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Shooting method applied to porous rotating disk: Darcy-Forchheimer flow of nanofluid

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Abstract. The characteristics of motile microorganism and three dimensional Darcy-Forchheimer nanofluid flow by a porous rotatable disk with heat generation/absorption is reported. Thermophoretic and Brownian motion aspects are included by utilizing Buongiorno model. Moreover, slip conditions are considered on velocity, thermal, concentration and microorganism. Shooting procedure is implemented to find the numerical results of physical quantities are evaluated parametrically. The different physical parameters like heat sink/source parameter, thermal, Brownian number, thermophoresis parameter, concentration, Peclet number, bioconvected Lewis number, microorganism on concentration and density of motile microorganism distributions is considered. Graphs of concentration and microorganism are plotted to examine the influence of distinct prominent flow parameters.

Keywords: bioconvected Lewis number; heat generation/absorption; microorganism; peclet number; shooting method

1. Introduction

Latest advancement in field of fluid dynamics has taken nanofluid under consideration which shows large thermal conductance and enlarges property of heat transformation in fluids. With quick advancement in technology, the challenges for increasing efficiency of energy exchanger and techniques for saving of energy are being met with the use of new materials. Moreover, conventional heat exchanger fluids could not meet exclusive conditions such as stable intensity of heat exchanger. Nanofluids contain the nano-size particles suspended in fluidic surface and the size of particles is less than 100nm. Nanoliquids have huge thermo-physical characteristics like viscidity, thermal conductivity and coefficient of heat exchanger in comparison to their base liquids. Thermal conductivity is basic feature of nanoliquids. Nanoliquids have great applications in different fields of technology and science. In addition, nanoliquids with special purpose are utilized in heat transfer devices, enhancing the efficiency of diesel generators, treatment of tumor, cooling/warming of home appliances and so on. The production of energetic nano-liquids with controlling micro framework is feasible because of chemical procedure. This is helpful to construct the micro framework which can make to control the chemical reactions that take place at a high speed and include a high degree of accuracy. Firstly, Wang 1988 studied the fluid behavior along the stretching cylinder. The detailed study of fluid flow along the stretched cylinder for the boundary layer was made (Ishak and Nazar 2009) regarding. Wang 2011 obtained asymptotic solutions for high Reynold number using slip flow condition. Mixed convection condition together with slip flow and obtained numerical solution for the boundary layer problem of Williamson fluid flow over a stretching cylinder (Salahuddin *et al*. 2017). Akbaş (2016a, b) studied the forced vibration analysis of a simple supported viscoelastic nanobeam based on modified couple stress theory (MCST). The nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. The elastic medium is considered as Winkler-Pasternak elastic foundation. The damping effect is considered by using the Kelvin–Voigt viscoelastic model. The cracked beam is modelled using a proper modification of the classical cracked-beam theory consisting of two sub-beams connected through a massless elastic rotational spring. Khan *et al*. (2022) investigated the free convection flow of Prabhakar fractional Jeffrey fluid on an oscillated vertical plate with homogenous heat flux. With the help of the Laplace transform and the Boussinesq's approximation, precise solutions for dimensionless momentum may be found.

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The temperature and velocity of Prabhakar fractional time free convection flows are compared to conventional thermal transport, as shown by Fourier's law. The effects of Soret and Dufour for the Casson fluid by considering the heat transfer along stretching cylinder was worked out (Mahdy, 2015). The mass and convective heat conditions for Casson fluid flow having nanoparticles along stretching cylinder was presented (Maria *et al.* 2016). A thorough numerical study of sisko fluid flow over stretching cylinder with effects of thermal conductivity and viscous dissipation was done (Malik *et al.*2016). Al-Maliki *et al.* (2020) carried out the dynamic analysis of functionally graded (FG) graphene-reinforced beams under thermal loading based on finite element approach. The presented formulation is based on a higher order refined beam element accounting for shear deformations. The graphene-reinforced beam is exposed to transverse periodic mechanical loading. Akbaş (2017a, b) investigated the free vibration analysis of edge cracked cantilever microscale beams composed of functionally graded material (FGM) based on the modified couple stress theory (MCST). The material properties of the beam are assumed to change in the height direction according to the exponential distribution. The FG nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. Mechanical properties of FG beam depends on the position. The Kelvin–Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory. Sun *et al.* (2022) explored the sequel to the biocompatibility, physico-chemical properties, high electrical conduction of copper nanoparticles over an inclined surface, little is known on the significance of nanoparticle radius, and inclined magnetic field on the dynamics of chemical reactive water conveying copper nanoparticles through a porous medium in the presence of Joule heating and spacing heating. In this case, there exists convective heating of the inclined surface due to thermal and significant concentration at the wall. The uniform suction/blowing effects together with transfer of heat outside the permeable stretching cylinder were considered (Ishaq *et al.*2008). Under convective boundary conditions, electrically conducting sisko fluid along the stretching cylinder in axial direction was probed (Khan and Malik 2015). They found the considerable boost in the flow parameters for shear thinning than thickening. The notable point about all the above mentioned studies is that the considered fluid is "Pure". Practically it is almost impossible to have such fluid which is free from any kind of impurity. Every naturally occurring fluid contains dust particles. Many engineering and industrial problems deal with dusty fluid such as powder mechanization and centrifugal technique to the detachment of particles from the fluid. Flow of dusty fluid can be viewed in many natural phenomena e.g. flow of mud in rivers, blood flow and atmospheric flow during haze. Initiative study of motion of dust particles in laminar flow has been carried out (Saffman 1962). An analysis for viscous, incompressible steady flow of dusty fluid flowing between two co-axial rotating cylinders under pressure gradient effect was carried out (Girishwar 1970). Akbaş (2018) investigated the forced vibration analysis of a cracked functionally graded microbeam using modified couple stress theory with damping effect. Mechanical properties of the functionally graded beam change vary along the thickness direction. The crack is modelled with a rotational spring. The Kelvin-Voigt model is considered in the damping effect. static bending of an edge cracked cantilever nanobeam composed of functionally graded material (FGM) subjected to transversal point load at the free end of the beam is investigated based on modified couple stress theory. Material properties of the beam change in the height direction according to exponential distributions. Dawar *et al.* (2022) presented the energy resources are among the essential objectives for the economic advancement of any developed country. Alternatively, fossil fuels that meet a large percentage of the world's energy needs are becoming increasingly rare and their quantity is dwindling. Solar systems which convert solar radiation into useable heat or electricity now play a vital part in the production of renewable energies. Akgoz and Civalek (2011) investigated geometrically the nonlinear free vibration analysis of thin laminated plates resting on non-linear elastic foundations. Winkler-Pasternak type foundation model is used. Governing equations of motions are obtained using the von Karman type nonlinear theory. The method of discrete singular convolution is used to obtain the discretised equations of motion of plates. Batou *et al.* (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. Sabu *et al.* (2021) proposed the numerical exploration of the hydromagnetic alumina–water nanoliquid flow due to a rotating rigid disk. The nanoliquid flow considering different nanoparticle shapes (namely sphere, platelet, cylinder, and brick) and the thermo-hydrodynamic slip constraints have been modeled utilizing the two-phase modified Buongiorno model (MBM). Baaskaran *et al.* (2018) studied the reliable and accurate method of computationally aided design processes of advanced thin walled structures in automotive industries for the efficient usage of smart materials, that possess higher energy absorption in dynamic compression loading. The most versatile components i.e., thin walled crash tubes with different geometrical profiles are introduced in view of mitigating the impact of varying cross section in crash behavior and energy absorption characteristics. Dusty gas flow in a region occupied by boundary layer was examined (Chakrabarti, 1974). The coefficient of friction and heat transfer for dusty boundary layer flow with pressure gradient was studied (Agranat 1988). Ramesh *et al.* (2022) studied the interaction of nanoparticles with fluids with onsiderable interest in the area of nanotechnology research. The purpose of this research is to see how a ternary nanofluid performs over a slippery surface. The energy equation is used to explain the heat source/sink effect. As a novel feature of the article, suction, slip effect, and convective boundary conditions are incorporated at the wall.

In addition to these studies for flow and transfer of heat for dusty fluid along sheet / surface, many researchers considered dusty fluid flow along cylinder. The viscous, incompressible gas flow having dust particles for an isothermal cylinder was discussed and results from various physical parameters were presented (Rebhi 2010). Chen *et al.* (2019a, b) carried the energy absorption characteristics of a lattice-web reinforced composite sandwich cylinder (LRCSC) which is composed of glass fiber reinforced polymer (GFRP) face sheets, GFRP lattice webs, polyurethane (PU) foam and ceramsite filler. The vortex-induced vibration of three circular cylinders (each of diameter D) in an equilateral triangular arrangement is investigated using the immersed boundary method. Abdulrazzaq *et al.* (2020) investigated the thermo-elastic buckling of small scale functionally graded material (FGM) nano-size plates with clamped edge conditions rested on an elastic substrate exposed to uniformly, linearly and non-linearly temperature distributions employing a secant function based refined theory. Material properties of the FGM nano-size plate have exponential gradation across the plate thickness. Civalek (2017) investigated the free vibration analysis of conical and cylindrical shells and annular plates made of composite laminated and functionally graded materials (FGMs). Carbon nanotubes reinforced (CNTR) composite case is also taken consideration for FGM. The equations of motion for conical shell are obtained via Hamilton's principle using the transverse shear deformation theory. Some valuable results regarding heat transfer of dusty fluid over a hollow stretching cylinder using multi-step DTM were reported (Rasekh *et al.* 2013). Conduction of dusty fluid flow along stretching cylinder with thermal conductivity and viscosity effects were dealt numerically (Konch and Hazarika, 2017). Derakhshandeh1a *et al.* (2020) investigated the Reynolds number Re (= 50-200) effects on the flows around a single cylinder and the two tandem (center-to-center spacing $L*= L/D= 4$) cylinders, each of a diameter D. Vorticity structures, Strouhal numbers, and time-mean and fluctuating forces are presented and discussed. Salah *et al.* (2019) employed a simple four-variable integral plate theory for examining the thermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. In some fresh attempts, the researchers have pondered over new dimensions of stretching i.e., exponentially stretching cylinder. The detailed study of flow and transfer of heat for hyperbolic tangent fluid over a stretching cylinder exponentially in vertical direction was carried out (Naseer *et al.* 2014). Shadravan *et al.* (2019) performed lateral load testing on seventeen wood wall frames in two sections. Section one included eight tests studying structural foam sheathing of shear walls subjected to monotonic loads following the ASTM E564 test method. Similarity solution has been derived for steady boundary layer and heat flow of Casson nanofluid (Malik *et al.* 2013) while cylinder was stretching exponentially along its radius. The flow of Micropolar fluid through vertical exponentially stretching cylinder along the axial direction and discussed heat transfer effects, too, were considered (Rehman *et al.* 2015). Williamson fluid flow along an exponentially stretching cylinder was examined and they found its numerical solution (Iqbal *et al.* 2018). Recently some researcher used different methods for nonlinear modeling (Eltaher *et al.* 2019, Ebrahimi *et al.* 2019, Safaei *et al.* 2019, Shahsavari *et al.* 2019, Benmansour *et al.* 2019).

All the above mentioned studies give us the incentive to work on a new research thought i.e. effects of of heat source/sink for Darcy-Forchheimer three dimensional nanofluid flow with gyrotactic microorganism by rotatable disk via porous media. No such investigation is reported till date. Shooting method is a tool for its numerical achievements. Many physical and mathematical problems yield highly non-linear differential equations and their exact results are not commonly possible. To evaluate such type of equations numerically, one of the powerful method is to obtained the solution of such problems is shooting technique. This technique is simple, elegant and without any difficult discretization approach. One of the exclusive quality of this approach is that missing boundary value conditions can be started by utilizing smart initial guesses. For better accuracy of solution convergence is investigated by utilizing this method. Impact of relevant physical parameters, like like heat sink/source parameter, thermal, Brownian number, thermophoresis parameter, concentration, Peclet number, bioconvected Lewis number, microorganism are studied & shown through plots.

2. Mathematical modeling

We investigate 3D steady Darcy-Forchheimer flow of viscous nanofluid through a porous rotatable disk with heat sink/source and motile microorganism. Further slips under deliberation are velocities, concentration, heat and microorganisms. At $z = 0$ disks is rotatable with constant angular velocity Ω. The influences of thermophoretic and Brownian diffusion are accounted for. Here (u, v, w) are the velocities in the directions of increasing (r, ϕ, z) separately.

The governing boundary layer PDE's are

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\begin{aligned} & u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} \\ & = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\nu}{k^*}u - Fu^2 \\ & + \frac{1}{\rho_f}\begin{bmatrix} \rho_f \beta g(1 - C_\infty)(T - T_\infty) - g(\rho_p - \rho_f)(C - C_\infty) \\ -\gamma g(\rho_m - \rho_f)(n - n_\infty) \end{bmatrix} \end{aligned} \tag{2}$$

$$\begin{aligned} & u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} \\ & = \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{\nu}{k^*}v - Fv^2 \end{aligned} \tag{3}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{\nu}{k^*}w - Fw^2 \tag{4}$$

$$\begin{aligned}&u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\\&=\alpha_m{}^{*}\left(\frac{\partial^2T}{\partial z^2}+\frac{\partial^2T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\right)+\frac{Q_\circ}{(\rho c)_f}(T-T_\infty)\\&+\frac{(\rho c)_p}{(\rho c)_f}\left(D_B\left(\frac{\partial T}{\partial z}\frac{\partial C}{\partial z}+\frac{\partial T}{\partial r}\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial z}\right)^2+\left(\frac{\partial T}{\partial r}\right)^2\right)\right)\end{aligned}\tag{5}$$

$$\begin{aligned}&u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}\\&=D_B\left(\frac{\partial^2C}{\partial z^2}+\frac{\partial^2C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2T}{\partial z^2}+\frac{\partial^2T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)\end{aligned}\tag{6}$$

$$\begin{aligned}&u\frac{\partial n}{\partial r}+w\frac{\partial n}{\partial z}+\frac{bW_c}{C_\infty}\left[\frac{\partial}{\partial z}\left(n\frac{\partial C}{\partial z}\right)\right]\\&=D_m{}^{*}\left(\frac{\partial^2n}{\partial z^2}+\frac{\partial^2n}{\partial r^2}+\frac{1}{r}\frac{\partial n}{\partial r}\right)\end{aligned}\tag{7}$$

The mathematical model is subjected to following boundary conditions

$$\begin{gathered}u=L_1\frac{\partial u}{\partial z},w=0,v=r\Omega+L_1'\frac{\partial v}{\partial z},\\T=T_w+L_2'\frac{\partial T}{\partial z},C=C_w+L_3'\frac{\partial C}{\partial z},n=n_w+L_4'\frac{\partial n}{\partial z}\\\text{when}\quad z=0\end{gathered}\tag{8}$$

$$u\to0,v\to0,T\to T_\infty,C\to C_\infty,n\to n_\infty\quad\text{when}\ z\to\infty\tag{9}$$

Velocity components are u,v and w along the directions of r,ϕ and z respectively, $v=\frac{\mu}{\rho_f}$ depict kinematic viscidity, ρ_f the density of base liquid, μ is dynamic viscidity, k^* denotes permeability of porous media, $\alpha_m{}^{*}=\frac{k}{(\rho c)_f}$ stands thermal diffusivity, k the thermal conductivity, $(\rho c)_f$ is liquid capacity of heat , $(\rho c)_p$ is effective heat capacity of nano size particles, $F=\frac{c_b}{r}k'^{\frac{1}{2}}$ is non-uniform inertia parameter, T represents temperature, D_B the Brownian coefficient, D_T is thermophoretic factor, C the nano size particles concentration, $Q_\circ$ is heat source/sink factor, $D_m{}^{*}$ is microorganism diffusivity factor, W_c symbolizes the maximum swimming speed, κ is Boltzmann constant, $C_w,C_\infty,T_w,T_\infty,n_w$ and n_∞ denotes the concentration, heat and motile microorganisms which exist at and far away from the surface. L_1,L_1',L_2',L_3',L_4' denotes the velocities, thermal, concentration and micro-organism slip coefficients.

We introduce the following variables:

$$\begin{gathered}u=r\Omega f'(\zeta),w=-(2\Omega\nu)^{\frac{1}{2}}f(\zeta),\\v=r\Omega g(\zeta),\zeta=\left(\frac{2\Omega}{\nu}\right)^{\frac{1}{2}}z\\\theta(\zeta)=\frac{T-T_\infty}{T_w-T_\infty},\phi(\zeta)=\frac{C-C_\infty}{C_w-C_\infty},\chi(\zeta)=\frac{n-n_\infty}{n_w-n_\infty}\end{gathered}\tag{10}$$

where $\zeta,f'(\zeta),g(\zeta),\theta(\zeta),\phi(\zeta)$ and $\chi(\zeta)$ depict the similarity variable, dimensionless velocities, non-dimensional temperature, dimensionless concentration and non-dimensional micro-organisms function. Eq (1) is trivially holds by Eq (10), while by applying Eq. (10) into Eqs. (2)-(9) yield

$$\begin{aligned}&2f'''-\lambda f'+2ff''-f'^2+g^2-Frf'^2+\\&\Gamma(\theta-Nr\phi-Nc\chi)=0\end{aligned}\tag{11}$$

$$2g''-\lambda g+2fg'-2f'g-Frg^2=0\tag{12}$$

$$\frac{1}{Pr''''''2}\tag{13}$$

$$\phi''+scf\phi'+\frac{Nt}{Nb}\theta''=0\tag{14}$$

$$\chi''+Lbf\chi'-Pe(\phi''(\chi+\omega)+\chi'\phi')=0\tag{15}$$

$$\begin{aligned}&f(0)=0,f'(0)=\alpha f''(0),g(0)=1+\alpha g'(0),\\&\theta(0)=1+\beta\theta'(0)\\&\phi(0)=1+\gamma\phi'(0),\chi(0)=1+\delta\chi'(0)\end{aligned}\tag{16}$$

$$\begin{aligned}&f'(\infty)\to0,g(\infty)\to0,\theta(\infty)\to0,\\&\phi(\infty)\to0,\chi(\infty)\to0\end{aligned}\tag{17}$$

where $Pr=\frac{\upsilon}{\alpha_m{}^{*}}$ Prandtl number, $\lambda=\frac{\upsilon_f}{\Omega k^*}$ is porosity factor, $Nb=\frac{D_B(\rho c)_p(C_w-C_\infty)}{(\rho c)_f\upsilon}$ is Brownian movement, $Nt=\frac{(\rho c)_pD_T(T_w-T_\infty)}{(\rho c)_f\nu T_\infty}$ thermophoresis coefficient , $\sigma=\frac{Q_\circ}{2\Omega(\rho c)_f}$ heat sink/source factor, $Sc=\frac{\nu}{D_B}$ Schmidt number, $Lb=\frac{\alpha}{D_m{}^{*}}$ is Lewis number, $Pe=\frac{bW_c}{\nu}$ is Peclet number, $Fr=\frac{c_b}{k'^{\frac{1}{2}}}$ is Forchheimer number, $\Gamma=\frac{g\beta(1-C_\infty)(T_w-T_\infty)}{r\Omega^2}$ is mixed convection factor, $Nr=\frac{(\rho_p-\rho_f)(C_w-C_\infty)}{\beta(1-C_\infty)(T_w-T_\infty)}$ Buoyancy ratio coefficient, $Nc=\frac{\gamma(\rho_m-\rho_f)(n_w-n_\infty)}{\beta(1-C_\infty)(T_w-T_\infty)}$ bio-convection Rayleigh number. Velocities, thermal, concentration microorganism slip parameters are $\alpha=L_1'\left(\frac{2\Omega}{\nu}\right)^{\frac{1}{2}},\beta=L_2'\left(\frac{2\Omega}{\nu}\right)^{\frac{1}{2}},\gamma=L_3'\left(\frac{2\Omega}{\nu}\right)^{\frac{1}{2}}$ and $\delta=L_4'\left(\frac{2\Omega}{\nu}\right)^{\frac{1}{2}}$ respectively. The dimensionless forms of skin frictions coefficients $\sqrt{Re_r}\ C_f=f''(0),\sqrt{Re_r}\ C_g=g'(0)$, Nusselt number $\frac{Nu}{\sqrt{Re_r}}=-\theta'(0)$, Sherwood number $\frac{Sh}{\sqrt{Re_r}}=-\phi'(0)$ and motile number $\frac{N_n}{\sqrt{Re_r}}=-\chi'(0)$, where $Re_r=\frac{2r(r\Omega)}{\nu}$ represents the Reynolds number.

In order to utilize the shooting method, the system of first order differential equations is attained:

$$\begin{aligned}&y_1'=y_2\\&y_2'=y_3\\&y_3{}'=0.5\left[y_2{}^2-2y_1y_3-y_4{}^2+\lambda y_2+Fry_2{}^2-\Gamma\left(y_6-Nry_8-Ncy_{10}\right)\right]\\&y_4'=y_5\\&y_5'=0.5\left[\lambda y_4-2y_1y_5+2y_2y_4+Fry_4{}^2\right]\\&y_6'=y_7\\&y_7'=-\Pr\left[y_1y_7+Nby_7y_9+Nty_7{}^2+\sigma y_6\right]\\&y_8'=y_9\\&y_9'=-Scy_1y_9-\left(\frac{Nt}{Nb}y_7'\right)\\&y_{10}'=y_{11}\\&y_{11}'=-Lby_1y_{11}+Pe\left[\left(y_{10}+\omega\right)y_9'+y_{11}y_9\right]\end{aligned}\tag{18}$$

Table 1 Numerical data of $-\chi'(0)$ for varying different parameters

Fr	λ	Lb	β	γ	Nt	Nb	ω	Pr	Pe	$-\chi'(0)$
0.0										0.3918
0.4	0.2	1.0	0.3	0.3	0.1	0.3	1.0	1.0	1.0	0.3708
0.8										0.3528
	0.1									0.4036
0.2	0.4									0.3369
	1.0									0.2270
		2.0								0.5092
		3.0								0.5831
		4.0								0.6395
			0.1							0.3816
			0.5							0.3801
			1.0							0.3785
				0.1						0.3901
				0.5						0.3724
				1.0						0.3543
					0.0					0.4109
					0.4					0.2927
					0.8					0.1958
						0.4				0.3914
						0.8				0.4089
						1.0				0.4132
							0.2			0.4171
							0.5			0.5275
							1.0			0.7162
								2.0		0.3490
								3.0		0.3307
								5.0		0.3097
									2.0	0.5384
									3.0	0.6758
									4.0	0.8037

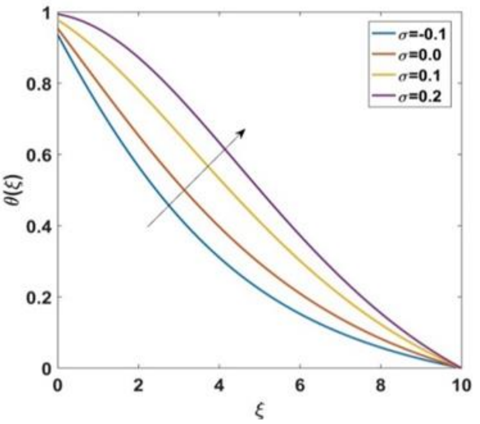

Fig. 1 Sketch of σ on $\theta(\zeta)$

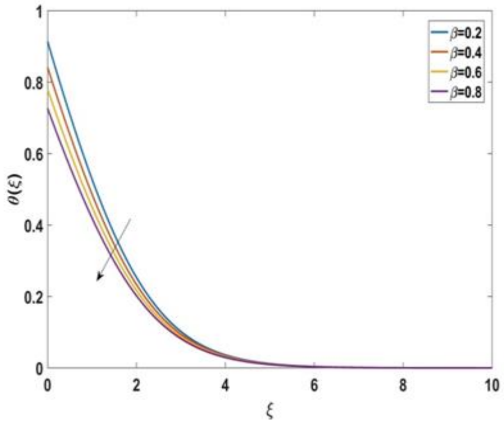

Fig. 2 Sketch of β on $\theta(\zeta)$

Subject to the non-dimensional boundary conditions are

$$\begin{gathered} y_1(0)=0, y_2(0)=\alpha y_3(0), \\ y_4(0)=1+\alpha y_5(0), y_6(0)=1+\beta y_7(0), \\ y_8(0)=1+\gamma y_9(0), y_{10}(0)=1+\delta y_{11}(0) \\ \text{as} \quad \zeta \to 0 \end{gathered} \tag{19}$$

$$\begin{gathered} y_2(\infty)\to 0, y_4(\infty)\to 0, y_6(\infty)\to 0, \\ y_8(\infty)\to 0, y_{10}(\infty)\to 0 \text{ as } \quad \zeta\to\infty \end{gathered} \tag{20}$$

For the residual of continuous outcomes, the faults control and mesh selection points are utilized for all the calculations.

3. Analytical procedure

The presentation of this section is to high light commitment of different physical parameters like heat sink/source parameter, β thermal, Brownian number Nb , thermophoresis parameter Nt, γ concentration, Peclet number Pe, bioconvected Lewis number Lb , δ micro-organism on concentration ϕ and density of motile micro-organism χ distributions. Table 1 present data of motile microorganism density for varying $Fr, \lambda, Nb, Nt, \beta, Sc,$ Pr, Pe, Lb, ω and γ. Impact of heat generation σ on temperature θ is displayed in Fig. 1. We noted that by increasing the values of σ, temperature θ and thermal layer

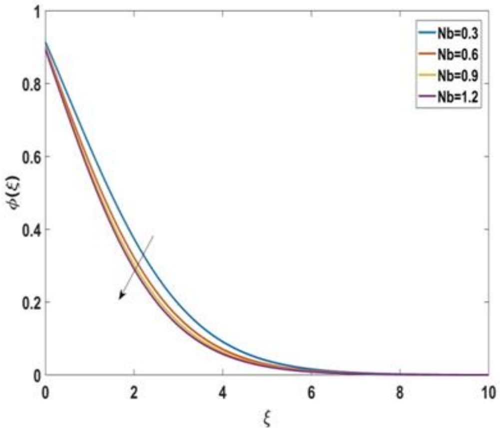

Fig. 3 Sketch of $\boldsymbol{Nb}$ on $\phi(\zeta)$

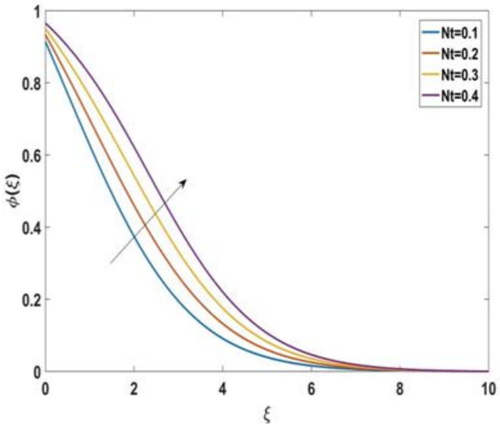

Fig. 4 Sketch of Nt on $\phi(\zeta)$

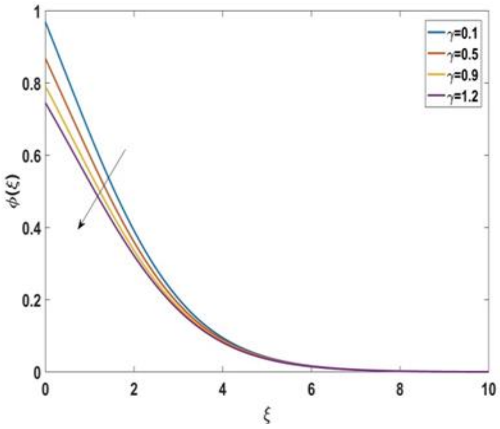

Fig. 5 Sketch of γ on $\phi(\zeta)$

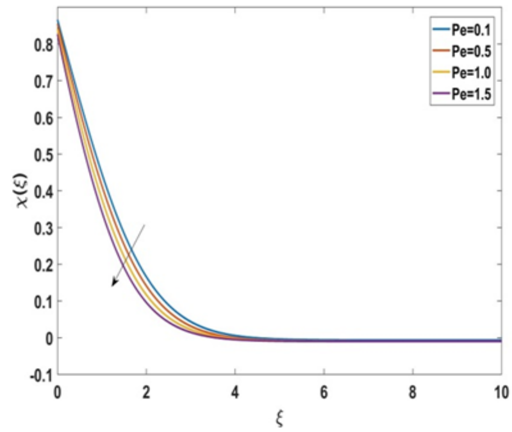

Fig. 6 Sketch of Pe on $\chi(\zeta)$

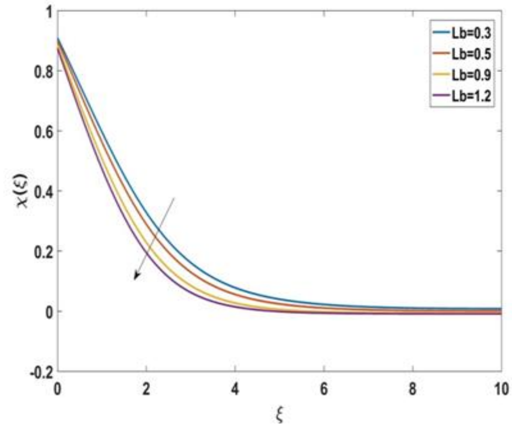

Fig. 7 Sketch of Lb on $\chi(\zeta)$

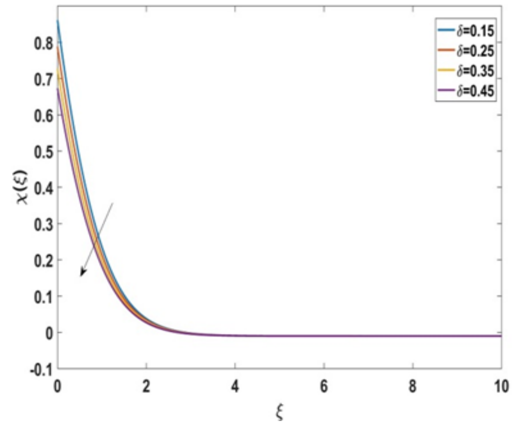

Fig. 8 Sketch of δ on $\chi(\zeta)$

shows expanding behavior. Fig. 2 demonstrates the differing of thermal slip coefficient β when utilized on temperature θ. Higher estimations of β, both the temperature θ and thickness of related thermal layer shows diminishing trend. The impact of Brownian motion Nb is displayed on concentration ϕ in Fig. 3. For larger estimations of Nb both the concentration ϕ and thickness of related concentration layer are degraded. We noted physically, a Brownian endeavor to drag the particles to the other direction of the concentration slop, nanoliquid becomes much homogeneous. After that higher concentration field, weaker the concentration inclination as greater the Brownian movement. Fig. 4 is prepared to check the curves of concentration ϕ for distinct values of Nt. It is clearly observed that for higher thermophoretic parameter implies the larger curves of concentration ϕ. For this purpose the nanoparticles are blend in base fluid according to modern science. This phenomenon is occur because migration of nanoparticles from hotter to colder surface far away from the surface. Fig. 5 represents the effect of solutal slip parameterγ on concentration ϕ. It is visualized that for distinct values of γ exhibit diminishing behavior for concentration ϕ. The influence of Peclet number Pe is shown on motile microorganism in Fig. 6. It is noted that for distinct values of Pe, microorganism χ decreases. Reason behind this enhancement in Peyields the increment in cell- swimming speed which cause the diminution in gyro-tactic micro-organisms thickness. Fig. 7 shows the impact of bio-convected Lewis number Lb on micro-organism χ. The decreasing behavior of χ is noted for larger values of Lb. This is due to less diffusivity of gyro-tactic micro-organisms. The less diffusivity takes place when values of Lb is increases as a result microorganism χ is retarded. The microorganism slip constraint δ produce weaker microorganism χ as displayed in Fig. 8.

4. Conclusions

Nanoliquids have acquired enormous importance in the current years because of their potential uses, not only as an enhancement of energy transfer but also their immense attention in applications like recovery of fuel and drug delivery. Therefore, the current communication scrutinized the 3D Darcy Forchheimer nanofluid flow due to porous rotatable disk with gyrotactic microorganism and heat source/sink. Thermophoretic and Brownian motion along mass and transfer of heat are also considered. Slip conditions of velocities, temperature, mass and micro-organism density are incorporated. For non-dimensional, we use the appropriate variables to handle the modeled equations. A shooting scheme is imposed to obtain the approximate solutions. The reduction is observed for larger like heat sink/source parameter, β thermal, Brownian number Nb, thermophoresis parameter Nt, γ concentration, Peclet number Pe, bioconvected Lewis number Lb, δ microorganism on concentration ϕ and density of motile microorganism χ distributions.

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