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A simplified directly determination of natural frequencies of CNT: Via aspect ratio

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Abstract. In this paper, a novel model is developed for frequency behavior of single walled carbon nanotubes. The governing equation of motion is constructed method based on the Sander theory using Rayleigh-Ritz's method The frequencies enhances on increasing the power law index using simply supported, clamped and clamped free end conditions. The frequency curve for C-F is less than other conditions. It is due to the physical constraints which are applied on the edge of the CNT. It is observed that the C-F boundary condition have less frequencies from the other two conditions. The frequency phenomena for zigzag are insignificant throughout the aspect ratio. Moreover when index of power law is increased then frequencies increases for all boundary conditions. The natural frequency mechanism for the armchair (10, 10) for various values of power law index with different boundary conditions is investigated. Here frequencies decrease on increases the aspect ratio for all boundary conditions. The frequency curves of SS-SS edge condition is composed between the C-C and C-F conditions. The curves of frequency are less significant from small aspect ratio (L/d = 4.86 ~ 8.47) and decreases fast for greater ratios. It is found that the frequencies via aspect ratios, armchair (10, 10) have higher values from zigzag (10, 0). It is due to the material structure which is made by the carbon nanotubes. The power law index have momentous effect on the vibration of single walled carbon nanotubes. The present frequency result is also compared numerically experimentally with Raman Spectroscopy.

Keywords: clamped-free; fraction law; natural frequency; Rayleigh-Ritz's method

1. Introduction

Extensive research has been conducted in the field of nanotubes, and as its use in commercial applications has increased; rapid development has occurred. The nanotubes are used for a number of scientific and technological applications.

In the last ten years, many researchers have studied the vibrations of SWCNTs using different beam theories. The nano- and micro-scale systems have wavy shape and there is a demand for a powerful tool to mathematically model waviness of those systems. In accordance with the above mentioned lack for the modeling of the waviness of the curved tiny structure, a novel approach is employed by implementing the Euler-Bernoulli beam theory. Owing to the small size of the micro beam, these structures are very appropriate for designing small instruments (Iijima 1991). Vibrations of the embedded MWCNTs were investigated in order to find their vibrational modes with frequencies. The beam model was utilized to model the carbon nanotubes and it was shown that the non-coaxial inter-tubular will be excited at the higher resonant frequency. Large deformation of CNTs related to morphological design corresponds to the release of energy in strains-stress curve. This idea provides an accurate roadmap of nanotube behavior for researchers. Lordi and Yao (1998) started the molecular dynamic simulation using inter atomic potential function to develop a formula for approximating the tube radii. The vibration of chiral SWCNTs was calculated by Timoshenko beam model (TBM), which involves rotary inertia and shear deformation. The calculations that were performed with the cylindrical shell model differ slightly from those obtained with the MD simulation. Second method that is experimental method was used by Krishnan *et al*. (1998) for studying the properties of CNTs. Harik (2002) studied the behaviors of CNTs using beam models. He concluded that beam model can be used for the qualitative exploration of CNTs with the condition of nanotube aspect ratio is greater than 10 nm. The behavior of elements become more complicated using experimental, molecular dynamics and continuum models. Akbaş (2016a, b) studied the forced vibration analysis of a simple supported viscoelastic nanobeam based on modified couple stress theory (MCST). The nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. The elastic medium is considered as Winkler-Pasternak elastic foundation. The damping effect is considered by using the Kelvin–Voigt viscoelastic model. The cracked beam is modelled using a proper modification

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of the classical cracked-beam theory consisting of two sub-beams connected through a massless elastic rotational spring. Zhao *et al*. (2002) predicted natural frequencies by applying MD simulations. Li and Chou (2003) established a connection between structural and molecular mechanics to produce deformation in CNTs. Akbaş (2017a, b) investigated the free vibration analysis of edge cracked cantilever microscale beams composed of functionally graded material (FGM) based on the modified couple stress theory (MCST). The material properties of the beam are assumed to change in the height direction according to the exponential distribution. The FG nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. Mechanical properties of FG beam depends on the position. The Kelvin- Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory. Wang *et al*. (2005) found the frequencies of cantilever SWCNT and the results were compared with earlier computations (Krishnan *et al*. 1998). Murmu and Pradhan (2009) investigated the vibrational frequencies with different modes along temperature change using nonlocal small scale effects. On the other side, for length scale coefficient and soft elastic medium with embedded carbon nanotube, the nonlocal frequencies are comparatively lower. It is also found that the frequencies of the nonlocal model at different stages of temperature are higher than the nonlocal with same temperature. Ahmed *et al*. (2019) investigated the nano-scaled beams constructed from porous functionally graded (FG) materials taking into account geometrical imperfection shape. Hence, two types of nanobeams which are perfect and imperfect have been studied. Porous FG materials are classified based on even or uneven porosity distributions. A higher order nonlinear refined beam theory is used in the present research. Both perfect and imperfect nanobeams are formulated based on this refined theory. Akbaş (2017a, b) investigated the forced vibration analysis of a cracked functionally graded microbeam using modified couple stress theory with damping effect. Mechanical properties of the functionally graded beam change vary along the thickness direction. The crack is modelled with a rotational spring. The Kelvin-Voigt model is considered in the damping effect. Static bending of an edge cracked cantilever nanobeam composed of functionally graded material (FGM) subjected to transversal point load at the free end of the beam is investigated based on modified couple stress theory. Material properties of the beam change in the height direction according to exponential distributions. Sakhaee-Pour *et al*. (2009) used formulism of FEM to explore the vibration of SWCNTs. By taking the different diameter and length side, the vibration of cantilever and bridge SWCNTs is calculated. The fundamental frequencies of different categories of SWCNTs are performed by atomistic simulation and a good agreement found with MSM. Yang *et al*. (2010) demonstrated the frequencies of CNTs using nonlocal theory and geometric theory. Vibrational frequencies of zigzag SWCNTs (5, 0), (8, 0), (9, 0) and (11, 0), with different boundary conditions are considered and calculated numerically through MD simulation. The influence of nonlocal parameter on height and radius is studied in detail. Rana *et al*. (2016) introduced the Buongiorno model analytically and numerically based on perturbation theory. The dispersion relation with applying normal modes, various parameters has been derived such as the Brownian diffusion parameters and the influence of thermophoresis. Akbaş (2018a, b, c) presented the forced vibration responses of a cantilever nanobeam with crack using modified couple stress theory with damping effect. The crack is modeled with a rotational spring. The Kelvin–Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory in the time domain. Influences of the geometry, crack and material parameters on forced vibration responses of cracked nanobeams are examined and discussed. Chawis *et al*. (2013) reported a nonlocal theory with scale length to conduct vibration of SWCNTs with Euler beam theory using nonlocal parameter. The results are obtained by classical solutions and compared with the results of FEM. In this study, effects of different geometrical boundary condition and tube chirality have been considered. They reported that different variation in the frequency observed with increasing the length, diameter and atomic arrangements. Moreover, a new pattern of frequencies observed with increasing the nonlocal parameter. Recently, CNTs are used on large scale applying synthesis method with various lengths, diameters, and chirality. There are many methods for the production of SWCNTs for the distribution of diameter and chiralities which are used in transistors and sensors (Smalley *et al*. 2006, Sanchez-Valencia *et al*. 2014). Besseghier *et al*. (2015) presented the nonlinear vibration of zigzag SWCNTs based on Winkler-type model. The energy-equivalent model was used for the derivation of general equation. Yang *et al*. (2010) demonstrated the frequencies of CNTs using nonlocal theory and geometric theory. Vibrational frequencies of zigzag SWCNTs (5, 0), (8, 0), (9, 0) and (11, 0), with different boundary conditions are considered and calculated numerically through MD simulation. The influence of nonlocal parameter on height and radius is studied in detail. Selim (2010) performed vibrational behavior of SWCNT with compression stresses using multilayers. The motion equation was derived for the vibration analysis of SWCNTs using wave propagation. However, the outcomes of this research noted for various structures and the overall electronic and structural properties. Khalaf *et al*. (2019) devoted to analyzing mechanical-thermal post-buckling behavior of a micro-size beam reinforced with graphene platelets (GPLs) based on geometric imperfection effects. Graphene platelets have three types of dispersion within the structure including uniform-type, linear-type and nonlinear-type. The micro-size beam is considered to be perfect (ideal) or imperfect. Buckling mode shape of the micro-size beam has been assumed as geometric imperfection. Akbaş (2019) presented axially forced vibration of a cracked nanorod under harmonic external dynamically load. In constitutive equation of problem, the nonlocal elasticity theory is used. The Crack is modelled as an axial spring in the crack section. In the axial spring model, the nonrod separates two sub-nanorods and the flexibility of the axial spring represents the effect of the crack. Boundary condition of the nanorod is selected as fixed-free and a harmonic load is subjected at the free end of the nanorod. Abdulrazzaq *et al*. (2020) exposed the thermo-elastic buckling of small scale functionally graded material (FGM) nano-size plates with clamped edge conditions rested on an elastic substrate exposed to uniformly, linearly and non-linearly temperature distribution employing a secant function based refined theory. Material properties of the FGM nano-size plate have exponential gradation across the plate thickness. Civalek and Jalaei, (2020) studied a geometric transformation method based on discrete singular convolution (DSC) to solve the buckling problem of a functionally graded carbon nanotube (FG-CNT)-reinforced composite skew plate. The straight-sided quadrilateral plate geometry is mapped into a square domain in the computational space using a four-node DSC transformation method. Civalek (2020) presented the free vibration characteristics of thick skew plates reinforced by functionally graded carbon nanotubes (CNTs) reinforced composite. Discrete singular convolution (DSC) method is used for the numerical solution of vibration problems via geometric mapping technique. Using the geometric transformation via a four-node element, the straight-sided quadrilateral physical domain is mapped into a square domain in the computational space. Bouadi *et al.* (2018) developed the new model displacement field for the nonlocal buckling properties of single graphene sheet. The Eringen relation was used for the theoretical formation with length scale parameter. Ebrahimi and Mahmoodi (2018) presented the static analysis of SWCNTs and vibration of CNTs using Eringen's beam theory. The bending moment and function of strain were performed with different boundary conditions. Recently some material researchers explained the axially vibration analysis, bukling response and effect of nanofluid with cracked nanorod using nonlocal theory with coupled nonlinear equation (Akbaş 2019). Ahmed *et al.* (2020) studied the cotangential shear strain function and nonlinear stability of nonlocal higherorder refined beams made of metal foams based on Chebyshev-Ritz method. Based on inverse cotangential beam model, it is feasible to incorporate shear deformations needless of shear correction factor. Metal foam is supposed to contain different distributions of pores across the beam thickness. Akbaş (2020) investigated the axially damped forced vibration responses of viscoelastic nanorods within the frame of the modal analysis. The nonlocal elasticity theory is used in the constitutive relation of the nanorod with the Kelvin-Voigt viscoelastic model. In the forced vibration problem, a cantilever nanorod subjected to a harmonic load at the free end of the nanorod is considered in the numerical examples. The thermal conductivity of carbon nanotubes, analysis of Bernoulli nanobeams and wave propagation in microbeam had been investigated in detail (Ansari *et al*, 2018, Attarnejad and Ershadbakhsh, 2016, Kocaturk and Akbas 2013). Ehyaei and Daman (2017) and Eltaher *et al.* (2019) investigated the vibration characteristics of SWCNTs and DWCNTs using initial perfection and continuum mechanics approach. The general equation of motion was obtained by Hamiltonian principle and energy equivalent model. The numerical frequencies of DWCNTs and SWCNTs were determined by Navier method and finite element method.

Many material researchers calculated the frequency of CNTs using different techniques, for example, Timoshenko beam element (Banerjee and Williams 1992), classical molecular dynamics (Han *et al.* 1997), strain gradient higher order shell and shear deformation theory (Karami *et al.* 2018, Zine *et al.* 2018); continuum models (Duan *et al.* 2007), Galerkin's method (Elishakoff and Pentaras, 2009), axially loaded double beam system (Natsuki *et al.* 2009), Ritz method (Emdadi *et al.* 2019), MD simulation (Rafiee and Mahdavi 2016), Euler-Bernoulli beam mode (Bensattalah *et al.* 2018), nonlocal Timoshenko beam theory (Wu *et al.* 2018), Euler-Bernoulli's elastic beams (Kumar 2018), nonlocal elastic theory (Lee and Chang 2008, Soltani *et al.* 2016), finite element method (Mungra and Webb 2015, Tserpes and Papanikos 2005), dynamic response for plates (Bakhadda *et al.* 2018, Medani *et al.* 2019, Draoui *et al.* 2019), nonlocal continuum mechanics (Narendar and Gopalakrishnan, 2011), and wave propagation (El-sherbiny *et al.* 2013). Recently some researcher used different methods for nonlinear modeling (Eltaher *et al.* 2019, Ebrahimi *et al.* 2019, Safaei *et al.* 2019, Shahsavari *et al.* 2019, Benmansour *et al.* 2019, Cao *et al.* 2020, Ahangarnazha *et al.* 2020, Si *et al.* 2020, Farokhian and Kolahchi 2020).

The use of Rayleigh's Ritz method (RRM) is important for the study of nanostructures to develop a new formulism with Sander's shell theory. In this method, eigenvalue form is developed with the help of axial modal function in matrix representation. With the help of computer software MATLAB, frequencies of SWCNTs are extracted. The formulation of WPA is given by Zhang *et al.* (2001), a brief yet simple explanation first time. To the best of knowledge, this new formulations for SWCNTs have not been explicitly reported in the open literature.

2. Theoretical assumptions

The tube geometry is shown in Fig. 1. The shell is assumed to have length L, thickness h and the radius $R$for cylindrical shell with its coordinate system (x, θ, t) as shown in Fig. 1. The x, θ co-ordinate are assumed to be along longitudinal and circumferential direction, respectively and z - co-ordinates are taken in its radial directions.

The strain-displacement relations from Budiansky and Sanders (1963) are furnished as:

$$e_{12} = \frac{\partial y}{\partial x}, \quad e_{22} = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} - w\right), \quad e_{12} = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta} \tag{1}$$

and the expressions for the curvature - displacement relations are represented as:

$$\begin{gathered} k_{11} = \frac{\partial^2 w}{\partial x^2}, \; k_{22} = \frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta}\right), \\ k_{12} = \frac{1}{R}\left(\frac{\partial^2 w}{\partial x \partial \theta} + \frac{3}{4}\frac{\partial v}{\partial x} - \frac{1}{4R}\frac{\partial u}{\partial \theta}\right) \end{gathered} \tag{2}$$

The strain energy, S of a vibrating CNT is expressed as:

$$\begin{aligned} S = {} & \frac{R}{2}\int_0^L \int_0^{2\pi} [A_{11}\, e_1^2 + A_{22} e_2^2 + 2A_{12} e_1 e_2 + A_{66} e_{12}^2 \\ & +2\begin{pmatrix} B_{11} e_1 k_1 + B_{11} e_1 k_1 + B_{11} e_1 k_1 \\ +B_{11} e_1 k_1 + 2B_{66} e_{12} k_{12} \end{pmatrix} + D_{11} k_1^2 \\ & +D_{22} k_2^2 + 2D_{12} k_1 k_2 + D_{66}^2 k_{12}^2] d\theta dx \end{aligned} \tag{3}$$

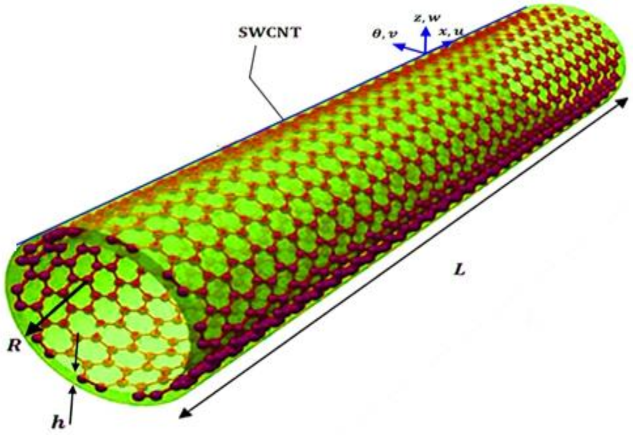

Fig. 1 Geometry of SWCNTs

where e_1, e_2 and e_3 designate the reference surface strains and k_1, k_2 and k_3 denote the reference surface curvatures respectively. The extensional stiffness, A_{ij} , coupling stiffness, B_{ij} and bending stiffness, D_{ij} are written as:

$$\{A_{ij}, B_{ij}, D_{ij}\} = \int_{h/2}^{h/2} Q_{ij}\{1, z, z^2\} dz (i, j = 1,2,6) \tag{4}$$

Here the reduced stiffness, Q_{ij}'s are written as:

$$Q_{11} = Q_{22} = \frac{E}{1-v^2}\ ,\ Q_{12} = \frac{vE}{1-v^2}\ ,\ Q_{66} = \frac{E}{2(1+v)} \tag{5}$$

for an isotropic cylindrical tube.

Making substitutions of these relations from the expression (1) and (2) into the formula (3), the tube strain energy, S is takes the following forms:

$$\begin{aligned} S = \frac{1}{2}\iint_{0\,0}^{2\pi L} & [A_{11}\left(\frac{\partial u}{\partial x}\right)^2 + A_{22}\frac{1}{R^2}\left(\frac{\partial v}{\partial \theta} + w\right)^2 \\ & +2A_{12}\frac{1}{R}\frac{\partial u}{\partial x}\left(\frac{\partial v}{\partial \theta} + w\right) + A_{66}\left(\frac{\partial v}{\partial \theta} + \frac{1}{R}\frac{\partial u}{\partial x}\right)^2 \\ & -2B_{11}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) - 2B_{12}\frac{1}{R^2}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) \\ & -2B_{12}\frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right)\left(\frac{\partial^2 w}{\partial x^2}\right) \\ & -2B_{22}\frac{1}{R^3}\left(\frac{\partial v}{\partial \theta} + w\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) \\ & -4B_{66}\frac{1}{R}\left(\frac{\partial v}{\partial \theta} + \frac{1}{R}\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{3}{4}\frac{\partial v}{\partial x} + \frac{1}{4R}\frac{\partial u}{\partial \theta}\right) \\ & +D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{D_{22}}{R^4}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right)^2 \\ & +2D_{12}\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) \\ & +4D_{66}\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{3}{4}\frac{\partial v}{\partial x} + \frac{1}{4R}\frac{\partial u}{\partial \theta}\right)^2]R dx d\theta \end{aligned} \tag{6}$$

The tube kinetic energy, K of the tube is written as:

$$K = \frac{1}{2}\iint_{0\,0}^{2\pi L} \rho_t \left[(\frac{\partial u}{\partial t})^2 + (\frac{\partial v}{\partial t})^2 + (\frac{\partial w}{\partial t})^2\right] R dx d\theta \tag{7}$$

Table 1 Comparison of frequencies of C-F first mode SWCNTs with MD simulation based on Sander shell theory

Aspect ratio	f (THz)	
	Zhang *et al.* (2009)	Present
4.67	0.23193	0.24041
6.47	0.12872	0.14431
7.55	0.1000	0.11868
8.28	0.07935	0.08943
10.07	0.05493	0.05728
13.69	0.03052	0.03640

at any time, t and ρ_t designates the mass density per unit length and is written as:

$$\rho_t = \int_{h/2}^{h/2} \rho dz \tag{8}$$

where ρ stands for the mass density and remains constant for an isotropic material. The Lagrange energy functional is tabulated as a difference of strain and kinetic energies:

$$\prod = K - S \tag{9}$$

The modal displacement relations is

$$\begin{aligned} u(x, \theta, t) &= P(x)\sin(n\theta)\sin(\omega t) \\ v(x, \theta, t) &= Q(x)\cos(n\theta)\sin(\omega t) \\ w(x, \theta, t) &= R(x)\sin(n\theta)\sin(\omega t) \end{aligned} \tag{10}$$

where the parameters $P(x)$, $Q(x)$, $R(x)$ in the relations (10) represents the vibration amplitudes in the x , θ and z directions correspondingly. where n gives number of circumferential wave modes, ω designates the natural circular frequency for a vibrating tube.

3. Use of the Rayleigh-Ritz procedure

The Rayleigh-Ritz technique has used has employed to frame the eigenvalue frequency equation in the generalized form. To obtain necessary extreme conditions the Lagrangian functional $\prod$ is differentiated with regard to the generalized

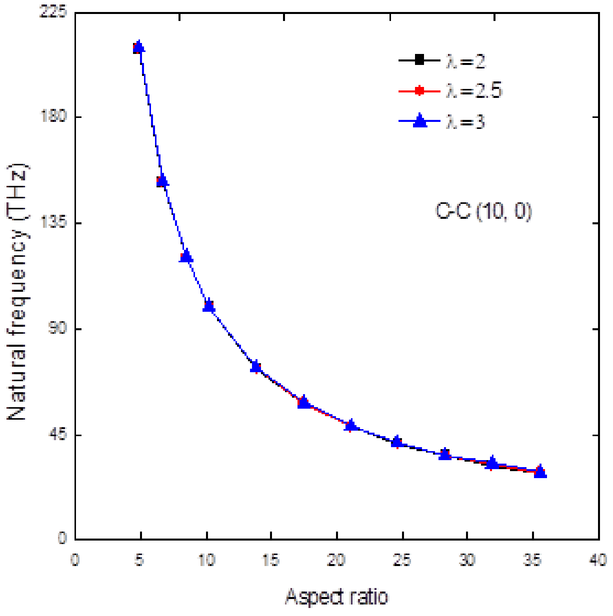

Fig. 2 Aspect ratio versus frequencies of C-C zigzag (10, 0) SWCNTs

Fourier coefficients a_i , $b_{i,}$ c_i. The flowing conditions are obtained:

$$\frac{\partial\Pi}{\partial a_i}=\frac{\partial\Pi}{\partial b_i}=\frac{\partial\Pi}{\partial c_i}=0 \text{ where } \quad i=1,2,\ldots\ldots\ldots N\,. \tag{11}$$

These equations are written in the following complete forms after concealing a huge amount of algebraic process:

After the ordering of above equations, the tube vibration frequency is expressed by the generalized eigenvalue relation as

$$\{[K]-\Delta[M]\}[\underline{x}]^T=0 \tag{12}$$

where [K] and [M] represent the stiffness and mass matrices for the tube and

$$\underline{x}]^T=(a_1,a_2,\ldots\ldots\ldots,a_N,\; b_2,\ldots\ldots b_N,c_1,c_2,\ldots\ldots\ldots..,c_N) \tag{13}$$

and

$$\Delta_1 = R^2\overline{\rho}_t\omega^2 \tag{14}$$

Eq. (19) yields tube frequencies and mode shapes for isotropic CNT. Coupling stiffness, B_{ij} become zero for isotropic materials. The eigenvalues represent with the tube frequencies and the corresponding eigenvectors designate the mode shapes. This procedure is used for any edge condition.

4. Volume fraction law

The material researcher Shen (2009) described the volume fraction law which is termed as power law:

$$V_{cnt}=\left[\frac{z}{h}+\frac{1}{2}\right]^{\lambda}V_{tcnt} \tag{15}$$

According to Shen (2009), V_{tcnt} is designated as total volume fraction of CNTs, respectively. The power law exponent is denoted as λ and h for thickness and z is the coordinate which varies from zero to infinity.

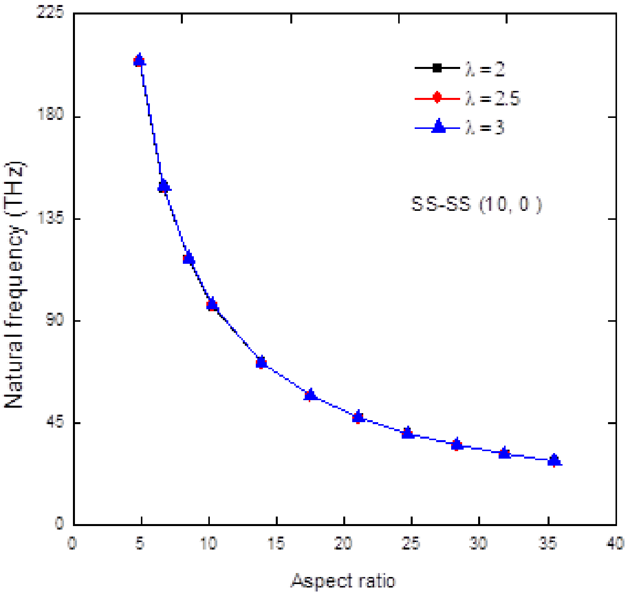

Fig. 3 Aspect ratio versus frequencies of SS-SS zigzag (10, 0) SWCNTs

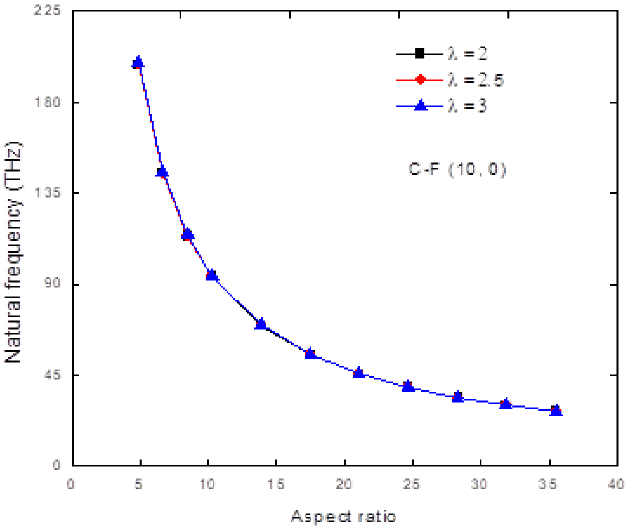

Fig. 4 Aspect ratio versus frequencies of C-F zigzag (10, 0) SWCNTs

5. Results and discussion

For obtaining new results, the appropriate tube thickness and material properties are a challenge. For present study, the material properties and tube thickness are chosen here, which was suggested by Shen (2009). The proposed model based on Ritz method can incorporate in order to accurately predict the acquired results of material data point. The present frequency result is also compared numerically (Zhang *et al.* 2009) experimentally with Raman Spectroscopy (Jorio *et al.* 2001). A good compliance is found for the

Table 2 Natural frequencies of the radial breathing mode between the Raman Spectroscopy (Jorio *et al*. 2001).

	f (THz)							
(*m*, *n*)	(8, 7)	(10, 5)	(11, 4)	(14, 1)	(18, 0)	(17, 2)	(11,11)	(12, 12)
Raman Spectroscopy (Jorio *et al*. 2001).	7.165	7.105	6.865	6.295	5.276	5.216	4.917	4.527
Present	7.018	7.093	5.781	6.103	5.161	5.142	4.805	4.465
Difference %	1.42	1.49	3.85	1.92	1.76	1.83	1.87	1.71

Table 3 Comparison of boundary conditions for frequencies of zigzag (19, 0) CNTs versus aspect ratio with different index of fraction law

Aspect ratio	C-C		SS-SS		C-F	
	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 2$	$\lambda = 2.5$
8.47	119.98	120.28	116.91	117.20	113.83	114.12
10.26	99.053	99.303	96.51	96.757	93.970	94.211
13.89	73.167	73.351	71.291	71.147	69.414	69.590
17.49	58.106	58.253	56.617	56.759	55.127	55.266
21.06	48.256	48.378	47.019	47.138	45.782	45.897

Table 4 Comparison of boundary conditions for frequencies of armchair (17, 17) CNTs versus aspect ratio with different index of fraction law

Aspect ratio	C-C		SS-SS		C-F	
	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 2$	$\lambda = 2.5$
8.47	245.82	377.92	372.41	373.35	376.22	377.17
10.26	242.53	373.01	367.83	368.76	371.83	372.77
13.89	239.28	368.16	365.78	366.71	367.51	368.44
17.49	237.83	365.99	364.69	365.61	365.58	366.50
21.06	237.06	364.84	364.04	364.95	364.55	365.47

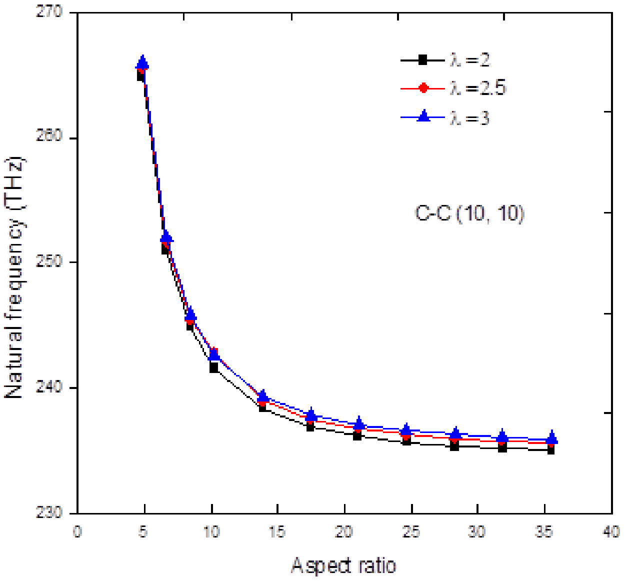

Fig. 5 Aspect ratio versus frequencies of C-C armchair (10, 10) SWCNTs

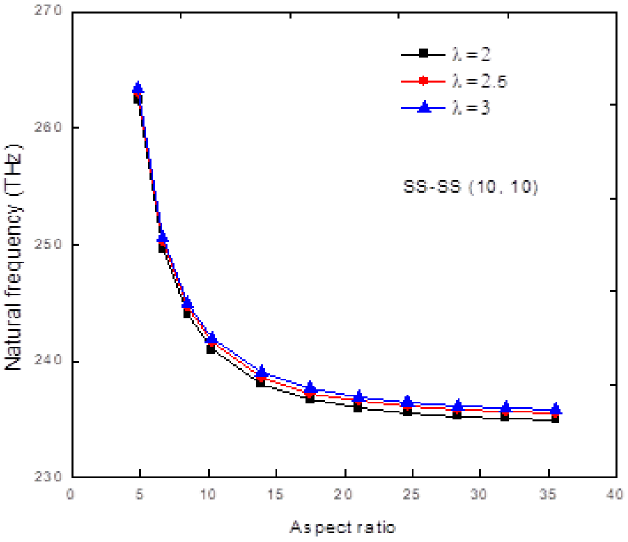

Fig. 6 Aspect ratio versus frequencies of SS-SS armchair (10, 10) SWCNTs

fundamental frequency it appears a satisfactory accuracy. Tables 3 and 4 describe the zigzag and armchair frequency comparison of three different boundary condition with volume fraction law. It is observed that the C-F boundary condition have less frequencies from the other two conditions. The frequencies enhances on increasing the power law index. Figs. 2-4 depicts the frequencies of zigzag

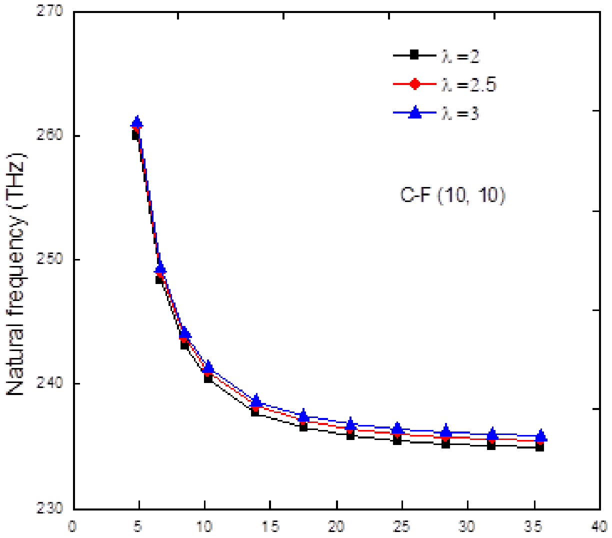

Fig. 7 Aspect ratio versus frequencies of C-F zigzag (10, 0) SWCNTs

(10, 0) with power law index $\lambda = 2,2.5,3$ versus aspect ratios (L/d: 4.86 ~ 35.53). The frequency of zigzag (10, 0) increases on decreasing the aspect ratios. The pattern of natural frequency for boundary conditions observed as C-C > SS-SS > C-F. In addition, when the tube aspect ratio increases from 5~10, the frequency decreases rapidly, while for aspect ratio (*L/d* = 10 ~ 15), the frequency is gently parallel. In present result, the frequencies are linear at aspect ratio from 15 ~ 35. The frequency curve for C-F is less than other conditions. It is due to the physical constraints which are applied on the edge of the CNT. The frequency phenomena for zigzag are insignificant throughout the aspect ratio. Moreover when index of power law is increases then frequencies increases for all boundary conditions as shown in the Figs 2-4. Figs 5-7 shows the natural frequency mechanism for the armchair (10, 10) for various values of power law index with different boundary conditions. Here frequencies decrease on increases the aspect ratio for all boundary conditions. The frequency curves of SS-SS edge condition is composed between the C-C and C-F conditions. The curves of frequency are less significant from aspect ratio (L/d = 4.86 ~ 8.47) and decreases fast for these ratios. The behavior occurred different for aspect ratio from 4.86 ~ 13.79 and the curves are parallel and more significant from (L/d = 4.86 ~ 8.47). The frequencies are much significant and linear at aspect ratio from 15 ~ 35. It can be seen that with the power law index, the frequencies increases slowly for specified boundary conditions. In all these indices of power law, the C-F is lower frequency curves than other two conditions. The power law index $\lambda = 2.5$ is sandwich between $\lambda = 2$ and $\lambda = 3$. The power law index $\lambda = 3$ have high frequency curves with all boundary conditions. It is due to the volume fraction for CNTs. In Figs. 2-7, the frequencies via aspect ratios, armchair (10, 10) have higher values from zigzag (10, 0). It is due to the material structure which is made by the carbon nanotubes. The power law index have momentous effect on the vibration of single walled carbon nanotubes.

6. Conclusions

The governing equation of motion using Rayleigh's method is written in the form of eigen value to extract the frequencies of CNTs. Vibrations of SWCNTs for armchair (10, 10) and zigzag (10, 0) have been analyzed SWCNTs invoking Sander shell theory. It is noted that the frequencies of C-C is higher than that of SS-SS and C-F. This modified model has less complication and has been compared with the earlier methods. It can be concluded that frequencies would increase by increasing the power law index. Hence, it is concluded that for each data point curve, frequency value for armchair is greater than that of zigzag and armchair values and frequency curves decreases as the aspect ratio with specific conditions. It can be seen that with the power law index, the frequencies increases slowly for specified boundary conditions. It is due to the volume fraction for CNTs. The present model can be used for the vibration of double walled carbon nanotubes with volume fraction laws, for future concern.

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