

# Conventional problem solving on the linear and nonlinear buckling of truncated conical functionally graded imperfect micro-tubes

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**Abstract.** This paper studies the buckling response of nonuniform functionally graded micro-sized tubes according to the high-order tube theory (HOTT) and classical beam theory (CBT) in addition to nonlocal strain gradient theory. The microtube is made of axially functionally graded material (AFGM). Both inner and outer tube radiuses are changed along the tube length; the microtube is the truncated conical type of tube. The nonlinear partial differential (PD) the formulations are obtained on the basis of the energy conservation method. Then, the linear and nonlinear results are computed via a powerful numerical approach. Finally, the impact of various parameters on the stability of axially functionally graded (AFG) microtube regarding the buckling analysis is discussed.

**Keywords:** buckling analysis; functionally graded materials; microstructure; nonuniform tubes; stability analysis

## 1. Introduction

As a type of composite material, the functionally graded materials are composed of two phases of metal and ceramic on either side of the structure and varying material exponentially between these two sides with no clear border (Azadi 2011). Because of their unique properties, these materials have extensive usage in extreme heat or force environments. The researchers have utilized to manufacture and design various devices with the aid of such composites. Thus, investigating their behavior in different shapes is essential. In this regard, it can refer to papers in which the FG material is utilized in plate (Zhao *et al.* 2021, Rouzegar and Abad 2015, Huang *et al.* 2021a, Jiao *et al.* 2021, Moradi *et al.* 2021, Xu *et al.* 2021), beam (Abo-bakr *et al.* 2021), shell (Mallek *et al.* 2019, Zhang *et al.* 2020a, Liu *et al.* 2021a, 2022, Li *et al.* 2022, Sun *et al.* 2022, Xiao *et al.* 2022, Zhou *et al.* 2022), and panel (Ma *et al.* 2021, Shahverdi and Khalafi 2016, Hou *et al.* 2021, Huang *et al.* 2021b, Liu *et al.* 2021c, Yu *et al.* 2022) shapes. Also, Wang and Wu (2016) investigated the buckling and dynamic behavior of AFG beams under thermal loading. The results in this paper were extracted via the Newmark procedure. Based on Timoshenko beam theory, the dynamic analysis associated with AFG beams, which are subjected to a moving load, was carried out (Xie *et al.* 2019). The dynamic and static characteristics corresponding to a functionally graded beam were examined by using Timoshenko's beam theory. By utilizing the DQ procedure as a solution method, the vibrational behavior of FG beams under a fluid was investigated through the added mass method (Katili *et al.* 2020). The buckling and post-buckling response associated with FG tubes, which are placed in a

nonlinear foundation, under mechanical and thermal loads were explored by Babaei *et al.* (2019a). In another paper, they modeled a functionally graded tube surrounded by a nonlinear medium affected by various types of temperature change to examine its buckling and post-buckling behavior (Babaei *et al.* 2019b).

The small-scale, nano or micro, structures have attracted so many scholars' attention in recent years (Guo *et al.* 2021a, Li *et al.* 2021, Wang *et al.* 2021a, Zhao and Yu 2021, Roudbari *et al.* 2022, Zhong and Liang 2022). They investigated such behaviors of these structures as vibration (Adamian *et al.* 2020, Al-Furjan *et al.* 2020a, Al-Furjan *et al.* 2020b, Li *et al.* 2020b, Zare *et al.* 2020, Dai *et al.* 2021, Naderi *et al.* 2022), wave propagation (Ghorbanpour Arani *et al.* 2015b, Al-Furjan *et al.* 2020c, d, f, Bai *et al.* 2020, Li *et al.* 2020a, Zhang *et al.* 2020b, Guo *et al.* 2021b, Liu *et al.* 2021b), and buckling (Mohammadgholiha *et al.* 2019, Hashemi *et al.* 2019, Al-Furjan *et al.* 2020e, Cheshmeh *et al.* 2020, Lori *et al.* 2020, Najaafi *et al.* 2020, Shariati *et al.* 2020c). In this regard, it can refer to a work that studies the vibration, buckling, as well as possibility of energy harvesting associated with piezoelectric nanobeams (Naderi *et al.* 2021). Also, Behdad *et al.* (2021) carried out the impact of crack on the vibrational response of a nanobeam which is placed on a two-parameter elastic foundation. They developed a model based on Euler Bernoulli's theory and two-phase elasticity. In addition, the vibrational behavior of a viscoelastic microdisk was analyzed by means of modified couple stress and the Kelvin-Voight model (Al-Furjan *et al.* 2021). Esmailpoor Hajilak *et al.* (2019), using modified strain gradient theory, studied forced and free vibrational analysis of reinforced composite nanoshells. The nanoshell is assumed to be affected by thermal loading. Also, the coupled axial and flexural vibration of a mass nanosensor, modeled on the basis of Timoshenko's theory, with an attached mass was studied by means of GDQM (Naderi *et al.* 2020).

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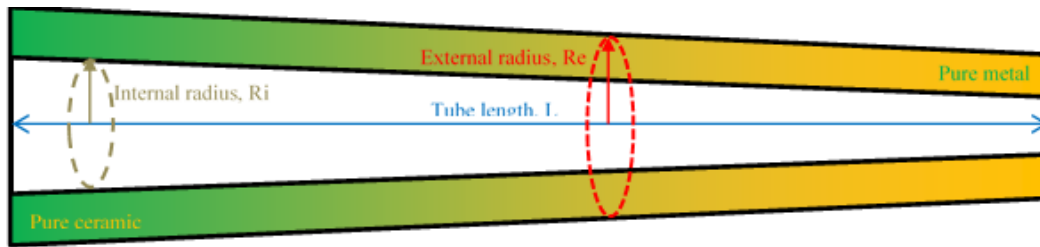


Fig. 1 Geometric and schematic of an axially functionally graded truncated conical tube

It was shown that the stability and buckling analysis of any structure must be examined before designing them. Given this, by considering the surface effect, the model for buckling analysis related to nanobeam was presented with the aid of NSGT and different beam theories (Hashemian *et al.* 2019). In addition, Tounsi *et al.* (2013) managed to present an investigation on the buckling of nanobeams through higher-order beam theory as well as nonlocal elasticity. Also, Fakher *et al.* (2020) presented a paper on the size-dependent thermal buckling related to nanobeams which are placed in a Winkler elastic medium. The static analysis, buckling in addition to bending, related to multi-wall nanotubes was explored (Falvo *et al.* 1997). Next, the paper by Ru (2001) can be cited which deals with the buckling of carbon nanotubes placed on an elastic medium with considering the van der Waals between two layers of the nanotube. Li and Kardomateas (2006) conducted research on the buckling of nanotubes modeled via nonlocal elasticity. In this paper, the closed-form solution was presented.

Nanotubes are one of the most utilized small-sized structures in various devices (Roudbari and Ansari 2020, Saffari *et al.* 2020). Nanotubes have been the topic of different papers, one of which is investigating the sensory capability of nanotubes (Zaporotskova *et al.* 2021). Next, it can refer to a paper in which the vibrational analysis associated with nanotubes modeled via Timoshenko's theory, which is placed on an elastic foundation, is presented (Ghorbanpour Arani *et al.* 2015a). Additionally, by using NSGT and Rayleigh theory, the vibrational behavior of nanotubes subjected to the moving particles was investigated (Roudbari and Doroudgar Jorshari 2018). In the abovementioned paper, this model is presented as a drug delivery device. The control vibration associated with a nanotube subjected to a moving load was carried out by Jorshari *et al.* (2019). The dynamic characteristics of two interacted nanotubes under a moving load were investigated through Newmark and Galerkin methods (Roudbari *et al.* 2020). With the help of the Euler–Bernoulli beam, the nonlinear vibrational response of double-walled nanotubes surrounded by an elastic foundation was explored (Ghorbanpour Arani *et al.* 2016).

In this investigation, the nonlinear buckling analysis associated with an AFG conical nanotube, modeled according to NSGT as well as two beam theories— HOTT and CBT—is carried out. The formulations related to this problem are acquired by means of the energy method. Then, GDQEM is employed to discretize the formulation. The nonlinear results are obtained through the iteration method.

The accuracy and credibility of the formulations and solution procedure are proven. Finally, the impact of such factors as the FG index, rate of radius change, and small-scale parameters on the system's stability is examined.

## 2. Definition of mathematic of problem

A nonuniform tube made of a composition of ceramic on the nanotube's left side and the metal on the right side of the tube is shown in Fig. 1, and the internal radius is denoted by 'Ri', the external radius is indicated by 'Re', and tube length is revealed via 'L'. In the current study, the Si<sub>3</sub>N<sub>4</sub> is assumed to be the ceramic phase, and the Nickel is considered the metal phase. The material and mechanical properties of AFG structures are changed in the tube length according to the following mathematical relation (Hashemi *et al.* 2019, Moayedi *et al.* 2019, 2020a, b, Oyarhossein *et al.* 2020, Shariati *et al.* 2020b).

$$E(x) = (E_N - E_A) \left(1 - \frac{x}{L}\right)^\eta + E_N \quad (1a)$$

$$\nu(x) = (\nu_A - \nu_N) \left(1 - \frac{x}{L}\right)^\eta + \nu_N \quad (1b)$$

'E' is Young's modulus, 'ν' is the Poisson ratio, which the values of them are presented in Table 1. Also, 'η' is the functionally graded parameter (functionally graded power index), '(<sub>A</sub>)' refers to the Al<sub>2</sub>O<sub>3</sub>, and '(<sub>N</sub>)' relates to the Nickel.

The radius of the tube is considered in the nonuniform function, in which the radius value is changed along the tube length based on the following equation for both internal radius as well as external radius.

$$R = R_L \left(1 - \chi \frac{x}{L}\right) \quad (2a)$$

where 'χ' is the parameter to show the rate of radius change, and 'R<sub>L</sub>' is the internal or external radius at the left side of the tube (x=0), moreover, for the internal radius,

$$R_i = R_{iL} \left(1 - \chi_i \frac{x}{L}\right) \quad (2b)$$

And for the external radius

$$R_e = R_{eL} \left(1 - \chi_e \frac{x}{L}\right) \quad (2c)$$

where 'χ<sub>i</sub>' and 'χ<sub>e</sub>' are the parameters to show the rate of internal and external radius change.

Table 1 Values of mechanical properties of ceramic and metal

	Al <sub>2</sub> O <sub>3</sub>	Nickel
Young's modulus, E (GPa)	323.393	205.097
Poisson ratio, ν	0.24	0.31

According to the energy principle, the formulation and associated boundary conditions of high-order microtube theory will be generated in the following.

$$\delta V + \delta S = 0 \tag{3}$$

Here ‘V’ and ‘S’ are the energy of the external work and the strain energy, respectively, and the virtual energy of the external work regarding the buckling force (F<sub>B</sub>) is calculated as follows (Habibi *et al.* 2019a, Safarpour *et al.* 2019b, Alipour *et al.* 2020, Ebrahimi *et al.* 2020a, Chen *et al.* 2022):

$$\delta V = \int_0^L F_B w_{,x} \delta(w_{,x}) dx \tag{4}$$

where ‘w’ is the lateral displacement. Moreover, the strain energy of the microtube on the basis of NSGT along with the nonlinear Von-Kármán theory is calculated as follows (Habibi *et al.* 2018a, 2019b, d, e, Pourjabari *et al.* 2019, Safarpour *et al.* 2019a):

$$2S = \iiint \sigma : \epsilon dv \tag{5}$$

where ‘ε’ is the strains that are considered as follows (Habibi *et al.* 2016, 2018b, Ebrahimi *et al.* 2019a, Esmailpoor Hajilak *et al.* 2019):

$$2\epsilon_{ij} = u_{i,j} + u_{j,i} \tag{6}$$

‘u’ is the displacement field regarding the high-order tube theory in addition to classical beam theory, which is assumed as follows (Ebrahimi *et al.* 2019b, c, 2020b, Mohammadi *et al.* 2019, Mohammadgholiha *et al.* 2019, Habibi *et al.* 2020, Shariati *et al.* 2020a, Shokrgozar *et al.* 2020):

$$u_x(x, y, z) = u(x) + \zeta [w_{,x}(x) + \psi(x)] - zw_{,x}(x) \tag{7a}$$

$$u_y(x, y, z) = 0 \tag{7b}$$

$$u_z(x, y, z) = w(x) \tag{7c}$$

‘ψ’ is the bending rotation, ‘w’ and ‘u’ represent the transverse as well as axial displacements. Moreover, for the high-order tube theory:

$$\zeta = r \sin(\theta) + r \sin(\theta) (R_0^2 R_i^2 r^{-2} - r^2/3)/(Ri^2 + Re^2) \tag{8a}$$

Classical beam theory:

$$\zeta = 0 \tag{8b}$$

The stresses are:

$$\sigma_{ij} = E \epsilon_{ij}, i = j \tag{9a}$$

$$\sigma_{ij} = G \epsilon_{ij}, i \neq j \tag{9b}$$

$$G = \frac{1}{2} E (1 + \nu)^{-1} \tag{9c}$$

Nonlocal strain gradient theory (NSGT) is one of the most accurate size-dependent theory which includes both softening and hardening phenomena is used in the present research. The presented theory which is introduced by Lim *et al.* (2015) impacts the stresses as (Habibi *et al.* 2017, 2019c, Safarpour *et al.* 2018, 2020, Ghazanfari *et al.* 2020):

$$[1 - (ea)^2 \nabla^2] \sigma_{ij} = (1 - l^2 \nabla^2) C : \epsilon_{ij} \tag{10}$$

where the tensor of elastic modulus is denoted by ‘C’, ‘l’ is the gradient strain parameter, ‘ea’ is the nonlocal parameter, and ‘∇’ is the Laplace operator. Based on NSGT, by substitution of the virtual strain energy (Eq.(5)) and virtual energy of external work (Eq.(4)) into the conservation energy principle (Eq.(3)), also, according to the Euler-Lagrange procedure, the following motion equations along with the related boundary conditions are obtained:

$$\begin{aligned} & -3l^2 \frac{d^2 A_{11}}{dx^2} \frac{\partial}{\partial x} \left( \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) + \frac{dA_{11}}{dx} \left( \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) \\ & \left[ \frac{d^3 A_{11}}{dx^3} \left( \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) + \frac{1}{2} A_{11} \frac{\partial^3}{\partial x^3} \left( \left( \frac{\partial w}{\partial x} \right)^2 \right) \right] \\ & + 3 \frac{dA_{11}}{dx} \left( \frac{\partial^3 u}{\partial x^3} + \frac{\partial^2}{\partial x^2} \left( \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) \right) + A_{11} \left( \frac{\partial^4 u}{\partial x^4} \right) \\ & + A_{11} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x} \left( \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) \right] - 3l^2 \frac{d^2 A_{11}}{dx^2} \frac{\partial^2 u}{\partial x^2} = 0 \end{aligned} \tag{11a}$$

$$\begin{aligned} & + H_{11} \frac{\partial^2 \psi}{\partial x^2} - B_{11} \left( \frac{\partial w}{\partial x} + \psi \right) + \frac{d\Sigma \partial^2 w}{dx \partial x^2} \\ & l^2 \left( -3 \frac{d^2 H_{11}}{dx^2} \frac{\partial^2 \psi}{\partial x^2} - \frac{d^3 H_{11}}{dx^3} \frac{\partial \psi}{\partial x} - \frac{d^2 \Sigma \partial^2 w}{dx^3 \partial x^2} + B_{11} \frac{\partial^2 \psi}{\partial x^2} \right) \\ & - 3 \frac{d\Sigma \partial^4 w}{dx \partial x^4} - \Sigma \frac{\partial^3 \psi}{\partial x^3} - 3 \frac{d^2 \Sigma \partial^3 w}{dx^2 \partial x^3} + \frac{d^2 B_{11} \partial w}{dx^2 \partial x} \\ & + \frac{d^2 B_{11}}{dx^2} \psi + 2 \frac{dB_{11}}{dx} \frac{\partial \psi}{\partial x} + 2 \frac{dB_{11}}{dx} \frac{\partial^2 w}{\partial x^2} + B_{11} \frac{\partial^3 w}{\partial x^3} \\ & + \Sigma \frac{\partial^3 w}{\partial x^3} + \frac{dH_{11}}{dx} \frac{\partial \psi}{\partial x} - l^2 \left( H_{11} \frac{\partial^4 \psi}{\partial x^4} + 3 \frac{dH_{11}}{dx} \frac{\partial^3 \psi}{\partial x^3} \right) = 0 \end{aligned} \tag{11b}$$

$$\begin{aligned} & Y \frac{\partial^4 w}{\partial x^4} + 2 \frac{dY \partial^3 w}{dx \partial x^3} + \frac{d^2 Y \partial^2 w}{dx^2 \partial x^2} - B_{11} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial \psi}{\partial x} \right) \\ & + 2 \frac{d\Sigma \partial^2 \psi}{dx \partial x^2} + \Sigma \frac{\partial^3 \psi}{\partial x^3} - \frac{dB_{11}}{dx} \left( \frac{\partial w}{\partial x} + \psi \right) + l^2 B_{11} \frac{\partial^3 \psi}{\partial x^3} \\ & - A_{11} \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{2} \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial x} \right)^2 \right) \frac{\partial w}{\partial x} + \frac{d^2 \Sigma \partial \psi}{dx^2 \partial x} + l^2 3 \frac{dB_{11} \partial^2 \psi}{dx \partial x^2} \\ & - A_{11} \left( \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) \frac{\partial^2 w}{\partial x^2} - \frac{dA_{11}}{dx} \left( \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) \frac{\partial w}{\partial x} \\ & \left[ 4 \frac{d^3 \Sigma \partial^2 \psi}{dx^3 \partial x^2} + \frac{d^4 \Sigma \partial \psi}{dx^4 \partial x} + Y \frac{\partial^6 w}{\partial x^6} + \frac{d^4 Y \partial^2 w}{dx^4 \partial x^2} \right] \\ & - l^2 \left[ 4 \frac{d^3 Y \partial^3 w}{dx^3 \partial x^3} + 6 \frac{d^2 Y \partial^4 w}{dx^2 \partial x^4} + 4 \frac{dY \partial^5 w}{dx \partial x^5} + \Sigma \frac{\partial^5 \psi}{\partial x^5} \right. \\ & \left. - B_{11} \frac{\partial^4 w}{\partial x^4} - 3 \frac{dB_{11}}{dx} \frac{\partial^3 w}{\partial x^3} - \frac{d^3 B_{11}}{dx^3} \frac{\partial w}{\partial x} + 6 \frac{d^2 \Sigma \partial^3 \psi}{dx^2 \partial x^3} \right. \\ & \left. - 3 \frac{d^2 B_{11} \partial^2 w}{dx^2 \partial x^2} - \frac{d^3 B_{11}}{dx^3} \psi - 3 \frac{d^2 B_{11} \partial \psi}{dx^2 \partial x} + 4 \frac{d\Sigma \partial^4 \psi}{dx \partial x^4} \right] \end{aligned} \tag{11c}$$

$$\begin{aligned}
 & + (ea)^2 \left[ \left( \frac{1}{2L} \frac{d^3 A_{11}}{dx^3} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx + \frac{3}{L} \frac{d^2 A_{11}}{dx^2} \int_0^L \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} dx \right) \frac{\partial w}{\partial x} \right. \\
 & \left. + \left( \frac{3}{L} \frac{dA_{11}}{dx} \int_0^L \frac{\partial^3 w}{\partial x^3} \frac{\partial w}{\partial x} dx + \frac{3}{L} \frac{dA_{11}}{dx} \int_0^L \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dx \right) \frac{\partial^2 w}{\partial x^2} \right. \\
 & \left. + \left( \frac{1}{2L} \frac{d^2 A_{11}}{dx^2} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx + \frac{2}{L} \frac{dA_{11}}{dx} \int_0^L \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} dx \right) \frac{\partial^2 w}{\partial x^2} \right. \\
 & \left. + F \frac{\partial^2 w}{\partial x^2} - (ea)^2 F \frac{\partial^4 w}{\partial x^4} + (ea)^2 \left( \frac{1}{L} A_{11} \int_0^L \frac{\partial^4 w}{\partial x^4} \frac{\partial w}{\partial x} dx \right. \right. \\
 & \left. \left. + \frac{3}{L} A_{11} \int_0^L \frac{\partial^3 w}{\partial x^3} \frac{\partial^2 w}{\partial x^2} dx \right) \frac{\partial w}{\partial x} \right] \\
 & = 0
 \end{aligned}$$

were

$$Y(x) = H_{11}(x) + D_{11}(x) - 2E_{11}(x) \tag{12a}$$

$$\Sigma(x) = H_{11}(x) - E_{11}(x) \tag{12b}$$

$$A_{11} = \iint_A E(x) dA \tag{12c}$$

For high-order tube theory:

$$(D_{11}, E_{11}, H_{11}) = \iint_A E(x) (z^2, z\zeta, \zeta^2) dA \tag{12d}$$

$$\begin{aligned}
 & (C_{11}, C_{12}, C_{13}, C_{14}) \\
 & = \iint_A E(x) (\zeta_x^2, \zeta_x, z\zeta_x, \zeta\zeta_x) dA = 0
 \end{aligned} \tag{12e}$$

$$B_{11} = \iint_A G(x) [\zeta_y^2 + \zeta_z^2] dA \tag{12f}$$

For the classical beam theory:

$$E_{11} = H_{11} = C_{11} = C_{12} = C_{13} = C_{14} = B_{11} = 0 \tag{12g}$$

The boundary conditions are (Shafiei and She 2018, Shafiei *et al.* 2019, 2020):

$$\begin{aligned}
 \delta(u): & A_{11} \frac{\partial u}{\partial x} - l^2 \frac{d^2 A_{11}(x)}{dx^2} \frac{\partial u}{\partial x} \\
 & - l^2 \left( A_{11}(x) \frac{\partial^3 u}{\partial x^3} + 2 \frac{dA_{11}(x)}{dx} \frac{\partial^2 u}{\partial x^2} \right) = 0
 \end{aligned} \tag{13a}$$

$$\begin{aligned}
 \delta(\psi): & -l^2 \left( \frac{d^2 E_{11}}{dx^2} \left( \frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) \right. \\
 & \left. + 2 \frac{dE_{11}}{dx} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3} \right) \right) \\
 E_{11} & \left( \frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) - C_{11} \frac{\partial^2 w}{\partial x^2} - l^2 E_{11} \left( \frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^4 w}{\partial x^4} \right) \\
 & + l^2 \left( \frac{d^2 C_{11}}{dx^2} \frac{\partial^2 w}{\partial x^2} + 2 \frac{dC_{11}}{dx} \frac{\partial^3 w}{\partial x^3} + C_{11} \frac{\partial^4 w}{\partial x^4} \right) = 0
 \end{aligned} \tag{13b}$$

$$\begin{aligned}
 \delta(w): & -l^2 B_{11} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^3 w}{\partial x^3} \right) \\
 & - \frac{dY}{dx} \frac{\partial^2 w}{\partial x^2} - Y \frac{\partial^3 w}{\partial x^3} - \frac{d\Sigma}{dx} \frac{\partial \psi}{\partial x} - \Sigma \frac{\partial^2 \psi}{\partial x^2} + B_{11} \left( \psi + \frac{\partial w}{\partial x} \right)
 \end{aligned} \tag{13c}$$

$$\begin{aligned}
 & + l^2 \left[ \begin{aligned} & Y \frac{\partial^5 w}{\partial x^5} + 3 \frac{dY}{dx} \frac{\partial^4 w}{\partial x^4} + 3 \frac{d^2 Y}{dx^2} \frac{\partial^3 w}{\partial x^3} + \frac{d^3 Y}{dx^3} \frac{\partial^2 w}{\partial x^2} \\ & + \Sigma \frac{\partial^4 \psi}{\partial x^4} + 3 \frac{d\Sigma}{dx} \frac{\partial^3 \psi}{\partial x^3} + 3 \frac{d^2 \Sigma}{dx^2} \frac{\partial^2 \psi}{\partial x^2} + \frac{d^3 \Sigma}{dx^3} \frac{\partial \psi}{\partial x} \\ & - \frac{d^2 B_{11}}{dx^2} \left( \psi + \frac{\partial w}{\partial x} \right) - 2 \frac{dB_{11}}{dx} \left( \frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) \end{aligned} \right] = 0 \\
 & \delta \left( \frac{\partial w}{\partial x} \right): Y \frac{\partial^2 w}{\partial x^2} + \Sigma \frac{\partial \psi}{\partial x} - l^2 \left( \frac{d^2 Y}{dx^2} \frac{\partial^2 w}{\partial x^2} + 2 \frac{dY}{dx} \frac{\partial^3 w}{\partial x^3} \right) \\
 & - l^2 \left( + \frac{d^2 \Sigma}{dx^2} \frac{\partial \psi}{\partial x} + 2 \frac{d\Sigma}{dx} \frac{\partial^2 \psi}{\partial x^2} + \Sigma \frac{\partial^3 \psi}{\partial x^3} \right) - l^2 \left( Y \frac{\partial^4 w}{\partial x^4} \right) = 0
 \end{aligned} \tag{13d}$$

### 3. Solution methodology

The generalized differential quadrature element method (GDQEM) is operated to solve nonlinear PDE's equations of the problem. Based on the GDQEM and by assembling the governing equation (Eq. (11)) in conjunction with the related boundary conditions (Eq. (13)), the linear buckling force of FG nanotube can be calculated as follows (Ebrahimi and Shafiei 2017, Ghadiri *et al.* 2017e, Mirjavadi *et al.* 2017a, Shafiei and Kazemi 2017a, Shafiei *et al.* 2017d, Azimi *et al.* 2018):

$$\begin{aligned}
 & \begin{bmatrix} [K_{aa}] & [K_{ab}] \\ [K_{ba}] & [K_{bb}] \end{bmatrix} \begin{Bmatrix} \{\lambda_a\} \\ \{\lambda_b\} \end{Bmatrix} \\
 & = F \begin{bmatrix} [M_{aa}] & [M_{ab}] \\ [M_{ba}] & [M_{bb}] \end{bmatrix} \begin{Bmatrix} \{\lambda_a\} \\ \{\lambda_b\} \end{Bmatrix}
 \end{aligned} \tag{14}$$

Also, according to the eigenvalue problems, and based on the GDQEM, for the present problem, we assumed (Ehyaei *et al.* 2017, Ghadiri *et al.* 2017c, d, Mirjavadi *et al.* 2017d, Shafiei and Kazemi 2017b, Shafiei *et al.* 2017c):

$$\begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ \vdots \\ K_{e-2} \\ K_{e-1} \\ K_e \end{bmatrix}^T = F \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ \vdots \\ M_{e-2} \\ M_{e-1} \\ M_e \end{bmatrix}^T \begin{Bmatrix} u_1 \\ \psi_1 \\ w_1 \\ \vdots \\ u_e \\ \psi_e \\ w_e \end{Bmatrix} \tag{15}$$

'K' and 'M' are the stiffness and mass matrices, and subscript 'e' means the element. In order to obtain the buckling response of the present engineering problem, we divided the beam into 'e' parts, so 'e'-element for the beam are required. Utilizing the Eq. (11), stiffness-matrices ( $K_e$ ) and mass-matrices ( $M_e$ ) are computed as (Ghadiri *et al.* 2017a, b, Mirjavadi *et al.* 2017b, c, Shafiei *et al.* 2017a, b):

$$\begin{bmatrix} \frac{dA_{11}(x)}{dx} \sum_{s=1}^n C_{rs}^{(1)} u_s + A_{11}(x) \sum_{s=1}^n C_{rs}^{(2)} u_s \\ -l^2 \left( \frac{d^3 A_{11}(x)}{dx^3} \sum_{s=1}^n C_{rs}^{(1)} u_s + 3 \frac{d^2 A_{11}(x)}{dx^2} \sum_{s=1}^n C_{rs}^{(2)} u_s \right) \\ + 3 \frac{dA_{11}(x)}{dx} \sum_{s=1}^n C_{rs}^{(3)} u_s + A_{11}(x) \sum_{s=1}^n C_{rs}^{(4)} u_s \end{bmatrix}_e = 0 \tag{16a}$$

$$\begin{aligned}
 & \left[ \begin{aligned}
 & \frac{d\Sigma}{dx} \sum_{s=1}^n C_{rs}^{(2)} w_s - B_{11} \left( \sum_{s=1}^n C_{rs}^{(1)} w_s + \psi_s \right) \\
 & + \Sigma \sum_{s=1}^n C_{rs}^{(3)} w_s + E_{11} \sum_{s=1}^n C_{rs}^{(2)} \psi + \frac{dE_{11}}{dx} \sum_{s=1}^n C_{rs}^{(1)} \psi \\
 & -l^2 \left( \begin{aligned}
 & \frac{d^3 \Sigma}{dx^3} \sum_{s=1}^n C_{rs}^{(2)} w_s + 3 \frac{d^2 \Sigma}{dx^2} \sum_{s=1}^n C_{rs}^{(3)} w_s \\
 & + 3 \frac{d \Sigma}{dx} \sum_{s=1}^n C_{rs}^{(4)} w_s + \Sigma \sum_{s=1}^n C_{rs}^{(5)} w_s \\
 & + \frac{d^3 E_{11}(x)}{dx^3} \sum_{s=1}^n C_{rs}^{(1)} \psi + 3 \frac{d^2 E_{11}(x)}{dx^2} \sum_{s=1}^n C_{rs}^{(2)} \psi \\
 & + 3 \frac{d E_{11}(x)}{dx} \sum_{s=1}^n C_{rs}^{(3)} \psi + E_{11}(x) \sum_{s=1}^n C_{rs}^{(4)} \psi
 \end{aligned} \right) \\
 & + l^2 \left( \begin{aligned}
 & B_{11}(x) \left( \sum_{s=1}^n C_{rs}^{(2)} \psi_s + \sum_{s=1}^n C_{rs}^{(3)} w_s \right) \\
 & + \frac{d^2 B_{11}(x)}{dx^2} \left( \psi_s + \sum_{s=1}^n C_{rs}^{(1)} w_s \right) \\
 & + 2 \frac{d B_{11}(x)}{dx} \left( \sum_{s=1}^n C_{rs}^{(1)} \psi_s + \sum_{s=1}^n C_{rs}^{(2)} w_s \right)
 \end{aligned} \right) \\
 & = 0
 \end{aligned} \right] \quad (16b)
 \end{aligned}$$

$$\begin{aligned}
 & \left[ \begin{aligned}
 & Y \sum_{s=1}^n C_{rs}^{(4)} w_s + 2 \frac{dY}{dx} \sum_{s=1}^n C_{rs}^{(3)} w_s + \frac{d^2 Y}{dx^2} \sum_{s=1}^n C_{rs}^{(2)} w_s \\
 & + \Sigma \sum_{s=1}^n C_{rs}^{(3)} \psi_s + \frac{d^2 \Sigma}{dx^2} \sum_{s=1}^n C_{rs}^{(1)} \psi + 2 \frac{d \Sigma}{dx} \sum_{s=1}^n C_{rs}^{(2)} \psi_s \\
 & -l^2 4 \frac{d \Sigma}{dx} \sum_{s=1}^n C_{rs}^{(4)} \psi_s - \frac{d B_{11}}{dx} \left( \sum_{s=1}^n C_{rs}^{(1)} w_s + \psi_s \right) - B_{11} \sum_{s=1}^n C_{rs}^{(1)} \psi_s \\
 & -l^2 \left( \begin{aligned}
 & Y \sum_{s=1}^n C_{rs}^{(6)} w_s + 4 \frac{dY}{dx} \sum_{s=1}^n C_{rs}^{(5)} w_s + 6 \frac{d^2 Y}{dx^2} \sum_{s=1}^n C_{rs}^{(4)} w_s \\
 & + 4 \frac{d^3 Y}{dx^3} \sum_{s=1}^n C_{rs}^{(3)} w_s + \frac{d^4 Y}{dx^4} \sum_{s=1}^n C_{rs}^{(2)} w_s + \frac{d^4 \Sigma}{dx^4} \sum_{s=1}^n C_{rs}^{(1)} \psi_s \\
 & + 4 \frac{d^3 \Sigma}{dx^3} \sum_{s=1}^n C_{rs}^{(2)} \psi_s + 6 \frac{d^2 \Sigma}{dx^2} \sum_{s=1}^n C_{rs}^{(3)} \psi_s + \Sigma \sum_{s=1}^n C_{rs}^{(5)} \psi_s \\
 & - 3 \left( \frac{d B_{11}(x)}{dx} \sum_{s=1}^n C_{rs}^{(2)} \psi_s + \frac{d^2 B_{11}(x)}{dx^2} \sum_{s=1}^n C_{rs}^{(1)} \psi_s \right) \\
 & - \left( \frac{d^3 B_{11}(x)}{dx^3} \sum_{s=1}^n C_{rs}^{(1)} w_s + B_{11}(x) \sum_{s=1}^n C_{rs}^{(4)} w_s \right) \\
 & + 3 \frac{d^2 B_{11}(x)}{dx^2} \sum_{s=1}^n C_{rs}^{(2)} w_s + 3 \frac{d B_{11}(x)}{dx} \sum_{s=1}^n C_{rs}^{(3)} w_s \\
 & - \left( B_{11}(x) \sum_{s=1}^n C_{rs}^{(3)} \psi_s + \frac{d^3 B_{11}(x)}{dx^3} \psi_s \right)
 \end{aligned} \right) \\
 & \left[ -B_{11} \sum_{s=1}^n C_{rs}^{(2)} w_s = F \sum_{s=1}^n C_{rs}^{(2)} w_s - F(ea)^2 \sum_{s=1}^n C_{rs}^{(4)} w_s = 0 \right] \quad (16c)
 \end{aligned}$$

In which ‘n’ is the number of grid points for each element, and ‘C’ is given as

$$C_{ij}^{(1)} = \Lambda(x_i)/(x_i - x_j)\Lambda(x_j) \quad (17)$$

where  $\tilde{M}(x)$  is:

$$\Lambda(x_i) = \prod_{j=1, j \neq i}^n (x_i - x_j) \quad (18)$$

The other derivative order of  $C^{(r)}$  can be obtained through (Ebrahimi and Shafiei 2016, Shafiei *et al.* 2016c, d, f, Ebrahimi *et al.* 2017, Shivanian *et al.* 2017):

$$C_{ij}^{(r)} = r[-C_{ij}^{(r-1)}/(x_i - x_j) + C_{ij}^{(1)} C_{ij}^{(r-1)}] \quad (19)$$

The grid points are distributed as shown in the following equation (Azimi *et al.* 2016, Ghadiri and Shafiei 2016a, c, Shafiei *et al.* 2016a, e, g):

$$2x_i = (1 - \cos \pi ((i - 1)/(N - 1)))L \quad (20)$$

Firstly, by using Eq. (15), the linear load and eigenvectors are obtained. Then by using the linear vectors the following nonlinear equation can be written

$$\begin{aligned}
 & \begin{bmatrix} [K_{dd}] & [K_{ab}] \\ [K_{bd}] & [K_{bb}] \end{bmatrix}_{Linear} \begin{Bmatrix} \{\lambda_d\} \\ \{\lambda_b\} \end{Bmatrix} \\
 & + \begin{bmatrix} [K_{dd}] & [K_{ab}] \\ [K_{bd}] & [K_{bb}] \end{bmatrix}_{Non-linear} \begin{Bmatrix} \{\lambda_d\} \\ \{\lambda_b\} \end{Bmatrix} \\
 & = \bar{F}_{nonlinear} \begin{bmatrix} [M_{dd}] & [M_{ab}] \\ [M_{bd}] & [M_{bb}] \end{bmatrix} \begin{Bmatrix} \{\lambda_d\} \\ \{\lambda_b\} \end{Bmatrix} \quad (21)
 \end{aligned}$$

Then, by using Eq. (21), the first nonlinear buckling load and first nonlinear eigenvector related to the problem are acquired. Again using this vector, a new equation similar to Eq. (21) can be written, based on which the new eigenvalue and vectors can be achieved. This process will be repeated until the error between two consecutive buckling loads is less than 1e-4, the error value is calculated as (Ghadiri and Shafiei 2016b, Ghadiri *et al.* 2016a, b, c, d, Shafiei *et al.* 2016b).

$$error = (F_{i+1} - F_i)/F_{i+1} \quad (22)$$

### 4. Results and discussion

Here, in two parts, the results associated with the nonlinear buckling of a conical AFG nanotube modeled via nonlocal strain gradient are presented. It should be stated that the nondimensional factors are:

Nonlinear amplitude or large deflection amplitude ( $\Delta$ ):

$$\Delta = \frac{a}{(r_g L)} \sqrt{\frac{I_L}{A_L}} \quad (23a)$$

where ‘ $r_g$ ’ is the gyration radius, ‘a’ is the nonlinear amplitude, ‘I’ and ‘A’ are the moment and area on the left side of the tube ( $x=0$ ). Nonlocal parameter ( $\beta$ ):

$$\beta^2 L = 2(ea)^2 \quad (23b)$$

Strain gradient parameter ( $\tau$ ):

$$\tau^2 L = 2l^2 \quad (23c)$$

Buckling load ( $\Xi$ ):

$$\Xi = \sqrt{FL^2(E_{AI}L)^{-1}} \quad (23d)$$

First, the buckling load associated with the nanotube for different boundary conditions,  $\chi$ , and nonlocal parameters

Table 2 Comparison of the computed buckling load ( $\Xi_2$ ) of presented results with the results of Wang *et al.* (2021b),  $L=30Re$ ,  $Re=5Ri$ 

		Local	ea=0, l=Re	ea=Re, l=0	ea=2Re, l=Re	ea=3Re, l=2Re
Fully pinned boundary conditions (PP)						
$\chi=+0.25$	Wang <i>et al.</i> (2021b)	4.778694	4.833018	4.721949	4.611971	4.513428
	Present, HOTT	4.778216131	4.832534698	4.721476805	4.611509803	4.512976657
	Present, CBT	4.779649739	4.833984604	4.72289339	4.612893394	4.514330686
$\chi=0.0$	Wang <i>et al.</i> (2021b)	8.197196	8.287088	8.108278	7.938851	7.788109
	Present, HOTT	8.19637628	8.286259291	8.107467172	7.938057115	7.787330189
	Present, CBT	8.198835439	8.288745418	8.109899656	7.94043877	7.789666622
$\chi=-0.25$	Wang <i>et al.</i> (2021b)	13.0385	13.18466	12.88905	12.6012	12.34363
	Present, HOTT	13.03719615	13.18334153	12.8877611	12.59993988	12.34239564
	Present, CBT	13.0411077	13.18729693	12.89162781	12.60372024	12.34609873
Fully clamped boundary conditions (CC)						
$\chi=+0.25$	Wang <i>et al.</i> (2021b)	18.3395	19.22296	17.50776	16.17372	15.89125
	Present, HOTT	18.33766605	19.2210377	17.50600922	16.17210263	15.88966088
	Present, CBT	18.3431679	19.22680459	17.51126155	16.17695474	15.89442825
$\chi=0.0$	Wang <i>et al.</i> (2021b)	31.88636	33.28505	30.54644	28.31663	26.87336
	Present, HOTT	31.88317136	33.2817215	30.54338536	28.31379834	26.87067266
	Present, CBT	31.89273727	33.29170701	30.55254929	28.32229333	26.87873467
$\chi=-0.25$	Wang <i>et al.</i> (2021b)	49.77018	52.08943	47.57653	44.00916	41.65587
	Present, HOTT	49.76520298	52.08422106	47.57177235	44.00475908	41.65170441
	Present, CBT	49.78013404	52.09984789	47.58604531	44.01796183	41.66420117

are obtained and compared to the results extracted from the reference in Table 2. It should be mentioned that the results are attained for two beam theories. As seen, the crucial point of this table is that the current results have little difference from the ones obtained from the other paper, proving the accuracy of the presented results. Also, it is evident that the higher nonlocality leads to a lower buckling load, as the system is softer.

Next, the influence of parameters which can affect the nonlinear buckling load of an AFG nanotube are investigated. First, in Fig. 2, the variation of buckling load for the nanotube is plotted against the rate of radius change ( $\chi$ ) for three different nanotube states—uniform internal radius, uniform external radius, and fully conical—and various material types—AFG, fully ceramic, and fully metal. The results in this figure indicate that, regardless of the shape of the nanotube and the rate of radius change, the system's buckling load can be higher if the effect of ceramic is more apparent, lower FG index. Additionally, it is evident that the buckling load of the nanotube is reduced by raising the rate of radius change, except the cases with uniform external radius,  $\chi_e=0$ .

In Fig. 3, similar to the previous one, the variation of the buckling load is plotted versus the rate of radius change for three different types of nanotube. However, in this figure, these results are extracted for different nonlinear

amplitudes. It can be concluded from this figure that the system is less stable in the linear state. In other words, the nanotube's buckling load is heightened by intensifying nonlinear amplitude. Also, the increasing effect related to the radius change in cases with  $\chi_i=\chi_e\neq 0$  and  $\chi_i=0$ ,  $\chi_e\neq 0$  can be observed.

Lastly, in Figs. 4 and 5, the impacts of the size-dependent theory on the stability of the conical nanotube are investigated. In these figures, the buckling loads are obtained for various nonlocal and strain gradient parameters. The results are shown in figures that the x-axes indicate the buckling load of the system, and the y-axes exhibit the rate of radius change for three different types of the nanotube. As expected, Fig. 4 shows the softening effect of the nonlocal parameter, as the higher the nonlocal parameter is, the lower the nonlinear buckling load of the system can be in any type of nanotube. Against this factor, intensifying the strain gradient factor cause the system to be more stable, having a higher buckling load. Also, the dominant effect of the outer radius in the system can be seen in both of the figures, since by changing the external radius, the buckling load is dropped, while it is heightened up when only the inner radius changes, and when both of them are changing the value of buckling is reduced by increasing  $\chi$ .

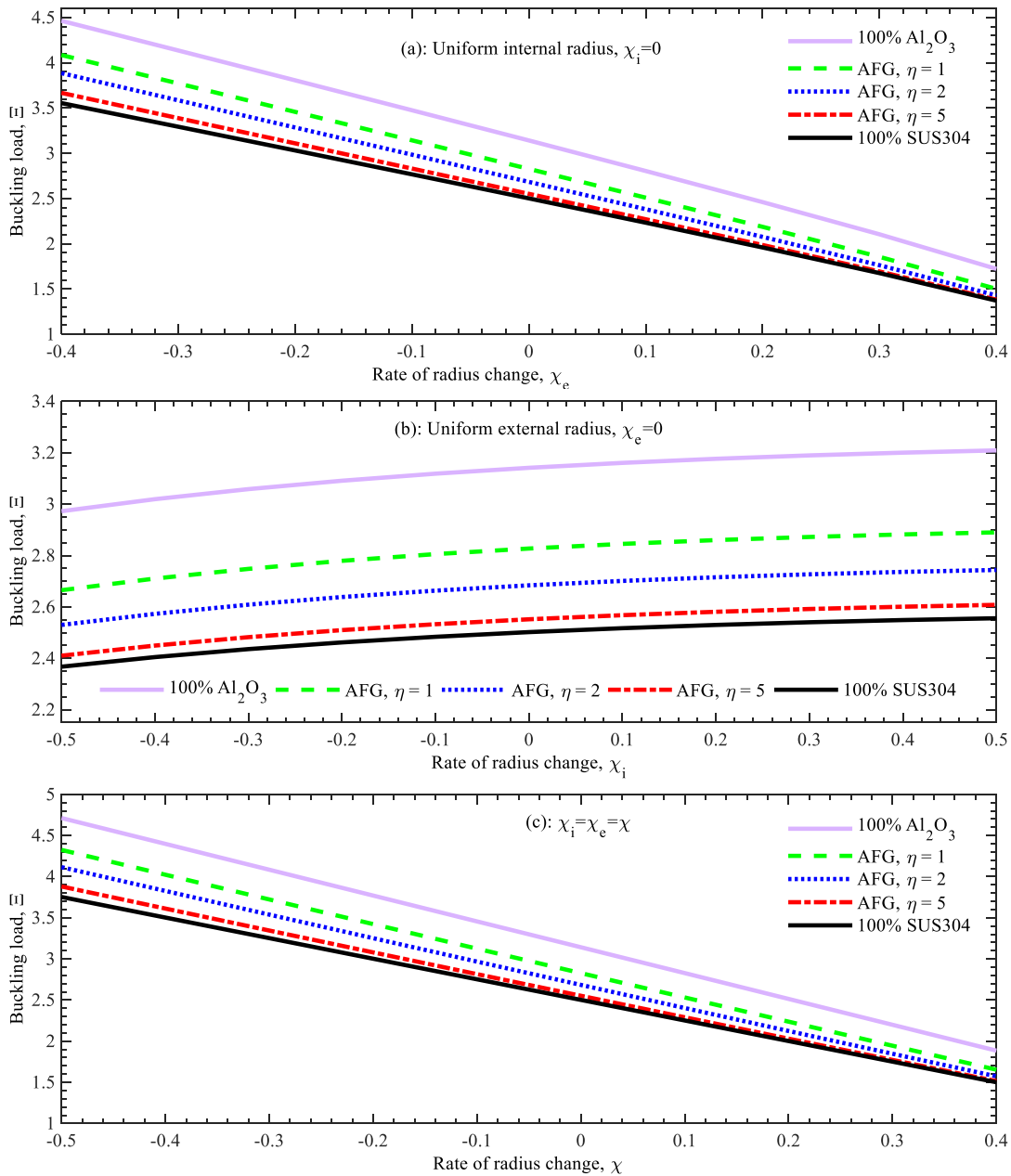
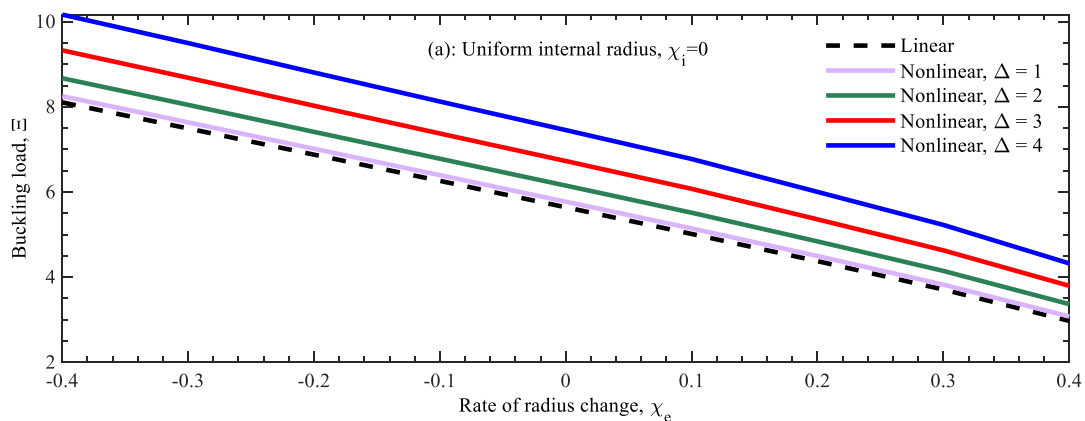


Fig. 2 Impact of AFG parameter ( $\eta$ ) on the buckling load ( $\Xi$ ) of fully pinned tube versus the rate of radius change ( $\chi$ ), a: when the internal radius is uniform (external radius is nonuniform,  $\chi_i=0, \chi_e \neq 0$ ), b: the external radius is uniform (internal radius is nonuniform,  $\chi_e=0, \chi_i \neq 0$ ), and c: both internal and external radiuses are nonuniform ( $\chi_i=\chi_e \neq 0$ ),  $Re=2Ri, L=40Re$



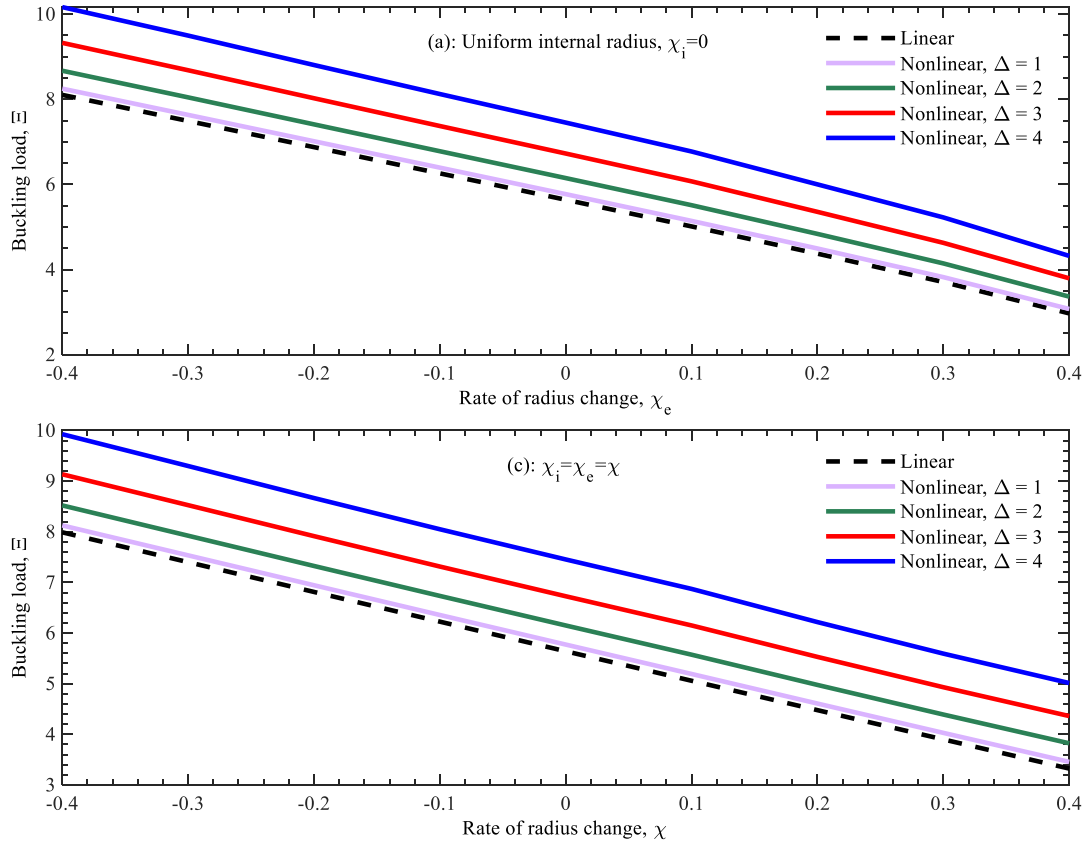
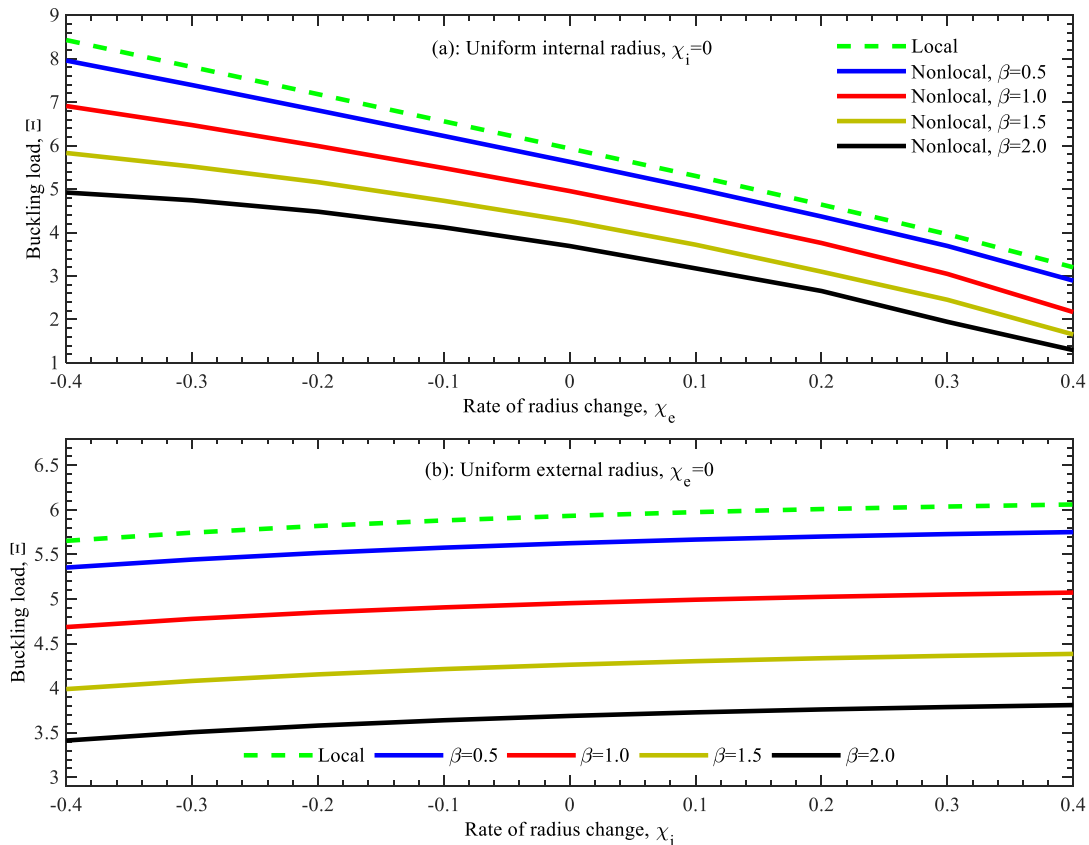


Fig. 3 Buckling load ( $\Xi$ ) of AFG clamped tube versus the rate of radius change ( $\chi$ ) along with the impact of nonlinear amplitude ( $\Delta$ ), a: when the internal radius is uniform (external radius is nonuniform,  $\chi_i=0, \chi_e \neq 0$ ), b: the external radius is uniform (internal radius is nonuniform,  $\chi_e=0, \chi_i \neq 0$ ), and c: both internal and external radiuses are nonuniform ( $\chi_i=\chi_e \neq 0$ ),  $\eta=1, Re=2Ri, L=40Re$



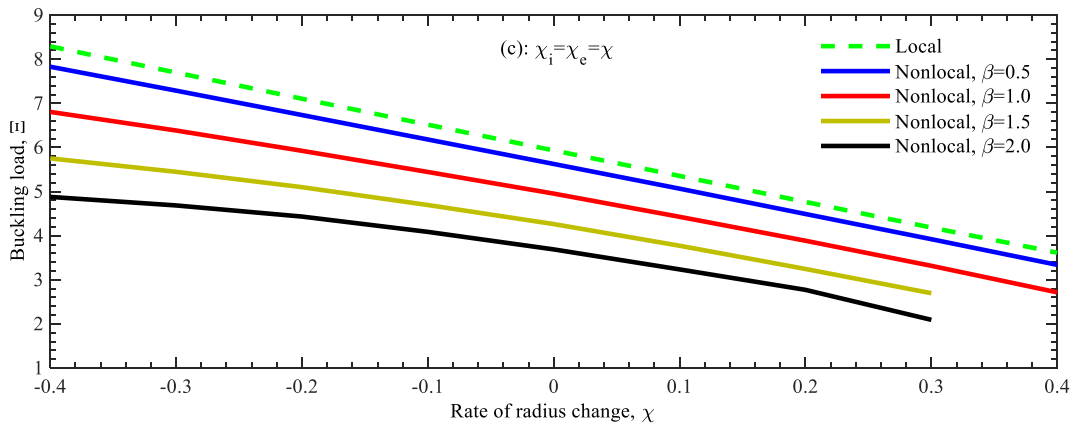


Fig. 4 Effect of the nonlocal parameter ( $\beta$ ) as well as the rate of radius change ( $\chi$ ) on the nonlinear buckling load ( $\Xi$ ) of AFG clamped nonuniform nanotube, a: when the internal radius is uniform (external radius is nonuniform,  $\chi_i=0$ ,  $\chi_e \neq 0$ ), b: the external radius is uniform (internal radius is nonuniform,  $\chi_e=0$ ,  $\chi_i \neq 0$ ), and c: both internal and external radii are nonuniform ( $\chi_i=\chi_e \neq 0$ ),  $\Delta=1.5$ ,  $\eta=1$ ,  $Re=2Ri$ ,  $L=40Re$

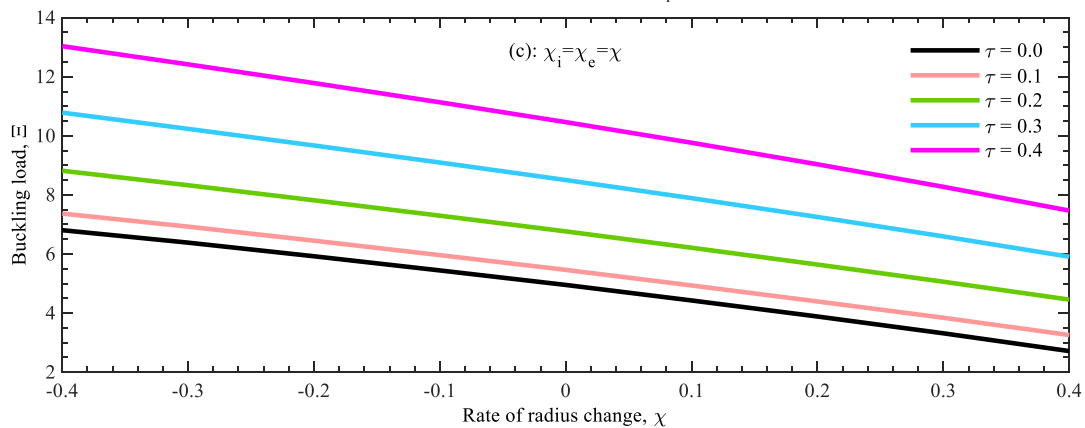
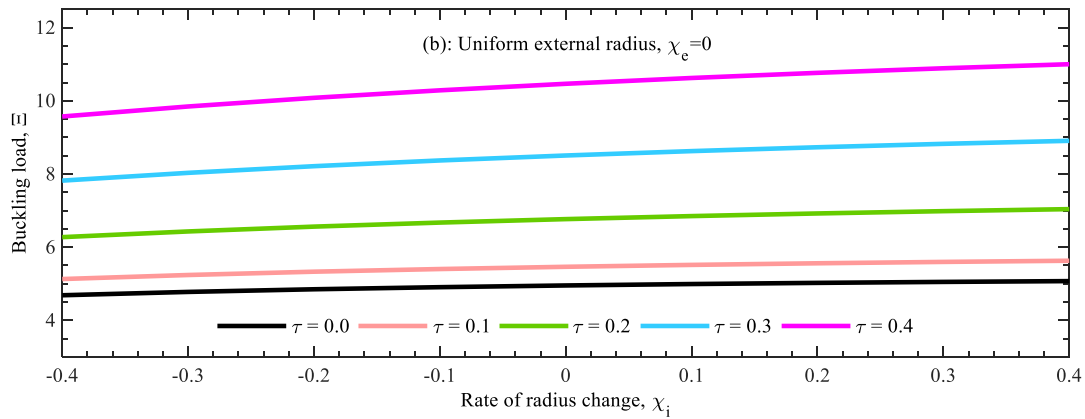
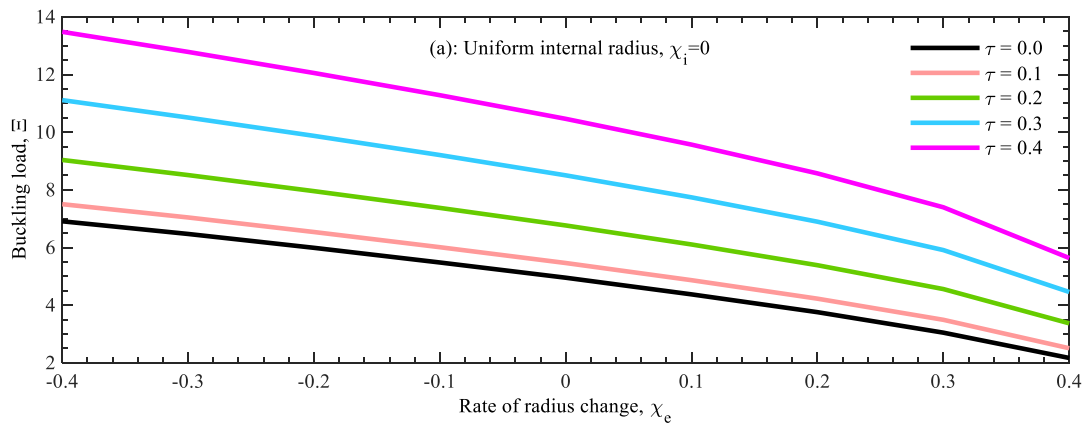


Fig. 5 Nonlinear buckling load ( $\Xi$ ) of the AFG nonuniform and clamped nonlocal nanotube for different rates of radius change ( $\chi$ ) versus the various values of strain gradient parameter ( $\tau$ ), a: when the internal radius is uniform (external radius is nonuniform,  $\chi_i=0$ ,  $\chi_e \neq 0$ ), b: the external radius is uniform (internal radius is nonuniform,  $\chi_e=0$ ,  $\chi_i \neq 0$ ), and c: both internal and external radiuses are nonuniform ( $\chi_i=\chi_e \neq 0$ ),  $\beta=1$ ,  $\Delta=1.5$ ,  $\eta=1$ ,  $Re=2Ri$ ,  $L=40Re$

## 5. Conclusions

The nonlinear buckling analysis associated with an AFG conical nanotube, which is modeled on the basis of nonlocal strain gradient and higher-order beam theory, is presented. The formulation is acquired through the energy method. Then by using GDQM coupled with the iteration method, the results are extracted. The results' accuracy and validity are proven by utilizing other published papers. The impact of different factors on the nonlinear buckling of the system is explored. The most important conclusion of the current study are:

- The higher the nonlocal parameter is, the lower the nonlinear buckling load of the system can be in any type of nanotube.
- Intensifying the strain gradient factor cause the system to be more stable.
- The nanotube's buckling load is heightened by intensifying nonlinear amplitude.
- The system's buckling load can be higher provided that the effect of ceramic is more apparent, lower FG index.

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