nuclear lamins in eukaryotic cells which is responsible for nuclear shape and structure and protects microorganisms inside the nuclear envelop (Gruenbaum *et al.* 2005), providing the attachment site for chromosomes but do break down during mitosis cell division (Ishikawa *et al.* 1968, Soltys *et al.* 1992, Osada *et al.* 2016, Goldman *et al.* 1996, Gittes *et al.* 1993). Organization of IFs and their involvement with plasma membranes suggests that their main function is to give shape and structure to the cell and perhaps the most important function is to provide mechanical support for the plasma membrane where it gets in touch with other cells or with extracellular matrix (Fuchs *et al.* 1994, Blobel *et al.* 2000). IFs can be considered as internal guy ropes that support the whole cytoskeleton. They are abundant in the most developed areas of the cell and are subjected to usual automatic pressure by the extracellular surroundings. IFs also provide support to MFs and MTs which can easily be broken and are connected to both types of cytoskeletal fibrous by plakin proteins. IFs are essential for stretching in epithelial cells (Ackbarow *et al.* 2007). Akbaş (2018b) composed the static bending of an edge cracked cantilever nanobeam of functionally graded material (FGM) subjected to transversal point load at the free end based on modified couple stress theory. Material properties of the beam change in the height direction according to exponential distributions. The cracked nanobeam is modelled using a proper modification of the classical cracked-beam theory consisting of two sub-nanobeams connected through a massless elastic rotational spring. The inclusion of an additional material parameter enables the new beam model to capture the size effect. The new non-classical beam model reduces to the classical beam model when the length scale parameter is set to zero. Civalek *et al.* (2010) formulated the equations of motion and bending of Euler-Bernoulli beam using the nonlocal elasticity theory for cantilever microtubules (MTs). The method of differential quadrature (DQ) has been used for numerical modeling. The size effect is taken into consideration using the Eringen's non-local elasticity theory. Before, in strain hardening and viscoelasticity at IFs level are checked and confirmed that these phenomena occur at molecular level

and firm association of dimmers provides high tensile strength to these filaments making sure the cell unity preventing the sharp crack of epithelial cell under tensile force by improving its resistance to compressive, twisting, stretching and bending forces which stabilize the extended axons of nerve cells as well as line the inside face of the nuclear envelope where they help harness and protect the cell's DNA (Lee *et al.* 2012, Goldman *et al.* 2012, Fudge *et al.* 2003). Akbaş (2017) presented the stability analysis of a non-homogeneous plate with porosity effect. Material properties of the plate vary in the thickness direction and depend on the porosity. In the solution of the problem, the Generalized Differential Quadrature method is used. In the porosity model, uniform porosity distribution is considered. The effects of the porosity and material distribution parameters on the critical buckling of the non-homogeneous plate are investigated. In another study (Gao *et al.* 2009) tensile properties of IFs are measured experimentally. As it was observed earlier, IFs are rigid and give support to both nuclear and cell membrane so there is a natural question that whether IFs may buckle while performing various functions. But still there is no theoretical work has been done to study the buckling behaviors of IFs. MFs also play role in the movement of the cell and this happens in all over the body and is important especially in an organism with entire body consists of only one cell, such as amoeba. Without MFs amoeba would not be motile. Actomyosins also play the same role as they play in the muscle cells. For the movement of cell, one end of MFs elongate while the other end must be shorten and at the same time myosin acts as a motor to make all this happen (Mullins *et al.* 1998). Abouelregal *et al.* (2021) established the influence of thermal conductivity on the dynamics of a rotating nanobeam in the context of nonlocal thermo-elasticity theory. To this end, the governing equations are derived using generalized heat conduction including phase lags on the basis of the Euler–Bernoulli beam theory. The thermal conductivity of the proposed model linearly changes with temperature and the considered nanobeam is excited with a variable harmonic heat source and exposed to a time-dependent load with exponential decay.

Akbaş (2017a, b) investigated the free vibration analysis of edge cracked cantilever microscale beams composed of functionally graded material (FGM) based on the modified couple stress theory (MCST). The material properties of the beam are assumed to change in the height direction according to the exponential distribution. The FG nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. Mechanical properties of FG beam depends on the position. The Kelvin-Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory. Considering the IFs as rod like as its structure suggests, here in the present article our aim is to explore the buckling behavior of IFs by using the Euler Bernoulli beam and Timoshenko beam theories. We also checked the effect of surface on critical buckling force because of Ramm *et al.* (2014) analyzed both experimentally and theoretically vimentin IFs for the folding and stability properties and observed that their physical and mechanical properties have a distinct dependence on the diameter/width

which shows that due to large ratio of surface area to volume at nanoscale, the surface effects may affect their physical and mechanical properties (Guzman *et al.* 2006). Experimentally, the surface effects on nanomaterials have been discussed by using atomic force microscopy AFM and found that these surfaces have profound effects on elastic properties of nanowires due to large diameter (Cuenot *et al.* 2004). Sae-Long presented a new analytical bar-substrate model for the analysis of an isotropic and homogeneous nanowire embedded in an elastic substrate. A fourth-order strain gradient model based on a thermodynamic approach is employed to represent the small-scale effect (nonlocal effect) while the Gurtin-Murdoch continuum model based on the surface elastic theory is used to account for the size-dependent effect (surface energy effect).

Koochi and Goharimanesh (2021) investigated the nonlinear oscillation of carbon nanotube manufactured nano-resonator. The governing equation of the nano-resonator is extracted in the context of the nonlocal elasticity. The impact of the Casimir force is also incorporated in the developed model. A closed-form solution based on the energy balance method is presented for investigating the oscillations of the nano-resonator. Sedighi *et al.* (2020) studied the hybrid nanotubes composed of carbon and boron-nitride nanotubes have manifested as innovative building blocks to exploit the exceptional features of both structures simultaneously. On the other hand, by mixing with other types of materials, the fabrication of relatively large nanotubes would be feasible in the case of macroscale applications. Ouakad *et al.* (2020) studied the effects of material properties, nonlocal parameter, Lorentz and electric forces on maximum static deflections and natural frequencies of actuated hybrid carbon/boron-nitride nanotubes (CBNNT) subjected to thermal loads are studied for the first time. The displacement field of the nanotube satisfies assumptions of the Bernoulli-Euler beam theory. The Green-Lagrange small strains and moderate rotations for geometric nonlinearity of the nanotube are taken into consideration. Sedighi and Malikan (2020) investigated the stress-driven nonlocal theory of elasticity, in its differential form, is applied to investigate the nonlinear vibrational characteristics of a hetero-nanotube in magneto-thermal environment with the help of finite element method. In order to more precisely deal with the dynamic behavior of size-dependent nanotubes, a two-node beam element with six degrees-of-freedom including the nodal values of the deflection, slope and curvature is introduced.

Theoretically in (Wang *et al.* 2009), the size dependence of the stretching bending and buckling behavior of such nanosized structural elements has been investigated. The nano mechanical properties of vimentin IFs dimers are investigated in (Qin *et al.* 2009, 2012) along with related deformation mechanisms. He employed an atomistic model of human vimentin dimer and carried out molecular dynamics simulations to estimate its reaction to mechanical stress under tensile loading. Miller and Shenoy observed the size dependence of stretching and bending behavior of such nano-sized structural elements.

In cytoplasmic streaming, MFs also play role which is the flow of cytoplasm in the whole cell. It also allows the

waste products, nutrients and cell organelles to move from one part of the cell to others. MFs can also attach to the organelles of the cell and then helps in contractions, pulling of the cell organelles. MFs also participated in cell division during mitosis and in the process of cytokinesis and separate the cell into two daughter cells. In cytokinesis, a ring of actin protein forms a ring around the cell that is separating and then myosin proteins pull on the actin protein and the cell to contract. The size of the ring gets narrower around the cell and dragging the cell membrane with it until it separates into two cells. After all these process, MFs get depolymerize causing the ring to dissemble when it is no longer required (Rudan *et al.* 2015, Xu *et al.* 2013).

As discussed above the role of MFs is very important within the cell like other filamentous, MTs and IFs. Many researchers have worked on the dynamics of these filaments. The dynamics of MTs and IFs are affected by the induction of nonlocal parameters (Civalek and Demir 2011, Miller *et al.* 2000).

Recently some researcher used different methods for nonlinear modeling (Tohidi *et al.* 2018, Arefi and Zenkour, 2017, Arani *et al.* 2018, Krommer *et al.* 2016, Yeh 2016) and for other structures (Boussoula *et al.* 2020, AlSaleh and Fuggini, 2020, Lee *et al.* 2019, Zahrai and Kakouei 2019, Poplawski *et al.* 2019). Recently some researcher used different methods for nonlinear modeling (Eltaher *et al.* 2019, Ebrahimi *et al.* 2019, Safaei *et al.* 2019, Shahsavari *et al.* 2019, Benmansour *et al.* 2019b).

Mehar and Panda (2016a, b) computed the vibration behavior, bending and dynamic response of FG reinforced CNT using shear deformation theory and finite element method. For the sake of generality, the mathematical model was presented with the mixture of Green Lagrange method. The convergence of these methodologies has been checked for the variety of results. The composite plates with different graded was investigated with isotropic and core phase.

The nonlocal theory of elasticity was pioneered by Eringen in 1972 (Eringen 1972a). According to this theory, the strain on the particular point is not only the function of stress acting on that point but also the function of the points in the neighborhood of that point. The results obtained by the applications of this theory are very near to the actual experimental verifications (Eringen 1984, Lazar *et al.* 2006, Lim *et al.* 2015, Zhang *et al.* 2005). Bending of protein MTs is also discussed in 2011 by (Civalek and Demir 2011) by using Euler Bernoulli beam theory (EBT) with nonlocal parametric effects. Inspired by the successful applications of EBT with nonlocal parametric effects on the dynamics of MTs, in the present study we investigated the bending and buckling of MFs by coupling the nonlocal effects.

2. Materials and methods

2.1 Euler beam theory

The structure of MF is like solid rods. Therefore Euler-Bernoulli's theory also termed as a linear theory of elastics can be used to calculate the bending deflection, bending

moment, and bending MF in homogenous loads (Timoshenko 1953). It is a particular example of Timoshenko beam theory, originally used and implemented on large scale in the late nineteenth century before the construction of the Eiffel Tower and the Ferris Wheel (Truesdell 1960, 1890). After those successful demonstrations, this is the cornerstone of technology. In the present study, we used this theory to calculate the bending and buckling of MF after the successful application of this theory in the study of the dynamics of proteins MTs and IFs (Domagala *et al.* 1986, Li 2008, Safeer *et al.* 2021). The mechanical properties of MFs that are used to study the bending, buckling, and persistent length, are $L = 6 \times 10^{-6}m$, radius, $r = 3.5 \times 10^{-9}m$, modulus of rigidity, $EI = 2.5 \times 10^{-26}Nm^2$ and young's modulus, $E = 2 \times 10^9Pa$ (Mofrad and Kamm 2006).

3. Mathematical formulations for the problem

3.1 Nonlocal constitutive relations

According to Eringen (Eringen 1972b, Reddy and Pang 2008), the nonlocal stress tensor τ is given by the relation,

$$\tau = \oint \kappa(|x' - x|, t) T(x') dx' \quad (1)$$

T is called the classical tensor at the point x and $\kappa(|x' - x|, t)$ represents the nonlocal modulus in which $|x' - x|$ is the distance, t is a material constant that depends upon the internal and external characteristics lengths. The stress T and the strain tensor ε are related by the relation,

$$T(x) = C(x) : \varepsilon(x) \quad (2)$$

$C(x)$ is the fourth order elasticity tensor, $:$ denotes the double dot product. The equivalent differential form of (1) and (2) is,

$$(1 - t^2 L_e^{-2} \nabla^2) * \tau = T, \quad t = \frac{e_0 L_i}{L_e} \quad (3)$$

e_0 is the material constant, L_i and L_e are the internal and external characteristics of the material respectively.

3.2 Stress resultants in nonlocal theories

Using (3), we can write the stress resultants for nonlocal Euler Bernoulli beam theory as,

$$\begin{aligned} \tau_{xx} - \mu * \tau''_{xx} &= E * \varepsilon_{xx}, \\ \tau_{xz} - \mu * \tau''_{xz} &= 2 * G * \varepsilon_{xz} \quad (\mu = e_0^2 * L_i^2) \end{aligned} \quad (3a)$$

where E is the young's modulus and G is the shear modulus. In all the theories, the relation between axial force F and strain ε is given by,

$$F - \mu F'' = E * A * \varepsilon_{xx}^0, \quad \varepsilon_{xx}^0 = u' \quad (4)$$

where u is the axial displacement. The displacement field for EBT is,

$$u_1 = u - z * w'', \quad u_2 = 0, \quad u_3 = w \quad (5)$$

w is the transverse displacement of the point $(x, 0)$ on the middle plane of the filament.

3.3 Governing equations for MFs

The governing equations for MFs along with the nonlocal parametric effects are,

$$M = -E^* * w'' + \mu * [(F * w')' - q + m_0 * \ddot{w} - m_2 * \ddot{w}''] \quad (6)$$

$$(-E^* * w'')'' + \mu * ((F * w')' - q + m_0 * \ddot{w} - m_2 * \ddot{w}'')'' = m_0 * \ddot{w} - m_2 * \ddot{w}'' \quad (7)$$

where,

$$]E^* = EI, \quad \ddot{w} = \frac{\partial^2 w}{\partial t^2}, \quad w'' = \frac{\partial^2 w}{\partial x^2}, \quad m_0 = \oint \rho dA, \quad m_2 = \oint \rho z^2 dA$$

BC's, involving specifying one variable of each of the following three pairs at $x = 0$ and $x = L$:

$$u \text{ or } F, \quad w \text{ or } M' \text{ and } w' \text{ or } M \quad (8)$$

The moment's equation for Euler Bernoulli Beam is:

$$\begin{aligned} M - \mu * M'' &= E^* * k \\ k &= -w'' \end{aligned} \quad (9)$$

The symbols, A and I are:

$$A = \oint dA, \quad I = \oint z^2 dA \quad (10)$$

Eqs., (6) and (7) are valid for all nonlocal beam theories. Equation of motion of conventional EBT can be obtained by setting, $\mu = 0$ in (6). The natural BC'S at the ends of MFs, $x = 0, L$ are specified as:

$$\bar{F} - E^* * A * u' - \mu * (m_0 * \dot{u}' - f') = 0 \quad (11)$$

$$\begin{aligned} \bar{V} - m_2 * \dot{w}' + F * w' + (E^* * w'')' - \mu \\ * \left((F * w')' - q + m_0 * \ddot{w} \right) &= 0 \\ * \left(-m_2 * \ddot{w}'' + q - (F * w')' \right) &= 0 \end{aligned} \quad (12)$$

$$-\bar{M} - E^* * w'' + \mu * \left((F * w')' - q + m_0 * \ddot{w} - m_2 * \ddot{w}'' \right) = 0 \quad (13)$$

Note that Eq. (12) is satisfied if the beam is fixed at the ends $x = 0$ and $x = l$. If the bar is fixed at the ends, the axial displacement and shear strain will be zero at each end (Rao 2019), thus

$$u(0, t) = 0, u(l, t) = 0, \frac{d^2 u(0, t)}{dx^2} = 0, \frac{d^2 u(l, t)}{dx^2} = 0 \quad (13)$$

where $\bar{F}, \bar{V}, \bar{M}$ are the specified generalized forces.

4. Results and discussions

4.1 Bending solutions

We consider MFs are isotropic and the geometric properties are same throughout the filament (Reddy 2006). We set here the nonlinear terms and time derivative to be zero, $f = 0$ and take the distributed transverse load to be

arbitrary, i.e., $q(x) = q$.

The equation for bending is,

$$(-E^* * w'')'' - \mu * q'' + q = 0 \quad (14)$$

On integrating the equation (14), it reduces to,

$$Q + E^* * w''' + q_0 * x + K_1 = 0 \quad (15)$$

$$M - E^* * w'' - \mu * q_0 + q_0 * \frac{x^2}{2} + K_1 * x + K_2 = 0 \quad (16)$$

$$\begin{aligned} E^* * w' + \mu * q_0 * x - q_0 * \frac{x^3}{6} \\ - K_1 * \frac{x^2}{2} - K_2 * x - K_3 = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} E^* * w + \mu * q_0 * \frac{x^2}{2} - \mu * q_0 * \frac{x^4}{24} \\ - K_1 * \frac{x^3}{6} - K_2 * \frac{x^2}{2} - K_3 * x - K_4 = 0 \end{aligned} \quad (18)$$

$K_1 - K_4$, are constants which can be determined by using the boundary conditions for uniformly distributed load of intensity, $q = q_0$.

The deflection and bending moment are calculated by using the four different boundary conditions, (i). MFs are simply supported at both the ends, (ii). The edges of MFs are clamped at the ends, (iii). The case of cantilever, (iv). The case of propped cantilever of MFs.

(i). when MFs are considered as simply supported, the boundary conditions are, $w = 0, M = -E^* * w'' - \mu * q_0$, at $x = 0$ and $x = L$. (ii). When both the edges are clamped at both the ends at $x = 0$ and $x = L$, the boundary conditions are, $w = 0, w' = 0$. (iii). For cantilever case, $w = 0, w' = 0$ at $x = 0$ and $V = M = 0$ at $x = L$. (iv). For the case of propped cantilever boundary conditions, $w = 0, w' = 0$ at $x = 0$ and $w = M = 0$ at $x = L$.

By using all these conditions, deflection and bending moment of IFs are calculated as,

(i). MFs are simply supported at both the ends,

$$\begin{aligned} w(x) = \frac{q_0 * L^4}{24 * E^*} \left[\left(\frac{x}{L} \right)^4 - 2 \left(\frac{x}{L} \right)^3 + \left(\frac{x}{L} \right) \right] \\ + \frac{\bar{\mu} * q_0 * L^4}{2 * E^*} \left[\left(\frac{x}{L} \right) - \left(\frac{x}{L} \right)^2 \right] \end{aligned} \quad (19)$$

$$M(x) = -\frac{q_0 * L^2}{2} \left[\left(\frac{x}{L} \right)^2 - \left(\frac{x}{L} \right) \right] \quad (20)$$

(ii). The edges of MFs are clamped at the ends,

$$w(x) = \frac{q_0 * L^4}{24 * E^*} \left[\left(\frac{x}{L} \right)^4 - 2 \left(\frac{x}{L} \right)^3 + \left(\frac{x}{L} \right)^2 \right] \quad (21)$$

$$M(x) = -\frac{q_0 * L^2}{12} \left[1 - 6 \left(\frac{x}{L} \right) + 6 \left(\frac{x}{L} \right)^2 \right] - \bar{\mu} * q_0 * L^2 \quad (22)$$

(iii). For cantilever case,

$$\begin{aligned} w(x) = \frac{q_0 * L^4}{24 * E^*} \left[\left(\frac{x}{L} \right)^4 - 4 \left(\frac{x}{L} \right)^3 + 6 \left(\frac{x}{L} \right)^2 \right] \\ - \frac{\bar{\mu} * q_0 * L^4}{2 * E^*} \left(\frac{x}{L} \right)^2 \end{aligned} \quad (23)$$

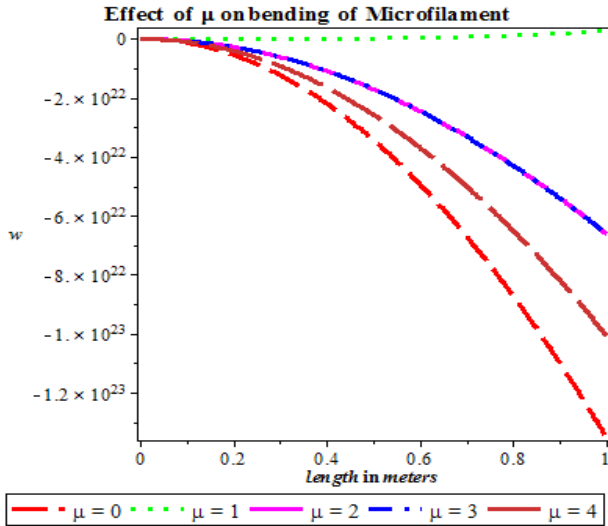


Fig. 1 Effect of bending with simply supported case

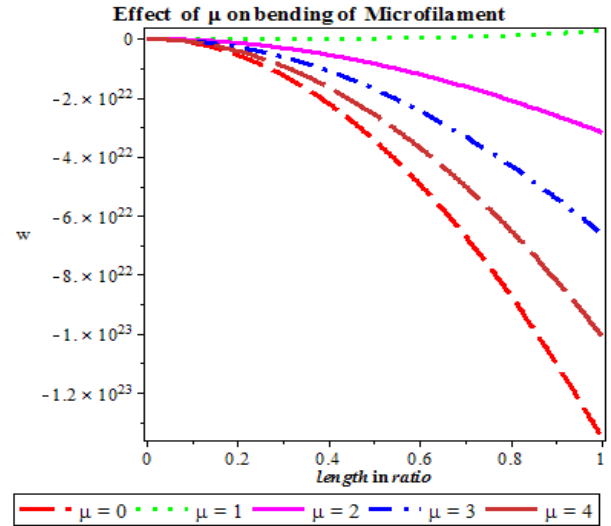


Fig. 3 Effect of bending with cantilever case

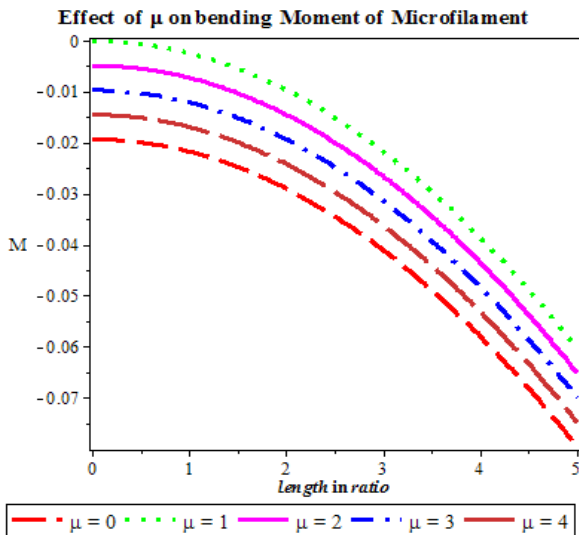


Fig. 2 Effect of bending with clamped case

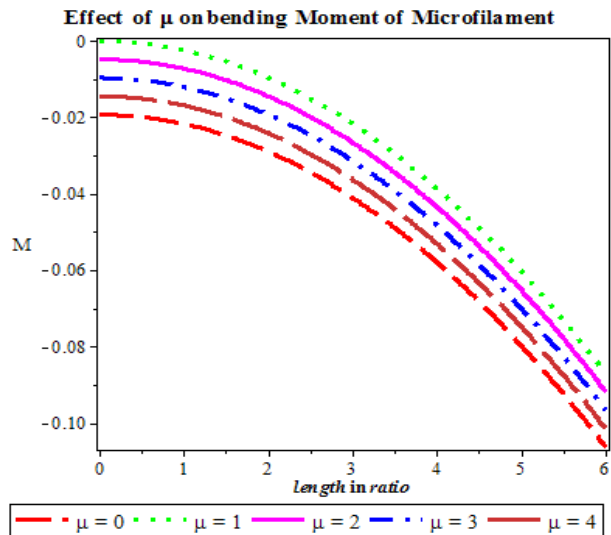


Fig. 4 Effect of bending with propped cantilever case

$$M(x) = \frac{q_0 * L^2}{2} \left[1 - \left(\frac{x}{L} \right)^2 \right] \quad (24)$$

(iv). For the case of propped cantilever,

$$w(x) = \frac{q_0 * L^4}{48 * E^*} \left[2 \left(\frac{x}{L} \right)^4 - 5 \left(\frac{x}{L} \right)^3 + 3 \left(\frac{x}{L} \right)^2 \right] \quad (25)$$

$$M(x) = -\frac{q_0 * L^2}{24} \left[3 - 15 \left(\frac{x}{L} \right) + 12 \left(\frac{x}{L} \right)^2 \right] - \bar{\mu} * q_0 * L^2 \quad (26)$$

where, $\bar{\mu} = \frac{\mu}{L^2}$

Figs. 1-4 shows the bending behavior of microfilaments with four different conditions. When filaments are considered as simply supported beam then the effects of μ is on the bending deflection of filaments and does not affect the bending moment as shown in Fig. 1. In Fig. 2, when MFs are clamped at the ends then the bending moment is affected by μ instead of bending deflections. In Fig. 3 for cantilever case, again the bending deflection of MFs are

affected by μ and bending moment is free from μ . In Fig. 4, for propped cantilever beam, the effects of μ is on the bending moment of MFs and bending deflection is free from μ .

5. Conclusions

Euler Bernoulli beam theory is used here to study the bending of MFs by considering the nonlocal parametric effects, μ . It is found that the impact of nonlocal parameter on bending of MFs is not ignorable and it affects the dynamics of the filaments. The four BC'S are used to study the dynamics of MFs and it is found that,

- When filaments are considered as simply supported beam then the effects of μ is on the bending deflection of filaments and does not affect the bending moment.
- In second case when MFs are clamped at the ends then the bending moment is affected by μ instead of bending

deflections.

- In the third case (cantilever case), again the bending deflection of MFs are affected by μ and bending moment is free from μ .

- In fourth case as propped cantilever beam, the effects of μ is on the bending moment of MFs and bending deflection is free from μ .

The graphical results show that the bending behavior of MFs is different when the effects of μ is considered.

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