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Bending of axially functionally graded carbon nanotubes reinforced composite nanobeams

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Abstract. This work presents a modified analytical model for the bending behavior of axially functionally graded (AFG) carbon nanotubes reinforced composite (CNTRC) nanobeams. New higher order shear deformation beam theory is exploited to satisfy parabolic variation of shear through thickness direction and zero shears at the bottom and top surfaces.A Modified continuum nonlocal strain gradient theoryis employed to include the microstructure and the geometrical nano-size length scales. The extended rule of the mixture and the molecular dynamics simulations are exploited to evaluate the equivalent mechanical properties of FG-CNTRC beams. Carbon nanotubes reinforcements are distributed axially through the beam length direction with a new power graded function with two parameters. The equilibrium equations are derived with associated nonclassical boundary conditions, and Navier's procedure are used to solve the obtained differential equation and get the response of nanobeam under uniform, linear, or sinusoidal mechanical loadings. Numerical results are carried out to investigate the impact of inhomogeneity parameters, geometrical parameters, loadings type, nonlocal and length scale parameters on deflections and stresses of the AFG CNTRC nanobeams. The proposed model can be used in the design and analysis of MEMS and NEMS systems fabricated from carbon nanotubes reinforced composite nanobeam.
Keywords: analytical solution; axially CNts distribution; new higher order shear deformation; nonlocal strain gradient theory; static bending and stress analyses

1. Introduction

Nowadays, Carbon nanotubes (CNTs) are proposed as the most candidate for reinforcement material for composites due to their remarkable mechanical, electrical and thermal characteristics such as high tensile modulus, high strength, low density, good conductivity and can sustain large elastic strain (Eltaher *et al*. 2020a, 2016, Mohamed *et al*. 2020, Wu *et al*. 2016, Daikh *et al* 2021a). Sometimes, instead of distributing CNTs in a uniform manner, they may be graded along a specific direction by different gradation functions. In this case, they are commonly called dimensions. In contrast to classical continuum theory, the stress of nonlocal theory at a reference point accounts for not only the strain at the reference point, but also the strains at all points in the whole body (Eringin 1983).

Based on the nonlocal theory of Eringen, many works have been performed to predict accurately the linear and nonlinear static, free vibration, and buckling responses of homogeneous or FGM nanobeams. Ghayesh and Farajpour (2019) presented a review on the analysis of FG micro/nano-scale structures. Chaht *et al*. (2015) analyzed the thickness stretching effect on size-dependent bending and buckling of FG nanobeams using SSDT. Abo-Bakr *et al*. (2020) investigated the pull-in and freestanding instability of actuated FG nanobeams including surface and stiffening effects. Esen *et al*. (2021) developed a mathematical model to study the vibration of FG cracked microbeam rested on an elastic foundation and exposed to thermal and magnetic fields. Hamed *et al*. (2019) developed a finite element model to study the bending behavior of FG porous nonlocal nanobeams using four types of porosity models.

Using the HSDT in conjunction with the Eringen nonlocal differential model (ENDM), Karami *et al*. (2019) examined a size-dependent buckling response of FG-CNTRC curved nanobeams. By employing the TBT and Kelvin-Voigt model with the help of NSGT, Zhen *et al*. (2019) studied the free vibration response of viscoelastic CNTs beams under a longitudinal magnetic field. Şimşek (2019) employed the NSGT to obtain some closed-form solutions for static bending, buckling, and free and forced vibration of simply supported FG nanobeams using the

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CBT.

More recently, Daikh *et al*. (2020) proposed a new hyperbolic shear deformation theory to study the static behavior of simply supported cross-ply CNTs reinforced composite laminated nanobeams. The same authors (Daikh *et al*. 2021c) presented in another work a comprehensive study on the free vibration, static stability, and bending of multilayer FG-CNTRC nanoplates using a 3D kinematic shear deformation theory. Based on Navier method, Talebizadehsardari *et al*. (2020) presented an analytical solution to study the static response of a simply supported FG-CNTRC curved nanobeam using the TBT with consideration of the NSGT. By using the isogeometric analysis (IGA) based on TBT, Li *et al*. (2020) carried out a size-dependent analysis of bi-directional FG graphene nanoplatelets reinforced composite microbeams using MCST. Abdelrahman *et al*. (2021) carried out a dynamic analysis of perforated nanobeams subject to moving mass using NSGT. Keshtegar *et al*. (2021) applied the exponential shear deformation beam theory (ESDBT) to investigate the dynamic stability behavior of hybrid nanocomposite polymer beams reinforced by carbon fibers and carbon nanotubes using differential quadrature method (DQM).

Later, Eltaher *et al*. (2020b) exploited doublet mechanics theory to study the buckling and vibration of curved CNTs beam structure. Belarbi *et al*. (2021a) developed a new finite element model to investigate the bending and buckling behavior of FG nanobeams using the nonlocal Eringen theory. Axially functionally graded (AFG) materials are a special kind of FGMs in which the material gradient varies in the direction of the length of the structures. Sarkar and Ganguli (2014) presented an exact solution for the free vibration response of clamped AFG Timoshenko beam. Şimşek (2015) detected the nonlinear free vibration of AFG microbeams by using the MCST and EBT. Abo-Bakr *et al*. (2020) predicted the optimum weight of AFG microbeams under buckling and vibration behaviors using the modified couple Timoshenko beam theory. Shafiei *et al*. (2016) proposed a semi-analytical method to study the nonlinear vibration behavior of AFG tapered microbeams with the help of Eringen's nonlocal theory and EBT. Later, the thermal effect on the dynamic response of AFG beam subjected to a moving harmonic load is discussed by Wang and Wu (2016) using both EBT and TBT. Li *et al*. (2017) investigated the buckling, bending and free vibration behavior of AFG nanobeams by using the NSGT and EBT.

By using Multi-objective shape optimization and Pareto, Abo-Bakr *et al*. (2021) studied the optimum weight of modified couple stress AFG nanobeam under buckling and frequency constraints. Cao *et al*. (2018) carried out a free vibration analysis of AFG beams using the asymptotic development method. Ebrahimi and Barati (2017) used a Galerkin-based solution technique to examine the buckling behavior of AFG nanobeams based on the NSGT. Based on the NSGT, Rajasekaran and Bakhshi Khaniki (2019) presented a comprehensive study on size-dependent mechanical behavior of AFG beams using finite element method. Akgoz (2019) discussed the static buckling problem of AFG tapered microcolumns with different boundary conditions. Similarly, Zheng *et al*. (2019) proposed a new nonlinear finite element formulation based on the MCST and EBT to study the free vibration response of AFG tapered microbeams. More recently, Khaniki and Ghayesh (2020) studied numerically the dynamics behavior of AFG-CNT beams using the generalized differential quadrature method (GDQM). Later, size-dependent vibration and stability of moving viscoelastic AFG nanobeams were carried out by Shariati *et al*. (2020) using both numerical and analytical formulations. Wang *et al*. (2020) examined the hygrothermal mechanical behaviors of AFG microbeams using a refined FSDT. Karamanli and Vo (2021) proposed a finite element model for the mechanical response of CNTs reinforced and graphene nanoplatelet reinforced composite beams by including the thickness stretching effect.

Based on couple stress theory and various beam theories, Civalek *et al*. (2021) studied the free vibration behavior of CNTRC microbeams by using Navier's solution procedure. El-Ashmawy and Xu (2021) analyzed the CNT gradation distribution and orientation on static and dynamic analysis of FG-CNTRC) beams using Timoshenko beam theory. It is observed from the aforementioned literature that there are limited studies for AFG nanobeams and no published work dealing with the static response of AFG-CNTRC nanobeams using modified HSDT. Therefore, this study fills this gap by proposing a new inverse trigonometric shear deformation theory, in conjunction with a modified nonlocal strain gradient theory, to examine the bending deflection and stresses of AFG CNTRC beams. Unlike any other theory, the number of unknown functions involved in the proposed model is only three, as against five in the case of other shear deformation theories. Further, the proposed theory provides an accurate parabolic distribution of transverse shear stress across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the AFG beam without using any shear correction factors.

The governing equations are derived by utilizing Hamilton’s principle and are solved by using Navier’s procedure. Moreover, a detailed parametric analysis checks for the sensitivity of the bending response of AFG-CNTRC nanobeams to the CNTs distribution, thickness ratio, and nonlocal and length scale parameters. The developed higher-order model is highly useful for the efficient bending analysis of thin and thick AFG-CNTRC nanobeams with different transverse loadings profiles. The present mathematical model is limited to uniform cross-sectional nanobeams with simply supported boundary conditions. While the proposed displacement field can be applied for both thin and thick beams. Therefore, this article tries to focus on these points and fill this gap in a comprehensive way.

2. Problem formulation

Consider a straight AFG composite nanobeam of length L and thickness h, made of polymer and reinforced by graded CNTs in the x-direction as shown in Fig. 1.

This work presents a new axially AFG CNTRC beam. The CNTs constituent is graded along the axial direction from right to the left extremity in accordance with a power-law function with two inhomogeneity parameters k and p. The first inhomogeneity parameter k $(0 < k \leq 1)$ insures the CNTs intensity along the axial direction as shown in Fig. 1, and the second inhomogeneity parameter p $(0 < p \leq 1)$ presents the parabolic distribution. Therefore, the volume fraction function of AFG CNTRC beams can be expressed as:

$$V_{CNT} = 2k\left(\frac{x}{a}\right)^p V_{CNT}^* \quad (1)$$

where, V_{cnt}^* indicate the volume fraction distribution of CNTs, and it can be written as (Daikh *et al*. 2020e,f):

$$V_{cnt}^* = \frac{W_{cnt}}{W_{cnt} + (\rho_{cnt}/\rho_m)(1 - W_{cnt})} \quad (2)$$

In which ρ_m, ρ_{cnt} and W_{cnt} denote polymer matrix mass density, CNTs mass density and CNTs mass fraction, respectively. The effective Young's modulus (E) and shear modulus (G) of CNTRC nanobeam can be evaluated by:

$$\begin{aligned} E_{11} &= \eta_1 V_{cnt} E_{11}^{cnt} + V_p E_p \\ \frac{\eta_2}{E_{22}} &= \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E_p} \\ \frac{\eta_3}{G_{12}^k} &= \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G_p} \end{aligned} \quad (3)$$

where Poisson's ratio ν_{12} and density ρ of a CNTRC beam can be expressed as:

$$\nu_{12} = V_{cnt}\nu_{12}^{cnt} + V_p \nu_p \quad (4)$$

$$\rho = V_{cnt}\rho_{cnt} + V_p \rho_p \quad (5)$$

In which E_{11}; E_{22} are Young's modulus across the length directions (x), and G_{12} represent the shear modulus of the composites. The superscripts p and cnt refer to the properties of the polymer and the CNTs, respectively. Here, the CNT efficiency parameters η_i $(i = 1,2,3)$ are given as

For $V_{cnt}^* = 0.12$; $\eta_1 = 1.2833$, $\eta_2 = \eta_3 = 1.0556$.
For $V_{cnt}^* = 0.17$; $\eta_1 = 1.3414$, $\eta_2 = \eta_3 = 1.7101$. (6)
For $V_{cnt}^* = 0.28$; $\eta_1 = 1.3238$, $\eta_2 = \eta_3 = 1.7380$.

3. Equilibrium equations for Nanobeams

In the current study, an efficient inverse trigonometric function is adopted to investigate the bending behavior of AFG carbon nanotubes reinforced composite nanobeams. The displacement field of the proposed theory is chosen based on the following assumptions:

1. The axial displacement consists of extension, bending, and shear components;
2. The bending component of axial displacement is similar to that given by the Euler–Bernoulli beam theory;
3. The shear component of axial displacement gives rise to the parabolic variation of shear strain and hence to shear stress through the thickness of the beam in such a way that shear stress vanishes on the top and bottom surfaces.

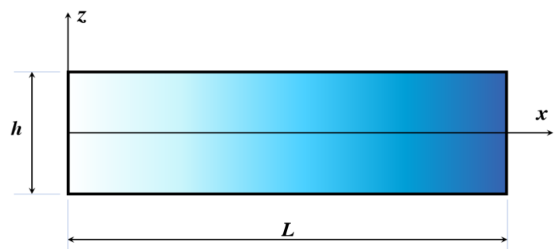

Fig. 1 Geometry of AFG-CNTRC beam

Based on the assumptions made above, the displacement field of the present theory can be described as (Daikh *et al*. 2019a):

$$\begin{aligned} u(x,z) &= u_0 - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x \\ w(x,z) &= w_0 \end{aligned} \quad (7)$$

where u_0 and w_0, represent the displacements along the x- and z-directions, respectively. While φ_x donates the rotation of the transverse normal on x-axis.

The shear deformation along the thickness direction can be expressed by an inverse trigonometricfunction as:

$$f(z) = 5h\tan^{-1}\left(\frac{z}{h}\right) - 4z \quad (8)$$

and the kinematic strain components associated with the displacement field are stated as:

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_{xx}^0 - z\varepsilon_{xx}^1 + f(z)\varepsilon_{xx}^2 \\ \gamma_{xz} &= \frac{df(z)}{dz}\varphi_x \end{aligned} \quad (9)$$

In which the strain components along the natural axis are defined as

$$\begin{aligned} \varepsilon_{xx}^0 &= \frac{\partial u}{\partial x} \\ \varepsilon_{xx}^1 &= \frac{\partial^2 w}{\partial x^2} \\ \varepsilon_{xx}^2 &= \frac{\partial \varphi_x}{\partial x} \end{aligned} \quad (10)$$

The constitutive stress–strain relations are formulated as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} \quad (11)$$

with

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{55} = G_{12} \quad (12)$$

Here, Q_{ij} is the equivalent stiffness. By the application of the principle of virtual work on the beams, we easily obtain:

$$\int_0^L \begin{bmatrix} N_{xx}\frac{d\delta u}{\partial x} - M_{xx}\frac{d^2\delta w}{\partial x^2} \\ +P_{xx}\frac{d\delta\varphi_x}{\partial x} + Q_{xz}\delta\varphi_x \end{bmatrix} \mathrm{d}x\mathrm{d}z - \int_0^L q(x)\mathrm{d}x = 0 \quad (13)$$

Here, *q(x)* denotes the applied transverse load. It is clear that the stress resultants of the present CNTRC nanobeam are related to the strains as the following equations:

$$\begin{Bmatrix} N_{xx} \\ M_{xx} \\ P_{xx} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} \sigma_{xx}\, \mathrm{d}z$$
$$Q_{xz} = \int_{-h/2}^{h/2} \frac{df(z)}{dz} \sigma_{xz}\, \mathrm{d}z \tag{14}$$

It is worth noting that the force and moment resultants of the CNTRC beam can be related to the total strains by:

$$\begin{Bmatrix} N_{xx} \\ M_{xx} \\ P_{xx} \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} & C_{11} \\ B_{11} & D_{11} & F_{11} \\ C_{11} & F_{11} & H_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^1 \\ \varepsilon_x^2 \end{Bmatrix}$$
$$Q_{xz} = A_{55}\gamma_{xz} \tag{15}$$

with

$$\{A_{11}, B_{11}, D_{11}, C_{11}, F_{11}, H_{11}\} = \int_{-h/2}^{h/2} Q_{11}\{1, z, z^2, f(z), zf(z), f(z)^2\} dz \tag{16}$$

$$A_{55} = \int_{-h/2}^{h/2} Q_{55} \left[\frac{df(z)}{dz}\right]^2 dz \tag{17}$$

It is also noticed that we can define the force and moment resultants in displacement fields as follows:

$$N_{xx} = A_{11}\frac{\partial u_0}{\partial x} - B_{11}\frac{\partial^2 w_0}{\partial x^2} + C_{11}\frac{\partial \varphi_x}{\partial x}$$
$$M_{xx} = B_{11}\frac{\partial u_0}{\partial x} - D_{11}\frac{\partial^2 w_0}{\partial x^2} + F_{11}\frac{\partial \varphi_x}{\partial x}$$
$$P_{xx} = C_{11}\frac{\partial u_0}{\partial x} - F_{11}\frac{\partial^2 w_0}{\partial x^2} + H_{11}\frac{\partial \varphi_x}{\partial x}$$
$$Q_{xz} = A_{55}\varphi_x \tag{18}$$

By the application of the virtual work principle, the equilibrium equations can be derived as:

$$\frac{\partial N_{xx}}{\partial x} = 0$$
$$\frac{\partial^2 M_{xx}}{\partial x^2} - q = 0$$
$$\frac{\partial P_{xx}}{\partial x} - Q_{xz} = 0 \tag{19}$$

4. Nonlocal strain gradient theory laminated nanobeam

Lim *et al*. (2015) proposed a function of stresses by including the strain gradient stress and nonlocal elastic stress fields which can be given as:

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \frac{d\sigma_{ij}^{(1)}}{dx} \tag{20}$$

where $\sigma_{ij}^{(0)}$ and $\sigma_{ij}^{(1)}$ denotes the classical stress corresponding to strain ε_{kl} and the higher-order stress $\sigma_{ij}^{(1)}$ corresponding to strain gradient $\varepsilon_{kl,x}$ respectively. Moreover, their forms can be written as:

$$\sigma_{ij}^{(0)} = \int_0^L C_{ijkl}\alpha_0(x, x', e_0 a)\varepsilon_{kl,x}(x')dx'$$
$$\sigma_{ij}^{(1)} = l^2 \int_0^L C_{ijkl}\alpha_1(x, x', e_1 a)\varepsilon_{kl,x}(x')dx' \tag{21}$$

In this case; C_{ijkl} is an elastic constant and l is the material length scale parameter introduced to consider the significance of the strain gradient stress field. $e_0 a$ and $e_1 a$ are the nonlocal parameters introduced to consider the significance of the nonlocal elastic stress field. Based on nonlocal kernel functions $\alpha_0(x, x', e_0 a)$ and $\alpha_1(x, x', e_1 a)$, the general constitutive relation of nonlocal strain constitutive relation can be written as, Eringen (1983):

$$[1 - (e_1 a)^2\nabla^2][1 - (e_0 a)^2\nabla^2]\sigma_{ij} = C_{ijkl}[1 - (e_1 a)^2\nabla^2]\varepsilon_{kl} - C_{ijkl}l^2[1 - (e_0 a)^2\nabla^2]\nabla^2\varepsilon_{kl} \tag{22}$$

∇^2 denote the Laplacian operator. In the present study, we consider that $e = e_0 = e_1$. The total nonlocal strain gradient constitutive relation can be written as below (Daikh and Zenkour 2020):

$$[1 - \mu\nabla^2]\sigma_{ij} = C_{ijkl}[1 - \lambda\nabla^2]\varepsilon_{kl} \tag{23}$$

where $\mu = (ea)^2$ and $\lambda = l^2$.

$$(1 - (e_0 a)^2\nabla^2)\sigma_{ij} = C_{ijkl}(1 - l^2\nabla^2)\varepsilon_{kl} \tag{24}$$

Therefore, Daikh and Zenkour (2020) utilized the constitutive relations for a nonlocal shear deformable axially CNTRC laminated nanobeam as:

$$\sigma_{xx} - \mu\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \bar{Q}_{11}^k\left(\varepsilon_{xx} - \lambda\frac{\partial^2 \varepsilon_{xx}}{\partial x^2}\right) \tag{25}$$

$$\sigma_{xz} - \mu\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \bar{Q}_{55}^k\left(\gamma_{xz} - \lambda\frac{\partial^2 \gamma_{xz}}{\partial x^2}\right) \tag{26}$$

By inserting the Eqs. (25) and (26) in Eq (18), the stress resultants relation can be obtained:

$$N_{xx} - \mu\frac{\partial^2 N_{xx}}{\partial x^2} = \left(1 - \lambda^2\frac{\partial^2}{\partial x^2}\right)\left[A_{11}\frac{\partial u_0}{\partial x} - B_{11}\frac{\partial^2 w_0}{\partial x^2} + C_{11}\frac{\partial \varphi_x}{\partial x}\right]$$
$$M_{xx} - \mu\frac{\partial^2 M_{xx}}{\partial x^2} = \left(1 - \lambda^2\frac{\partial^2}{\partial x^2}\right)\left[B_{11}\frac{\partial u_0}{\partial x} - D_{11}\frac{\partial^2 w_0}{\partial x^2} + F_{11}\frac{\partial \varphi_x}{\partial x}\right]$$
$$P_{xx} - \mu\frac{\partial^2 P_{xx}}{\partial x^2} = \left(1 - \lambda^2\frac{\partial^2}{\partial x^2}\right)\left[C_{11}\frac{\partial u_0}{\partial x} - F_{11}\frac{\partial^2 w_0}{\partial x^2} + H_{11}\frac{\partial \varphi_x}{\partial x}\right]$$
$$Q_{xz} - \mu\frac{\partial^2 Q_{xz}}{\partial x^2} = \left(1 - \lambda\frac{\partial^2}{\partial x^2}\right)[A_{55}\varphi_x] \tag{27}$$

The equilibrium equations for AFG CNTRC nonlocal strain gradient nanobeam can be expressed as:

$$\left(1 - \lambda\frac{\partial^2}{\partial x^2}\right)\left(A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_0}{\partial x^3} + C_{11}\frac{\partial^2 \varphi_x}{\partial x^2}\right) = 0$$
$$\left(1 - \lambda\frac{\partial^2}{\partial x^2}\right)\left(B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} + F_{11}\frac{\partial^3 \varphi_x}{\partial x^3}\right) - \left(1 - \mu\frac{\partial^2}{\partial x^2}\right)q = 0$$
$$\left(1 - \lambda\frac{\partial^2}{\partial x^2}\right)\left(C_{11}\frac{\partial^2 u_0}{\partial x^2} - F_{11}\frac{\partial^3 w_0}{\partial x^3} + H_{11}\frac{\partial^2 \varphi_x}{\partial x^2} - J_{66}\varphi_x^1\right) = 0 \tag{28}$$

5. Exact solutions for FG-CNTRC nanobeams

The functions of the displacements field that satisfy the simply-simply cross-ply laminated beams boundary conditions can be written as (Daikh *et al*. 2019b):

$$u_0 = \sum_{m=1}^{\infty} U_m \cos(\beta x)$$
$$w_0 = \sum_{m=1}^{\infty} W_m \sin(\beta x) \tag{29}$$
$$\varphi_x = \sum_{m=1}^{\infty} X_m \cos(\beta x)$$

where U_{mn}, V_{mn}, and X_{mn} are arbitrary parameters and $\beta = m\pi/L$. By substitution of Eq. (29) into Eq. (28), gives the system stiffness matrix as:

$$\begin{bmatrix} -A_{11}(\beta^2+\lambda\beta^4) & B_{11}(\beta^3+\lambda\beta^5) & -C_{11}(\beta^2+\lambda\beta^4) \\ 0 & -D_{11}(\beta^4+\lambda\beta^6) & F_{11}(\beta^3+\lambda\beta^5) \\ symmetric & 0 & -H_{11}(\beta^2+\lambda\beta^4)-A_{55}(1+\lambda\beta^2) \end{bmatrix} \begin{Bmatrix} U_m \\ W_m \\ X_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ (1+\mu\beta^2)Qm \\ 0 \end{Bmatrix} \tag{30}$$

6. Profile of applied loads

In the present analysis, three types of transverse mechanical loadings are proposed to evaluate the nanobeam response (see Figs. 2(a)-(c)). The transverse loadings can be expressed by Fourier expansion, which varies according to the load type.

The first example of transverse loading is a uniform one. The load is applied on the top surface ($z = h/2$). The mathematical formulation of this load can be expressed as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\beta x) \tag{31}$$

where the coefficients of the Fourier expansion are given by:

$$Q_m = \frac{4q_0}{m\pi} \text{ for } m = 1,3,5,\ldots$$
$$Q_m = 0 \quad \text{for } m = 2,4,6,\ldots \tag{32}$$

The second loading has a sinusoidal distribution as shown in Fig. 2. The load form can be written as:

$$q(x) = \sum_{m=1}^{\infty} q_0 \sin(\beta x) \tag{33}$$

The third one is the linearly varying loads, given by $q(x) = \frac{q_0 x}{L}$

$$Q_m = -\frac{2q_0}{m\pi}\cos(m\pi) \text{ for } m = 1,3,5,\ldots$$
$$Q_m = 0 \quad \text{for } m = 2,4,6,\ldots \tag{34}$$

7. Numerical results

The present work aims to study the bending deflection and stresses of AFG CNTRC nanobeams using a new inverse trigonometric higher order shear deformation beam theory. The analyzed nanobeam is assumed to be simply supported from both ends. The nanobeam is made from Polymethylmethacrylate (PMMA) as a matrix and the Armchair (10,10) SWCNTs as a reinforcement. Therefore, the material properties of the matrix are $\nu^p = 0.3$, $\rho^p = 1190\ \mathrm{Kg/m^3}$ and $E^p = 2.5\mathrm{GPa}$, and for the CNTs reinforcement are $\nu^{cnt} = 0.19$, $\rho^{cnt} = 1400\ \mathrm{Kg/m^3}$, $E_{11}^{cnt} = 600\mathrm{GPa}$, $E_{22}^{cnt} = 10\mathrm{GPa}$and $G_{12}^{cnt} = 17.2\mathrm{GPa}$.

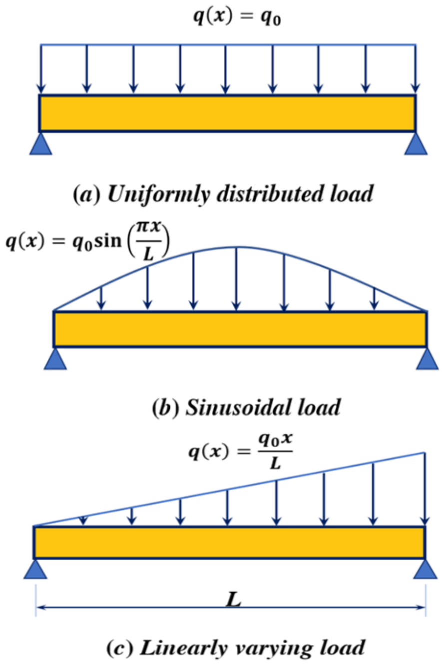

Fig. 2 Types of loads applied in the beam formulation

To present the numerical results in terms of dimensionless deflection and stresses parameters, the following formulas are used:

It is also noticed that we can define the force and moment resultants in displacement fields as follows:

$$\bar{w} = \frac{10^2 E_p h^2}{L^4 q_0} w\left(\frac{L}{2}\right) \tag{35}$$

$$\bar{\sigma}_{xx} = -\frac{h}{Lq_0}\sigma_{xx}\left(\frac{L}{2}, \frac{h}{2}\right) \tag{36}$$

$$\bar{\tau}_{xz} = \frac{h}{Lq_0}\tau_{xz}(0,0) \tag{37}$$

A comparison study is performed in Table 1 to validate the accuracy of the applied inverse trigonometric HSDT with the results obtained by Wattanasakulpong and Ungbhakorn (2013) using the third order shear deformation theory (TSDT) of Reddy $\left(f(z) = z - \frac{4}{3}\frac{z^3}{h^2}\right)$. It is observed from the results presented in Table 1, that the proposed theory agrees well and is very close to the obtained results by Wattanasakulpong and Ungbhakorn.

Table 2 presents the impact of two inhomogeneity parameters p and k on the dimensionless central deflection of axially CNTRC beam subjected to various transverse loads.

The inhomogeneity parameters p and k are taken as 0.2,

Table 1 Nondimensional deflection and stresses in uniformly distributed CNTRC beam

V_{cnt}^{*}	L/h	$\bar{w}$		$\bar{\sigma}_{xx}$		$\bar{\tau}_{xz}$	
		TSDT [86]	Present	TSDT [86]	Present	TSDT [86]	Present
Uniform load							
0,12	10	0.704	0.7027	8.399	8.4562	0.701	0.7183
	15	0.524	0.5237	11.849	11.8874	0.716	0.7371
	20	0.461	0.4605	15.448	15.4781	0.725	0.7464
0,17	10	0.449	0.4484	8.268	8.3177	0.725	0.7225
	15	0.344	0.3436	11.762	11.7951	0.719	0.7399
	20	0.307	0.3067	15.384	15.4088	0.726	0.7485
0,28	10	0.325	0.3244	8.562	8.6298	0.697	0.7134
	15	0.235	0.2345	11.959	12.0032	0.714	0.7338
	20	0.203	0.2027	15.530	15.5649	0.723	0.7440
Sinusoidal load							
0,12	10	0.562	0.5606	6.970	7.0232	0.472	0.4868
	15	0.416	0.4160	9.716	9.7527	0.475	0.4902
	20	0.365	0.3649	12.608	12.6351	0.476	0.4915
0,17	10	0.358	0.3575	6.842	6.8880	0.473	0.4877
	15	0.273	0.2727	9.630	9.6614	0.476	0.4907
	20	0.243	0.2428	12.543	12.5663	0.476	0.4917
0,28	10	0.260	0.2590	7.130	7.1920	0.472	0.4856
	15	0.187	0.1864	9.824	9.8670	0.475	0.4897
	20	0.161	0.1607	12.689	12.7213	0.476	0.4912

Table 2 Effect of inhomogeneity parameters on the dimensionless central deflection of axially CNTRC beam

p	Uniform Load			Sinusoidal load			Linear load		
	k								
	0.2	0.5	1	0.2	0.5	1	0.2	0.5	1
0.2	1.4545	0.7955	0.5328	1.1544	0.6340	0.4259	0.3636	0.1989	0.1332
0.5	1.6848	0.9034	0.5981	1.3361	0.7193	0.4777	0.4212	0.2258	0.1495
1	2.0626	1.0819	0.7018	1.6340	0.8603	0.5599	0.5157	0.2705	0.1755

Table 3 Effect of nonlocal and length scale parameters on the dimensionless central deflection of axially CNTRC beam

μ	λ	Uniform Load			Sinusoidal load			Linear load		
		V_{CNT}^{*}								
		0.12	0.17	0.28	0.12	0.17	0.28	0.12	0.17	0.28
0	0	0.7019	0.4470	0.3180	0.5599	0.3563	0.2538	0.1755	0.1118	0.0795
	1	0.6427	0.4092	0.2912	0.5096	0.3243	0.2310	0.1607	0.1023	0.0728
	2	0.5911	0.3763	0.2679	0.4676	0.2976	0.2120	0.1478	0.0941	0.0670
1	0	0.7644	0.4871	0.3462	0.6152	0.3915	0.2789	0.1911	0.1218	0.0865
	1	0.7019	0.4470	0.3180	0.5599	0.3563	0.2538	0.1755	0.1118	0.0795
	2	0.6465	0.4117	0.2929	0.5138	0.3269	0.2329	0.1616	0.1029	0.0732
2	0	0.8269	0.5271	0.3743	0.6704	0.4266	0.3039	0.2067	0.1318	0.0936
	1	0.7611	0.4849	0.3447	0.6102	0.3883	0.2766	0.1903	0.1212	0.0862
	2	0.7019	0.4470	0.3180	0.5599	0.3563	0.2538	0.1755	0.1118	0.0795

0.5 and 1. As shown at specific p and k, the highest deflection has been observed in the case of uniform load, however, the lowest deflection has been observed in the case of linear load. For a specific load type, by increasing

Table 4 Axial stresses $\bar{\sigma}_{xx}$ of axially CNTRC beam under sinusoidal transverse load

V^*_{CNT}	L/h	*k*=0.2			*k*=0.5			*k*=1		
		p								
		0.2	0.5	1	0.2	0.5	1	0.2	0.5	1
0.12	10	6.8676	6.7854	6.3162	6.8317	7.2015	6.6074	7.0088	7.7608	7.0341
	20	13.1662	13.0968	12.2820	13.1638	13.3436	12.4362	13.2662	13.6444	12.6647
	30	19.5906	19.5131	18.3253	19.6065	19.7188	18.4368	19.6894	19.9377	18.6027
0.17	10	7.3545	6.7588	6.2872	7.2447	7.1163	6.5455	7.5516	7.5587	6.9002
	20	13.4421	13.1023	12.2696	13.3930	13.3100	12.4101	13.5544	13.5478	12.6056
	30	19.8078	19.5377	18.3195	19.7825	19.7070	18.4250	19.8963	19.8805	18.5733
0.28	10	7.9877	6.9114	6.3938	7.7235	7.3960	6.7665	7.9508	7.8375	7.1760
	20	13.7761	13.1959	12.3278	13.6438	13.4615	12.5324	13.7626	13.6983	12.7637
	30	20.0428	19.6188	18.3634	19.9579	19.8189	18.5184	20.0400	19.9900	18.6987

Table 5 Shear stresses $\bar{\tau}_{xz}$ of axially CNTRC beam under sinusoidal transverse load

V^*_{CNT}	L/h	*k*=0.2			*k*=0.5			*k*=1		
		p								
		0.2	0.5	1	0.2	0.5	1	0.2	0.5	1
0.12	10	0.4726	0.4763	0.4803	0.4653	0.4517	0.4613	0.4476	0.4108	0.4292
	20	0.4743	0.4778	0.4814	0.4668	0.4547	0.4638	0.4498	0.4158	0.4334
	30	0.4746	0.4780	0.4816	0.4671	0.4553	0.4642	0.4502	0.4167	0.4342
0.17	10	0.4426	0.4704	0.4758	0.4243	0.4365	0.4495	0.3805	0.3796	0.4043
	20	0.4462	0.4716	0.4767	0.4272	0.4391	0.4516	0.3843	0.3834	0.4076
	30	0.4469	0.4719	0.4769	0.4278	0.4395	0.4520	0.3850	0.3841	0.4082
0.28	10	0.3937	0.4558	0.4646	0.3565	0.3999	0.4207	0.2688	0.3036	0.3412
	20	0.3993	0.4577	0.4660	0.3606	0.4033	0.4235	0.2725	0.3075	0.3449
	30	0.4003	0.4580	0.4663	0.3614	0.4040	0.4241	0.2732	0.3082	0.3456

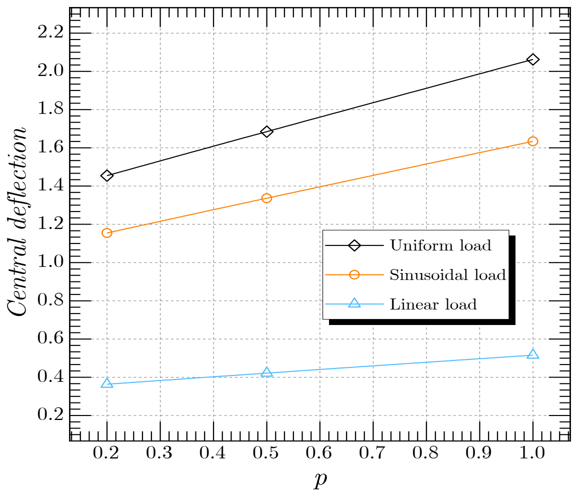

Fig. 3 Effect of inhomogeneity parameter p on the central deflection for various loading types ($k = 1$, $L/h = 10$, $\mu = \lambda = 0$)

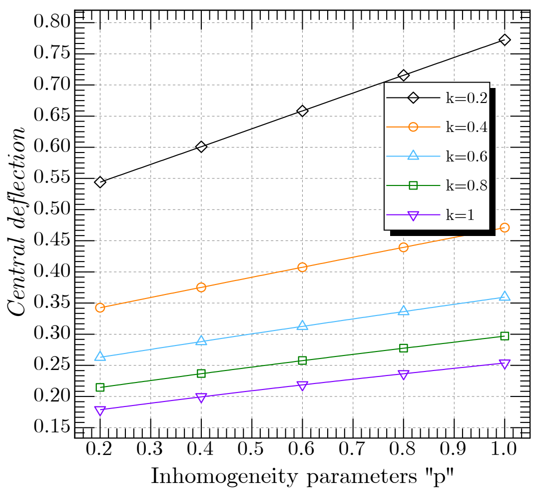

Fig. 4 Central deflections versus the inhomogeneity parameter p ($V^*_{cnt} = 0.28$, $L/h = 10$, $\mu = \lambda = 0$)

the parameter "p" or decreasing the parameter "k", the deflection increases. Therefore, the deflection is proportional with the parameter "p" and inversely with the parameter "k".

In Table 3, The effect of nonlocal parameter μ and length scale parameter λ on central deflection is examined by considering various CNTs volume fraction values and loads. The parameters μ and λ are varied from 0 to 2, wherever the CNTs volume fraction V^*_{CNT} are taken as

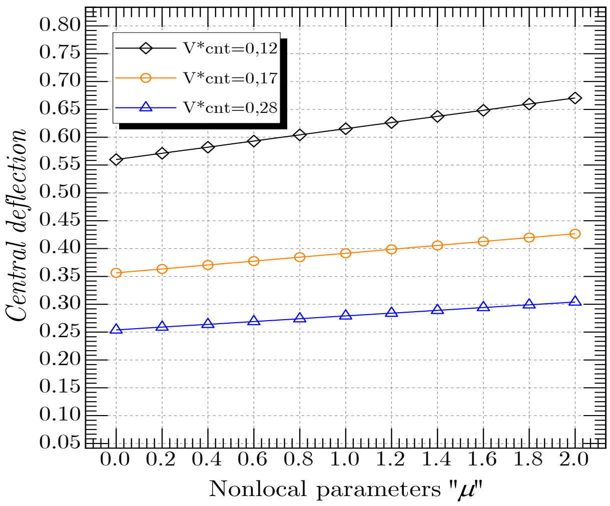

Fig. 5 Central deflection versus the nonlocal parameter "μ" for various volume fraction V_{cnt}^{*} (L/h =10,$p = k = 1$, $\lambda = 0$).

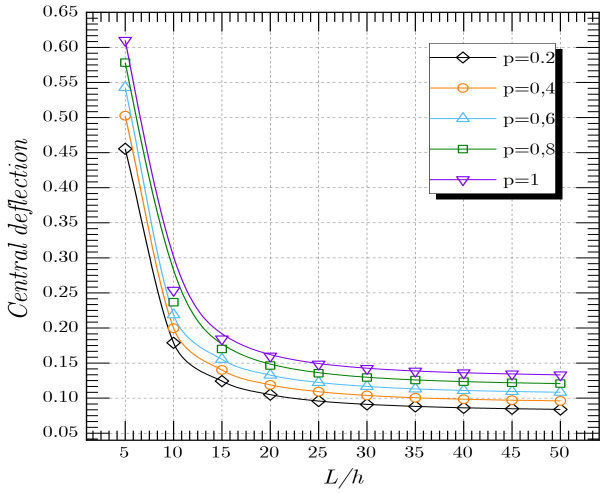

Fig. 8 Central deflection versus the thickness ratio L/h for various values of parameter p ($V_{cnt}^{*} = 0.28$, L/h =10, $k = 1$, $\mu = \lambda = 0$)

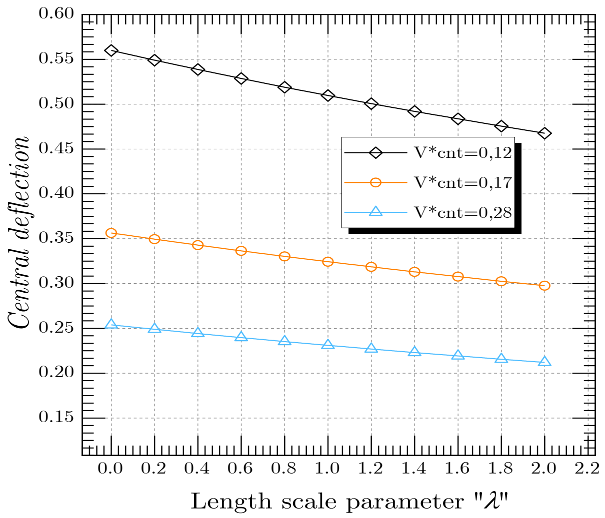

Fig. 6 Central deflection versus the length scale parameter (L/h =10,$p = k = 1$, $\mu = 0$).

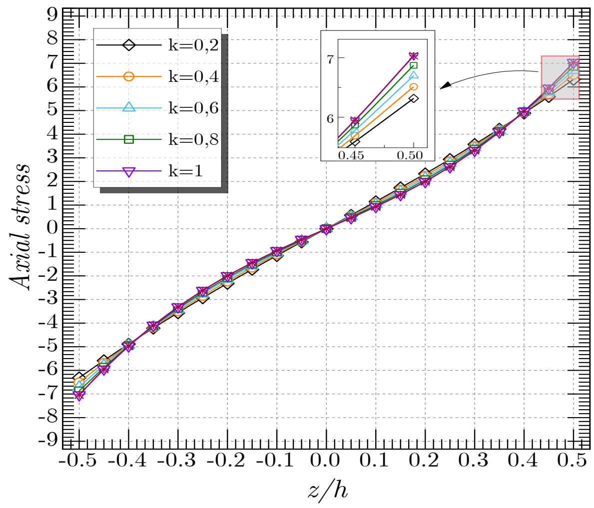

Fig. 9 Effect of inhomogeneity parameter k on the axial stresses ($V_{cnt}^{*} = 0.28$, $p = 1$, L/h =10, $\mu = \lambda = 0$)

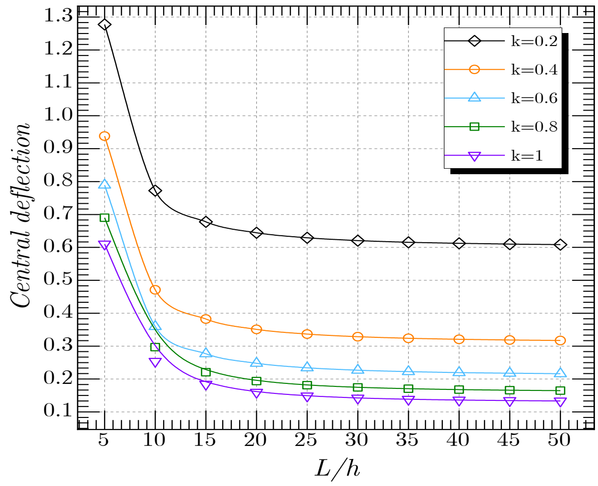

Fig. 7 Central deflection versus the thickness ratio L/h for various values of parameter k ($V_{cnt}^{*} = 0.28$, L/h =10, $p = 1$, $\mu = \lambda = 0$)

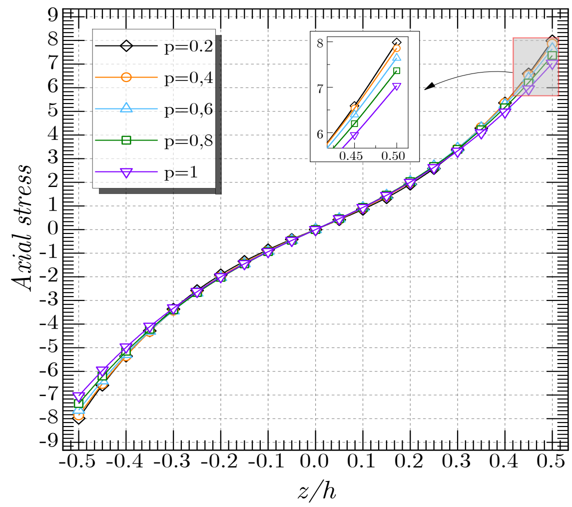

Fig. 10 Effect of inhomogeneity parameter p on the axial stresses ($V_{cnt}^{*} = 0.28$, $k = 1$, L/h =10, $\mu = \lambda = 0$)

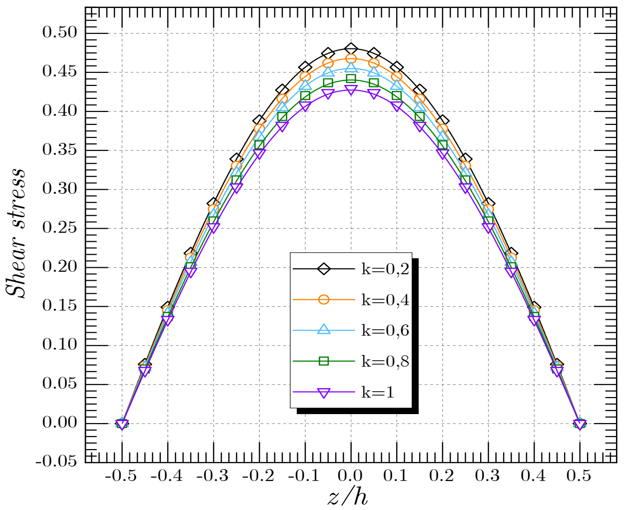

Fig. 11 Effect of inhomogeneity parameter k on the shear stresses ($V_{cnt}^{*} = 0.28,\ p = 1,\ L/h = 10,\ \mu = \lambda = 0$

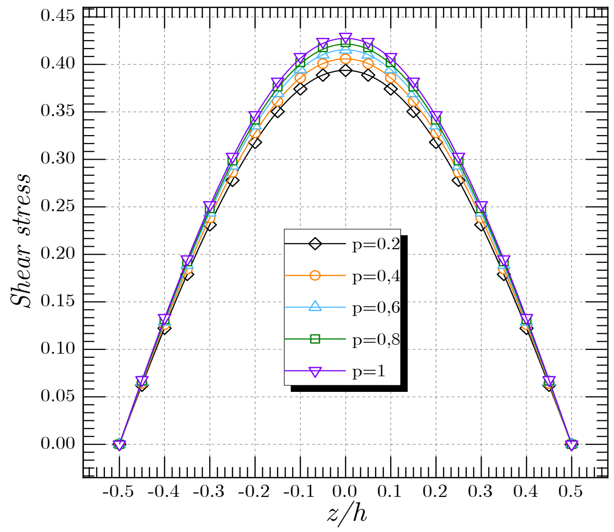

Fig. 12 Effect of inhomogeneity parameter p on the shear stresses ($V_{cnt}^{*} = 0.28,\ k = 1,\ L/h = 10,\ \mu = \lambda = 0$).

12%, 17% and 28%. As seen, by increasing the volume fraction of CNTs, the stiffness of nanobeam is increased and hence the deflection decreased. Increasing the volume fraction of CNTs from 0.12 to 0.17, tends to decrease the deflection by 35%. It is also observed that the nonlocal size effect μ tends to soften the structure and therefore, the deflection increased by increasing μ . However, the microstructure parameter λ tends to streng then the structure and thus, the deflection decreased by increasing λ.

Numerical results related to axial stresses $\bar{\sigma}_{xx}$ and shear stresses $\bar{\tau}_{xz}$ are tabulated in Tables 4 and 5, respectively. The effect of nonlocal parameter μ, length scale parameter λ, CNTs volume fration V_{CNT}^{*}, thickness ration L/h and two inhomogeneity parameters are taken into account. It is noted that the effects of length scale, microstructure length, the volume fraction of CNTs and distribution parameters on the stresses are the same as on the deflection, as observed in the previous paragraph.

Fig. 3 shows the effect of inhomogeneity parameter "*p*" on the central deflection for various loading types. As seen, as the parameter "*p*" increases, the central deflection increases for all loading types. So, it can be concluded that the beam stiffness reduces with increasing the inhomogeneity parameter "*p*".

Since the analysis here is focused on the central point of the beam, the displacement of this point is the lowest for the linear loading, because the highest loads are concentrated at the right edge (x=L), whereas the highest deflections are for the uniform loading.

The central deflection versus the inhomogeneity parameters "p" and "k" is plotted in Fig. 4. It can be seen from this figure that the increase of the inhomogeneity parameter "*p*" leads to a decrement in the rigidity of the beam, therefore, an increase in the central deflection. Moreover, as observed, with an increase in the inhomogeneity parameter "k" the central deflection decrease. One can easily find that the inhomogeneity parameters "p" and "k" have an important effect on the stiffness of CNTRC beams.

The effect of nonlocal parameter "μ" for various volume fraction "V_{CNT}^{*}" on central deflection, is illustrated in Fig. 5. As seen, as the nonlocal parameter "μ" increases, the central deflection increases permanently regardless of the volume fraction. This is caused by the decrement of the beam stiffness. In addition, increasing the volume fraction decreases the central deflection of the CNTRC beam.

It is clear the beam stiffness improves by increasing of CNTs volume fraction, and reduces by increasing the nonlocal parameter "μ".

Fig. 6 describes the effects of the length scale parameter "λ" on the central deflection for various volume fractions. Unlike the nonlocal parameter effect, it is noted that as the length scale parameter "λ" increases, the central deflection decreases, at the same time, the central deflection decrease with the increase of volume fraction.

As shown in Fig. 7, the central deflection versus the thickness ratio "*L/h*" for various values of the parameter "k" is illustrated. It is apparent that the central deflection reduces brutally with *L/h* ratio of less than 20, but, for *L/h* ratio of more than 20 (thin beam), the thickness of the beam has become ineffective, and the variation of the central deflection is almost constant. In contrast, for the cases of inhomogeneity parameter "k". It is concluded that on increasing the *k*, the central deflection is decreased.

Fig. 8 presents the influence of the combination of thickness ratio and the inhomogeneity parameter p on the central deflection. As we mentioned before, the central deflection decreases by increasing the thickness ratio "*L/h*", Moreover, with an increase in the inhomogeneity parameter *p*, the central deflection increase. It is clear that the central deflection decreases quickly in the first part when $p \leq 20$, and it is constant in the second part when $20 \leq p \leq 50$.

The graphs of Figs. 9 and **10** show the effect of inhomogeneity parameters k and p on the axial stresses along the thickness direction of the axially FG-CNTRC beam, respectively. One can see that the axial stresses are tensile at the top surface and compressive at the bottom surface. The axially FG-CNTRC beam with the parameter's values *p*=0.2 and k=1 yields the maximum compressive and tensile stresses at the bottom and top surfaces respectively.

Figs. 11 and 12 present the dimensionless shear stress along the thickness direction of the axially FG-CNTRC beams for several values of parameters *p* and *k*. The maximum results occur at a point on the midplane of the beam. As seen, the largest magnitude is obtained for *p*=1 and *k*=0.2.

8. Conclusions

In the current study, the bending response of axially functionally graded CNTRC beams with different transverse loadings profiles is investigated for the first time using a new inverse trigonometric shear deformation theory. Unlike any other theory, the number of unknown functions involved in the proposed model is only three, as against five in the case of other shear deformation theories. The proposed model can provide an accurate parabolic distribution of transverse shear stress through the thickness direction satisfying the traction-free boundary conditions needless of any shear correction factor. A Modified continuum nonlocal strain gradient theory is employed to include the microstructure and the geometrical nano-size length scales. By adopting Hamilton's principle, the equilibrium equations are derived and solved using Navier's technique.

The reliability and the accuracy of the proposed model are validated by comparing the author's results with various analytical solutions available in the literature. The important key points that can be concluded from this investigation are summarized as follows:

• The beam stiffness reduces by increasing of inhomogeneity parameter "k" and decreasing the inhomogeneity parameter "p".

• The deflections are high when the beam is shorter.

• The inclusion of the nonlocal parameter leads to the reduction of the stiffness of the axially CNTRC nanobeams, therefore increasing the dimensionless deflection.

• Unlike the nonlocality effect, the dimensionless deflections decrease by increasing of the length scale parameter λ.

• Finally, the structure presented here provides benchmark results, which can be employed in the design of composite structures, and the proposed method of solution can be compared in the future by using other approximate methods such as the finite-element method.

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