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Elasticity-based analysis of bigraded FGM beams subjected to shear loading

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Abstract. This paper explores the plane stress problem for a cantilever beam under shear loading that causes bending, specifically focusing on functionally graded materials (FGMs). These materials exhibit a gradual change in mechanical properties, promoting enhanced performance over traditional homogeneous materials. The authors introduce a new formulation for the elastic modulus of FGM, incorporating two graduation parameters: one for material composition variation along the beam and another for the longitudinal elastic modulus ratio, which characterizes stiffness variation. The study formulates the static equilibrium equations that relate external forces, stresses, and internal deformations. Analytical expressions derived from these equations predict stress and displacement distributions within the beam, particularly under a tangential load at the free end, which induces shear and bending deformations. The analysis reveals how the gradation parameters influence the beam's structural response, demonstrating the effect of elastic modulus variation on stress distribution and deflection behavior. The study culminates in an analytical framework for assessing FGM cantilever beams under shear loading, enhancing understanding of material gradation impacts on structural behavior and aiding in the design and optimization of advanced engineering structures utilizing FGMs.

Keywords: anisotropy; cantilever beams; elastic properties; shear loads; static analysis; structural materials

1. Introduction

A bi-material system comprises two dissimilar material layers or components bonded together, commonly used in applications such as adhesive joints, coatings, and beams or plates. This configuration allows for the combination of desirable properties of different materials; however, it also introduces stress discontinuities at the interface, which can lead to damage or crack propagation, threatening structural integrity. To address these challenges, functionally graded materials (FGMs) have been developed, featuring a continuous and gradual variation in mechanical and physical properties. This variation mitigates stress concentrations and enhances structural performance by providing a smooth transition between material phases, improving layer compatibility, and increasing durability. The innovation of FGMs permits the design of advanced structures, including coating systems with graded intermediate layers, FGM beams and plates, and sandwich structures with graded cores. The integration of FGM layers or cores significantly

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enhances various mechanical properties, including residual stress distribution, impact resistance, buckling load, wear resistance, and crack propagation. Thus, FGMs represent a valuable solution for creating high-performance, reliable structures across diverse engineering applications.

The studies of these new structures were firstly on thermal and residual stresses. Brahim [1] adopted experimental methods and used a numerical model to investigate the effect of a graded intermediate layer on the thermal and residual stresses in the coating structure. Using the classical beam theory, Hassaine Daouadji [2] analyzed the thermal stress in two materials joined by a graded layer, and applied the higher order deformation theory to study the static behavior of a functionally graded metal–ceramic beam under the ambient temperature. Based on the asymptotic solution approach. Sankar [3] studied functionally graded beams, assuming elastic compliance parameters are proportional to exponential (kz), where k is a constant and thickness coordinate (z). Hassaine Daouadji [4] assumed elastic compliance parameters are proportional to a polynomial of z, which cannot be obtained using the Fourier series expansion method. They used the Galerkin method for an approximate solution. However, Sankar's [3] method cannot be used for anisotropic beams, even for homogeneous ones. Hadj Henni [5] used the trial-and-error method to study nonhomogeneous orthotropic cantilever beams, assuming elastic compliance parameters are functions of the thickness coordinate. The stress expressions were simple and usable, demonstrating the complexity of these methods [6-24]. The paper presents a generalized version of Hassaine Daouadji's [25]. method for obtaining elasticity solutions of plane anisotropic functionally graded beams. The method considers body force varying with coordinates and does not assume variations in elastic compliance parameters along the beam thickness. The solutions degenerate to those for homogeneous beams, with new solutions for fixed-fixed homogeneous anisotropic beams subjected to uniform load. Comparing these solutions with existing elasticity solutions, a good agreement is obtained, except for some mistakes. Numerical results of a functionally graded anisotropic beam, where only one elastic compliance coefficient varies with the thickness coordinate, are shown in figure form to demonstrate the effect of material inhomogeneity parameter on the displacement and stress field in the beam [26-46].

Among structural elements, beams made of functionally graded materials are frequently used in engineering designs. Understanding their mechanical behavior under different loading conditions is therefore essential for ensuring structural reliability and optimizing performance. In particular, the analysis of cantilever beams subjected to shear loading that produces bending effects is an important problem in structural mechanics. Accurate prediction of stresses and displacements in such beams is necessary for safe and efficient design. In this context, the present paper investigates the plane stress problem of a cantilever beam subjected to shear loading in bending. A new formulation for the elastic modulus of FGM materials is proposed in order to better represent the variation of mechanical properties within the beam. This formulation introduces two graduation parameters: the first parameter describes the material gradation, while the second parameter is related to the ratio of the longitudinal elastic modulus, allowing a more precise representation of the stiffness variation along the beam. To analyze the structural response, a mathematical formulation based on the static equilibrium equations is developed. This formulation makes it possible to determine the distribution of stresses and displacements in a cantilever beam subjected to a tangential load applied at its free end. The proposed approach contributes to improving the analytical modeling of FGM structures and provides useful insights for the design and optimization of beams made of functionally graded materials.

This article presents a novel formulation of the elastic modulus for functionally graded materials, incorporating two gradient parameters to achieve a more accurate and flexible representation of spatial property variation. This approach offers distinct advantages in practical engineering applications, including enhanced modeling precision, improved structural design optimization, and a more realistic characterization of material behavior under service conditions. Compared with existing models, which often rely on simplified gradient assumptions, the proposed formulation provides more reliable and refined predictions. Consequently, this work highlights the significance and innovative contribution of the new expression, demonstrating its potential to effectively address practical engineering problems.

2. Theoretical formulations

2.1 Properties of the FGM material used in this study

Functionally graded materials (FGMs) are engineered materials characterized by a gradual and continuous variation in their constituent phases according to a pre-defined spatial profile. Unlike conventional homogeneous materials, FGMs exhibit non-uniform microstructures, which result in continuously graded macroscopic properties. The defining feature of an FGM is the variation in the volume fraction of its constituents, typically described mathematically using either a power-law function or an exponential function.

In this study, the focus is on FGM beams whose material gradation is described using a sigmoid function—a method that has received relatively limited attention in the existing literature. The primary objective is to provide a comprehensive understanding of the distinctive features of FGMs and the influence of their graded structure on mechanical behavior.

Specifically, the study examines an elastic cantilever beam composed of a combination of ceramic and metal phases. The material properties, including Young's modulus and Poisson's ratio, differ between the upper and lower surfaces of the beam. In the thickness direction, these properties vary continuously: while Poisson's ratio is assumed to remain constant throughout the thickness, the Young's modulus follows an exponential distribution, representing a particular type of FGM known as a power-law FGM (P-FGM). This continuous variation in material properties allows the beam to exhibit tailored mechanical responses, which can be optimized for specific engineering applications.

In this approach, Young's modulus is reformulated by introducing two distinct gradation parameters: k_1, representing the intrinsic material gradient, and k_2, reflecting the variation in the longitudinal elastic modulus. This formulation enhances the flexibility and accuracy in modelling complex material distributions and mechanical responses along the structure.

$$E(y) = E_2\left[(k_2 - 1)(\frac{y}{h} + \frac{1}{2})^{k_1} + 1\right] \tag{1}$$

The Young's modulus for aluminum is denoted by E_2, while k_1 and k_2 represent the material gradation and the longitudinal elastic modulus ratio, respectively.

2.2 Elasticity solution

A functionally graded material (FGM) cantilever beam of uniform thickness h is considered and described within a Cartesian coordinate system, as illustrated in Fig. 1. The beam is oriented such that its longitudinal axis coincides with the xxx-direction, while the thickness is measured

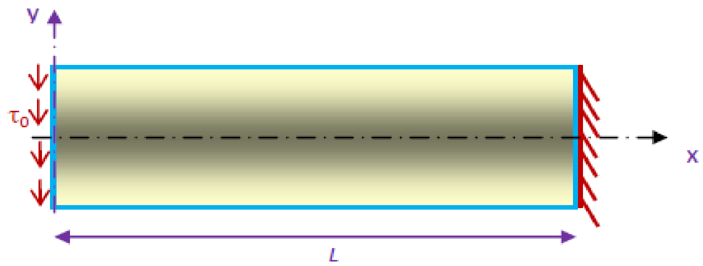

Figure 1. Geometries of the cantilever beam loaded by a shear load

along the y-direction. The upper and lower surfaces of the beam are located at y=+h/2 and y=−h/2, respectively, defining a symmetric thickness distribution about the mid-plane. The geometric dimensions of the beam are characterized by its length L and width b, with the width extending along the out-of-plane z-direction. The beam is assumed to operate under plane stress conditions, meaning that the stress components normal to the x–y plane (i.e., along the z-direction) are negligible. In addition, the beam is made of a functionally graded material, implying that its mechanical properties such as Young's modulus, density, and possibly Poisson's ratio are not constant but vary continuously through the thickness direction (y). This gradation is typically defined by a prescribed material distribution law, allowing for a smooth transition between different constituent materials and enhancing the structural performance under mechanical and dynamic loading conditions.

In the absence of body forces the equilibrium equations are given as:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \tag{2a}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0. \tag{2b}$$

where σ_x, σ_y, τ_{xy} are stress components.

Considering that the upper and lower surfaces of the beam are traction-free, the boundary conditions impose that no external loads act on these faces. As a direct consequence, the normal stress component in the thickness direction ($\sigma_y = 0$) vanishes throughout the beam. Starting from the governing equilibrium equations (Eq. (2)), and taking into account this stress condition, integration across the thickness leads to a set of constraint relations. These expressions establish the connections between the stress components and ensure that the internal stress distribution satisfies both equilibrium and the prescribed boundary conditions.

$$\tau_{xy} = f(y), \tag{3a}$$

$$\sigma_x = -xf^{'}(y) + g(y) \tag{3b}$$

where $f(y)$ and $g(y)$ are unknown functions.

The relationships between strains and displacements are:

$$\varepsilon_x = \frac{1}{E(y)}\sigma_x, \tag{4a}$$

$$\varepsilon_y = \frac{-\nu}{E(y)}\sigma_x, \tag{4b}$$

$$\gamma_{xy} = 2\frac{1+\nu}{E(y)}\tau_{xy} \tag{4c}$$

where ε_x, ε_y, γ_{xy} are strain components that should satisfy the following strain compatibility equation:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x\,\partial y} = 0 \tag{5}$$

Determining the unknown functions *f*(*y*) and *g*(*y*) by solving Eq. (5), we obtain the following:

$$\left[\frac{g(y)}{E(y)}\right]'' - x\left[\frac{f'(y)}{E(y)}\right]'' = 0 \tag{6}$$

From Eq. (6), we deduce:

$$g(y) = c_1 yE(y) + c_2 E(y) \tag{7a}$$

$$f'(y) = c_3 yE(y) + c_4 E(y) \tag{7b}$$

$$f(y) = c_3 \int_{-h/2}^{y} yE(y)dy + c_4 \int_{-h/2}^{y} E(y)dy + c_5 \tag{7c}$$

where c_i: i=1 à 5 are constants of integration.

The boundary conditions of elasticity at the upper and lower surfaces are:

$$\tau_{xy}(x, -h/2) = f(-h/2) = 0, \tag{8a}$$

$$\tau_{xy}(x, +h/2) = f(+h/2) = 0, \tag{8b}$$

$$c_5 = 0 \tag{8c}$$

The boundary conditions at the left end (free) of the FGM beam are:

$$N_0 = 0, \tag{9a}$$

$$M_0 = 0, \tag{9b}$$

$$Q_0 = \tau_0 h \tag{9c}$$

where N_0 and M_0 denote the axial force, moment and shear force at $x = 0$. The boundary conditions for the fixed end at the right end (fixed) of the beam are taken as:

$$u = v = 0, \tag{10a}$$

$$\frac{\partial v}{\partial x} = 0 \quad \text{at } x = L,\ y = 0 \tag{10b}$$

Finally the integration of Eq. (4), we could obtain the displacement components as follows:

$$u = x\,S_{11}g(y) - \frac{x^2}{2}S_{11}f'(y) + \int_{-h/2}^{y}(y-\xi)\,S_{12}f'(y)d\xi + \int_{-h/2}^{y} S_{66}f(y)d\xi - \delta\,\mathrm{y} + u_0, \tag{11a}$$

$$v = \int_{-h/2}^{y} s_{12}g(y)d\xi - x\int_{-h/2}^{y} s_{12}f'(y)d\xi + \frac{c_1}{6}x^3 - \frac{c_3}{2}x^2 + \delta x + v_0 \tag{11b}$$

Where: u_0, v_0 and δ are integral constants.

Substituting the previous equations, we obtain the following matrix system:

$$\begin{bmatrix} \lambda_2(h/2) & \beta_2(h/2) & 0 & 0 \\ \lambda_1(h/2) & \beta_1(h/2) & 0 & 0 \\ 0 & 0 & \lambda_2(h/2) & \beta_2(h/2) \\ 0 & 0 & \lambda_1(h/2) & \beta_1(h/2) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tau_0 h \\ 0 \end{bmatrix} \tag{12}$$

Or:

$$\lambda_1(h/2) = \int_{-h/2}^{+h/2} \xi E(\xi) d\xi \tag{13a}$$

$$\lambda_2(h/2) = \int_{-h/2}^{+h/2} (\frac{h}{2} - \xi) \xi E(\xi) d\xi \tag{13b}$$

$$\beta_1(h/2) = \int_{-h/2}^{+h/2} E(\xi) d\xi \tag{13c}$$

$$\beta_2(h/2) = \int_{-h/2}^{+h/2} (\frac{h}{2} - \xi) E(\xi) d\xi \tag{13d}$$

3. Numerical results and discussion

In this section, a numerical study is conducted on a functionally graded cantilever beam with length L=1 mL =1\ \text{m}L=1 m and height h=0.1 mh =0.1\ \text{m}h=0.1 m, subjected to a shear load of amplitude τ0=108 N/m^2\tau_0=10^8\ \text{N/m}^{2}τ0=108 N/m^2. The analysis considers both homogeneous (pure) materials and P-FGM materials with different values of the gradation parameters k_1 and k_2. For the P-FGM beams, the Poisson's ratio is ν= 0.3, applied at the upper surface of the beam, and the Young's modulus for aluminum is E_2=70 GPa.

The mterial variation is assumed to follow the graded function defined in Eq. (1) for P-FGM beams, which captures continuous property variation along the thickness. The numerical study will be carried out using the analytical solution derived in the previous section, allowing evaluation of stresses and displacements for different combinations of material parameters.

The key calculation parameters to be considered in the analysis include:

- Values of gradation parameters k_1 and k_2 for P-FGM materials,
- Material properties of the constituent phases, including Young's modulus and Poisson's ratio,
- Load amplitude $\tau_0=10^8$ N/ml and beam geometry (L=1 m, height h=0.1 m),
- Distribution of Young's modulus and Poisson's ratio along the beam thickness according to the P-FGM model.

$$\bar{v} = \frac{10 E_2 v h^3}{\tau_0 L^4} \tag{14a}$$

$$\bar{\sigma}_x = \frac{\sigma_x}{\tau_0} \tag{14b}$$

$$\bar{\tau} = \frac{\tau_{xy}}{\tau_0} \tag{14c}$$

Table 1. Comparison of dimensionless stresses and displacements, for different gradations of FGM material in a cantilever beam

k_1	k_2	σ_{xx}/τ_0 (L/2; h/2)	τ_{xy}/τ_0 (L/2;0)	$\bar{v}(0,0)$
	1	29,999	1,500	-4,00
0	2	29,999	1, 500	-1,999
	5	29,999	1, 500	-0,799
	1	29,999	1,500	-4,00
1	2	36,923	1,499	-2,769
	5	45,652	1,499	-1,565
	1	29,999	1,500	-4,00
2	2	39,252	1,472	-2,990
	5	52,817	1,426	-1,972

This setup enables a comprehensive evaluation of the effects of material gradation on beam behavior under shear loading, providing insight into the mechanical performance of both homogeneous and functionally graded beams.

The Table 1 presents the mid-span stresses and the displacements at the free end of the beam for different material gradations (Table 1). For homogeneous materials (k_1=0), the normal stresses remain constant along the beam, as expected, since the material properties do not vary. However, the displacements at the free end still change depending on the gradation values (from 0 to 5), reflecting the influence of longitudinal elastic modulus variations on beam deformation.

For graded materials, the behavior differs depending on the type of material gradation:

- Normal stresses vary along the beam, reaching a maximum value, particularly noticeable for gradation types (2-5), which corresponds to a quadratic material profile (Q-FGM).
- Shear stresses reach a minimum value for the same type (2-5), further highlighting the influence of a quadratic gradation on stress distribution.
- In contrast, the displacements show a more stable minimum value for gradation type (1-5), which represents a linear material profile (L-FGM). This indicates that linear gradation leads to a more uniform and predictable deflection pattern.

Overall, the results confirm that the expected mechanical logic is respected: the normal stresses reach the highest values, while displacements and shear stresses behave consistently with the type and degree of material gradation. This demonstrates the effectiveness of using graded material models to control stress and displacement distributions in beams.

Figs. 2 and 3 illustrate the distributions of normal and shear (tangential) stresses across the thickness of the cantilever beam for both homogeneous and functionally graded materials (FGMs). The analysis highlights several key observations:

a) In homogeneous materials, the normal stress distribution is linear through the thickness, reflecting uniform material properties, while the shear stresses follow a symmetric parabolic pattern, consistent with classical beam theory.

b) In FGM materials, the behavior is more complex and insightful:

- The normal stress distribution becomes nonlinear, with particularly high values observed for the quadratic FGM (Q-FGM, type 2-5), demonstrating the effect of nonlinear material gradation on stress concentration.

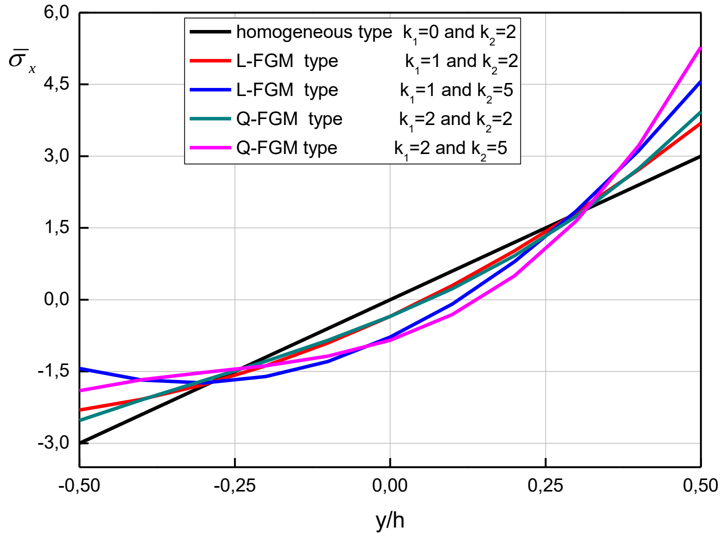

Figure 2. Dimensionless normal stress variation through the thickness and at mid-span of the beam for homogeneous and L-FGM and Q-FGM materials

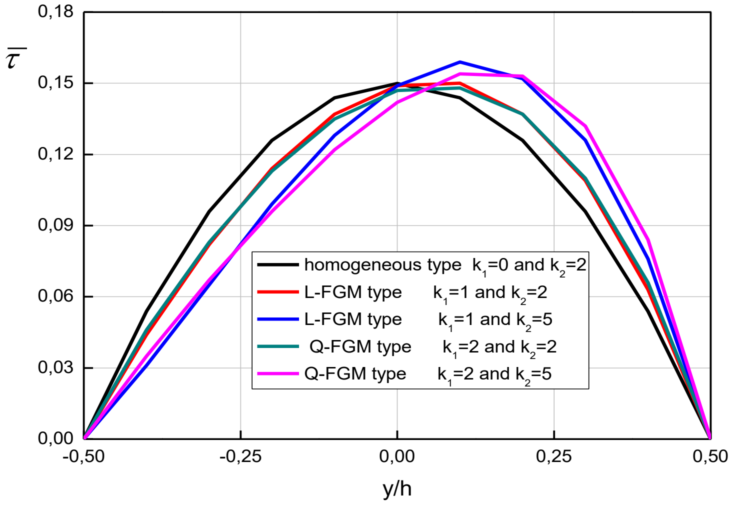

Figure 3. Variation of dimensionless tangential stresses across the thickness and at mid-span of the beam for homogeneous materials and L-FGM and Q-FGM

- Shear (tangential) stresses maintain a generally parabolic profile but are shifted toward the stiffer face of the beam, with the maximum shear stress recorded for the linear FGM (L-FGM, type 1-5). This indicates that material gradation can be strategically used to control stress localization and improve mechanical performance.

These results underscore the advantages of using FGM designs, as they allow tailored stress distributions, enhance load-bearing efficiency, and offer the ability to manipulate peak stress locations according to engineering requirements.

The variation of dimensionless tangential (shear) stresses across the thickness and at the mid-

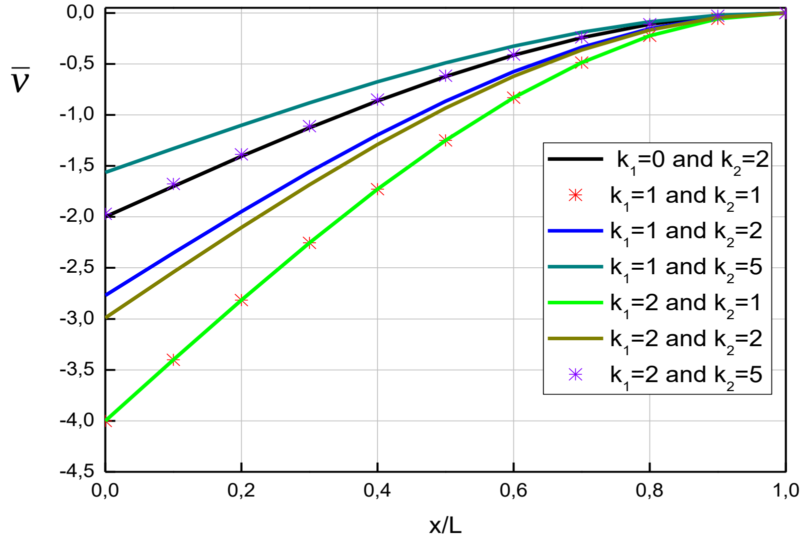

Figure 4. Variation of dimensionless displacements along and at the free end of the beam for different graduations

span of the beam is analyzed for homogeneous materials, linear FGMs (L-FGM), and quadratic FGMs (Q-FGM). Key observations include:

- For homogeneous materials, the tangential stress distribution is parabolic and symmetric through the thickness, reflecting uniform material properties. The mid-span stresses follow this pattern consistently.
- In L-FGM beams, the shear stresses remain generally parabolic but are shifted toward the stiffer face of the beam, resulting in slightly lower maximum values than in Q-FGM materials. This indicates that a linear material gradation provides more uniform stress control.
- In Q-FGM beams, the shear stress distribution exhibits a nonlinear asymmetry, with a noticeable peak near the stiffest layer. This reflects the effect of quadratic gradation, which concentrates shear stresses differently along the thickness.

Overall, comparing the three types highlights that material gradation can be strategically used to control shear stress distribution, with L-FGM providing smoother and more predictable behavior, while Q-FGM allows tailoring of peak stress locations for enhanced structural performance.

Figs. 4 and 5 illustrate the variation of dimensionless displacements along the beam and at its free end for different material gradations, emphasizing the role of flexible versus rigid homogeneous materials, particularly in Fig. 5.

- For functionally graded materials (FGMs), both linear (L-FGM) and quadratic (Q-FGM) types, which incorporate material gradation and higher rigidity, provide enhanced structural stability, exhibiting minimal displacements compared to less rigid materials, as clearly shown in Fig. 4. This highlights the advantage of tailored material distribution in controlling deformation.
- For homogeneous materials, the analysis in Fig. 5 shows that higher elastic modulus ratios correspond to improved stability. Rigid homogeneous materials (gradation 0-5) display

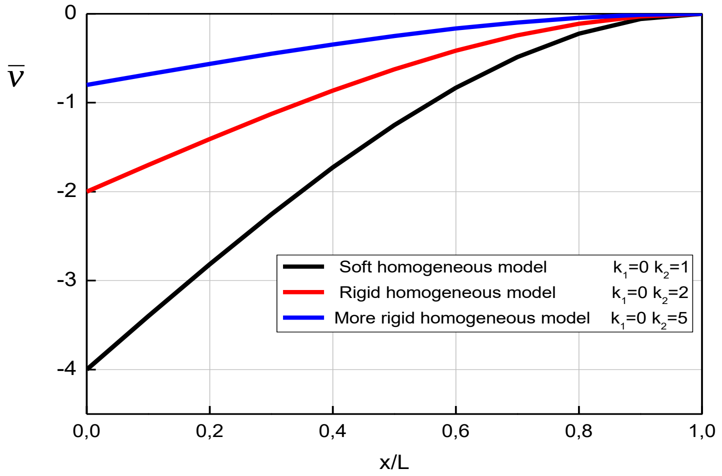

Figure 5. Variation of dimensionless displacements along and at the free end of the beam for flexible and rigid homogeneous materials

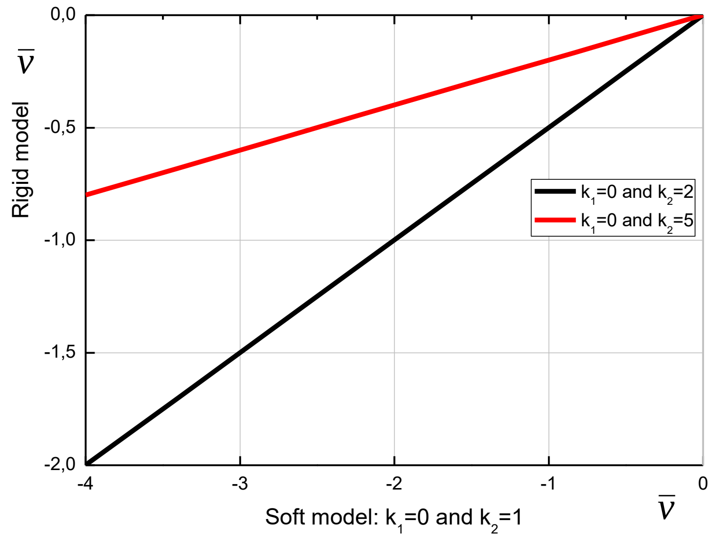

Figure 6. Displacement of the rigid models as a function of the flexible model

significantly lower displacements than less rigid (0-2) or flexible (0-1) materials, demonstrating that material stiffness plays a crucial role in minimizing beam deflection.

Overall, these results emphasize that both material gradation and stiffness are key factors in optimizing beam performance, with FGMs offering superior control over displacement and mechanical response compared to homogeneous materials.

The study demonstrates that the relationship between rigid and flexible homogeneous models remains linear, confirming that the mechanical behavior follows expected theoretical trends and validating the modeling approach, as shown in Fig. 6. Additionally, the comparison between FGM models and homogeneous models reveals consistent and logical behavior: linear FGMs (L-FGM) exhibit lower response values than quadratic FGMs (Q-FGM) under the same testing conditions,

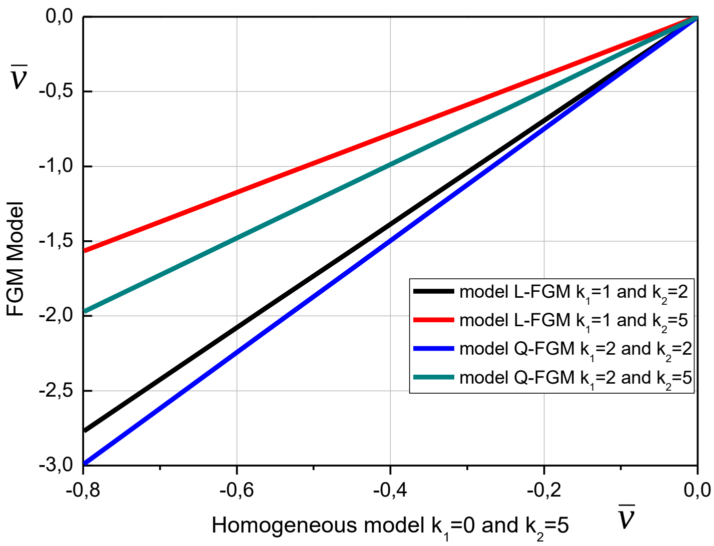

Figure 7. Displacement of FGM models as a function of the homogeneous model

as illustrated in Fig. 7.

These results highlight the robustness of the study, confirming that the behavioral trends of both homogeneous and functionally graded materials are well captured and that the models reliably reflect the influence of material gradation and stiffness on the structural response. This consistency reinforces the credibility and predictive capability of the proposed modeling framework.

The displacement of rigid models can be evaluated relative to the corresponding flexible models to assess the effect of stiffness on beam deformation (Fig. 6). In this comparison:

- Rigid models consistently exhibit lower displacements than flexible models, reflecting their higher resistance to bending under the same loading conditions.
- The relationship between rigid and flexible models is linear, indicating that increasing stiffness produces predictable reductions in displacement.
- This correlation confirms that the mechanical behavior of the beam responds logically to variations in material rigidity, providing a clear framework for designing structures with tailored stiffness profiles.

Overall, analyzing rigid model displacements as a function of flexible model displacements demonstrates the controlled influence of material rigidity and supports the validity of the modeling approach.

The displacement of FGM models can be analyzed relative to their corresponding homogeneous models to understand the effect of material gradation on beam behavior (Fig. 7). In this comparison:

- Linear FGMs (L-FGM) consistently show lower displacements than their homogeneous counterparts with equivalent stiffness, indicating that a gradual material distribution can improve structural stability.
- Quadratic FGMs (Q-FGM), with a more pronounced gradation, generally exhibit slightly higher displacements than L-FGMs but still follow a predictable trend relative to the homogeneous models.

- Overall, plotting FGM displacements as a function of homogeneous model displacements demonstrates a clear and logical correlation, confirming that the influence of material gradation on deformation is both consistent and controllable.

This approach highlights the advantages of FGM design, showing that material gradation can be strategically tailored to optimize beam performance compared to conventional homogeneous materials.

4. Conclusions

In this study, a plane elasticity solution has been developed for a functionally bigraded cantilever beam using a two-dimensional elasticity framework. By assuming that the material properties vary continuously along the thickness, the model provides a more realistic representation of functionally graded materials (FGMs), capturing subtle variations in mechanical behavior that simpler models might overlook. A general analytical solution was derived for beams subjected to concentrated shear loads, accurately accounting for both bending and shear effects. The formulation introduces a novel expression of the elastic modulus incorporating two gradation parameters, which allows the model to precisely characterize material variations in both directions. This dual-parameter approach ensures that the mechanical behavior of bigraded FGMs is captured more faithfully than with traditional single-gradient models. The results show that the proposed formulation provides reliable predictions of both stresses and displacements, as well as beam stability, under tangential loading at the free end. The analytical model successfully reproduces the stress distributions and deformation patterns, demonstrating its accuracy and predictive capability.

Overall, this work confirms that the bigraded FGM approach combined with a 2D elasticity solution is a powerful and practical tool for the design and analysis of advanced structural components. It offers enhanced accuracy in stress prediction and a more comprehensive evaluation of structural performance under complex loading conditions, highlighting the advantages of FGMs in achieving optimized mechanical behavior.

Although the present study provides a comprehensive elasticity solution for bigraded FGM beams subjected to shear loads, several important research directions remain open and deserve further investigation to enhance the applicability and robustness of the proposed model.

- First, future work should extend the current formulation to nonlinear regimes by incorporating material nonlinearity, geometric nonlinearity, and progressive damage mechanisms. Such developments would enable a more realistic prediction of the structural response of bigraded FGM beams under high loading levels and complex service conditions.
- Second, the inclusion of thermomechanical coupling effects represents a crucial extension. Since functionally graded materials are frequently employed in high-temperature environments, accounting for temperature-dependent material properties, thermal gradients, and induced stresses would significantly improve the model’s practical relevance.
- Third, multi-scale modeling approaches should be explored to bridge the gap between microstructural characteristics and macroscopic behavior. Considering the influence of phase distribution, porosity, and interfacial effects at the microscale could lead to more accurate predictions of the global response of bigraded FGM beams.

In addition, the optimization of bidirectional gradation profiles remains a promising research avenue. Advanced optimization techniques, such as genetic algorithms and machine learningbased methods, could be employed to identify optimal material distributions that enhance stiffness, reduce weight, or improve shear resistance. Moreover, the development and application of advanced numerical methods such as meshless methods, extended finite element methods (XFEM), and isogeometric analysis could improve computational efficiency and accuracy, particularly for complex geometries and boundary conditions. Another essential direction is the experimental validation of the proposed models. Designing and testing bigraded FGM beam specimens would provide valuable data to assess the accuracy and reliability of the theoretical and numerical predictions.

Finally, the present work can be extended to more complex structural configurations, including plates, shells, and fully three-dimensional bigraded structures. Furthermore, investigating dynamic behavior (free and forced vibrations), stability (buckling), and fracture characteristics under various loading conditions would significantly broaden the scope of this research.

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