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Evaluation method for local mode extraction of beam string structures based on modal assurance criterion

Zeyu Wang[1a], Yuxin Zhang[*1], Tao Zhang[2b], Hexin Zhang[3c] and Lu Zhang[4d]

[1]School of Civil Engineering, Shanghai Normal University, Shanghai 201400, P.R. China
[2]Housing Security and Property Management Bureau, Shanghai 201400, P.R. China
[3]School of Computing, Engineering and the Built Environment, Edinburgh Napier University, Edinburgh EH105DT, Scotland, UK
[4]Myers-Lawson School of Construction, Virginia Tech, Blacksburg, VA 24061, USA

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Abstract. Because of its simplicity, quickness, and high accuracy, the frequency-based method has become one of the most used techniques for cable force identification in engineering. Several scholars have developed cable force identification methods, such as the "Three Criteria method", based on the traditional frequency method. These methods are appropriate for complex cable systems, the majority of which have short, thick cables and relatively complex supporting circumstances. The proposed methodology aims to extract local modal information of cables from global modal information, thereby establishing a cable force-frequency correlation model that incorporates the influence of the overall structural system. Based on the "Three Criteria method", this study conducts numerical experiments under a range of operating conditions to systematically investigate the threshold setting criteria for the Modal Assurance Criterion (MAC) values for the purpose of more precisely extracting local vibration modes of the cables. Ultimately, practical engineering was used to confirm the efficacy of the criteria establishing process.

Keywords: Beam String Structure (BSS); cable force identification; frequency domain method; Modal Assurance Criterion (MAC)

1. Introduction

Over the past ten years, the long-span beam string structure has developed rapidly and become widely used, making it a revolutionary sort of self-balanced spatial structure (Zhang *et al*. 2022). Beam string structures have gained considerable favor in many engineering projects and have been widely used because of their sensible force distribution, ease of production and transportation, and ease of construction (Guo *et al.* 2024).

The total stiffness and safety of the structural system are directly influenced by the stress condition of the tensile cables, which are the main load-bearing elements of a cable-supported

*Corresponding author, Professor, E-mail: zyx@shnu.edu.cn
[a]Graduate Student, E-mail: wangzeyu_edu@163.com
[b]M.Eng., E-mail: zhang_t@shmh.gov.cn
[c]Associate Professor, E-mail: j.zhang@napier.ac.uk
[d]Associate Professor, E-mail: luzhang@vt.edu

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structure. The cable forces interact during the batch tensioning construction of prestressed cables: the forces in previously tensioned cables are nearly always changed by each tensioning operation. Such variations could result in a loss of control as the cable forces deviate from the intended values. Unpredictable internal structural force redistribution or even the collapse of the entire structure might result from uncontrolled cable forces (Chen *et al*. 2011). Continuous exposure to dynamic loads and severe climatic conditions during the service period of cable-supported structures causes a variety of problems, such as corrosion, fatigue, prestress loss, and wire breakage. These damages present serious safety hazards and may influence the structural integrity. A sudden change in cable force happens when a cable breaks (Shang *et al*. 2025). Although there are many different ways to measure cable forces at the moment, the vibration frequency method has emerged as the most popular in engineering (Guo *et al*. 2024) (Syamsi *et al*. 2022) because of its benefits, including the equipment's capacity for repeated installation, the high precision of measurement data, and the convenience of instrument operation. The frequency approach is nearly the only practical choice when it comes to measuring the tension of in-service cables.

However, the majority of current research on cable tension identification is predominantly based on the single-cable models. This means that, in both theoretical studies and practical engineering applications, the single-cable model remains the most widely used model for cable tension identification. The correlation between cable force and frequency in a beam string system is different from that of the single-cable models because the cables are affected by the overall structural behavior. Investigating the cable tension-frequency relationship while taking the structure's overall influence into account is crucial.

Numerous academics have carried out comparable studies in this field so far. A useful formula for determining cable force based on the fundamental frequency was proposed by Ren *et al.* (2005), who used the energy approach and numerical simulation techniques while accounting for the effects of bending stiffness and sag. A new strategy for estimating cable tension force from recorded natural frequencies was presented by Kim *et al*. (2007). The suggested approach can concurrently determine a cable system's axial rigidity, flexural rigidity, and cable tension. By developing pertinent models that take into account the effects of cables by elements including sag, bending stiffness, and elastic boundary conditions, a workable calculation formula for cable force and frequency was derived and fitted Li *et al*. (2009). Chen *et al*. (2011) established an analytical mechanical model and used Newton's iterative method to numerically solve the vibration equations in order to overcome the challenges associated with cable force determination for beam string systems. Eventually, they were able to determine the force-frequency relationship by curve fitting. Zhang *et al*. (2017) analyzed the displacement of the beam string structure through experimental testing in order to determine the cable force after analyzing the overall stress characteristics of the beam string structure. Zhang *et al*. (2020) proposed a two-step process to determine the cable force, bending moment, etc., while taking the cable's boundary conditions, bending stiffness, and droop effect into account. Hou *et al*. (2021) proposed a method for cable force identification based on the sub-structure isolation method to assess cable damage by giving each cable virtual supports so that the cables have the same length and boundary conditions. The relationship between cable forces and natural frequency can then be used to identify the cable forces. For in-service beam string structures in engineering applications, Guo *et al*. (2022) presented a workable cable force detection technique based on the calibration concept. This approach considers how the overall behavior of the structure affects the measurement of cable force inside the structure. Gai *et al*. (2023) proposed a vibration-based approach to cable force identification in order to circumvent the issue of indistinguishable boundary conditions and the effect of inadequate low-order natural frequencies on cable force determination. In order to precisely calculate the cable force, this method uses finite element simulation data in conjunction with cable length, linear density, bending stiffness, and input frequency as input parameters. A frequency-based technique for accurate cable tension prediction under ambiguous boundary conditions—such as unknown rotational stiffness and support constraints—was presented by Zhang *et al*. (2023). This method explicitly takes into account the impact of end rotational stiffness and boundary limitations in addition to standard factors (such as inclination angle, sag, and bending stiffness) by creating a nonlinear vibration model of cables. Yang *et al.* (2024) calculated the time-varying cable force using the axially loaded beam theory while taking bending stiffness into account. In order to overcome the difficulty of real-time time-varying cable force identification, Liu *et al.* (2024) creatively presented a real-time monitoring technique based on the Scale-Space Peak Picking (SSPP) algorithm and Adaptive Chirp Mode Decomposition (ACMD) algorithm. This allowed for a more precise determination of time-varying frequencies and cable forces.

Zhang *et al*. (2023) conducted finite element numerical simulation to perform a modal analysis of the entire beam string structure. To separate the local vibration modes of cables from the global structural modes, they suggested a Three Criteria approach. By using this technique, a more precise cable force-frequency correlation model for cable force identification is established by obtaining the local modal information of the cables under the impact of the overall structure. Among these, figuring out the Modal Assurance Criterion (MAC) threshold value is essential for precisely identifying the local vibration modes of cables.

The present paper explores the rational threshold for the MAC value building on the "Three Criteria" assessment procedure. The research results are helpful in effectively extracting the local vibration modes of the cables in beam string structures.

2. Local mode extraction technique using finite element analysis for BSS

2.1 Fundamentals of dynamic finite element analysis for BSS

This study uses the subspace iteration approach (Hjelmstad 2022) to analyze the beam-string structure dynamically using finite element method. By using this technique, the system's natural vibration periods and mode shapes can be obtained in accordance with various precision requirements. The following equation is satisfied by the first p natural frequencies and mode shapes in the subspace iteration method:

$$[K][\varphi]=[M][\varphi][\lambda] \tag{1}$$

$$[\lambda]=diag\left(\lambda_1=\omega_1^2\right) \tag{2}$$

$$[\varphi]=\left[\{\varphi\}_1 \quad \{\varphi\}_2 \quad \cdots \quad \{\varphi\}_p\right] \tag{3}$$

$$[\varphi]_i=\left[\varphi_{1i} \quad \varphi_{2i} \quad \cdots \quad \varphi_{ni}\right]^T \tag{4}$$

where $[K]$ represents the structural stiffness matrix, consisting of the linear stiffness matrix which is commonly used for small deformation structural analysis, and the geometrical stiffness matrix induced by the pre-stressed state; $[M]$ represents the mass matrix; ω_i denotes the *i*-th natural frequency; φ is the mode shape. Based on the principle of modal orthogonality, it can be established that

$$[\varphi]^T[K][\varphi]=[\lambda][\varphi]^T[M][\varphi]=[\lambda][I] \tag{5}$$

2.2 Introduction to the "Three Criteria" method *for obtaining local modes of cables*

All the vibration mode and frequency information of the entire structure can be obtained by performing a finite element analysis of the beam string structure. Some of these modes reflect the structural overall vibration, while others show the local vibration of cables. According to Zhang *et al.* (2022), if the local modes of a particular cable can be identified, the associated mode shape reflects the n-order vibration mode of the cable under the overall structural influence, and the corresponding frequency of this mode represents the nth order vibration frequency of cable while accounting for the influence of the overall structure.

Criterion 1: Search for the dominate vibration cable segment.

Find the node that corresponds to the largest amplitude and the cable segment where the node is placed to identify the dominant vibration cable segment of the mode. This will help you establish which cable segment might be the dominant vibration cable for a particular order of vibration.

Criterion 2: Recognize the local vibration mode.

Based on the Criterion 1, Criterion 2 will determine whether a mode is the local vibration mode of the cable segment:

(1) If the amplitude of one node of the cable segment is the largest, and the amplitude of each node of the other cable segments is 0 or less than the set small value, it can be determined as a local mode of a certain order of the cable segment.

(2) In the case of similar cable lengths and similar positions, when the amplitude of the cable segment is the largest, except for the cable elements, the amplitude of each node of the other elements is close to 0 or less than a set small value, which may be a local mode of a certain order of the cable segment.

Criterion 3: Judge the local modal order based on the MAC value.

When a global mode is identified as a local mode of a cable segment using Criterion 2, the order of the modes that correspond to the cable segment is determined by whether the magnitude of the MAC value between the local mode of the cable segment and the vibration mode of the single cable model reaches the assessment threshold.

2.3 Determination of the MAC value

The MAC is a common tool for evaluating structural dynamic characteristics. It is used to evaluate the degree of correlation between two sets of mode shape vectors. The definition of MAC is shown (Nguyen *et al*. 2017)

$$MAC_{ij}=\frac{\left(\Phi_i{}^T\Phi_j\right)^2}{\left(\Phi_i{}^T\Phi_i\right)\left(\Phi_j{}^T\Phi_j\right)} \tag{6}$$

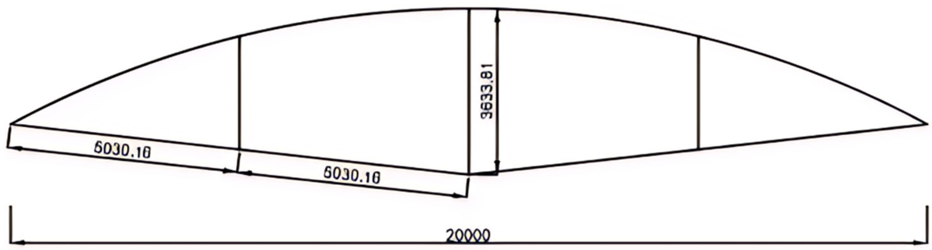

Fig. 1 Dimensional details of the beam string structure model

Where MAC_{ij} is the value of the modal confidence criterion of the two mode shape vectors; Φ_i、Φ_j are two sets of mode shape vectors, respectively. The value range of MAC_{ij} is [0,1].

In the "Three criteria" method for determining local modes of the cable of the beam string structure, the selection of the appropriate threshold for the MAC value to ensure accurate modal discrimination is a critical issue that must be carefully considered. The relevant literature suggests that (Zhang 2023), based on conventional empirical practices, we posit that: when $MAC_{ij} < 0.05$, Φ_i、Φ_j are independent; when $MAC_{ij} > 0.85$, Φ_i、Φ_j are related. The mode represented by the extracted mode shape vector corresponds to the desired local mode of a specific order for the cable segment.

The choice of the MAC value threshold is a crucial consideration when using the previously mentioned "Three Criteria" technique to separate the local modes of the cable segment from the overall modes of the beam string structure. In order to provide recommendations for the selection of MAC threshold values, this study analyzes the MAC-based assessment standards for local modes of cables in beam string structures under a variety of working conditions using numerical experiments.

3. Numerical experiments

3.1 MAC threshold for local mode extraction in an equal-span beam string structure

3.1.1 Introduction to the structural parameters of the model

The structural model used in this section is a one-way equal-span beam string structure. A circular tube with an inner radius of 95.5 mm and an outside radius of 101.5 mm makes up the beam element of the structure. In contrast to the cable element section, which is likewise circular in shape and has a cross-sectional area of 1600 mm^2, the strut element section has a cross-sectional area of 346 mm^2. Cables have a linear mass density (m) of 2.716 kg/m and a bending stiffness flexural rigidity (EI) of 1810 N·m^2. Beam and strut steel has an elastic modulus of 2.06×10^5 MPa, while cable steel has a lower elastic modulus of 1.9×10^5 MPa. The stated ultimate tensile capacity of the cable is 612.42 kN.

Fig. 1 lists the structural dimensions. Fig. 2 displays the cable number and finite element model. The cable segments S1-1 and S1-4, as well as S1-2 and S1-3, have the same parameters because of the structural symmetry of the beam string structure model used in this investigation.

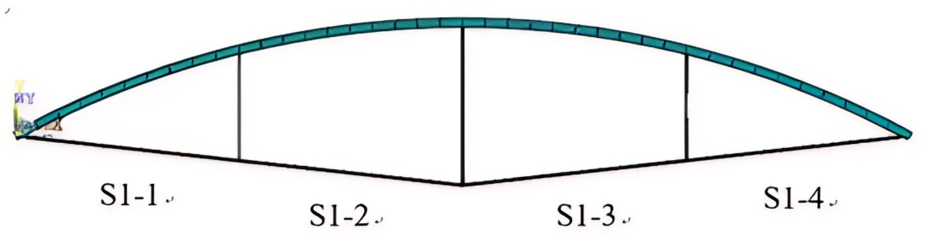

Fig. 2 Finite element model of the beam string structure and cable numbering

Table 1 MAC values of the first four modes in S1-1 corresponding to different cable forces

$A(mm^2)$	$T(kN)$	first-order	second-order	third-order	fourth-order
1600	137.22	0.9917	0.9252	0.9622	0.9370
1600	160.14	0.9892	0.9562	0.9643	0.9441
1600	183.08	0.9900	0.9754	0.9670	0.9331
1600	206.03	0.9906	0.9826	0.9692	0.9271
1600	229.01	0.9911	0.9850	0.9709	0.9160
1600	252.01	0.9913	0.9910	0.9723	0.8742
1600	275.03	0.9908	0.9894	0.9735	0.9130
1600	298.08	0.9845	0.9832	0.9745	0.9220
1600	321.15	0.9939	0.9878	0.9754	0.9282
1600	344.26	0.9943	0.9881	0.9761	0.9329
1600	367.40	0.9945	0.9883	0.9768	0.9367
1600	390.57	0.9947	0.9885	0.9775	0.9397
1600	413.78	0.9949	0.9886	0.9820	0.9425
1600	437.02	0.9951	0.9887	0.9770	0.9449
1600	460.31	0.9953	0.9887	0.9773	0.9473
1600	483.63	0.9955	0.9884	0.9773	0.9495

3.1.2 MAC threshold for local mode extraction corresponding to different cable forces

To investigate the MAC threshold of the local modality of the cable segment in the beam string construction under various cable forces, the cable force range is set to be 0.22 Tu~0.79 Tu when prestress is applied, which translates to 137.42 kN~487.37 kN. Each cable segment's local modal information is then extracted from the overall structural modes using the "Three Criteria" approach for extracting local mode information. The MAC values between the associated single-cable vibration modes and the extracted local modes are then computed. Due to the structural symmetry of the model, this study solely looks into the two cable segments S1-1 and S1-2.

As can be seen from the above Tables, when only the fundamental frequency is used under different cable forces, the corresponding local modes can be extracted when the MAC value threshold is set to 0.95 ($MAC_{ij} > 0.95$). As the order increases, the mode shapes show greater complexity, which results in a corresponding decrease in the MAC values. Setting the MAC

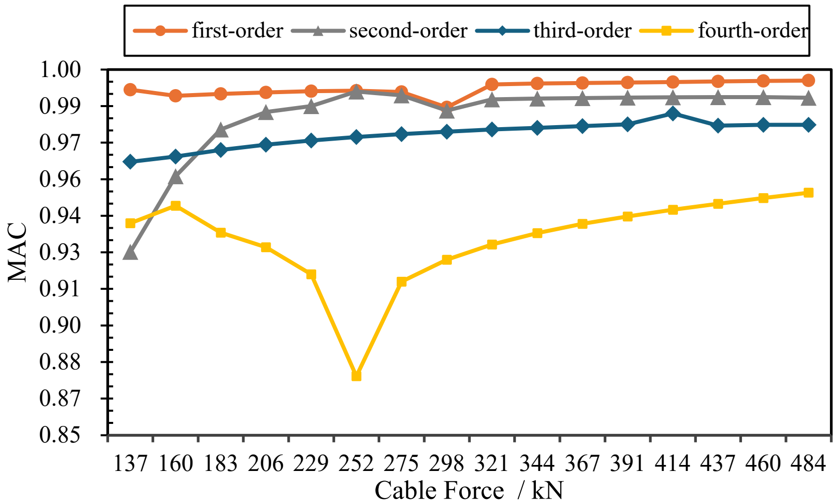

Fig. 3 MAC values of the first four modes in S1-1 corresponding to different cable forces

Table 2 MAC values of the first four local modes of S1-2 corresponding to different cable forces

$A(mm^2)$	$T(kN)$	first-order	second-order	third-order	fourth-order
1600	137.22	0.9917	0.9252	0.96221	0.9372
1600	160.14	0.9894	0.9562	0.9643	0.9441
1600	183.08	0.9900	0.9754	0.9670	0.9331
1600	206.03	0.9906	0.9826	0.9692	0.9271
1600	229.01	0.9911	0.9850	0.9709	0.9160
1600	252.01	0.9913	0.9910	0.9723	0.8742
1600	275.03	0.9908	0.9893	0.9735	0.9131
1600	298.08	0.9845	0.9832	0.9745	0.9220
1600	321.15	0.9938	0.9878	0.9754	0.9282
1600	344.26	0.9943	0.9881	0.9761	0.9329
1600	367.40	0.9945	0.9883	0.9768	0.9367
1600	390.57	0.9947	0.9885	0.9775	0.9397
1600	413.78	0.9949	0.9886	0.9820	0.9425
1600	437.02	0.9951	0.9887	0.9771	0.9449
1600	460.31	0.9953	0.9886	0.9773	0.9473
1600	483.63	0.9955	0.9884	0.9773	0.9495

threshold to 0.9 usually allows for the extraction of the corresponding local modes for the second- and third-order modes. By using a MAC threshold of 0.85 as the modal correlation criterion, it is typically possible to successfully identify the local modes of the cable segments while examining the fourth-order mode.

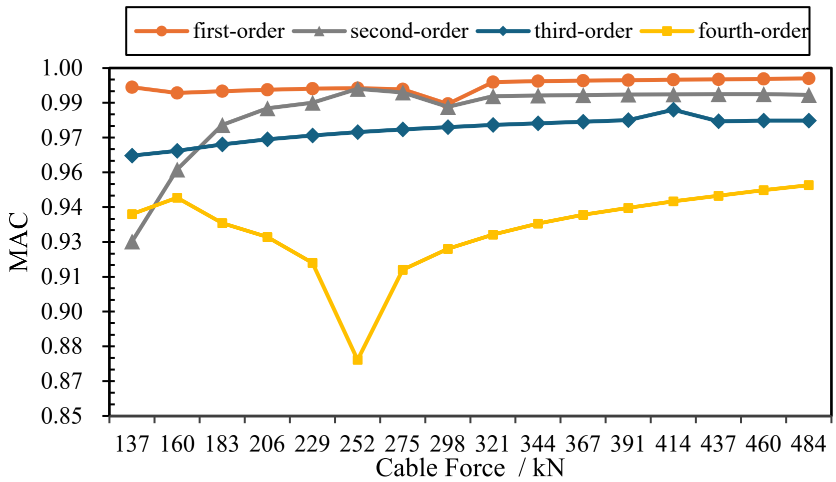

Fig. 4 MAC values of the first four modes in S1-2 corresponding to cable force variation

3.1.3 MAC threshold for extracting local modes corresponding to different cable cross-sectional areas

The cross-sectional area (A) of cables varies between 1200 and 3800 mm². The MAC values between the recovered local modes and the theoretical single-cable mode are computed when the model is examined in finite element modes. Tables 3 and 4 provide a summary of the findings.

Tables 3 and 4 show that by setting the MAC threshold to 0.95, the appropriate local modes may be successfully retrieved when using only the fundamental frequency under various cable cross-sectional areas. The MAC value threshold for the second-order and third-order modes should be 0.9, and for the fourth mode, a threshold of 0.85 is still suitable.

3.1.4 MAC threshold for extracting local modes corresponding to different L/D ratios

Based on the equal-span string beam model, this section examines the MAC threshold for extracting local modes of cable segments in the beam string structure under different slenderness ratios (L/D). A circular tube with an inner radius of 335 mm and an outer radius of 351 mm serves as the beam element section. Every other parameter stays the same as the ones mentioned in the section above.

Make sure that as the cross-sectional diameter of the cables is modified in the model, the length of the cables remains consistent. The work is done using a global structural modal analysis with slenderness ratios between 40 and 120. The MAC values between the theoretical single-cable mode and the retrieved local modes are then computed, and Tables 5 and 6 provide a summary of the findings.

Tables 5 and 6 show that the results are similar to the findings of the previous investigation within the slenderness ratio L/D range of 40 to 120. Additionally, it is clear that the curvature of the cable gradually resembles that of a flexible cable as the value of L/D increases, reducing the impact of the global structural system on the cable. As a result, the local vibration mode shapes of the cable segments progressively resemble those of the single-cable model, which causes the MAC values continuously increase.

Table 3 MAC values of the first four local modes of S1-1 corresponding to different cross-sectional areas

$A(mm^2)$	$T(kN)$	first-order	second-order	third-order	fourth-order
1200	359.39	0.9966	0.9699	0.9805	0.9708
1400	359.45	0.9954	0.9900	0.9805	0.9534
1600	359.52	0.9944	0.9882	0.9765	0.9355
1800	359.64	0.9933	0.9626	0.9739	0.9081
2000	359.79	0.9729	0.9858	0.9716	0.9132
2200	359.98	0.9660	0.9832	0.9698	0.9354
2400	360.01	0.9836	0.9823	0.9684	0.9376
2600	360.10	0.9851	0.9810	0.9673	0.9354
2800	360.16	0.9852	0.9379	0.9665	0.9370
3000	360.28	0.9849	0.9394	0.9660	0.9386
3200	360.38	0.9839	0.9333	0.9656	0.9404
3400	360.46	0.9897	0.9398	0.9652	0.9428
3600	360.53	0.9863	0.9402	0.9648	0.9469
3800	360.59	0.9855	0.9412	0.9645	0.8732

Table 4 MAC values of the first four local modes of S1-2 corresponding to different cross-sectional areas

$A(mm^2)$	$T(kN)$	first-order	second-order	third-order	fourth-order
1200	359.39	0.9997	0.9619	0.9702	0.9344
1400	359.45	0.9892	0.9509	0.9585	0.9629
1600	359.52	0.9941	0.9115	0.9487	0.9603
1800	359.64	0.9157	0.9862	0.9697	0.9453
2000	359.79	0.9994	0.9785	0.9349	0.9444
2200	359.98	0.9995	0.9322	0.9303	0.9373
2400	360.01	0.9990	0.9424	0.9269	0.9325
2600	360.10	0.9259	0.9183	0.9245	0.9329
2800	360.16	0.9759	0.9015	0.9230	0.9283
3000	360.28	0.9924	0.8995	0.9221	0.9268
3200	360.38	0.9913	0.9751	0.9218	0.9276
3400	360.46	0.9917	0.9717	0.9221	0.9219
3600	360.53	0.9923	0.9688	0.9227	0.9240
3800	360.59	0.9929	0.9665	0.9236	0.9255

3.2 MAC threshold for local mode extraction of a cable in unequal-span beam string structures

3.2.1 Introduction to the structural parameters of the model

An unequal-span beam string structure model is developed based on the equal-span beam string structure model and the MAC threshold for extracting local modes of the cables in a one-way beam string structure is explored.

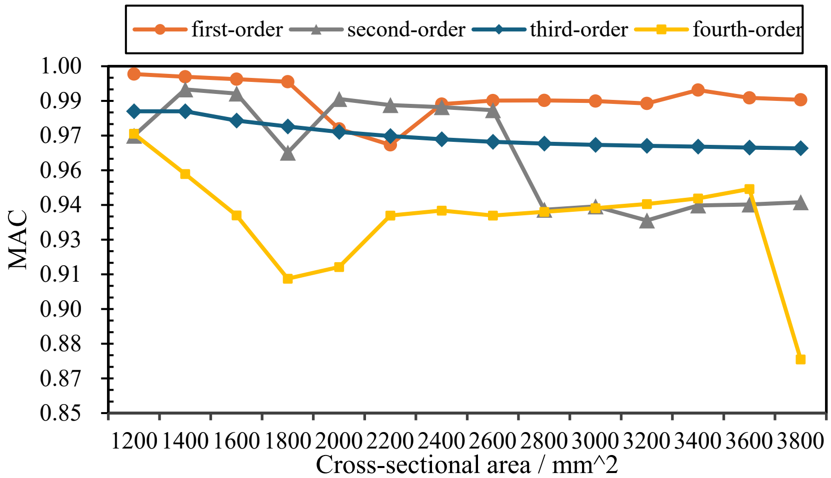

Fig. 5 MAC values of the first four modes in S1-1 corresponding to different cross-sectional areas

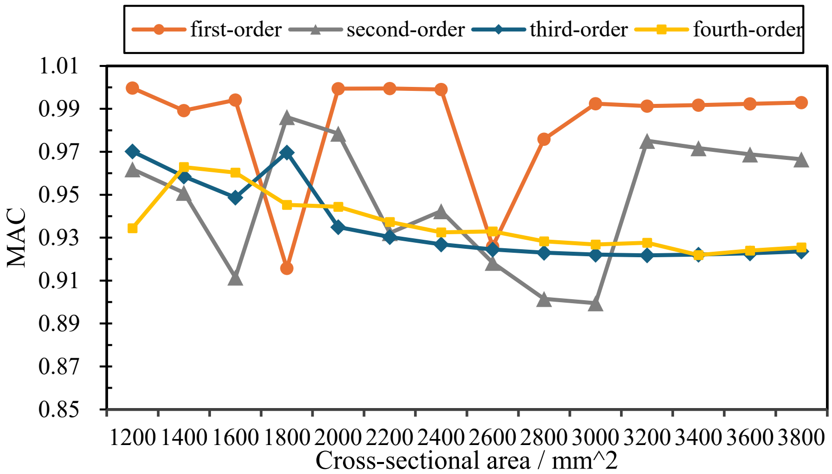

Fig. 6 MAC values of the first four local modes of S1-2 corresponding to different cross-sectional areas

The material parameters and cross-sectional parameters in the equal-span beam string structure model in Section 3.1.1 and the unequal-span beam string structure model are identical. Fig. 9 shows the structural dimensions. Fig. 10 displays the cable number and finite element model.

3.2.2 MAC threshold for local modes extraction corresponding to different strut positions

The spans of cable segments S2-1 and S2-2 will alter with the change of strut placements in the model. The length of cable S2-1 varies between 1 and 9 meters. The unequal-span beam string structure model is used for modal analysis in order to determine the structural overall vibration modes. After that, local modes are extracted from the overall modes using the "Three Criteria"

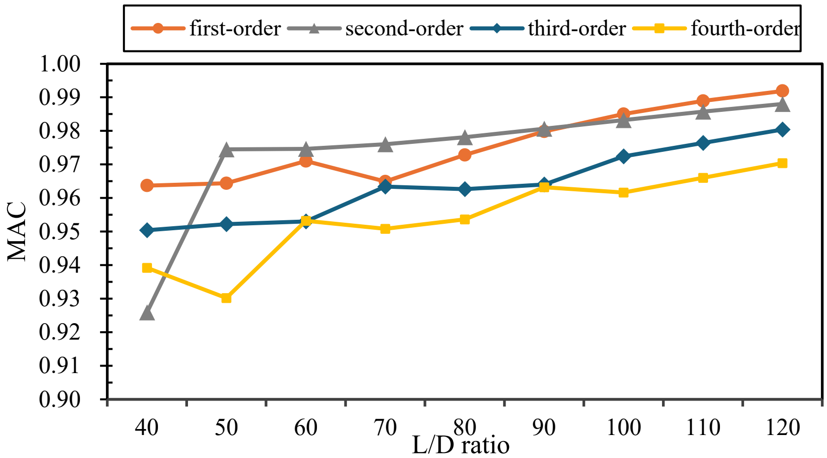

Fig. 7 MAC values of the first four local modes of Cable S1-1 corresponding to different L/D ratios

Table 5 MAC values of the first four local modes of Cable S1-1 corresponding to different L/D ratios

L/m	D/mm	S/mm^2	L/D	first-order	second-order	third-order	fourth-order
5.037	125.93	12454.14	40	0.9637	0.9259	0.9504	0.9392
5.037	100.74	7970.65	50	0.9644	0.9745	0.9522	0.9302
5.037	83.95	5535.17	60	0.9710	0.9746	0.9530	0.9532
5.037	71.96	4033.66	70	0.9649	0.9760	0.9634	0.9508
5.037	62.96	3113.54	80	0.9728	0.9781	0.9626	0.9536
5.037	55.97	2460.08	90	0.9798	0.9806	0.9640	0.9632
5.037	50.37	1992.66	100	0.9850	0.9832	0.9724	0.9616
5.037	45.79	1646.83	110	0.9889	0.9857	0.9764	0.9660
5.037	41.98	1383.79	120	0.9919	0.9880	0.9804	0.9704

Table 6 MAC values of the first four local modes of Cable S1-3 corresponding to different L/D ratios

L/m	D/mm	S/mm^2	L/D	first-order	second-order	third-order	fourth-order
5.037	125.93	12454.14	40	0.9850	0.9779	0.9697	0.9674
5.037	100.74	7970.65	50	0.9978	0.9468	0.9762	0.9758
5.037	83.95	5535.17	60	0.9975	0.9854	0.9774	0.9684
5.037	71.96	4033.66	70	0.9976	0.9869	0.9778	0.9718
5.037	62.96	3113.54	80	0.9980	0.9889	0.9804	0.9726
5.037	55.97	2460.08	90	0.9984	0.9906	0.9825	0.9746
5.037	50.37	1992.66	100	0.9988	0.9920	0.9845	0.9766
5.037	45.79	1646.83	110	0.9990	0.9932	0.9862	0.9786
5.037	41.98	1383.79	120	0.9964	0.9942	0.9877	0.9804

approach, and the MAC values of each order of modes are determined. Tables 7 and 8 present the findings.

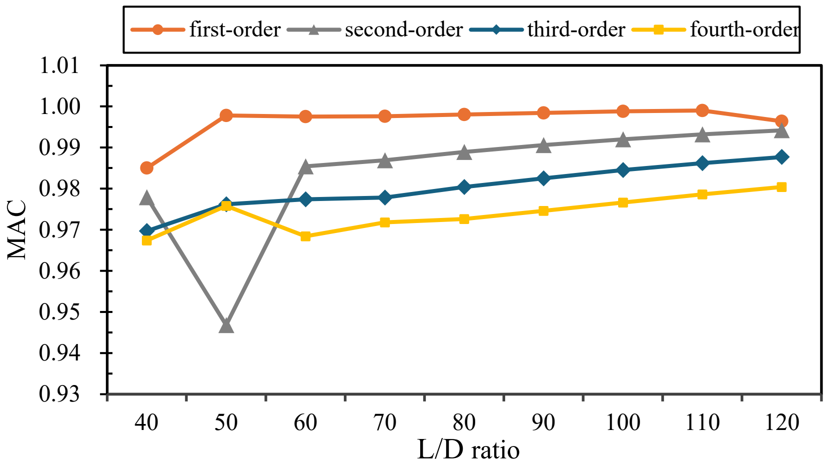

Fig. 8 MAC values of the first four local modes of Cable S1-3 corresponding to different L/D ratios

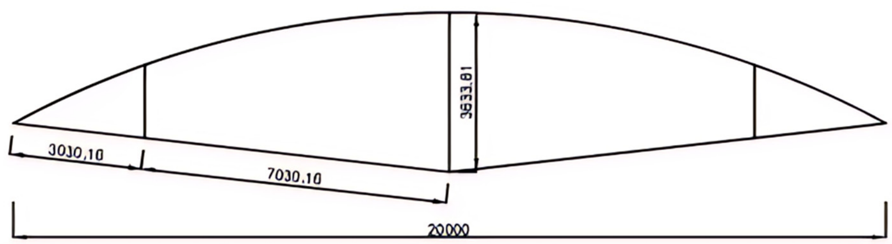

Fig. 9 Dimensional details of the beam string structure model

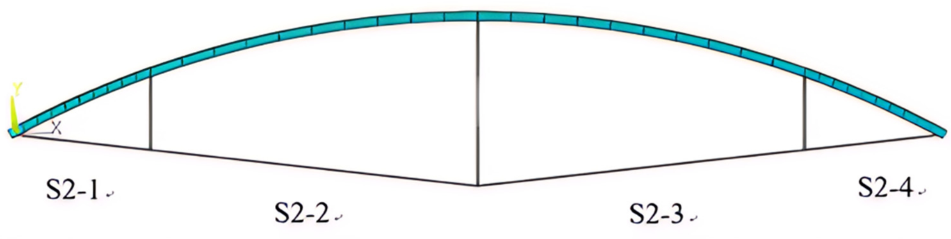

Fig. 10 Finite element model of the beam string structure

Tables 7 and 8 show that the MAC values for the fundamental frequency stay around 0.98 regardless of the strut position. The minimal MAC value is 0.95 when higher-order frequencies are taken into account. It goes without saying that it is simpler to identify the local mode in unequal-span beam string structures than in equal-span ones since the self-vibration frequency of each cable differs significantly from that of the neighboring cable segments in unequal-span beam string structure.

Table 7 MAC values of the first four local modes of cable S2-2 corresponding to different strut positions

L(m)	$T(kN)$	first-order	second-order	third-order	fourth-order
1	296.60	0.9921	0.9866	0.9307	0.9691
2	304.09	0.9878	0.9941	0.9901	0.9830
3	308.79	0.9992	0.9880	0.9961	0.9938
4	311.24	0.9932	0.9936	0.9989	0.9759
5	311.22	0.9996	0.9962	0.9984	0.9989
6	308.75	0.9991	0.9988	0.9994	0.9996
7	304.57	0.9975	0.9983	0.9983	0.9993
8	300.21	0.9960	0.9933	0.9988	0.9962
9	297.52	0.9897	0.9977	0.9991	0.9949

Table 8 MAC values of the first four local modes of cable S2-1 corresponding to different strut positions

L(m)	$T(kN)$	first-order	second-order	third-order	fourth-order
9	305.02	0.9996	0.9986	0.9915	0.9967
8	310.75	0.9989	0.9992	0.9994	0.9904
7	314.87	0.9978	0.9983	0.9994	0.9813
6	316.93	0.9906	0.9983	0.9949	0.9817
5	316.45	0.9999	0.9985	0.9882	0.9910
4	313.16	0.9990	0.9938	0.9985	0.9967
3	307.71	0.9981	0.9972	0.9972	0.9808
2	301.68	0.9989	0.9991	0.9577	0.9788
1	297.17	0.9968	0.9912	0.9912	0.9506

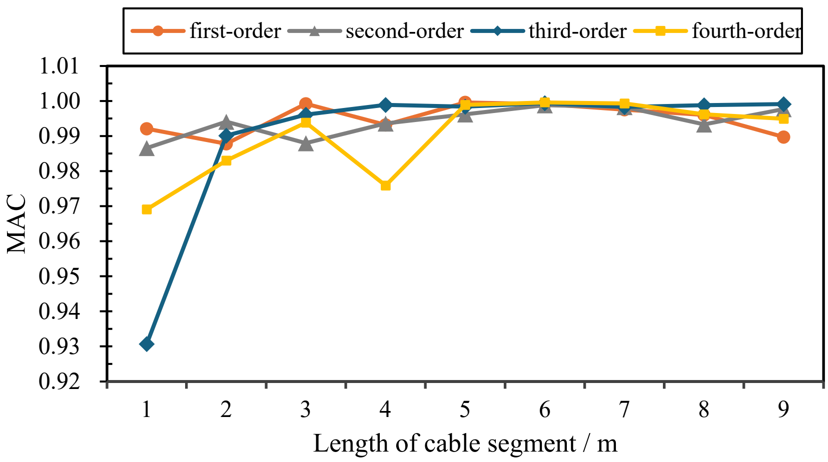

Fig. 11 MAC values of the Cable S2-2 corresponding to different strut positions

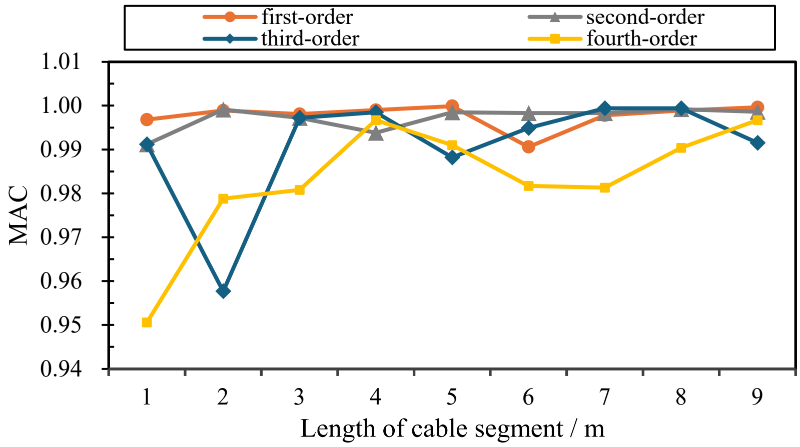

Fig. 12 MAC values of the first four local modes of cable S2-1 corresponding to different strut positions

Obviously, the larger the threshold value of MAC_{ij} we stipulate, the higher the correlation between the two vibration mode vectors, and the greater the correctness of the local mode extraction. This study suggests that, provided Criterion 1 and 2 are satisfied, the MAC threshold for the first mode can be set at 0.95 to reliably ensure local mode extraction, improving the accuracy of local mode extraction in the "Three Criteria" method while maintaining engineering applicability. The MAC thresholds for the second, third, and fourth modes can be adjusted to 0.90 and 0.85, respectively.

4. Practical engineering applications

The Yellow River Estuary Model Test Hall is situated in Dongying City, Shandong Province. Its main purpose is to replicate the dynamic circumstances of rivers and oceans, allowing for the processing and real-time observation of hydrological factors, including sediment and water flow. Marine Space A Hall, Marine Area B Hall, and River Channel Hall are the three components that make up the testing hall, which has a total floor space of 45,333 m². Fig. 14 displays the floor plan.

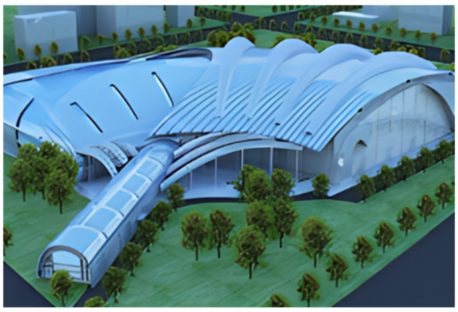

Fig. 13 Rendering of the Yellow River Estuary Model Test Hall

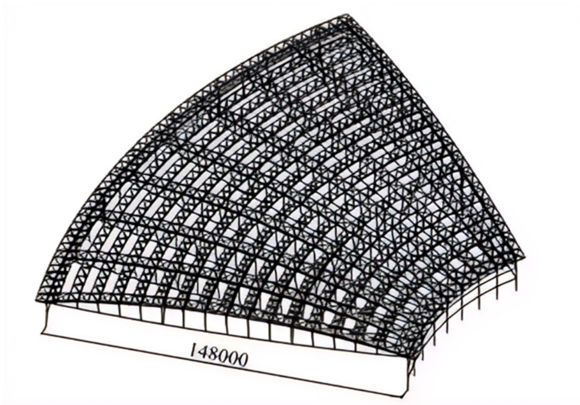

Fig. 14 Floor Plan

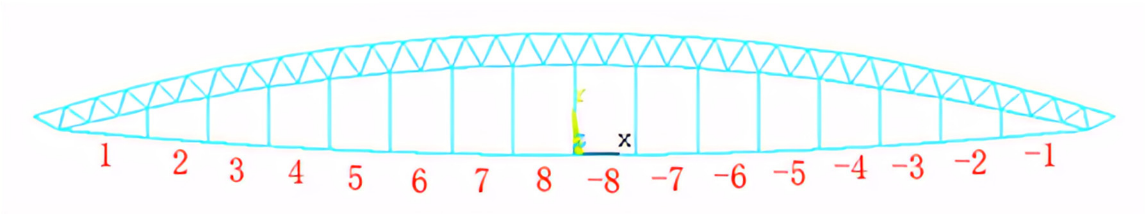

Fig. 15 Finite Element Model and Cable Numbering of the One-Way Beam String Truss

Table 9 Parameters of Cables 1 to 8

Cable Numbering	L (m)	Cross-Sectional Area of Cables(m^2)	Linear Mass Density (kg/m)	Flexural Rigidity (EI) $(N \cdot m^2)$
1	12.83			
2	8.69			
3	8.74			
4	8.77	1.30E-02	101.76	2540592.50
5	8.80			
6	8.82			
7	8.84			
8	8.84			

The finite element model is created and the structural overall modal analysis is conducted in order to verify the suitability of the suggested MAC thresholds in real-world engineering. Fig. 15 shows the cable numbering scheme and the finite element model. Cable Nos. 1 and -1 have the same characteristics since the structure is symmetrically positioned on the left and right. Table 9 lists specific cable parameters that were obtained from pertinent technical documents.

Table 10 MAC Values of No.1-8 Cable Segments

Cable Numbering	MAC			
	first-order	second-order	third-order	fourth-order
1	0.9993	0.9973	0.9946	0.9857
2	0.9997	0.9989	0.9943	0.9981
3	0.9994	0.9974	0.9992	0.9920
4	0.9996	0.9999	0.9998	0.9907
5	0.9998	0.9994	0.9983	0.9982
6	0.9977	0.9966	0.9885	0.9862
7	0.9997	0.9988	0.9970	0.9947
8	0.9966	0.9982	0.9916	0.9897

Table 11 Comparison between the extracted and measured natural frequencies for a few chosen cables

Cable Numbering	measured	identified	error
2	9.750	10.310	5.74%
4	9.875	10.140	2.68%
5	9.875	10.170	2.99%
6	9.880	9.9800	1.01%
8	9.750	9.8540	1.07%

Similarly, ANSYS is used to create a finite element model and conduct an overall modal analysis on a single one-way string truss. The local modes of cables 1 through 8 are then separated from the global modes using the "Three Criteria" judgment procedure. MATLAB is used to determine the MAC values for each cable. Table 10 provides a summary of the findings.

The MAC values of the local modes taken from the modal analysis of a one-way string truss in the Test Hall for cables 1 through 8 all meet the previously indicated findings, as shown in Table 10.

As a finished project, the cable segments in the Test Hall have measured natural frequency data. This study compares the natural frequencies extracted by our method with those obtained experimentally for a subset of cable segments in order to thoroughly validate the 'Three Criteria' method and the accuracy of the suggested methodology.

The results of comparing the identified and measured frequencies for a few chosen cables are shown in Table 11. With a maximum error of just 5.74%, the findings reveal that the fundamental frequencies extracted using the suggested method correlate very well with the measured values, proving the accuracy of our methodology.

5. Conclusions

The correctness of the “Three Criteria” method is greatly impacted by the MAC threshold decision in Criterion 3. If the threshold I in the condition MAC > I is set too low, nearly most of mode shape vectors will meet this requirement but some of these modes do not correspond to the intended local modes of the cables. On the other hand, a larger threshold I results in more precise computational analysis of local modes of cables but may cause some true local modes failure to be considered. By calculating the local modes of the cables of the beam string structure under different working conditions, it can be essentially concluded that the local modes of cables can be relatively effectively extracted by balancing redundancy and failure probability when the MAC threshold values are set as 0.95, 0.9, and 0.85 respectively corresponding to different orders of modes.

By combining the methodology in this work, the "Three Criteria" method efficiently extracts the local vibration modes of cables from global structural modes, making it a more accurate frequency extraction technique for beam string structures. The approach increases the precision of the subsequent cable force calculation by accounting for the impact of the overall structure on the local mode extraction of cables.

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