Advances in Nano Research, *Vol. 14, No. 6 (2023) 495-506*
https://doi.org/10.12989/anr.2023.14.6.495

Wave propagation in double nano-beams in thermal environments using the Reddy's high-order shear deformation theory

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(*Received June 6, 2021, Revised November 3, 2022, Accepted December 22, 2022*)

Abstract. We study the bending wave, shear wave and longitudinal wave characteristics in the double nanobeams in this paper for the first time, in the process of research, based on the Reddy's higher-order shear deformation theory and considering shear layer stiffness, linear stiffness, inter-laminar stiffness, the pore volume fraction, temperature variation, functionally graded index influence on wave propagation, based on the nonlocal strain gradient theory and Hamilton variational principle, the wave equation of the double-nanometer beams are derived. Since there are three different motion states for the double nanobeams, which includes the cases of "in phase", "out of phase" and "one nanobeam fixed", the propagation characteristics of shear-, bending-, and longitudinal- waves in these three cases are discussed respectively, and some valuable conclusions are obtained.
Keywords: double nanobeams; elastic foundations; porosities; thermal effects; wave propagation

1. Introduction

With the continuous development of nanotechnology, micro-electro-mechanical systems with miniaturization, intelligence, integration and high sensitivity emerge at the historic moment and become the hot direction of contemporary science and technology (Demir and Civalek 2017, Civalek *et al*. 2021, Abdelmalek *et al*.2019). Generally speaking, nano devices have the advantages of small size, light weight, and high resonant frequency. Nanostructures have a wide range of applications, and have brought new progress for the scientific and technological revolution. Scientists have found that nano-materials do not satisfy simple Newtonian mechanics. A large number of experiments have shown that nano-materials have size dependent properties (Lu *et al*. 2021, Zhang *et al*. 2021, She *et al*. 2021). Since classical mechanics cannot describe the new characteristics of nanostructures at the nano-scale, some nonclassical mechanical theories are proposed to reveal the size effect, including the nonlocal theory (Eringen 1998), surface effect/elasticity (Abo-Bakr *et al*. 2022, Akgöz and Civalek 2015, Almitani *et al*. 2020), and nonlocal strain gradient theory (Lim *et al*. 2015).

Because the nonlocal theory only contains one size parameter and is relatively simple, it is widely used in the mechanical behavior analysis of nanostructures. For example, Ahmed *et al*. (2018) used the nonlocal theory to analyze the thermal and small-scale effects on free vibration of embedded armchair single-walled carbon nanotubes. Employed the nonlocal theory, Aissani *et al*. (2015) proposed a new nonlocal hyperbolic shear deformation theory for the analysis of nanobeams resting on elastic mediums. Bensattalah *et al*. (2020) obtained the critical buckling load of Triple-walled carbon nanotube using the nonlocal theory. Based on the nonlocal theory, Benahmed *et al*. (2019) performed the buckling analysis of functionally graded nanobeams with porosities using different higher order beam theories. Belmahi *et al*. (2019) utilized the nonlocal theory to deal with the forced vibration of nanobeams resting on elastic foundations. Malikan and Eremeyev (2020) used a new theory to analyze the thick beams with imperfection. Malikan and Eremeyev (2022) also discussed the vibrating response of multi-physic composite beam-like actuators. Malikan *et al*. (2019) discussed transient response of a carbon nanotubes with a damping. Malikan *et al*. (2022) studied the thermal buckling of piezomagnetic micro- and nano- beams including flexomagnetic effect. Under the hypothesis of nonlocal theory, Dastjerdi *et al*. (2020) employed the semi-analytical polynomial method to the time-dependent deflection of viscoelasticity functionally graded nano-gyroscopes in hygro-thermal environments. Gafour *et al*. (2020) studied the porosity-dependent free vibration analysis of functionally graded nanobeams using non-local theory. Guessas *et al*. (2018) obtained the critical buckling loads for the reinforced nanocomposite porous nanoplates with different porosity types. Jalaei and Civalek (2019) used the nonlocal theory to study the dynamic instabilities of viscoelastic porous functionally graded nanobeams resting on visco-Pasternak mediums under the action of the axially oscillating loadings and magnetic forces. Tayeb *et al*. (2019) applied the nonlocal theory to obtain the critical buckling loads for the zigzag triple walled carbon nanotubes under axial loads. Based on nonlocal theory, Hamed *et al*. (2019) used finite element method to study the static bending behaviors of size dependent functionally graded nanobeams. Bouhadra *et al*. (2021) studied the buckling characteristics of porous functionally graded nanobeams with different boundary conditions. Salami *et al*. (2021) performed the geometrically nonlinear bending analysis of

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http://www.techno-press.org/?journal=journal=anr&subpage=5
ISSN: 2287-237X (Print), 2287-2388 (Online)

composite panels reinforced by CNTs under thermal effects. Hadji and Avca (2021) used the nonlocal theory to analyze the free vibration characteristics of porous nanobeams using hyperbolic shear deformation beam theory. Considering the effect of Winkler-Pasternak foundation, Singh and Azam (2021) paid attention to the size-dependent vibrations of functionally graded nanoplates under hygrothermal effects by Rayleigh-Ritz method. Khosravi *et al*. (2020) discussed the size dependent axial forced vibrations of carbon nanotubes with the help of different rod models and nonlocal theory. Based on nonlocal theory, Malikan *et al*. (2019) studied the transient response of oscillated carbon nanotubes with an internal and external damping using Euler beam theory.

The nonlocal strain gradient theory (NSGT) has attracted extensive research interest in recent years because it can simultaneously explain the stiffness strengthening and stiffness softening effects of nanostructures. For example, Daikh and his partners (2021a) performed the static buckling analysis of curved sandwich nanobeams under the action of thermal loadings. With the help of NSGT, Daikh *et al*. (2021b) studied the static bending analysis of sigmoid functionally graded sandwich nanoplates with simply supported ends. Ebrahimi *et al*. (2020) utilized the Chebyshev-Ritz method and NSGT to analyze the static buckling and free vibration behaviors of nanobeams. Under the hypothesis of NSGT, Esen *et al*. (2022a) studied the free vibrations and buckling behaviors of functionally graded nanobeams under magnetic and thermal fields using the Timoshenko beam theory. Esen *et al*. (2022b) investigated the dynamic response of carbon nanotubes reinforced nanobeams within the framework of NSGT. Abdelrahman *et al*. (2021) analyzed the dynamic response of perforated nanobeams under the action of moving mass using the NSGT. On the basis of NSGT, Dai and Safarpour (2021), Dai *et al*. (2021) obtained the vibration frequency and thermal buckling loads of doubly curved nanoshells. Fenjan *et al*. (2020) employed the differential quadrature method to solve the functionally graded nanobeams. She *et al*. (2018) employed the Reddy' beam theory to discuss the phase velocity behaviors in functionally graded nanobeams. Esmaeilzadeh *et al*. (2021) studied the geometrically nonlinear thermo-mechanical bending of nanoplates using the Newmark technique. Shariati *et al*. (2020) studied the linear bending characteristics of smart magneto-electro-piezoelectric nanobeams system.

Micro structures and nano structure are not existed alone, its dynamic characteristics are influenced by the factors such as the surrounding medium (Safari *et al*. 2021, Timesli 2020), and sometimes, medium have crucial influence on the buckling, post-buckling, bending, vibration and wave propagation problems, ignoring these influences will lead to the fundamental error (Mousavi *et al*. 2021, Eyvazian *et al*. 2021). To characterize this, researchers began to study the mechanical properties of the double micro and nano scale structures. For example, Ebrahimi and Ali (2021) based on the Euler beam model to analyze the thermal-magneto-elastic waves in the double nanobeams, however, they used classical beam theory and did not consider shear deformation, besides, they do not consider the thermal effects. Jazi (2020) discussed the nonlinear vibrations of the double nanobeams under moving loadings, in their work, the Timosheko beam model is adopted, but they also do not consider thermal effects and they only discussed the vibration problems. Ghafarian *et al*. (2020) presented the vibration analysis of the rotating double axially functionally graded beams via the Euler beam theory and the nonlocal theory. Bahaadini *et al*. (2019) discussed the bucking problems of the double nanobeams under axial forces using the nonlocal theory. Ebrahimi and Ali (2018a) implemented the NSGT to model the wave propagation of double nanobeams under thermal-magnetically loadings, however, they only used the Euler beam model and they do not investigate the behaviors of the shear waves and longitudinal waves. Ebrahimi and Ali (2018b) also applied the NSGT to study the bending wave in the piezoelectric double nanobeams, this paper has the same issues that shear wave and longitudinal wave are not studied. In addition, the role of pores and thermal effects are not considered in this paper. Using the Timoshenko beam and Euler beam models, Zhou *et al*. (2018) studied the free vibration frequency characteristics of the double nanometer beams on an elastic medium. Rahmani *et al*. (2017) used the Euler beam theory to study the buckling properties of the double nanometer beams considering different boundary conditions. She *et al*. (2019) studied the bending wave in the double nanotubes with porosities subjected the Hygro-thermal loads, however, they did not study the characteristics of the shear wave and the longitudinal wave in the double nanotubes in their paper. Barati and Zenkour (2017) studied the bending wave of the double nanobeams, however, they only used the Euler beam model, and they did not study the shear wave and the longitudinal wave in the double nanobeams, besides, they did not consider the thermal effects in their paper.

In addition, the mechanical behavior of structures under thermal environment has also aroused the interest of researchers in recent years, Ding and She (2021) used a higher-order beam model for the thermal snap-buckling analysis of functionally graded pipes conveying hot fluid. She (2021) illustrated the effect of thermal loading on the guided wave propagation of porous functionally graded plates. She *et al*. (2022) discussed the wave propagation in a functionally graded circular plate via the physical neutral surface concept incorporating thermal effects. Taking thermal effects into account, Zhang and She (2022) investigated the wave propagations and vibrations of functionally pipes conveying hot fluid. Zhao *et al*. (2022) discussed the vibration behaviors of functionally graded carbon nanotubes reinforced double-beams in thermal environments. In addition, these years, She and his partners published a set of papers (Chen *et al*. 2022, Ding and She 2021, Ding *et al*. 2022a, 2022b, She 2021, She and Ding 2023, She *et al*. 2021, 2022, She and Li 2022, Lu *et al*. 2021, Xu and She 2022, Zhang *et al*. 2021, 2022, 2023, Zhang and She 2022, Zhao *et al*. 2022a, 2022b), which systematically studied the wave propagation, vibration and buckling of structures, which can provide reference and guidance for the preset study.

From what has been discussed above, it can be found that, there is no literature studying the wave propagation characteristics of double nanobeams in thermal environments, and there are no publishing papers investigating the shear wave and the longitudinal wave in the double nanobeams. Therefore, it is necessary to study this problem. In this paper, for the first time, we use Reddy beam theory to study the propagation characteristics of longitudinal wave, shear wave and bending wave in the double nanobeams, and also, we consider the influence of thermal effect. In addition, we also consider the influence of shear layer stiffness coefficient and linear layer stiffness coefficient on wave propagation. Our study finds that temperature, interlayer stiffness, shear layer stiffness coefficient and linear layer stiffness coefficient have very important effects on the problems.

2. Mathematical modeling

Illustrated in Fig. 1 is a functionally graded porous double nanobeams, the two constituents are metal and ceramic. "*m*" denotes the metal and "*c*" denotes the ceramics. The constant thickness is shown by h, L is the length. The material properties for the nanobeam I and the nanobeam II are the same, which are defined as (Hadji and Avcar 2021, Matouk *et al*. 2020, She *et al*. 2018)

$$P_f(z) = P_m + \left(\frac{z}{h} + \frac{1}{2}\right)^K (P_c - P_m) - \frac{\beta}{2}(P_c + P_m) \quad for \quad K \in [0, +\infty], \quad 0 \le \beta << 1 \tag{1}$$

in which, P can stand for μ (Poisson ratio), E (Young's modulus), ρ (mass density), α (thermal expansion coefficient), K denotes the power law index, β is the porosity volume fraction. Considering the different beam model for the beams, the displacement for the nanobeam I and the nanobeam II have the following form, as

$$U_1 = u_1 - z\frac{\partial w_1}{\partial x} + f(z)\left[\frac{\partial w_1}{\partial x} - \psi_1\right], \quad W_1 = w_1 \tag{2a}$$

$$U_2 = u_2 - z\frac{\partial w_2}{\partial x} + f(z)\left[\frac{\partial w_2}{\partial x} - \psi_2\right], \quad W_2 = w_2 \tag{2b}$$

with u_1 being the axial displacement, ψ_1 being the rotation, w_1 being the deflection for the nanobeam I, u_2 being the axial displacement, ψ_2 being the rotation, w_2 being the deflection for the nanobeam II. For Euler beam, f(z)=0; for Timoshenko beam, f(z)=z; for Reddy beam, f(z)= $z\left(1 - \frac{4z^2}{3h^2}\right)$. It is worth noting that Euler beam theory do not consider shear deformation, so the Euler beam theory is only valid for slender beams. Timoshenko beam theory considers shear deformation, but it needs the shear correction coefficient. The Reddy beam theory takes into account the shear deformation, but it does not need to introduce the shear correction coefficient. Therefore, compared with Euler beam and Timoshenko beam theories, Reddy beam theory has more advantages. Therefore, in the following dynamic modeling, we adopt the Reddy beam theory to derive motion equations.

Thus, the linear strain for the nanobeam I and the nanobeam II can be obtained as,

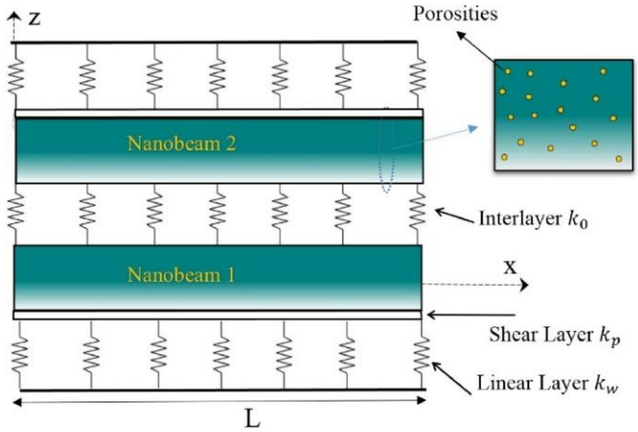


Fig. 1 The double nanobeams with porosities (from Barati and Zenkour 2017), Barati and Zenkour (2017) studied the bending wave of the double nanobeams, however, they only used the Euler beam model, and they did not study the shear wave and the longitudinal wave in the double nanobeams, besides, they did not consider the thermal effects in their paper

$$(\varepsilon_{xx})_1 = \frac{\partial u_1}{\partial x} - z\frac{\partial^2 w_1}{\partial x^2} + f(z)\left(\frac{\partial^2 w_1}{\partial x^2} - \frac{\partial \psi_1}{\partial x}\right), \quad (\gamma_{xz})_1 = \frac{df}{dz}\left(\frac{\partial w_1}{\partial x} - \psi_1\right). \tag{3a}$$

$$(\varepsilon_{xx})_2 = \frac{\partial u_2}{\partial x} - z\frac{\partial^2 w_2}{\partial x^2} + f(z)\left(\frac{\partial^2 w_2}{\partial x^2} - \frac{\partial \psi_2}{\partial x}\right), \quad (\gamma_{xz})_2 = \frac{df}{dz}\left(\frac{\partial w_2}{\partial x} - \psi_2\right). \tag{3b}$$

At this time, the variation of the total energy functional for the nanobeam I and the nanobeam II can be written as

$$\begin{aligned}
\delta\Pi = &\int_0^L \begin{bmatrix} N_1\frac{\partial \delta u_1}{\partial x} + (P_1 + Q_1 - M_1)\frac{\partial^2 \delta w_1}{\partial x^2} \\ +P_1\left(-\frac{\partial \delta\psi_1}{\partial x}\right) + Q_1(-\delta\psi_1) \end{bmatrix} dx \\
&+ \left[\begin{matrix} \left(N^{(1)}\right)_1\frac{\partial \delta u_1}{\partial x} + \left(\left(P^{(1)}\right)_1 + \left(Q^{(1)}\right)_1 - \left(M^{(1)}\right)_1\right)\frac{\partial^2 \delta w_1}{\partial x^2} \\ +\left(P^{(1)}\right)_1\left(-\frac{\partial \delta\psi_1}{\partial x}\right) + \left(Q^{(1)}\right)_1(-\delta\psi_1) \end{matrix}\right]\Bigg|_0^L \\
&- \int_0^L (N_T)_1\frac{\partial w_1}{\partial x}\delta\left(\frac{\partial w_1}{\partial x}\right)dx + \int_0^L [-K_c(w_1 - w_2)\delta w_1]\,dx \\
&+ \int_0^L K_p\frac{\partial w_1}{\partial x}\delta\left(\frac{\partial w_1}{\partial x}\right)dx + \int_0^L [K_w w_1 \delta w_1]\,dx \\
&+ \int_0^L \Bigg\{(I_0)_1\left[\frac{\partial u_1}{\partial t}\delta\left(\frac{\partial u_1}{\partial t}\right) + \frac{\partial w_1}{\partial t}\delta\left(\frac{\partial w_1}{\partial t}\right)\right] \\
&-(I_1)_1\left[\left(\frac{\partial^2 w_1}{\partial x\partial t}\right)\delta\left(\frac{\partial u_1}{\partial t}\right) + \left(\frac{\partial u_1}{\partial t}\right)\delta\left(\frac{\partial^2 w}{\partial x\partial t}\right)\right] + (I_2)_1\frac{\partial^2 w_1}{\partial x\partial t}\delta\left(\frac{\partial^2 w_1}{\partial x\partial t}\right) \\
&+(I_3)_1\begin{bmatrix} \left(\frac{\partial^2 w_1}{\partial x\partial t}\right)\delta\left(\frac{\partial u_1}{\partial t}\right) - \left(\frac{\partial \psi_1}{\partial t}\right)\delta\left(\frac{\partial u_1}{\partial t}\right) \\ +\left(\frac{\partial u_1}{\partial t}\right)\delta\left(\frac{\partial^2 w_1}{\partial x\partial t}\right) - \left(\frac{\partial u_1}{\partial t}\right)\delta\left(\frac{\partial \psi_1}{\partial t}\right) \end{bmatrix} \\
&+(I_4)_1\left[\left(\frac{\partial \psi_1}{\partial t}\right)\delta\left(\frac{\partial^2 w_1}{\partial x\partial t}\right) - 2\left(\frac{\partial^2 w_1}{\partial x\partial t}\right)\delta\left(\frac{\partial^2 w_1}{\partial x\partial t}\right) + \left(\frac{\partial^2 w_1}{\partial x\partial t}\right)\delta\left(\frac{\partial \psi_1}{\partial t}\right)\right] \\
&+(I_5)_1\begin{bmatrix} \left(\frac{\partial^2 w_1}{\partial x\partial t}\right)\delta\left(\frac{\partial^2 w_1}{\partial x\partial t}\right) - \left(\frac{\partial \psi_1}{\partial t}\right)\delta\left(\frac{\partial^2 w_1}{\partial x\partial t}\right) \\ -\left(\frac{\partial^2 w_1}{\partial x\partial t}\right)\delta\left(\frac{\partial \psi_1}{\partial t}\right) + \left(\frac{\partial \psi_1}{\partial t}\right)\delta\left(\frac{\partial \psi_1}{\partial t}\right) \end{bmatrix}\Bigg\}dx
\end{aligned} \tag{4a}$$

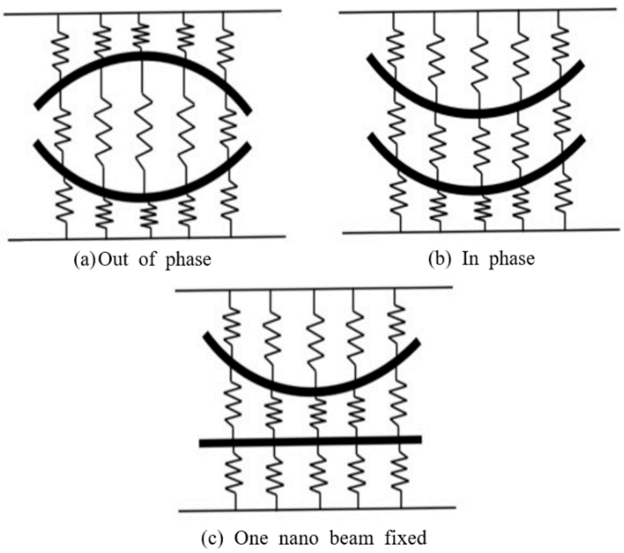


Fig. 2 Three types of motion for the double nanobeams (from Barati and Shahverdi 2017), (a) Out of phase; (b) In phase; (c) One nano beam fixed

$$
\begin{aligned}
\delta\prod = &\int_0^L \begin{bmatrix} N_2 \dfrac{\partial \delta u_2}{\partial x} - M_2 \dfrac{\partial^2 \delta w_2}{\partial x^2} \\ + P_2\left(\dfrac{\partial^2 \delta w_2}{\partial x^2} - \dfrac{\partial \delta \psi_2}{\partial x}\right) + Q_2\left(\dfrac{\partial \delta\psi_2}{\partial x} - \delta\psi_2\right) \end{bmatrix} dx \\
&+ \begin{bmatrix} \left(N^{(1)}\right)_2 \dfrac{\partial \delta u_2}{\partial x} - \left(M^{(1)}\right)_2 \dfrac{\partial^2 \delta w_2}{\partial x^2} \\ + \left(P^{(1)}\right)_2\left(\dfrac{\partial^2 \delta w_2}{\partial x^2} - \dfrac{\partial \delta \psi_2}{\partial x}\right) + \left(Q^{(1)}\right)_2\left(\dfrac{\partial \delta\psi_2}{\partial x} - \delta\psi_2\right) \end{bmatrix}_0^L \\
&- \int_0^L (N_T)_2 \frac{\partial w_2}{\partial x}\delta\left(\frac{\partial w_2}{\partial x}\right) dx + \int_0^L [K_c(w_1 - w_2)\delta w_2]\, dx \\
&+ \int_0^L K_p \frac{\partial w_2}{\partial x}\delta\left(\frac{\partial w_2}{\partial x}\right) dx + + \int_0^L [K_w w_2 \delta w_2]\, dx \\
&+ \int_0^L \Bigg\{ (I_0)_2 \left[\frac{\partial u_2}{\partial t}\delta\left(\frac{\partial u_2}{\partial t}\right) + \frac{\partial w_2}{\partial t}\delta\left(\frac{\partial w_2}{\partial t}\right)\right] \\
&- (I_1)_2 \left[\left(\frac{\partial^2 w_2}{\partial x \partial t}\right)\delta\left(\frac{\partial u_2}{\partial t}\right) + \left(\frac{\partial u_2}{\partial t}\right)\delta\left(\frac{\partial^2 w_2}{\partial x \partial t}\right)\right] \\
&+ (I_2)_2 \frac{\partial^2 w_2}{\partial x \partial t}\delta\left(\frac{\partial^2 w_2}{\partial x \partial t}\right) \\
&+ (I_3)_2 \begin{bmatrix} \left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right)\delta\left(\dfrac{\partial u_2}{\partial t}\right) - \left(\dfrac{\partial \psi_2}{\partial t}\right)\delta\left(\dfrac{\partial u_2}{\partial t}\right) \\ + \left(\dfrac{\partial u_2}{\partial t}\right)\delta\left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right) - \left(\dfrac{\partial u_2}{\partial t}\right)\delta\left(\dfrac{\partial \psi_2}{\partial t}\right) \end{bmatrix} \\
&+ (I_4)_2 \begin{bmatrix} \left(\dfrac{\partial \psi_2}{\partial t}\right)\delta\left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right) - 2\left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right)\delta\left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right) \\ + \left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right)\delta\left(\dfrac{\partial \psi_2}{\partial t}\right) \end{bmatrix} \\
&+ (I_5)_2 \begin{bmatrix} \left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right)\delta\left(\dfrac{\partial^2 w_1}{\partial x \partial t}\right) - \left(\dfrac{\partial \psi_2}{\partial t}\right)\delta\left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right) \\ - \left(\dfrac{\partial^2 w_2}{\partial x \partial t}\right)\delta\left(\dfrac{\partial \psi_2}{\partial t}\right) + \left(\dfrac{\partial \psi_2}{\partial t}\right)\delta\left(\dfrac{\partial \psi_2}{\partial t}\right) \end{bmatrix} \Bigg\} dx
\end{aligned} \tag{4b}
$$

Here, K_c is the stiffness of the connection between the nanobeam I and the nanobeam II, K_w is the linear elastic foundation, K_p is the shear doundation, other symbols are defined as follows,

$$
\begin{aligned}
& I_0 = (I_0)_1 = (I_0)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)\, dz, \\
& I_1 = (I_1)_1 = (I_1)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\rho(z)\, dz, \\
& I_2 = (I_2)_1 = (I_2)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\rho(z)\, dz, \\
& I_3 = (I_3)_1 = (I_3)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} f\rho(z)\, dz, \\
& I_4 = (I_4)_1 = (I_4)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} zf\rho(z)\, dz, \\
& I_5 = (I_5)_1 = (I_5)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} f^2\rho(z)\, dz, \\
& N = N_1 = N_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx} dz, M = M_1 = M_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_{xx} dz, \\
& P = P_1 = P_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} f\sigma_{xx} dz, Q = Q_1 = Q_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\partial f}{\partial z}\tau_{xz} dz, \\
& N_T = (N_T)_1 = (N_T)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z,T)\alpha\Delta T\, dz, \\
& N^{(1)} = \left(N^{(1)}\right)_1 = \left(N^{(1)}\right)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} {\sigma_{xx}}^{(1)} dz, \\
& M^{(1)} = \left(M^{(1)}\right)_1 = \left(M^{(1)}\right)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} z{\sigma_{xx}}^{(1)} dz, \\
& P^{(1)} = \left(P^{(1)}\right)_1 = \left(P^{(1)}\right)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} f{\sigma_{xx}}^{(1)} dz, \\
& Q^{(1)} = \left(Q^{(1)}\right)_1 = \left(Q^{(1)}\right)_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\partial f}{\partial z}{\tau_{xz}}^{(1)} dz.
\end{aligned} \tag{5}
$$

Since the variation of the total energy functional is equal to 0 in the time domain according to the Hamiltonian variational principle, we can obtain the following governing equation for the nanobeam I and the nanobeam II, as

$$
\begin{aligned}
& \frac{dN_1}{dx} = I_0\frac{\partial^2 u_1}{\partial t^2} - I_1\frac{\partial^3 w_1}{\partial x \partial t^2} + I_3\left(\frac{\partial^3 w_1}{\partial x \partial t^2} - \frac{\partial^2 \psi_1}{\partial t^2}\right), \\
& \frac{dP_1}{dx} - Q_1 = I_3\frac{\partial^2 u_1}{\partial t^2} - I_4\frac{\partial^3 w_1}{\partial x \partial t^2} + I_5\left(\frac{\partial^3 w_1}{\partial x \partial t^2} - \frac{\partial^2 \psi_1}{\partial t^2}\right), \\
& \frac{d^2 P_1}{dx^2} - \frac{d^2 M_1}{dx^2} - \frac{dQ_1}{dx} + K_c(w_1 - w_2) + K_w w_1 \\
& + \left(N_T - K_p\right)\frac{\partial^2 w_1}{\partial x^2} = -I_0\frac{\partial^2 w_1}{\partial t^2} - I_1\frac{\partial^3 u_1}{\partial x \partial t^2} + I_2\frac{\partial^4 w_1}{\partial x^2 \partial t^2} \\
& + I_3\frac{\partial^3 u_1}{\partial x \partial t^2} + I_4\left(\frac{\partial^3 \psi_1}{\partial x \partial t^2} - 2\frac{\partial^4 w_1}{\partial x^2 \partial t^2}\right) \\
& + I_5\left(\frac{\partial^4 w_1}{\partial x^2 \partial t^2} - \frac{\partial^3 \psi_1}{\partial x \partial t^2}\right).
\end{aligned} \tag{6a}
$$

$$
\begin{aligned}
& \frac{dN_2}{dx} = I_0\frac{\partial^2 u_2}{\partial t^2} - I_1\frac{\partial^3 w_2}{\partial x \partial t^2} + I_3\left(\frac{\partial^3 w_2}{\partial x \partial t^2} - \frac{\partial^2 \psi_2}{\partial t^2}\right), \\
& \frac{dP_2}{dx} - Q_2 = I_3\frac{\partial^2 u_2}{\partial t^2} - I_4\frac{\partial^3 w_2}{\partial x \partial t^2} + I_5\left(\frac{\partial^3 w_2}{\partial x \partial t^2} - \frac{\partial^2 \psi_2}{\partial t^2}\right), \\
& \frac{d^2 P_2}{dx^2} - \frac{d^2 M_2}{dx^2} - \frac{dQ_2}{dx} - K_c(w_1 - w_2) + K_w w_2 \\
& + \left(N_T - K_p\right)\frac{\partial^2 w_2}{\partial x^2} = -I_0\frac{\partial^2 w_2}{\partial t^2} - I_1\frac{\partial^3 u_2}{\partial x \partial t^2} + I_2\frac{\partial^4 w_2}{\partial x^2 \partial t^2} \\
& + I_3\frac{\partial^3 u_2}{\partial x \partial t^2} + I_4\left(\frac{\partial^3 \psi_2}{\partial x \partial t^2} - 2\frac{\partial^4 w_2}{\partial x^2 \partial t^2}\right)
\end{aligned} \tag{6b}
$$

$$+I_5\left(\frac{\partial^4 w_2}{\partial x^2\partial t^2}-\frac{\partial^3\psi_2}{\partial x\partial t^2}\right).$$

Using NSGT to describe the nanometer effect, which incorporate the nonlocal parameter *ea* and strain gradient parameter *l*, then we can get

$$N_1-(ea)^2\frac{\partial^2 N_1}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\times\left[A_{11}\frac{\partial u_1}{\partial x}-B_{11}\frac{\partial^2 w_1}{\partial x^2}+C_{11}\left(\frac{\partial^2 w_1}{\partial x^2}-\frac{\partial\psi_1}{\partial x}\right)-(N_T)_1\right],$$

$$M_1-(ea)^2\frac{\partial^2 M_1}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\times\left[B_{11}\frac{\partial u_1}{\partial x}-D_{11}\frac{\partial^2 w_1}{\partial x^2}+E_{11}\left(\frac{\partial^2 w_1}{\partial x^2}-\frac{\partial\psi_1}{\partial x}\right)-(M_T)_1\right],\quad (7a)$$

$$P_1-(ea)^2\frac{\partial^2 P_1}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\times\left[C_{11}\frac{\partial u_1}{\partial x}-E_{11}\frac{\partial^2 w_1}{\partial x^2}+H_{11}\left(\frac{\partial^2 w_1}{\partial x^2}-\frac{\partial\psi_1}{\partial x}\right)-(P_T)_1\right],$$

$$Q_1-(ea)^2\frac{\partial^2 Q_1}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)A_{55}\left(\psi_1-\frac{\partial w_1}{\partial x}\right).$$

$$N_2-(ea)^2\frac{\partial^2 N_2}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\times\left[A_{11}\frac{\partial u_2}{\partial x}-B_{11}\frac{\partial^2 w_2}{\partial x^2}+C_{11}\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial\psi_2}{\partial x}\right)-(N_T)_2\right],$$

$$M_2-(ea)^2\frac{\partial^2 M_2}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\times\left[B_{11}\frac{\partial u_2}{\partial x}-D_{11}\frac{\partial^2 w_2}{\partial x^2}+E_{11}\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial\psi_2}{\partial x}\right)-(M_T)_2\right],\quad (7b)$$

$$P_2-(ea)^2\frac{\partial^2 P_2}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\times\left[C_{11}\frac{\partial u_2}{\partial x}-E_{11}\frac{\partial^2 w_2}{\partial x^2}+H_{11}\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial\psi_2}{\partial x}\right)-(P_T)_2\right],$$

$$Q_2-(ea)^2\frac{\partial^2 Q_2}{\partial x^2}=\left(1-l^2\frac{\partial^2}{\partial x^2}\right)A_{55}\left(\psi_2-\frac{\partial w_2}{\partial x}\right).$$

Here,

$$[A_{11},B_{11},D_{11},C_{11},E_{11},H_{11},A_{55}]=\int_A E_f(z)\left[1,z,z^2,f,zf,f^2,\frac{1}{2[1+v_f(z)]}\left(\frac{df}{dz}\right)^2\right]dA,$$

$$M_T=(M_T)_1=(M_T)_2=\int_{-\frac{h}{2}}^{\frac{h}{2}} zE(z,T)\alpha\Delta T\,dz,\quad (8)$$

$$P_T=(P_T)_1=(P_T)_2=\int_{-\frac{h}{2}}^{\frac{h}{2}} fE(z,T)\alpha\Delta T\,dz.$$

Using Eq. (7) and Eq. (5), then Eq. (6) can be written as,

$$\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left[A_{11}\frac{\partial^2 u_1}{\partial x^2}-B_{11}\frac{\partial^3 w_1}{\partial x^3}+C_{11}\left(\frac{\partial^3 w_1}{\partial x^3}-\frac{\partial^2\psi_1}{\partial x^2}\right)\right]\quad (9a)$$
$$=\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[I_0\frac{\partial^2 u_1}{\partial t^2}-I_1\frac{\partial^3 w_1}{\partial x\partial t^2}+I_3\left(\frac{\partial^3 w_1}{\partial x\partial t^2}-\frac{\partial^2\psi_1}{\partial t^2}\right)\right],$$

$$\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left[\begin{gathered}C_{11}\frac{\partial^2 u_1}{\partial t^2}-E_{11}\frac{\partial^3 w_1}{\partial x^3}\\+H_{11}\left(\frac{\partial^3 w_1}{\partial x^3}-\frac{\partial^2\psi_1}{\partial x^2}\right)-A_{55}\left(\frac{\partial w_1}{\partial x}-\psi_1\right)\end{gathered}\right]=$$
$$\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[I_3\frac{\partial^2 u_1}{\partial t^2}-I_4\frac{\partial^3 w_1}{\partial x\partial t^2}+I_5\left(\frac{\partial^3 w_1}{\partial x\partial t^2}-\frac{\partial^2\psi_1}{\partial t^2}\right)\right],$$

$$\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left[\begin{gathered}C_{11}\frac{\partial^3 u_1}{\partial x^3}-E_{11}\frac{\partial^4 w_1}{\partial x^4}+H_{11}\left(\frac{\partial^4 w_1}{\partial x^4}-\frac{\partial^3\psi_1}{\partial x^3}\right)\\-B_{11}\frac{\partial^3 u_1}{\partial x^3}+D_{11}\frac{\partial^4 w_1}{\partial x^4}-E_{11}\left(\frac{\partial^4 w_1}{\partial x^4}-\frac{\partial^3\psi_1}{\partial x^3}\right)\\-A_{55}\left(\frac{\partial^2 w_1}{\partial x^2}-\frac{\partial\psi_1}{\partial x}\right)\end{gathered}\right]+\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\times\left(K_c(w_1-w_2)+K_w w_1+(N_T-K_p)\frac{\partial^2 w_1}{\partial x^2}\right)$$
$$=\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[-I_0\frac{\partial^2 w_1}{\partial t^2}-I_1\frac{\partial^3 u_1}{\partial x\partial t^2}+I_2\frac{\partial^4 w_1}{\partial x^2\partial t^2}+I_3\frac{\partial^3 u_1}{\partial x\partial t^2}-I_4\left(2\frac{\partial^4 w_1}{\partial x^2\partial t^2}-\frac{\partial^3\psi_1}{\partial x\partial t^2}\right)+I_5\left(\frac{\partial^4 w_1}{\partial x^2\partial t^2}-\frac{\partial^3\psi_1}{\partial x\partial t^2}\right)\right].$$

$$\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left[A_{11}\frac{\partial^2 u_2}{\partial x^2}-B_{11}\frac{\partial^3 w_2}{\partial x^3}+C_{11}\left(\frac{\partial^3 w_2}{\partial x^3}-\frac{\partial^2\psi_2}{\partial x^2}\right)\right]$$
$$=\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[I_0\frac{\partial^2 u_2}{\partial t^2}-I_1\frac{\partial^3 w_2}{\partial x\partial t^2}+I_3\left(\frac{\partial^3 w_2}{\partial x\partial t^2}-\frac{\partial^2\psi_2}{\partial t^2}\right)\right],$$

$$\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left[\begin{gathered}C_{11}\frac{\partial^2 u_2}{\partial t^2}-E_{11}\frac{\partial^3 w_2}{\partial x^3}\\+H_{11}\left(\frac{\partial^3 w_2}{\partial x^3}-\frac{\partial^2\psi_2}{\partial x^2}\right)-A_{55}\left(\frac{\partial w_2}{\partial x}-\psi_2\right)\end{gathered}\right]=$$
$$\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[\begin{gathered}I_3\frac{\partial^2 u_2}{\partial t^2}-I_4\frac{\partial^3 w_2}{\partial x\partial t^2}\\+I_5\left(\frac{\partial^3 w_2}{\partial x\partial t^2}-\frac{\partial^2\psi_2}{\partial t^2}\right)\end{gathered}\right],\quad (9b)$$

$$\left(1-l^2\frac{\partial^2}{\partial x^2}\right)\left[\begin{gathered}C_{11}\frac{\partial^3 u_2}{\partial x^3}-E_{11}\frac{\partial^4 w_2}{\partial x^4}+H_{11}\left(\frac{\partial^4 w_2}{\partial x^4}-\frac{\partial^3\psi_2}{\partial x^3}\right)\\-B_{11}\frac{\partial^3 u_2}{\partial x^3}+D_{11}\frac{\partial^4 w_2}{\partial x^4}-E_{11}\left(\frac{\partial^4 w_2}{\partial x^4}-\frac{\partial^3\psi_2}{\partial x^3}\right)\\-A_{55}\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial\psi_2}{\partial x}\right)\end{gathered}\right]+\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left(\begin{gathered}-K_c(w_1-w_2)+K_w w_2\\+(N_T-K_p)\frac{\partial^2 w_2}{\partial x^2}\end{gathered}\right)$$
$$=\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[-I_0\frac{\partial^2 w_2}{\partial t^2}-I_1\frac{\partial^3 u_2}{\partial x\partial t^2}+I_2\frac{\partial^4 w_2}{\partial x^2\partial t^2}+I_3\frac{\partial^3 u_2}{\partial x\partial t^2}-I_4\left(2\frac{\partial^4 w_2}{\partial x^2\partial t^2}-\frac{\partial^3\psi_2}{\partial x\partial t^2}\right)+I_5\left(\frac{\partial^4 w_2}{\partial x^2\partial t^2}-\frac{\partial^3\psi_2}{\partial x\partial t^2}\right)\right]$$

3. Solution

In this paper, we should consider three cases (See Fig. 2). The first case is called as "out of phase", which has $w = w_1 - w_2 \neq 0$. It is shown that the propagation characteristics of longitudinal wave, shear wave and bending wave in the nanobeam I and the nanobeam II are not identical. Subtract Eq. (9a) from Eq. (9b), and then we have

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[A_{11}\frac{\partial^2(u_1-u_2)}{\partial x^2} - B_{11}\frac{\partial^3(w_1-w_2)}{\partial x^3} + C_{11}\left(\frac{\partial^3(w_1-w_2)}{\partial x^3} - \frac{\partial^2(\psi_1-\psi_2)}{\partial x^2}\right)\right] = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right] \times \left[\begin{matrix} I_0\frac{\partial^2(u_1-u_2)}{\partial t^2} - I_1\frac{\partial^3(w_1-w_2)}{\partial x\partial t^2} \\ +I_3\left(\frac{\partial^3(w_1-w_2)}{\partial x\partial t^2} - \frac{\partial^2(\psi_1-\psi_2)}{\partial t^2}\right)\end{matrix}\right],$$

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[\begin{matrix} C_{11}\frac{\partial^2(u_1-u_2)}{\partial t^2} - E_{11}\frac{\partial^3(w_1-w_2)}{\partial x^3} \\ +H_{11}\left(\frac{\partial^3(w_1-w_2)}{\partial x^3} - \frac{\partial^2(\psi_1-\psi_2)}{\partial x^2}\right) \\ -A_{55}\left(\frac{\partial(w_1-w_2)}{\partial x} - (\psi_1-\psi_2)\right)\end{matrix}\right] = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[\begin{matrix} I_3\frac{\partial^2(u_1-u_2)}{\partial t^2} - I_4\frac{\partial^3(w_1-w_2)}{\partial x\partial t^2} \\ +I_5\left(\frac{\partial^3(w_1-w_2)}{\partial x\partial t^2} - \frac{\partial^2(\psi_1-\psi_2)}{\partial t^2}\right)\end{matrix}\right], \tag{10}$$

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[\begin{matrix} C_{11}\frac{\partial^3(u_1-u_2)}{\partial x^3} - E_{11}\frac{\partial^4(w_1-w_2)}{\partial x^4} \\ +H_{11}\left(\frac{\partial^4(w_1-w_2)}{\partial x^4} - \frac{\partial^3(\psi_1-\psi_2)}{\partial x^3}\right) \\ -B_{11}\frac{\partial^3(u_1-u_2)}{\partial x^3} + D_{11}\frac{\partial^4(w_1-w_2)}{\partial x^4} \\ -E_{11}\left(\frac{\partial^4(w_1-w_2)}{\partial x^4} - \frac{\partial^3(\psi_1-\psi_2)}{\partial x^3}\right)\end{matrix} -A_{55}\left(\frac{\partial^2(w_1-w_2)}{\partial x^2} - \frac{\partial(\psi_1-\psi_2)}{\partial x}\right)\right] + \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left(\begin{matrix}(2K_c+K_w)(w_1-w_2) \\ +(N_T-K_p)\frac{\partial^2(w_1-w_2)}{\partial x^2}\end{matrix}\right) = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[-I_0\frac{\partial^2(w_1-w_2)}{\partial t^2} - I_1\frac{\partial^3(u_1-u_2)}{\partial x\partial t^2} + I_2\frac{\partial^4(w_1-w_2)}{\partial x^2\partial t^2} + I_3\frac{\partial^3(u_1-u_2)}{\partial x\partial t^2} - I_4\left(2\frac{\partial^4(w_1-w_2)}{\partial x^2\partial t^2} - \frac{\partial^3(\psi_1-\psi_2)}{\partial x\partial t^2}\right) + I_5\left(\frac{\partial^4(w_1-w_2)}{\partial x^2\partial t^2} - \frac{\partial^3(\psi_1-\psi_2)}{\partial x\partial t^2}\right)\right].$$

For this case, we adopt the following harmonic series, that is,

$$(u_1-u_2) = U_m\, e^{[i(\kappa x - \omega t)]}, \quad (\psi_1-\psi_2) = \Phi_m\, e^{[i(\kappa x - \omega t)]}, \quad (w_1-w_2) = W_m\, e^{[i(\kappa x - \omega t)]}. \tag{11}$$

With U_m, Φ_m, and W_m being the wave amplitudes, κ being the wave number, ω being the circular frequency. Putting Eq. (11) into Eq. (10), we can find the expressions of the longitudinal wave $(u_1 - u_2)$, the shear wave $(\psi_1 - \psi_2)$ and the bending wave $(w_1 - w_2)$.

The second case is called as "in phase", in this case, $w_1 = w_2$, and the propagation characteristics of longitudinal wave, shear wave and bending wave in the nanobeam I and the nanobeam II are identical. In this case, the wave equations become

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[A_{11}\frac{\partial^2 u}{\partial x^2} - B_{11}\frac{\partial^3 w}{\partial x^3} + C_{11}\left(\frac{\partial^3 w}{\partial x^3} - \frac{\partial^2\psi}{\partial x^2}\right)\right] = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[I_0\frac{\partial^2 u}{\partial t^2} - I_1\frac{\partial^3 w}{\partial x\partial t^2} + I_3\left(\frac{\partial^3 w}{\partial x\partial t^2} - \frac{\partial^2\psi}{\partial t^2}\right)\right],$$

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[\begin{matrix} C_{11}\frac{\partial^2 u}{\partial t^2} - E_{11}\frac{\partial^3 w}{\partial x^3} + H_{11}\left(\frac{\partial^3 w}{\partial x^3} - \frac{\partial^2\psi}{\partial x^2}\right) \\ -A_{55}\left(\frac{\partial w}{\partial x} - (\psi_1 - \psi_2)\right)\end{matrix}\right] = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[I_3\frac{\partial^2 u}{\partial t^2} - I_4\frac{\partial^3 w}{\partial x\partial t^2} + I_5\left(\frac{\partial^3 w}{\partial x\partial t^2} - \frac{\partial^2\psi}{\partial t^2}\right)\right], \tag{12}$$

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[\begin{matrix} C_{11}\frac{\partial^3 u}{\partial x^3} - E_{11}\frac{\partial^4 w}{\partial x^4} + H_{11}\left(\frac{\partial^4 w}{\partial x^4} - \frac{\partial^3\psi}{\partial x^3}\right) \\ -B_{11}\frac{\partial^3 u}{\partial x^3} + D_{11}\frac{\partial^4 w}{\partial x^4} - E_{11}\left(\frac{\partial^4 w}{\partial x^4} - \frac{\partial^3\psi}{\partial x^3}\right)\end{matrix} -A_{55}\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial\psi}{\partial x}\right)\right] + \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left(K_w w + (N_T-K_p)\frac{\partial^2 w}{\partial x^2}\right) = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[-I_0\frac{\partial^2 w}{\partial t^2} - I_1\frac{\partial^3 u}{\partial x\partial t^2} + I_2\frac{\partial^4 w}{\partial x^2\partial t^2} + I_3\frac{\partial^3 u}{\partial x\partial t^2} - I_4\left(2\frac{\partial^4 w}{\partial x^2\partial t^2} - \frac{\partial^3\psi}{\partial x\partial t^2}\right) + I_5\left(\frac{\partial^4 w}{\partial x^2\partial t^2} - \frac{\partial^3\psi}{\partial x\partial t^2}\right)\right].$$

Obviously, in the present case, the effect of interlayer stiffness disappears, we use the following harmonic series,

$$u_1 = u_2 = U_m\, e^{[i(\kappa x - \omega t)]}, \quad \psi_1 = \psi_2 = \Phi_m\, e^{[i(\kappa x - \omega t)]}, \quad w_1 = w_2 = W_m\, e^{[i(\kappa x - \omega t)]}. \tag{13}$$

Putting Eq. (13) into Eq. (12), we also can find the expressions of longitudinal wave, the shear wave and the bending wave.

The third case is called as "one nanobeam fixed", we assume that the nanobeam II is fixed, then, the nanobeam II no longer propagate waves, that is, $w_2 = 0$, That is to say, only the nanobeam I has wave propagation, then the equations become

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[A_{11}\frac{\partial^2 u_1}{\partial x^2} + (C_{11}-B_{11})\frac{\partial^3 w_1}{\partial x^3} + C_{11}\left(-\frac{\partial^2\psi_1}{\partial x^2}\right)\right] = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[I_0\frac{\partial^2 u_1}{\partial t^2} + (I_3 - I_1)\frac{\partial^3 w_1}{\partial x\partial t^2} + I_3\left(-\frac{\partial^2\psi_1}{\partial t^2}\right)\right],$$

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[\begin{matrix} C_{11}\frac{\partial^2 u_1}{\partial t^2} + (H_{11}-E_{11})\frac{\partial^3 w_1}{\partial x^3} - H_{11}\left(\frac{\partial^2\psi_1}{\partial x^2}\right) \\ -A_{55}\left(\frac{\partial w_1}{\partial x} - \psi_1\right)\end{matrix}\right] = \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[I_3\frac{\partial^2 u_1}{\partial t^2} - I_4\frac{\partial^3 w_1}{\partial x\partial t^2} + I_5\left(\frac{\partial^3 w_1}{\partial x\partial t^2} - \frac{\partial^2\psi_1}{\partial t^2}\right)\right], \tag{8}$$

$$\left(1 - l^2 \frac{\partial^2}{\partial x^2}\right)\left[\begin{matrix} C_{11}\frac{\partial^3 u_1}{\partial x^3} - E_{11}\frac{\partial^4 w_1}{\partial x^4} + H_{11}\left(\frac{\partial^4 w_1}{\partial x^4} - \frac{\partial^3\psi_1}{\partial x^3}\right) \\ -B_{11}\frac{\partial^3 u_1}{\partial x^3} + D_{11}\frac{\partial^4 w_1}{\partial x^4} - E_{11}\left(\frac{\partial^4 w_1}{\partial x^4} - \frac{\partial^3\psi_1}{\partial x^3}\right)\end{matrix} -A_{55}\left(\frac{\partial^2 w_1}{\partial x^2} - \frac{\partial\psi_1}{\partial x}\right)\right] + \left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left(\begin{matrix}(K_c+K_w)w_1 \\ +(N_T-K_p)\frac{\partial^2 w_1}{\partial x^2}\end{matrix}\right)$$

$$=\left[1-(ea)^2\frac{\partial^2}{\partial x^2}\right]\left[-I_0\frac{\partial^2 w_1}{\partial t^2}-I_1\frac{\partial^3 u_1}{\partial x\partial t^2}+I_2\frac{\partial^4 w_1}{\partial x^2\partial t^2}\right.$$
$$\left.+I_3\frac{\partial^3 u_1}{\partial x\partial t^2}-I_4\left(2\frac{\partial^4 w_1}{\partial x^2\partial t^2}-\frac{\partial^3 \psi_1}{\partial x\partial t^2}\right)+I_5\left(\frac{\partial^4 w_1}{\partial x^2\partial t^2}-\frac{\partial^3 \psi_1}{\partial x\partial t^2}\right)\right]$$

In this case, we use the following harmonic series,

$$\begin{aligned} u_1 &= U_m\, e^{[i(\kappa x-\omega t)]}, \quad u_2 = 0,\\ \psi_1 &= \Phi_m\, e^{[i(\kappa x-\omega t)]}, \quad \psi_2 = 0,\\ w_1 &= W_m\, e^{[i(\kappa x-\omega t)]}, \quad w_2 = 0. \end{aligned} \tag{15}$$

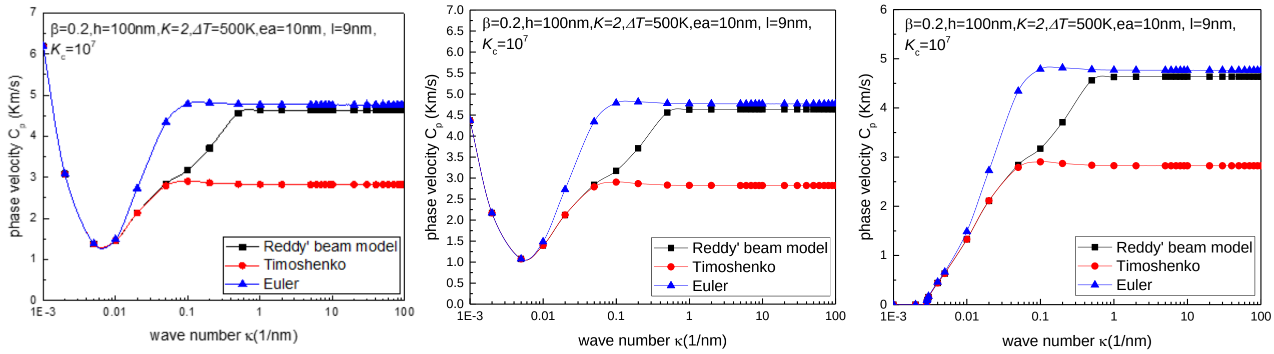
Putting Eq. (15) into Eq. (14), then we also can find the expressions of the longitudinal wave, the shear wave and the bending wave. Based on the differences between the mechanism of wave propagation problems, stress waves in the double nanobeams can be divided into the longitudinal wave, the shear wave and the bending wave.

4. Analysis and calculation

In the process of calculation, the materials used are Tri-silicon tetranitride (Si_3N_4) and stainless steel 304 (SUS304). Their material properties can be obtained from Reddy and Chin (1998). In addition, the correctness of this paper can be verified by the contrast study (See Fig. 3). It should be mentioned that in Figs. 3-8, K_w=K_p=0.

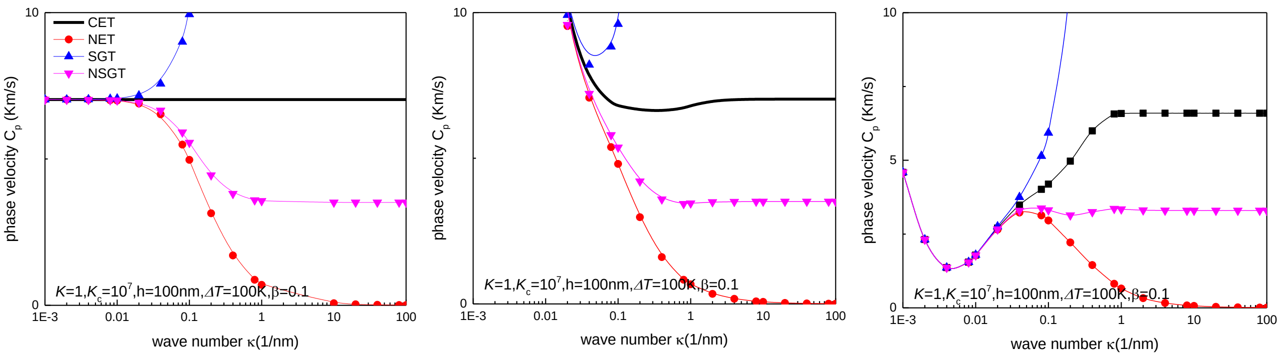
Illustrated in Fig. 4 shows the effects of nonlocal parameter and strain gradient parameter on the phase velocity of the double nanobeams. The acronyms CET (*ea*=*l*=0), NET(*ea*=10nm,*l*=0), SGT(*ea*=0,*l*=10nm), NSGT (*ea*=10nm,*l*=5nm) stand for classical continuum theory, nonlocal theory, strain gradient theory and nonlocal strain gradient theory respectively. It shows that, when the wave number is very small (e.g., κ<0.1 1/nm), the size parameters have little effect on the dispersion relation. In contrast, when the wave number is relatively large (e.g., κ>0.1 1/nm), the size parameters have remarkable effects on the dispersion relation. The second point, the dispersion relation of different wave propagation is different. Small scale parameters are only valid for wave propagation in the case of large wave number. Stiffness strengthening effect and stiffness softening effect can be clearly characterized by the values of *ea* and *l*. This shows that only when the wave number is relatively large, the size effect will play a role in the wave propagation of the nanostructures. If the wave number is relatively small, the size effect has very little influence on the wave propagation of nanostructures. In other words, the size effect only works when the wave number is relatively large.

Figs. 5-7 respectively study the influence of functionally graded index, temperature changes and porosity volume fraction on wave propagation. It can be seen that, as functionally graded index, temperature changes and porosity volume fraction rise, the stiffness of the beam will be reduced, thus leading to the decrease of phase velocity. This is because when the functionally graded index increases, the



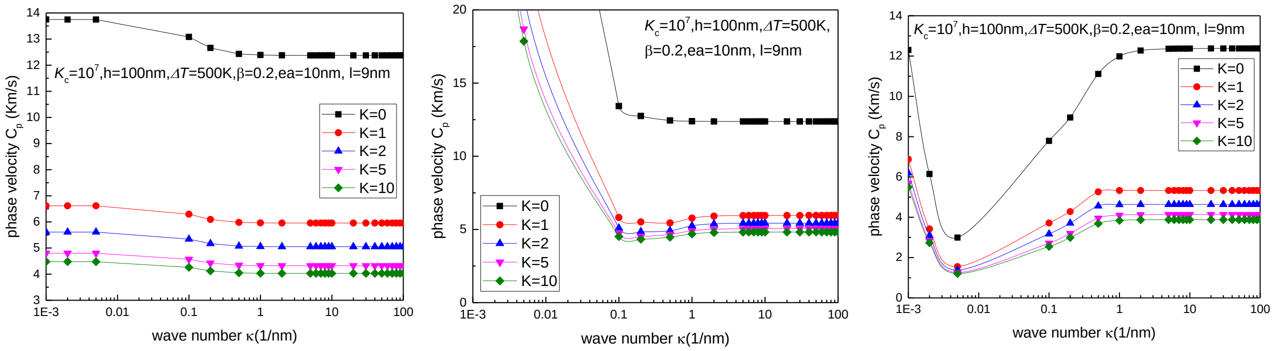
(a) out of phase (b) one nanobeam fixed (c)in phase

Fig. 3 Phase velocity of the bending wave for different beam models



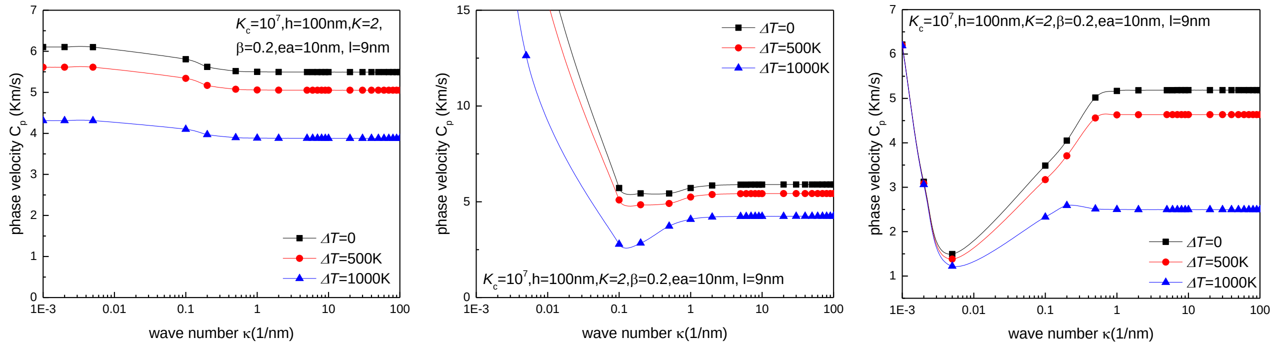
(a) longitudinal wave (b) shear wave (c) bending wave

Fig. 4 Wave propagation of the double nanobeams for "one nanobeam fixed": the size effect (for CET, *ea*=*l*=0, for NET *ea*=10nm, *l*=0, for SGT, *ea*=0, *l*=10nm, for NSGT *ea*=10nm, *l*=5nm)



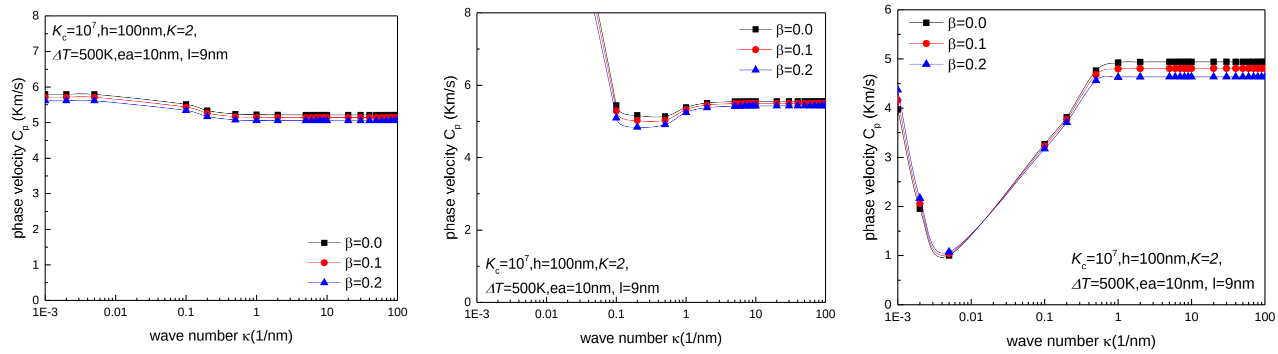
(a) longitudinal wave (b) shear wave (c) bending wave

Fig. 5 Wave propagation of the double nanobeams for "out of phase": the effect of power law index



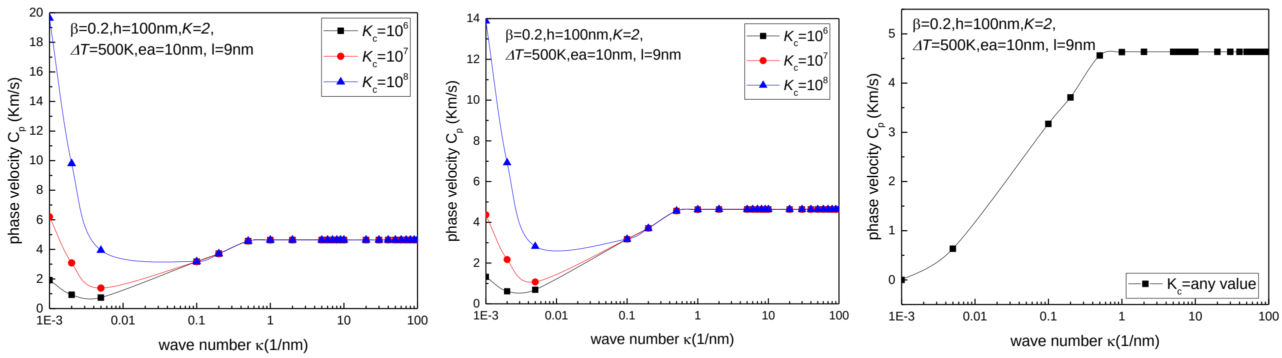
(a) longitudinal wave (b) shear wave (c) bending wave

Fig. 6 Wave propagation of the double nanobeams for "out of phase": the effect of temperature



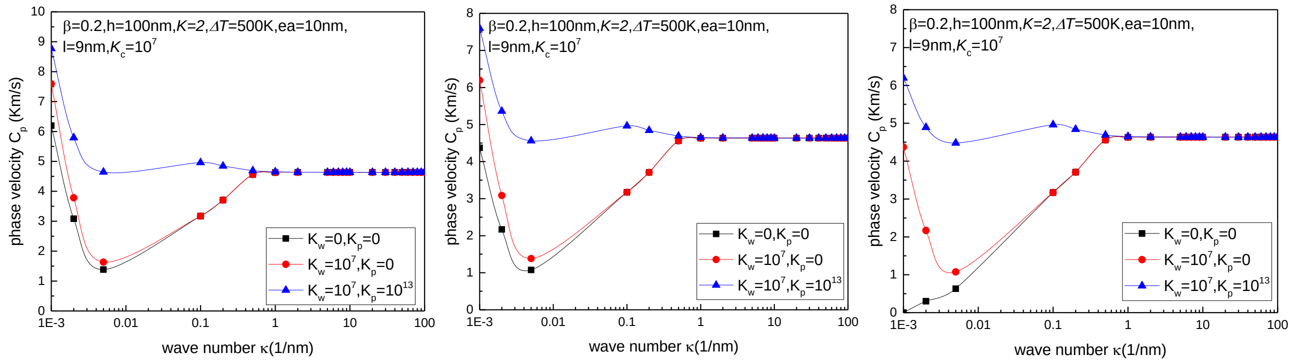
(a) longitudinal wave (b) shear wave (c) bending wave

Fig. 7 Wave propagation of the double nanobeams for "one nanobeam fixed": the effect of porosity



(a) out of phase (b) one nanobeam fixed (c) in phase

Fig. 8 The bending wave of the double nanobeams: the influence of interlayer stiffness, (a) for "out of phase"; (b) for "one nanobeam fixed", (c) for "in phase"



(a) out of phase (b) one nanobeam fixed (c) in phase

Fig. 9 The bending wave of the double nanobeams: the influence of the shear layer stiffness Kp and the linear stiffness Kw, (a) for "out of phase"; (b) for "one nanobeam fixed", (c) for "in phase"

volume fraction of ceramics in the nanobeam decreases, and the metal content increases, leading to a decrease in the stiffness of the structure. In addition, as the temperature rises, the thermal expansion effect will take place in the double nanobeams, which will lead to the corresponding thermal loadings, thus reducing the stiffness of the double nanobeams. Meanwhile, once the porosity volume fraction increases, the double nanobeams will contain more porosities, and the more porosities, the less rigid the double nanobeams will be. It is obvious that these three parameters change the phase velocity in the wave propagation problems by changing the stiffness coefficient of the doubly nanobeams.

As shown in Fig.8, interlayer stiffness K_c only works for the case of "out of phase" and "one nanobeam fixed", and the interlayer stiffness can improve the phase velocity of wave propagation, especially when the wave number is very small. For the case of "in phase", no matter how the interlayer stiffness varies, it has no effect on the relationship between phase velocity and wave number. As for the cause of this phenomenon, we can understand from Fig. 2 that for the case of "out of phase", the nanobeam I is bent upward and the nanobeam II is bent downward. Therefore, the deformation of the spring Kc is the largest, and the wave propagation velocity is more significant. For the case of "in phase", the nanobeam I and nanobeam II bend along the same direction. Therefore, it can be considered that the spring Kc has no deformation at this time. Therefore, no matter how the stiffness coefficient Kc changes, it has no effect on the wave propagation problems. For the case of "one nanobeam fixed", we can see that the deformation of spring Kc is between the case of "out of phase" and the case of "in phase". Obviously, in this case, only nanobeam I is deformed, due to the fact that the nanobeam II is fixed, so there is no deformation for the nanobeam II for the case of "one nanobeam fixed".

As shown in Fig. 9, both the shear layer stiffness K_p and the linear layer stiffness K_w can improve the phase velocity of the bending wave for "out of phase", "one nanobeam fixed" and "in phase". This is because the elastic coefficient can improve the rigidity of the double nanobeams.

5. Conclusions

Based on Reddy's high-order shear deformation theory, this paper studies the propagation characteristics of shear wave, bending wave and compressional wave in the double-nanobeams. At the same time, considering the influence of temperature, shear layer stiffness, linear stiffness and interlayer stiffness on wave propagation, the influence of different parameters on the problems are systematically analyzed and the following conclusions are drawn:

- Stiffness strengthening effect and stiffness softening effect can be changed by the values of *ea* and *l*.
- It seems that, the temperature rises, or the functionally graded index rises, or the porosity volume fraction rises will lead to the phase velocity go down.
- Both the shear layer stiffness and the linear stiffness can improve the phase velocity of the bending wave for the double nanobeams.

The interlayer stiffness has only effects on the cases of the "out of Phase" and "one nanobeam fixed". For "in phase", the interlayer stiffness coefficient has no effect, and the interlayer stiffness can improve the phase velocity more for the case of the "out of phase" compared with the case of "one nanobeam fixed".

Acknowledgement

The authors acknowledge this work is supported by the first-rate talent introduction project of Chongqing University (02090011044159).

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