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Fundamental and conventional computer simulation for the stability of non-uniform systems

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Abstract. The accurate assessment of the performance of nonuniform systems requires a thorough understanding of stability analysis. As a result, the theoretical modeling of the influence of various variables on the performance of small-scale nonuniform structures with conventional and non-conventional geometries is presented in this paper. According to the fundamental computer simulation based on mathematical and mechanical principles, the stability of the nonuniform structures is examined. Thus, a numerical procedure is used to simulate the stability and instability characteristics of the nonuniform small-scale structures via computer aid. Theoretic simulation methods provide a great deal of the design and production of small-scale structures at a low cost compared to experimental simulations. Thus, this paper provides a good presentation of the stability analysis of the nonuniform nanoscale structures with high accuracy without actual experimental.

Keywords: computer simulation; nonuniform structures; numerical procedure; small-scale structures; stability analysis

1. Introduction

As one of the finest structures for meeting the demands of goods with their special qualities in non-homogeneous structures, functionally graded materials (FGM) are an excellent choice (Habibi *et al*. 2016, 2018b, Ebrahimi *et al*. 2019a, Esmailpoor Hajilak *et al*. 2019). When two or more distinct materials are mixed, the resulting construction retains both phases' useful characteristics (Habibi *et al*. 2018a, 2019b, d, e, Pourjabari *et al*. 2019, Safarpour *et al*. 2019a). On the other hand, although metal and ceramics are both formable, they have poor heat resistance and formability, respectively (Habibi *et al*. 2019a, Safarpour *et al*. 2019b, Alipour *et al*. 2020, Ebrahimi *et al*. 2020a, Liu *et al*. 2022). As a result, the heat resistance and formability of the functionally graded ceramic and metal material are complementary (Habibi *et al*. 2017, 2019c, Safarpour *et al*. 2018, 2020, Ghazanfari *et al*. 2020). Academics and designers are increasingly interested in using this important content into their work (Ebrahimi *et al*. 2019b, c, Mohammadgholiha *et al*. 2019, Mohammadi *et al*. 2019, Ebrahimi *et al*. 2020b, Habibi *et al*. 2020, Shariati *et al*. 2020a, Shokrgozar *et al*. 2020). Luat Doan *et al*. (2021) used shear deformation beam theory and an analytical technique to investigate the stability of a two-dimensional functionally graded nanoscale sandwich beam based on Eringen's theory. While conducting stability research for nonlocal functionally graded plates, Kumar *et al*. (2021) focused on the vibration behavior of porosity-dependent nonlocal plates. Bouhadra *et al*. (2021) used the analytical technique to investigate the buckling characteristics of a two-dimensional functionally graded nanobeam according to first-order shear deformation beam theory under various boundary conditions. Hadji and Avcar (2021) devised the hyperbolic shear deformation theory of beam to study the vibrational behavior of an imperfect functionally grade nanobeam. Nejadi Mohamad *et al*. (2021) used the numerical differential quadratic approach to investigate the stability regarding the buckling and vibration analysis of functionally graded nanosized plates supported on an elastic basis. Khadir Adnan *et al*. (2021) developed a unique theory for examining the stability of functionally graded laminated nanoplates using a static and dynamic analysis technique. Wang *et al*. (2021) used buckling analysis in conjunction with high-order shear deformation theory to determine the effect of shear correction on the stability of a first-order functionally graded nanotube, including its porosity. Luat Doan *et al*. (2021) investigated the buckling and vibration properties of a new 2D-FG sandwich shear deformation beam using nonlocal theory and the Hamilton technique. Shen and Xiang (2021) investigated the effect of temperature on the pre- and post-buckling characteristics of laminated functionally graded cylindrical panels using molecular dynamics and mathematical modelling based on the high-order theory of shells, while also taking into account the nonlinear effects of nonlinear strains via the perturbation method.

Despite the fact that the application of small-scale structures has advanced in recent years (Hashemi *et al*. 2019, Moayedi *et al*. 2019, 2020a, b, Oyarhossein *et al*. 2020, Shariati *et al*. 2020b), many groups of researchers have concentrated their efforts on theoretical inquiries due to the time-consuming and expensive nature of experimental analysis (Liu *et al*. 2020, Wang *et al*. 2020, Zhou *et al*. 2020, Dai *et al*. 2021a, Guo *et al*. 2021, Shao *et al*. 2021, Wu and Habibi 2021). Theory that is reliant on the size of

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the system is known as size-dependent theory (Adamian *et al*. 2020, Al-Furjan *et al*. 2020a, b, Li *et al*. 2020, Zare *et al*. 2020, Dai *et al*. 2021b). Examples of size-dependent theories include the strain gradient theory (Zhao *et al*. 2021, Huang *et al*. 2021a, Jiao *et al*. 2021, Moradi *et al*. 2021, Xu *et al*. 2021a), the Eringen theory, the modified couple stress theory, and the nonlocal strain gradient theory (Azimi *et al*. 2016, Ghadiri and Shafiei 2016a, c, Shafiei *et al*. 2016a, e, g). The fundamental assumptions of the various size-dependent theories are distinct, and these additional deductions result in a wide range of behavior as a result of the small-scale consequences (Ma *et al*. 2021, Hou *et al*. 2021a, Huang *et al*. 2021b, Liu *et al*. 2021b, Yu *et al*. 2022). The hardening phenomenon is predicted by some theories, such as the modified coupled stress theory, strain gradient theory, and so on, because of the impact on a small scale (Ghadiri *et al*. 2016a, b, c, d, Ghadiri and Shafiei 2016b, Shafiei *et al*. 2016b). The length-scale factors in these theories influence the stiffness part of the governing equation, while other theories (Du *et al*. 2022, Lv *et al*. 2022, Wang *et al*. 2022, Zheng *et al*. 2022a, c, d), such as the nonlocal theory, worked on mass matrices and predicted the softening phenomenon (Gong *et al*. 2022, Jiang *et al*. 2022, Li *et al*. 2022, Liu *et al*. 2022, Zheng *et al*. 2022b, Zhong *et al*. 2022). The nonlocal theory has been combined with the strain gradient theory in recent years, leading to the introduction of the nonlocal strain gradient theory, which encompasses both softening and hardening phenomena (Lim *et al*. 2015). According to this theory, the small-scale factor in this theory works on both the mass and stiffness parts of the governing equations. Because of the nonlocal strain gradient theory, the stability of small-scale structures can be better controlled by investigators, and this has resulted in a greater number of researchers from a variety of professional backgrounds gravitating into this topic. Arshid *et al*. (2021) Yang used the modified couple stress theory to conduct a stability study of an imperfect FG viscous curved beam using high-order beam theory and the Navier solution. Kong (2022) reviewed the mathematical simulation of microstructures regarding stability, vibrational and buckling examination involving microbeam and microplate, based on the modified couple stress theory. Shanab and Attia (2021) mathematically simulated the stability of a bi-directional functionally graded non-uniform microbeam using the analytical Navier approach in conjunction with first-order beam theory and modified couple stress theory. Zhang *et al*. (2022) numerically investigated the static and dynamic behavior of a two-dimensional FG microbeam, and the governing equations were formulated using first-order shear deformation beam theory in conjunction with couple stress theory. On the basis of modified couple stress theory, Teng *et al*. (2022) employed the differential transform technique to examine the free vibration and buckling characteristics of microscale porosity-dependent functionally graded beams. According to the modified couple stress theory, Gorji Azandariani *et al*. (2021) applied the Euler-Bernoulli beam theory coupled with nonlinear effects for the statically buckling analysis of a two-dimensional functionally graded microbeam. Hosseini and Arvin (2021) studied the pre and post-buckling analysis of spinning FG microscale beam based on the classical beam theory along with the modified couple stress theory considering the thermal impacts of the thermal environment. Moradifard *et al*. (2021) examined the nonlinear free vibration of a 2D-FG classical micro beam surrounded by an elastic foundation based on the modified couple stress theory operating the Galerkin method. Akbarzadeh Khorshidi (2021) studied the post-buckling of viscoelastic microbeam embedded on a visco-elastic foundation according to the modified couple stress theory. Kumar and Mukhopadhyay (2021) considered the dual-phase-lag heat condition for the stability regarding the dynamic analysis of micro scale Timoshenko beam in the thermal conditions and on the basis of modified couple stress theory. An investigation of piezoelectric microbeam dynamics based on the modified couple stress theory and differential quadrature technique was reported by Li and Xiao (2021). Al-Furjan *et al*. (2021) employed the modified couple stress theory in order to investigate the dynamic behavior of composite circular microplate entrenched in a viscoelastic foundation.

Static analysis of micro and nanobeams, micro and nanotubes, as well as other relevant structures such as micro and nanoplates, has been extensively studied (Ebrahimi and Shafiei 2016, Shafiei *et al*. 2016c, d, f, Ebrahimi *et al*. 2017, Shivanian *et al*. 2017), nevertheless, the stability of micro-structures beams still needs to be developed (Chen *et al*. 2021, Liu *et al*. 2021a, Long *et al*. 2021, Tang *et al*. 2021, Wei *et al*. 2021, Xu *et al*. 2021b). A micro-beam with axially functionally graded aluminum oxide and nickel composition is studied in this research for the mechanical buckling characteristics. The modified couple stress theory and the first-order shear deformation beam theory are used for the mathematical modeling of a micro-sized beam. The GDQ technique determines the numerical results after the partial differential equations have been generated using the conservation approach.

2. Mathematical simulation of microstructures

A nonuniform beam is considered in this paper made of functionally graded material that the beam thickness changes in the length direction regarding the exponential function. In the current study, a uniform thickness as well as a nonuniform exponential function are assumed as the beam thickness function regarding the following mathematical equation (Ghadiri *et al*. 2017a, b, Mirjavadi *et al*. 2017b, c, Shafiei *et al*. 2017a, b).

Uniform beam thickness: $h = h0$ (1a)

Non-uniform beam thickness: $h = h0e^{\left(-\varsigma\frac{x}{L}\right)}$ (1b)

where ‘h0’ is the initial thickness which is shown in Fig. 1. The material composition is varied in the beam length direction that the left side is made of pure Aluminum oxide and the right side is made of pure Nickel shown in Fig. 1. The Young’s modulus (E) and Poisson ratio (ν) are the required mechanical properties for the stability analysis for both Aluminum oxide (Al_2O_3) and Nickel, these properties are listed in Table 1.

Table 1 Poisson ratio and Young's modulus of Nickel and Aluminum oxide (Reddy and Chin 1998)

	Aluminum oxide	Nickel
Young's modulus (GPa)	349.55	223.95
Poisson ratio	0.24	0.31

The mathematical formulation of the material composition in the axially functionally graded structures is presented in the following equation (Ehyaei *et al.* 2017, Ghadiri *et al.* 2017c, d, Mirjavadi *et al.* 2017d, Shafiei and Kazemi 2017b, Shafiei *et al.* 2017c).

$$E(x) = E_N + (E_{Al} - E_N)\left(1 - \frac{x}{L}\right)^{\eta} \tag{2a}$$

$$\nu(x) = \nu_N + (\nu_{Al} - \nu_N)\left(1 - \frac{x}{L}\right)^{\eta} \tag{2b}$$

The mechanical material properties are changed in the beam length, which '$()_{Al}$' refers to the Al_2O_3, and '$()_N$' indicates the Nickel.

In this research, the first-order shear deformation beam theory is utilized along with the modified couple stress theory for mathematical modeling of the stability of an AFG beam based on the following energy conservation principle (Ebrahimi and Shafiei 2017, Ghadiri *et al.* 2017e, Mirjavadi *et al.* 2017a, Shafiei and Kazemi 2017a, Shafiei *et al.* 2017d, Azimi *et al.* 2018).

$$\delta W + \delta P = 0 \tag{3}$$

where

$$\delta W = \iiint \left(\frac{1}{2} F\delta\left(\frac{\partial w}{\partial x}\right)^2 \delta w\right) dv \tag{4}$$

'F' is the buckling load. Also, 'P' is the strain or potential energy regarding the modified couple stress theory that is summed as follows:

$$P = \frac{1}{2}\iiint (\sigma\colon \varepsilon + m\colon \chi) dv \tag{5}$$

'm', 'ε', 'χ', and 'σ' are the deviatoric matrices, strains, symmetric curvature, and stress tensors, respectively. On the basis of Timoshenko beam theory, the displacement fields are supposed as follows:

$$u_x(x, y, z) = z\varphi(x) + u(x) \tag{6a}$$

$$u_y(x, y, z) = 0 \tag{6b}$$

$$u_z(x, y, z) = w(x) \tag{6c}$$

where 'φ', 'w', and 'u' are the rotation, lateral, and axial beam displacement. Also, the total movement along the z-axis is 'u_z', the y-axis is 'u_y', and the x-axis is 'u_x'. Utilizing the obtained displacement fields, the strains are calculated as follows:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = z\frac{\partial \varphi}{\partial x} + \frac{\partial u}{\partial x} \tag{7a}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{yz} = \varepsilon_{zy} = 0 \tag{7b}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \psi\right) \tag{7c}$$

Also, 'm' is considered as follows:

$$m = 2l^2\mu\chi \tag{8}$$

'μ' is the Lamé constant, and 'l' is the size-dependent parameter based on the modified couple stress theory. Also, 'χ' is defined as follows:

$$\chi_{xy} = \chi_{yx} = \frac{1}{4}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - \frac{1}{2}\frac{\partial^2 w}{\partial x^2} \tag{9a}$$

$$\chi_{xz} = \chi_{zx} = \chi_{yz} = \chi_{zy} = \chi_{xx} = \chi_{yy} = \chi_{zz} = 0 \tag{9b}$$

The potential energy of the microstructure beam is calculated by substituting Eqs. (7)-(9) into Eq. (5) (See Appendix I).

$$Appemdix\ I \tag{10}$$

where

$$(A_{11}, A_{10}, C_{10}, D_{11}) = \int_A E(x)\left(1, \frac{1}{2(1+\nu(x))}, \frac{K_S}{2(1+\nu(x))}, z^2\right) dA \tag{11}$$

'KS' is the shear correction factor. The following Euler-Lagrange equations will be obtained by substituting Eqs. (4) and (10) into Eq. (3).

$$\delta(u)\colon \frac{\partial}{\partial x}\left(A\frac{\partial u}{\partial x}\right) = 0 \tag{12a}$$

$$\begin{gathered}\delta(\varphi)\colon C_{10}\left(\varphi + \frac{\partial w}{\partial x}\right) - \frac{\partial}{\partial x}\left(D\frac{\partial \varphi}{\partial x}\right) \\ -\frac{1}{4}l^2 A_{10}\left(\frac{\partial \varphi}{\partial x} - \frac{\partial^2 w}{\partial x^2}\right) = 0\end{gathered} \tag{12b}$$

$$\begin{gathered}\delta(w)\colon \frac{\partial^2}{\partial x^2}\left(\frac{1}{4}l^2 A_{10}\left(\frac{\partial \varphi}{\partial x} - \frac{\partial^2 w}{\partial x^2}\right)\right) + \\ \frac{\partial}{\partial x}\left(C_{10}\left(\varphi + \frac{\partial w}{\partial x}\right)\right) + F\frac{\partial^2 w}{\partial x^2} = 0\end{gathered} \tag{12c}$$

Also, the boundary conditions are:

$$\delta(u)\colon A\frac{\partial u}{\partial x} = 0 \tag{12d}$$

$$\delta(\varphi)\colon \frac{1}{4}l^2 A_{10}\left(\frac{\partial \varphi}{\partial x} - \frac{\partial^2 w}{\partial x^2}\right) + D\frac{\partial \varphi}{\partial x} = 0 \tag{12e}$$

$$\begin{gathered}\delta(w)\colon \frac{1}{4}l^2\frac{\partial}{\partial x}\left(A_{10}\left(\frac{\partial \varphi}{\partial x} - \frac{\partial^2 w}{\partial x^2}\right)\right) \\ +C_{10}\left(\varphi + \frac{\partial w}{\partial x}\right) = 0\end{gathered} \tag{12f}$$

3. Numerical procedure of solution

In the present paper, the generalized differential quadratic method is employed to calculate the numerical results, according to this numerical procedure (Shafiei and She 2018, Shafiei *et al.* 2019, 2020), the derivative function of 'Y' is reformed to the following format.

$$\left.\frac{\partial^b Y(x)}{\partial x^b}\right|_{x=x_P} = \sum_{j=1}^{n} Q_{ij}^{(b)} Y(x_i) \tag{13}$$

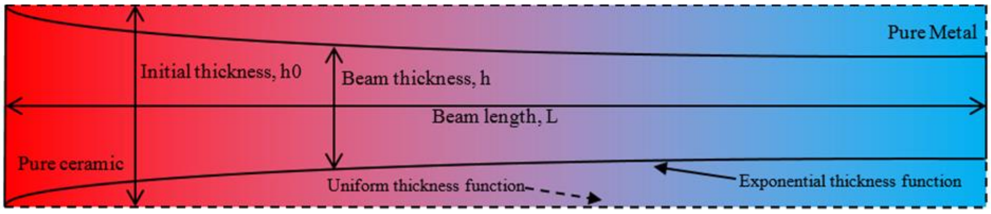

Fig. 1 Geometric details and material composition of functionally graded uniform and nonuniform beam

Table 2 A comparison between the estimated buckling load (FL^2/EI) and the findings of Li *et al.* (2021) and Hou *et al.* (2021b)

	l/h0=0	l/h0=0.2	l/h0=0.4	l/h0=0.6	l/h0=0.8	l/h0=1.0
Present study	9.854839933	115.6843338	167.0979416	252.7997549	372.7687569	527.0173076
Li *et al.* (2021)	9.8696	115.8581	167.349	253.1795	373.3288	527.809
Hou *et al.* (2021b)	9.8696	115.859	167.3501	253.1811	373.3333	527.8097

Table 3 The buckling load (Y) of the microscale beam versus the small-scale parameter (β) along with the uniform and nonuniform section for different boundary conditions

	Fully pinned		Clamped-pinned		Fully clamped	
	Uniform	Exponential	Uniform	Exponential	Uniform	Exponential
β=0.0	6.283185	1.419037	12.85382	2.971023	25.13274	5.845369
β=0.1	6.445332	1.506162	13.18553	3.154707	25.78133	6.208559
β=0.2	6.931772	1.764173	14.18066	3.696189	27.72709	7.27671
β=0.3	7.742506	2.186936	15.83922	4.577792	30.97002	9.010812
β=0.4	8.877533	2.770453	18.1612	5.787621	35.51013	11.38525
β=0.5	10.33685	3.513359	21.1466	7.321282	41.34741	14.39094
β=0.6	12.12047	4.415668	24.79543	9.178506	48.48187	18.0276
β=0.7	14.22837	5.477849	29.10768	11.36051	56.9135	22.298
β=0.8	16.66058	6.70042	34.08335	13.86876	66.6423	27.20534
β=0.9	19.41707	8.083821	39.72244	16.70456	77.66828	32.75241
β=1.0	22.49786	9.628388	46.02496	19.86891	89.99143	38.94139
β=1.1	25.90294	11.33438	52.9909	23.3626	103.6118	45.77394
β=1.2	29.63231	13.20197	60.62026	27.18621	118.5293	53.25128
β=1.3	33.68598	15.2313	68.91304	31.34017	134.7439	61.37431
β=1.4	38.06394	17.42247	77.86925	35.82478	152.2558	70.14372
β=1.5	42.7662	19.77556	87.48888	40.64031	171.0648	79.56002
β=1.6	47.79275	22.29062	97.77194	45.78692	191.171	89.62358
β=1.7	53.14359	24.9677	108.7184	51.26476	212.5743	100.3347
β=1.8	58.81872	27.80682	120.3283	57.07394	235.2749	111.6936
β=1.9	64.81815	30.80802	132.6016	63.21453	259.2726	123.7005
β=2.0	71.14187	33.97132	145.5384	69.68662	284.5675	136.3555

where

$$if\ i \neq j,\quad Q_{ij}^{(b)} = b\left[Q_{ij}^{(b-1)}Q_{ij}^{(1)} - \frac{Q_{ij}^{(b-1)}}{(x_i - x_j)}\right] \tag{14a}$$

$$ifi = j,\ Q_{ii}^{(b)} = -\sum_{j=1,i\neq j}^{n} Q_{ij}^{(b)} \tag{14b}$$

and

$$if\ i \neq j,\quad Q_{ij}^{(1)} = \frac{R(x_i)}{(x_i - x_j)R(x_j)} \tag{14c}$$

$$ifi = j,\ Q_{ij}^{(1)} = -\sum_{j=1,i\neq j}^{n} Q_{ij}^{(1)} \tag{14d}$$

Also

$$R(x_i) = \prod_{j=1,j\neq i}^{n} (x_i - x_j) \tag{15}$$

Substituting derivative matrices into the governing and related boundary conditions, the eigenvalue equations will be obtained (See Appendix II) in which the buckling load is the eigenvalues of the equation (Li *et al.* 2017, Che and Wang 2021, Kong *et al.* 2021, Meng *et al.* 2021, Yu *et al.* 2021).

$$Appendix\ II \tag{16}$$

Table 4 The buckling load (Υ) of the microscale beam versus the volume fraction parameter (η) along with the uniform and nonuniform section for different boundary conditions, β=1

	Fully pinned		Clamped-pinned		Fully clamped	
	Uniform	Exponential	Uniform	Exponential	Uniform	Exponential
Al_2O_3, η=0	22.4979	9.62839	46.025	19.8689	89.9914	38.9414
η=1	17.8606	7.9201	36.4135	16.1427	71.0932	31.5299
η=2	16.0966	7.17022	33.545	14.9747	66.057	29.3666
η=3	15.2623	6.76507	31.9797	14.2859	63.6453	28.2214
η=4	14.8169	6.52616	31.0098	13.8215	62.2861	27.5223
η=5	14.5568	6.37511	30.3739	13.4921	61.4344	27.0631
η=6	14.3946	6.27438	29.9405	13.2513	60.8498	26.7437
η=7	14.2882	6.20438	29.6357	13.0711	60.4162	26.5095
η=8	14.2157	6.15419	29.4159	12.934	60.0742	26.3294
η=9	14.1646	6.11729	29.2543	12.8278	59.7919	26.1851
η=10	14.1276	6.08947	29.1329	12.7443	59.551	26.0654
η=11	14.0998	6.06793	29.0395	12.6773	59.3407	25.963
η=12	14.0782	6.05077	28.9657	12.6226	59.1542	25.8736
η=13	14.061	6.03676	28.9059	12.577	58.9869	25.7941
η=14	14.0472	6.02517	28.8567	12.5387	58.8354	25.7223
η=15	14.0364	6.01562	28.816	12.5063	58.6974	25.6568
η=16	14.0283	6.00792	28.7829	12.4792	58.5711	25.5965
η=17	14.0228	6.00199	28.757	12.4569	58.4551	25.5407
η=18	14.0199	5.99779	28.7378	12.4394	58.3482	25.4888
η=19	14.0192	5.99522	28.7252	12.4263	58.2497	25.4404
Nickel, η=∞	13.7188	5.83097	28.0652	12.0368	54.8751	23.5935

4. Results presentation

The buckling analysis of the first-order shear deformation microstructure beam based on the modified couple stress theory is investigated in this paper, as previously stated, the motion equations were solved via a numerical approach, so Table 2 compared the calculated numerical results of the presented work with the results of Li *et al.* (2021) and Hou *et al.* (2021b) to validate the solution procedure and also, the generated governing equations.

Before examining the obtained findings, it is necessary to establish various dimensionless parameters, such as the dimensionless buckling load (Υ) and small-scale parameters (β).

$$\Upsilon = \frac{2FL^2}{E_{Al}I\pi} \tag{17a}$$

$$l^2 = \beta^2 \mathrm{h0} \tag{17b}$$

Table 3 present the impact of small-scale parameter on the mechanical buckling loaf of microstructure beam made of Aluminum oxide for both uniform and nonuniform exponential section under the different types of boundary conditions involving the pinned and clamped. It is seen in all presented boundary conditions and both types of sections, the small-scale parameter improves the buckling load because this parameter directly impacts the stresses and improves the stability of the beam by an increment of beam stiffness. It is also noticed that the clamped beam has a higher buckling load and the pinned beam has a lower buckling load because the rigidity of the clamped beam is higher than the pinned beam, so the clamped beam is more stable than the pinned beam. Another result presented in this table is the nonuniformity effect on the buckling and stability of the microbeam, it is caught that the nonuniform beam is more unstable than the uniform beam and the buckling load of the uniform beam is higher than the exponential beam.

Table 4 explores the effect of functionally graded factors on the stability of the microtube when subjected to buckling force. The results are explored for both uniform and nonuniform small-scale beam configurations, as well as for various boundary conditions. An increase in functionally graded parameters reduces the buckling load and degrades the stability of microbeam. Because Aluminum oxide is stiffer than nickel, raising the FG parameter increases the Nickel volume in the microbeam structure, decreasing beam stability. As previously stated, uniform beams are more stable than exponential beams at all volume fraction values. Furthermore, the clamped-pinned beam is more stable than the completely pinned beam while being less stable than the fully clamped beam.

Fig. 2 illustrates the effect of the small-scale parameter, as well as the volume fraction parameter, on the stability of the microstructure exponential non-uniform beam under buckling stress for doubly clamped, doubly pinned, and clamped-pinned boundary conditions. The buckling load rises when the small-scale parameter of the modified couple

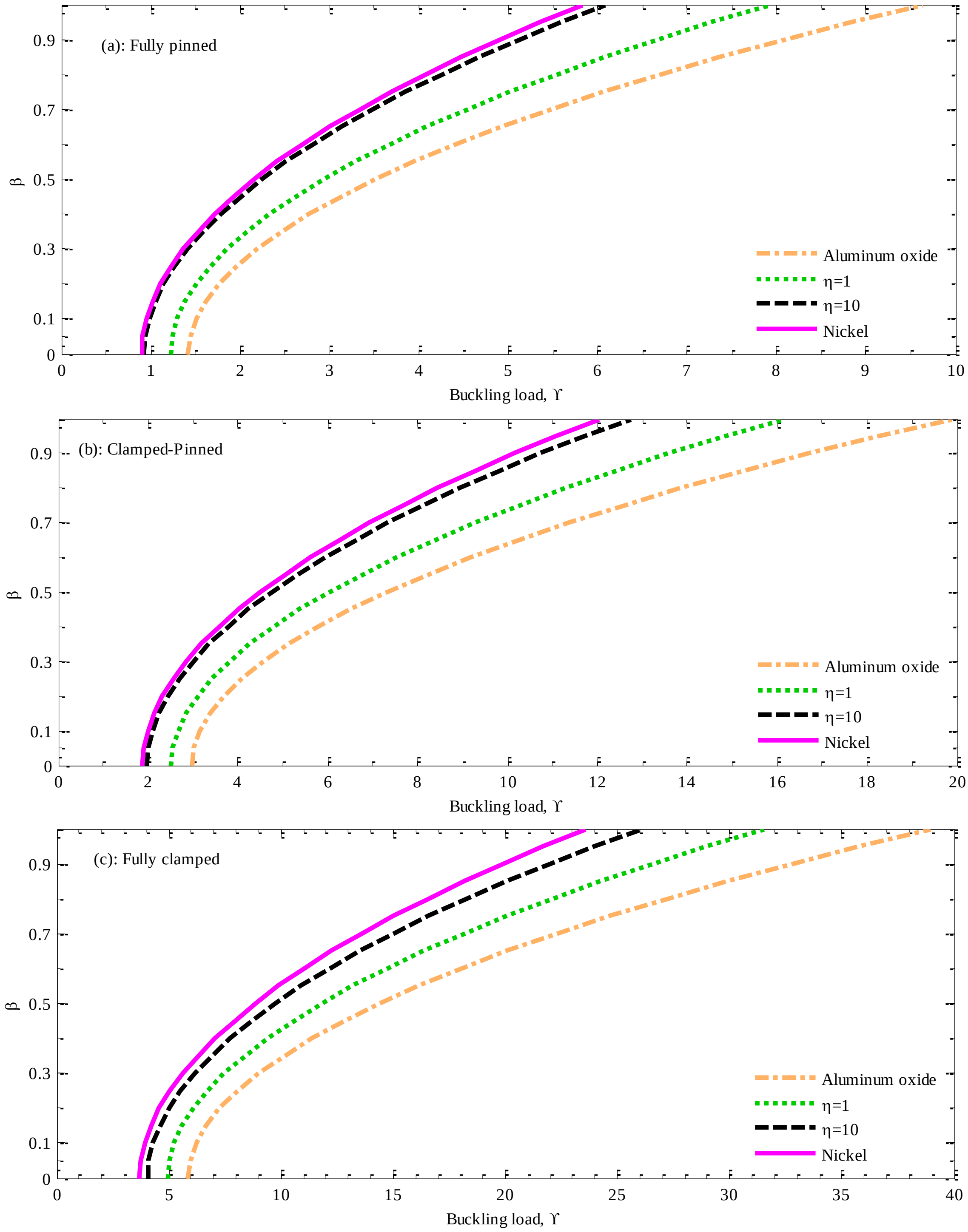

Fig. 2 Buckling load (Y) of microstructure beam versus small-scale parameter (β) as well as FG parameter (η) for a: fully pinned, b: clamped-pinned, c: fully clamped non-uniformbeam

stress theory is increased, implying that the microbeam's stability increases as the length scale parameter is increased. Additionally, the volume fraction restricts the buckling Additionally, the volume fraction restricts the buckling stress and reduces beam stability. Additionally, it is shown that the effect of FG parameters is more pronounced for the more extended length scale parameter. All possible boundary conditions are taken into account while drawing these conclusions.

5. Conclusions

The primary framework of the present study was to examine the buckling analysis of an AFG microbeam using the Timoshenko beam theory and the modified couple stress theory. The microbeam was constructed using functionally graded material. The material parameters varied over the beam length, and the partial differential governing equations were obtained using the conservation principle technique. In addition, the numerical method was used to solve the produced equations. The following main conclusion was found regarding the discussed results.

- An increment of the small-scale parameter of the modified couple stress theory improves the stability and the buckling load of the microstructures beam.
- The volume fraction parameter decreases the buckling load in the microstructures beam and leads to an unstable beam in higher FG parameter values.
- The small-scale clamped-pinned beam is more stable than the pinned beam and more unstable than clamped beam.
- Clamped microbeams provide more rigidity and stability than pinned microbeams.
- The non-uniformity of the section decreases the beam stability, and the nonuniform exponential microbeam is an unstable structure than the uniform beam for all types of boundary conditions.

References

Adamian, A., Safari, K.H., Sheikholeslami, M., Habibi, M., Al-Furjan, M. and Chen, G. (2020), "Critical temperature and frequency characteristics of GPLs-reinforced composite doubly curved panel", *Appl. Sci.*, **10**(9), 3251. https://doi.org/10.3390/app10093251.

Akbarzadeh Khorshidi, M. (2021), "Postbuckling of viscoelastic micro/nanobeams embedded in visco-Pasternak foundations based on the modified couple stress theory", *Mech. Time Depend. Mat.*,. **25**(2), 265-278. https://doi.org/10.1007/s11043-019-09439-8.

Al-Furjan, M., Dehini, R., Khorami, M., Habibi, M. and won Jung, D. (2020a), "On the dynamics of the ultra-fast rotating cantilever orthotropic piezoelectric nanodisk based on nonlocal strain gradient theory", *Compos. Struct.*, 112990. https://doi.org/10.1016/j.compstruct.2020.112990.

Al-Furjan, M., Fereidouni, M., Habibi, M., Abd Ali, R., Ni, J. and Safarpour, M. (2020b), "Influence of in-plane loading on the vibrations of the fully symmetric mechanical systems via dynamic simulation and generalized differential quadrature framework", *Eng. Comput.*, 1-23. https://doi.org/10.1007/s00366-020-01177-7.

Al-Furjan, M.S.H., Samimi-Sohrforozani, E., Habibi, M., Jung, D.w. and Safarpour, H. (2021), "Vibrational characteristics of a higher-order laminated composite viscoelastic annular microplate via modified couple stress theory", *Compos. Struct.*, **257**, 113152. https://doi.org/10.1016/j.compstruct.2020.113152.

Alipour, M., Torabi, M.A., Sareban, M., Lashini, H., Sadeghi, E., Fazaeli, A., Habibi, M. and Hashemi, R. (2020), "Finite element and experimental method for analyzing the effects of martensite morphologies on the formability of DP steels", *Mech. Based Des. Struct.*, **48**(5), 525-541. https://doi.org/10.1080/15397734.2019.1633343.

Arshid, E., Arshid, H., Amir, S. and Mousavi, S.B. (2021), "Free vibration and buckling analyses of FG porous sandwich curved microbeams in thermal environment under magnetic field based on modified couple stress theory", *Arch. Civil Mech. Eng.*, **21**(1), 6. https://doi.org/10.1007/s43452-020-00150-x.

Azimi, M., Mirjavadi, S.S., Shafiei, N. and Hamouda, A.M.S. (2016), "Thermo-mechanical vibration of rotating axially functionally graded nonlocal Timoshenko beam", *Appl. Phys. A.*, **123**(1), 104. https://doi.org/10.1007/s00339-016-0712-5.

Azimi, M., Mirjavadi, S.S., Shafiei, N., Hamouda, A.M.S. and Davari, E. (2018), "Vibration of rotating functionally graded Timoshenko nano-beams with nonlinear thermal distribution", *Mech. Adv. Mater. Struct.*, **25**(6), 467-480. https://doi.org/10.1080/15376494.2017.1285455.

Bouhadra, A., Menasria, A. and Rachedi Mohamed, A. (2021), "Boundary conditions effect for buckling analysis of porous functionally graded nanobeam", *Adv. Nano Res.*, **10**(4), 313-325. https://doi.org/10.12989/ANR.2021.10.4.313.

Che, H. and Wang, J. (2021), "A two-timescale duplex neurodynamic approach to mixed-integer optimization", *IEEE T. Neural Networ.*, **32**(1), 36-48. https://doi.org/10.1109/TNNLS.2020.2973760.

Chen, H., Miao, Y., Chen, Y., Fang, L., Zeng, L. and Shi, J. (2021), "Intelligent model-based integrity assessment of nonstationary mechanical system", *J. Web Eng.*, **22**(2), 253-280. https://doi.org/10.13052/jwe1540-9589.2022.

Dai, Z., Jiang, Z., Zhang, L. and Habibi, M. (2021a), "Frequency characteristics and sensitivity analysis of a size-dependent laminated nanoshell", *Adv. Nano Res.*, **10**(2), 175. https://doi.org/10.12989/anr.2021.10.2.175.

Dai, Z., Zhang, L., Bolandi, S.Y. and Habibi, M. (2021b), "On the vibrations of the non-polynomial viscoelastic composite open-type shell under residual stresses", *Compos. Struct.*, 113599. https://doi.org/10.1016/j.compstruct.2021.113599.

Du, Y., Zhu, Y., Zhao, C., Liao, F. and Chan, C.Y. (2022), "A novel direct trajectory planning approach based on generative adversarial networks and rapidly-exploring random tree", *IEEE T. Intell. Transp.*, 1-12. https://doi.org/0.1109/TITS.2022.3164391.

Ebrahimi, F., Habibi, M. and Safarpour, H. (2019a), "On modeling of wave propagation in a thermally affected GNP-reinforced imperfect nanocomposite shell", *Eng. Comput.*, **35**(4), 1375-1389. https://doi.org/10.1007/s00366-018-0669-4.

Ebrahimi, F., Hajilak, Z.E., Habibi, M. and Safarpour, H. (2019b), "Buckling and vibration characteristics of a carbon nanotube-reinforced spinning cantilever cylindrical 3D shell conveying viscous fluid flow and carrying spring-mass systems under various temperature distributions", *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, **233**(13), 4590-4605. https://doi.org/10.1177/0954406219832323.

Ebrahimi, F., Hashemabadi, D., Habibi, M. and Safarpour, H. (2020a), "Thermal buckling and forced vibration characteristics of a porous GNP reinforced nanocomposite cylindrical shell", *Microsyst. Technol.*, **26**(2), 461-473. https://doi.org/10.1007/s00542-019-04542-9.

Ebrahimi, F., Mohammadi, K., Barouti, M.M. and Habibi, M. (2019c), "Wave propagation analysis of a spinning porous graphene nanoplatelet-reinforced nanoshell", *Wave. Random Complex Med.*, 1-27. https://doi.org/10.1080/17455030.2019.1694729.

Ebrahimi, F. and Shafiei, N. (2016), "Application of Eringen's nonlocal elasticity theory for vibration analysis of rotating functionally graded nanobeams", *Smart Struct. Syst.*, **17**(5), 837-857. https://doi.org/10.12989/sss.2016.17.5.837.

Ebrahimi, F. and Shafiei, N. (2017), "Influence of initial shear stress on the vibration behavior of single-layered graphene sheets embedded in an elastic medium based on Reddy's higher-order shear deformation plate theory", *Mech. Adv. Mater. Struct.*, **24**(9), 761-772. https://doi.org/10.1080/15376494.2016.1196781.

Ebrahimi, F., Shafiei, N., Kazemi, M. and Mousavi Abdollahi, S.M. (2017), "Thermo-mechanical vibration analysis of rotating nonlocal nanoplates applying generalized differential quadrature method", *Mech. Adv. Mater. Struct.*, **24**(15), 1257-1273. https://doi.org/10.1080/15376494.2016.1227499.

Ebrahimi, F., Supeni, E.E.B., Habibi, M. and Safarpour, H. (2020b), "Frequency characteristics of a GPL-reinforced composite microdisk coupled with a piezoelectric layer", *Eur. Phys. J. Plus*, **135**(2), 144. https://doi.org/10.1140/epjp/s13360-020-00217-x.

Ehyaei, J., Akbarshahi, A. and Shafiei, N. (2017), "Influence of porosity and axial preload on vibration behavior of rotating FG nanobeam", *Adv. Nano Res.*, **5**(2), 141. https://doi.org/10.12989/anr.2017.5.2.141.

Esmailpoor Hajilak, Z., Pourghader, J., Hashemabadi, D., Sharifi Bagh, F., Habibi, M. and Safarpour, H. (2019), "Multilayer GPLRC composite cylindrical nanoshell using modified strain gradient theory", *Mech. Based Des. Struct.*, **47**(5), 521-545. https://doi.org/10.1080/15397734.2019.1566743.

Ghadiri, M., Hosseini, S.H.S. and Shafiei, N. (2016a), "A power series for vibration of a rotating nanobeam with considering thermal effect", *Mech. Adv. Mater. Struct.*, **23**(12), 1414-1420. https://doi.org/10.1080/15376494.2015.1091527.

Ghadiri, M., Mahinzare, M., Shafiei, N. and Ghorbani, K. (2017a), "On size-dependent thermal buckling and free vibration of circular FG Microplates in thermal environments", *Microsyst. Technol.*, **23**(10), 4989-5001. https://doi.org/10.1007/s00542-017-3308-x.

Ghadiri, M. and Shafiei, N. (2016a), "Nonlinear bending vibration of a rotating nanobeam based on nonlocal Eringen's theory using differential quadrature method", *Microsyst. Technol.*, **22**(12), 2853-2867. https://doi.org/10.1007/s00542-015-2662-9.

Ghadiri, M. and Shafiei, N. (2016b), "Vibration analysis of a nano-turbine blade based on Eringen nonlocal elasticity applying the differential quadrature method", *J. Vib. Control*, **23**(19), 3247-3265. https://doi.org/10.1177/1077546315627723.

Ghadiri, M. and Shafiei, N. (2016c), "Vibration analysis of rotating functionally graded Timoshenko microbeam based on modified couple stress theory under different temperature distributions", *Acta Astronaut.*, **121**, 221-240. https://doi.org/10.1016/j.actaastro.2016.01.003.

Ghadiri, M., Shafiei, N. and Akbarshahi, A. (2016b), "Influence of thermal and surface effects on vibration behavior of nonlocal rotating Timoshenko nanobeam", *Appl. Phys. A*. **122**(7), 673. https://doi.org/10.1007/s00339-016-0196-3.

Ghadiri, M., Shafiei, N. and Alavi, H. (2017b), "Thermo-mechanical vibration of orthotropic cantilever and propped cantilever nanoplate using generalized differential quadrature method", *Mech. Adv. Mater. Struct.*, **24**(8), 636-646. https://doi.org/10.1080/15376494.2016.1196770.

Ghadiri, M., Shafiei, N. and Alavi, H. (2017c), "Vibration analysis of a rotating nanoplate using nonlocal elasticity theory", *J. Solid Mech.*, **9**(2), 319-337.

Ghadiri, M., Shafiei, N. and Alireza Mousavi, S. (2016c), "Vibration analysis of a rotating functionally graded tapered microbeam based on the modified couple stress theory by DQEM", *Appl. Phys. A*. **122**(9), 837. https://doi.org/10.1007/s00339-016-0364-5.

Ghadiri, M., Shafiei, N. and Babaei, R. (2017d), "Vibration of a rotary FG plate with consideration of thermal and Coriolis effects", *Steel Compos. Struct.*, **25**(2), 197-207. https://doi.org/10.12989/SCS.2017.25.2.197.

Ghadiri, M., Shafiei, N. and Safarpour, H. (2017e), "Influence of surface effects on vibration behavior of a rotary functionally graded nanobeam based on Eringen's nonlocal elasticity", *Microsyst. Technol.*, **23**(4), 1045-1065. https://doi.org/10.1007/s00542-016-2822-6.

Ghadiri, M., Shafiei, N., Salekdeh, S.H., Mottaghi, P. and Mirzaie, T. (2016d), "Investigation of the dental implant geometry effect on stress distribution at dental implant–bone interface", *J. Brazil. Soc. Mech. Sci. Eng.*, **38**(2), 335-343. https://doi.org/10.1007/s40430-015-0472-8.

Ghazanfari, A., Soleimani, S.S., Keshavarzzadeh, M., Habibi, M., Assempuor, A. and Hashemi, R. (2020), "Prediction of FLD for sheet metal by considering through-thickness shear stresses", *Mech. Based Des. Struct.*, **48**(6), 755-772. https://doi.org/10.1080/15397734.2019.1662310.

Gong, X., Wang, L., Mou, Y., Wang, H., Wei, X., Zheng, W. and Yin, L. (2022), "Improved four-channel PBTDPA control strategy using force feedback bilateral teleoperation system", *Int. J. Control Autom.*, **20**(3), 1002-1017. https://doi.org/10.1007/s12555-021-0096-y.

Gorji Azandariani, M., Gholami, M., Vaziri, E. and Nikzad, A. (2021), "Nonlinear static analysis of a bi-directional functionally graded microbeam based on a nonlinear elastic foundation using modified couple stress theory", *Arab. J. Sci. Eng.*, **46**(12), 12641-12651. https://doi.org/10.1007/s13369-021-06053-0.

Guo, J., Baharvand, A., Tazeddinova, D., Habibi, M., Safarpour, H., Roco-Videla, A. and Selmi, A. (2021), "An intelligent computer method for vibration responses of the spinning multi-layer symmetric nanosystem using multi-physics modeling", *Eng. Comput.*, 1-22. https://doi.org/10.1007/s00366-021-01433-4.

Habibi, M., Hashemi, R., Sadeghi, E., Fazaeli, A., Ghazanfari, A. and Lashini, H. (2016), "Enhancing the mechanical properties and formability of low carbon steel with dual-phase microstructures", *J. Mater. Eng. Perform.*, **25**(2), 382-389. https://doi.org/10.1007/s11665-016-1882-1.

Habibi, M., Ghazanfari, A., Assempour, A., Naghdabadi, R. and Hashemi, R. (2017), "Determination of forming limit diagram using two modified finite element models", *Mech Eng*. **48**(4), 141-144. https://doi.org/10.22060/MEJ.2016.664.

Habibi, M., Hashemi, R., Ghazanfari, A., Naghdabadi, R. and Assempour, A. (2018a), "Forming limit diagrams by including the M–K model in finite element simulation considering the effect of bending", *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, **232**(8), 625-636. https://doi.org/10.1177/1464420716642258.

Habibi, M., Hashemi, R., Tafti, M.F. and Assempour, A. (2018b), "Experimental investigation of mechanical properties, formability and forming limit diagrams for tailor-welded blanks produced by friction stir welding", *J. Manuf. Proc.*, **31**, 310-323. https://doi.org/10.1016/j.jmapro.2017.11.009.

Habibi, M., Hashemabadi, D. and Safarpour, H. (2019a), "Vibration analysis of a high-speed rotating GPLRC nanostructure coupled with a piezoelectric actuator", *Eur. Phys. J. Plus*, **134**(6), 307. https://doi.org/10.1140/epjp/i2019-12742-7.

Habibi, M., Mohammadgholiha, M. and Safarpour, H. (2019b), "Wave propagation characteristics of the electrically GNP-reinforced nanocomposite cylindrical shell", *J. Brazil. Soc. Mech. Sci. Eng.*, **41**(5), 221. https://doi.org/10.1007/s40430-019-1715-x.

Habibi, M., Mohammadi, A., Safarpour, H. and Ghadiri, M. (2019c), "Effect of porosity on buckling and vibrational characteristics of the imperfect GPLRC composite nanoshell", *Mech. Based Des. Struct.*, 1-30. https://doi.org/10.1080/15397734.2019.1701490.

Habibi, M., Mohammadi, A., Safarpour, H., Shavalipour, A. and Ghadiri, M. (2019d), "Wave propagation analysis of the laminated cylindrical nanoshell coupled with a piezoelectric actuator", *Mech. Based Des. Struct.*, 1-19. https://doi.org/10.1080/15397734.2019.1697932.

Habibi, M., Taghdir, A. and Safarpour, H. (2019e), "Stability analysis of an electrically cylindrical nanoshell reinforced with graphene nanoplatelets", *Compos. Part B Eng.*, **175**, 107125. https://doi.org/10.1016/j.compositesb.2019.107125.

Habibi, M., Safarpour, M. and Safarpour, H. (2020), "Vibrational characteristics of a FG-GPLRC viscoelastic thick annular plate using fourth-order Runge-Kutta and GDQ methods", *Mech.*

Based Des. Struct., 1-22. https://doi.org/10.1080/15397734.2020.1779086.

Hadji, L. and Avcar, M. (2021), "Nonlocal free vibration analysis of porous FG nanobeams using hyperbolic shear deformation beam theory", *Adv. Nano Res.*, **10**(3), 281-293. https://doi.org/10.12989/ANR.2021.10.3.281.

Hashemi, H.R., Alizadeh, A.a., Oyarhossein, M.A., Shavalipour, A., Makkiabadi, M. and Habibi, M. (2019), "Influence of imperfection on amplitude and resonance frequency of a reinforcement compositionally graded nanostructure", *Wave Random Complex Med.*, 1-27. https://doi.org/10.1080/17455030.2019.1662968.

Hosseini, S.M.H. and Arvin, H. (2021), "Thermo-rotational buckling and post-buckling analyses of rotating functionally graded microbeams", *Int. J. Mech. Mater. Des.*, **17**(1), 55-72. https://doi.org/10.1007/s10999-020-09509-7.

Hou, F., Wu, S., Moradi, Z. and Shafiei, N. (2021a), "The computational modeling for the static analysis of axially functionally graded micro-cylindrical imperfect beam applying the computer simulation", *Eng. Comput.*, 1-19. https://doi.org/10.1007/s00366-021-01456-x.

Hou, F., Wu, S., Moradi, Z. and Shafiei, N. (2021b), "The computational modeling for the static analysis of axially functionally graded micro-cylindrical imperfect beam applying the computer simulation", *Eng. Comput.*, 1-19. https://doi.org/10.1007/s00366-021-01456-x.

Huang, X., Zhang, Y., Moradi, Z. and Shafiei, N. (2021a), "Computer simulation via a couple of homotopy perturbation methods and the generalized differential quadrature method for nonlinear vibration of functionally graded non-uniform micro-tube", *Eng. Comput.*, 1-18. https://doi.org/10.1007/s00366-021-01395-7.

Huang, X., Zhu, Y., Vafaei, P., Moradi, Z. and Davoudi, M. (2021b), "An iterative simulation algorithm for large oscillation of the applicable 2D-electrical system on a complex nonlinear substrate", *Eng. Comput.*, **38**(4), 3137-3149. https://doi.org/10.1007/s00366-021-01320-y.

Jiang, J., Guo, Y., Huang, Z., Zhang, Y., Wu, D. and Liu, Y. (2022), "Adjacent surface trajectory planning of robot-assisted tooth preparation based on augmented reality", *Eng. Sci. Technol.*, **27**, 101001. https://doi.org/10.1016/j.jestch.2021.05.005.

Jiao, J., Ghoreishi, S.M., Moradi, Z. and Oslub, K. (2021), "Coupled particle swarm optimization method with genetic algorithm for the static–dynamic performance of the magneto-electro-elastic nanosystem", *Eng. Comput.*, 1-15. https://doi.org/10.1007/s00366-021-01391-x.

Khadir Adnan, I., Daikh Ahmed, A. and Eltaher Mohamed, A. (2021), "Novel four-unknowns quasi 3D theory for bending, buckling and free vibration of functionally graded carbon nanotubes reinforced composite laminated nanoplates", *Adv. Nano Res.*, **11**(6), 621-640. https://doi.org/10.12989/ANR.2021.11.6.621.

Kong, H., Lu, L., Yu, J., Chen, Y. and Tang, F. (2021), "Continuous authentication through finger gesture interaction for smart homes using WiFi", *IEEE T. Mobile Comput.*, **20**(11), 3148-3162. https://doi.org/10.1109/TMC.2020.2994955.

Kong, S. (2022), "A review on the size-dependent models of micro-beam and micro-plate based on the modified couple stress theory", *Arch. Comput. Meth. Eng.*, **29**(1), 1-31. https://doi.org/10.1007/s11831-021-09567-w.

Kumar, H. and Mukhopadhyay, S. (2021), "Response of deflection and thermal moment of Timoshenko microbeams considering modified couple stress theory and dual-phase-lag heat conduction model", *Compos. Struct.*, **263**, 113620. https://doi.org/10.1016/j.compstruct.2021.113620.

Kumar, Y., Gupta, A. and Tounsi, A. (2021), "Size-dependent vibration response of porous graded nanostructure with FEM and nonlocal continuum model", *Adv. Nano Res.*, **11**(1), 1-17. https://doi.org/10.12989/ANR.2021.11.1.001.

Li, J., Xu, K., Chaudhuri, S., Yumer, E., Zhang, H. and Guibas, L. (2017), "GRASS: generative recursive autoencoders for shape structures", *ACM T. Graphic.*, **36**(4), 1-14. https://doi.org/10.1145/3072959.3073637.

Li, X., Wang, T., Liu, F. and Zhu, Z. (2021), "Computer simulation of the nonlinear static behavior of axially functionally graded microtube with porosity", *Adv. Nano Res.*, **11**(4), 437-451. https://doi.org/10.12989/anr.2021.11.4.437.

Li, Y., Li, S., Guo, K., Fang, X. and Habibi, M. (2020), "On the modeling of bending responses of graphene-reinforced higher order annular plate via two-dimensional continuum mechanics approach", *Eng. Comput.*, 1-22. https://doi.org/10.1007/s00366-020-01166-w.

Li, Y.S. and Xiao, T. (2021), "Free vibration of the one-dimensional piezoelectric quasicrystal microbeams based on modified couple stress theory", *Appl. Math. Modell.*, **96**, 733-750. https://doi.org/10.1016/j.apm.2021.03.028.

Li, Z., Chen, L., Nie, L. and Yang, S.X. (2022), "A novel learning model of driver fatigue features representation for steering wheel angle", *IEEE T. Veh. Technol.*, **71**(1), 269-281. https://doi.org/10.1109/TVT.2021.3130152.

Lim, C.W., Zhang, G. and Reddy, J.N. (2015), "A higher-order nonlocal elasticity and strain gradient theory and its applications in wave propagation", *J. Mech. Phys. Solids*, **78**, 298-313. https://doi.org/10.1016/j.jmps.2015.02.001.

Liu, L., Xiang, H. and Li, X. (2021a), "A novel perturbation method to reduce the dynamical degradation of digital chaotic maps", *Nonlinear Dynam.*, **103**(1), 1099-1115. https://doi.org/10.1007/s11071-020-06113-4.

Liu, Y., Wang, W., He, T., Moradi, Z. and Larco Benítez, M.A. (2021b), "On the modelling of the vibration behaviors via discrete singular convolution method for a high-order sector annular system", *Eng. Comput.*, 1-23. https://doi.org/10.1007/s00366-021-01454-z.

Liu, Z., Su, S., Xi, D. and Habibi, M. (2020), "Vibrational responses of a MHC viscoelastic thick annular plate in thermal environment using GDQ method", *Mech. Based Des. Struct.*, 1-26. https://doi.org/10.1080/15397734.2020.1784201.

Liu, Z., Wu, S., Jin, S., Liu, Q., Ji, S., Lu, S. and Cheng, L. (2022), "Investigating pose representations and motion contexts modeling for 3d motion prediction", *IEEE T. Pattern Anal.*, 1-1. https://doi.org/10.1109/TPAMI.2021.3139918.

Long, X., Jia, Q., Shen, Z., Liu, M. and Guan, C. (2021), "Strain rate shift for constitutive behaviour of sintered silver nanoparticles under nanoindentation", *Mech. Mater.*, **158**, 103881. https://doi.org/10.1016/j.mechmat.2021.103881.

Luat Doan, T., Thom Do, V., Thanh Tran, T., Minh Phung, V., Ke Tran, V. and Vinh Pham, V. (2021), "Mechanical analysis of bi-functionally graded sandwich nanobeams", *Adv. Nano Res.*, **11**(1), 55-71. https://doi.org/10.12989/ANR.2021.11.1.055.

Lv, Z., Guo, J. and Lv, H. (2022), "Safety Poka Yoke in zero-defect manufacturing based on digital twins", *IEEE T. Ind. Inform.*, 1-1. https://doi.org/10.1109/TII.2021.3139897.

Ma, L., Liu, X. and Moradi, Z. "On the chaotic behavior of graphene-reinforced annular systems under harmonic excitation", *Eng. Comput.*, 1-25. https://doi.org/10.1007/s00366-020-01210-9.

Meng, Q., Lai, X., Yan, Z., Su, C.Y. and Wu, M. (2021), "Motion planning and adaptive neural tracking control of an uncertain two-link rigid-flexible manipulator with vibration amplitude constraint", *IEEE T. Neural Networ.*, 1-15. https://doi.org/10.1109/TNNLS.2021.3054611.

Mirjavadi, S.S., Afshari, B.M., Shafiei, N., Hamouda, A., Kazemi, M. and Structures, C. (2017a), "Thermal vibration of two-dimensional functionally graded (2D-FG) porous Timoshenko nanobeams", *Steel Compos. Struct.*, **25**(4), 415-426. https://doi.org/10.12989/scs.2017.25.4.415.

Mirjavadi, S.S., Matin, A., Shafiei, N., Rabby, S. and Mohasel Afshari, B. (2017b), "Thermal buckling behavior of two-dimensional imperfect functionally graded microscale-tapered porous beam", *J. Therm. Stress.*, **40**(10), 1201-1214. https://doi.org/10.1080/01495739.2017.1332962.

Mirjavadi, S.S., Mohasel Afshari, B., Shafiei, N., Rabby, S. and Kazemi, M. (2017c), "Effect of temperature and porosity on the vibration behavior of two-dimensional functionally graded micro-scale Timoshenko beam", *J. Vib. Control*, **24**(18), 4211-4225. https://doi.org/10.1177/1077546317721871.

Mirjavadi, S.S., Rabby, S., Shafiei, N., Afshari, B.M. and Kazemi, M. (2017d), "On size-dependent free vibration and thermal buckling of axially functionally graded nanobeams in thermal environment", *Appl. Phys. A*. **123**(5), 315. https://doi.org/10.1007/s00339-017-0918-1.

Moayedi, H., Aliakbarlou, H., Jebeli, M., Noormohammadiarani, O., Habibi, M., Safarpour, H. and Foong, L. (2020a), "Thermal buckling responses of a graphene reinforced composite micropanel structure", *Int. J. Appl. Mech.*, **12**(1), 2050010. https://doi.org/10.1142/S1758825120500106.

Moayedi, H., Ebrahimi, F., Habibi, M., Safarpour, H. and Foong, L.K. (2020b), "Application of nonlocal strain–stress gradient theory and GDQEM for thermo-vibration responses of a laminated composite nanoshell", *Eng. Comput.*, 1-16. https://doi.org/10.1007/s00366-020-01002-1.

Moayedi, H., Habibi, M., Safarpour, H., Safarpour, M. and Foong, L. (2019), "Buckling and frequency responses of a graphene nanoplatelet reinforced composite microdisk", *Int. J. Appl. Mech.*, **11**(10), 1950102. https://doi.org/10.1142/S1758825119501023.

Mohammadgholiha, M., Shokrgozar, A., Habibi, M. and Safarpour, H. (2019), "Buckling and frequency analysis of the nonlocal strain–stress gradient shell reinforced with graphene nanoplatelets", *J. Vib. Control*, **25**(19-20), 2627-2640. https://doi.org/10.1177/1077546319863251.

Mohammadi, A., Lashini, H., Habibi, M. and Safarpour, H. (2019), "Influence of viscoelastic foundation on dynamic behaviour of the double walled cylindrical inhomogeneous micro shell using MCST and with the aid of GDQM", *J. Solid Mech.*,. **11**(2), 440-453. https://doi.org/10.22034/JSM.2019.665264.

Moradi, Z., Davoudi, M., Ebrahimi, F. and Ehyaei, A.F. (2021), "Intelligent wave dispersion control of an inhomogeneous micro-shell using a proportional-derivative smart controller", *Waves Random Complex Med.*, 1-24. https://doi.org/10.1080/17455030.2021.1926572.

Moradifard, R., Gholami, M. and Zare, E. (2021), "Nonlinear free vibration analysis of a bi-directional functionally graded microbeam on nonlinear elastic foundation using modified couple stress theory", *Int. J. Comput. Mater. Sci. Eng.*, **10**(1), 2150001. https://doi.org/10.1142/S2047684121500019.

Nejadi Mohamad, M., Mohammadimehr, M. and Mehrabi, M. (2021), "Free vibration and buckling of functionally graded carbon nanotubes / graphene platelets Timoshenko sandwich beam resting on variable elastic foundation", *Adv. Nano Res.*, **10**(6), 539-548. https://doi.org/10.12989/ANR.2021.10.6.539.

Oyarhossein, M.A., Alizadeh, A.a., Habibi, M., Makkiabadi, M., Daman, M., Safarpour, H. and Jung, D.W. (2020), "Dynamic response of the nonlocal strain-stress gradient in laminated polymer composites microtubes", *Sci. Rep.*, **10**(1), 1-19. https://doi.org/10.1038/s41598-020-61855-w.

Pourjabari, A., Hajilak, Z.E., Mohammadi, A., Habibi, M. and Safarpour, H. (2019), "Effect of porosity on free and forced vibration characteristics of the GPL reinforcement composite nanostructures", *Comput. Math. Appl.*, **77**(10), 2608-2626. https://doi.org/10.1016/j.camwa.2018.12.041.

Reddy, J.N. and Chin, C.D. (1998), "Thermomechanical analysis of functionally graded cylinders and plates", *J. Therm. Stress.*, **21**(6), 593-626. https://doi.org/10.1080/01495739808956165.

Safarpour, H., Ghanizadeh, S.A. and Habibi, M. (2018), "Wave propagation characteristics of a cylindrical laminated composite nanoshell in thermal environment based on the nonlocal strain gradient theory", *Eur. Phys. J. Plus*, **133**(12), 532. https://doi.org/10.1140/epjp/i2018-12385-2.

Safarpour, H., Hajilak, Z.E. and Habibi, M. (2019a), "A size-dependent exact theory for thermal buckling, free and forced vibration analysis of temperature dependent FG multilayer GPLRC composite nanostructures restring on elastic foundation", *Int. J. Mech. Mater. Des.*, **15**(3), 569-583. https://doi.org/10.1007/s10999-018-9431-8.

Safarpour, H., Pourghader, J. and Habibi, M. (2019b), "Influence of spring-mass systems on frequency behavior and critical voltage of a high-speed rotating cantilever cylindrical three-dimensional shell coupled with piezoelectric actuator", *J. Vib. Control*, **25**(9), 1543-1557. https://doi.org/10.1177/1077546319828465.

Safarpour, M., Ebrahimi, F., Habibi, M. and Safarpour, H. (2020), "On the nonlinear dynamics of a multi-scale hybrid nano-composite disk", *Eng. Comput.*, 1-20. https://doi.org/10.1007/s00366-020-00949-5.

Shafiei, N., Ghadiri, M. and Mahinzare, M. (2019), "Flapwise bending vibration analysis of rotary tapered functionally graded nanobeam in thermal environment", *Mech. Adv. Mater. Struct.*, **26**(2), 139-155. https://doi.org/10.1080/15376494.2017.1365982.

Shafiei, N., Ghadiri, M., Makvandi, H. and Hosseini, S.A. (2017a), "Vibration analysis of Nano-Rotor's Blade applying Eringen nonlocal elasticity and generalized differential quadrature method", *Appl. Math. Modell.*, **43**, 191-206. https://doi.org/10.1016/j.apm.2016.10.061.

Shafiei, N., Hamisi, M. and Ghadiri, M. (2020), "Vibration analysis of rotary tapered axially functionally graded Timoshenko nanobeam in thermal environment", *J. Solid Mech.*, **12**(1), 16-32.

Shafiei, N. and Kazemi, M. (2017a), "Buckling analysis on the bi-dimensional functionally graded porous tapered nano-/micro-scale beams", *Aerosp. Sci. Technol.*, **66**, 1-11. https://doi.org/10.1016/j.ast.2017.02.019.

Shafiei, N. and Kazemi, M. (2017b), "Nonlinear buckling of functionally graded nano-/micro-scaled porous beams", *Compos. Struct.*, **178**, 483-492. https://doi.org/10.1016/j.compstruct.2017.07.045.

Shafiei, N., Kazemi, M. and Fatahi, L. (2017b), "Transverse vibration of rotary tapered microbeam based on modified couple stress theory and generalized differential quadrature element method", *Mech. Adv. Mater. Struct.*, **24**(3), 240-252. https://doi.org/10.1080/15376494.2015.1128025.

Shafiei, N., Kazemi, M. and Ghadiri, M. (2016a), "Comparison of modeling of the rotating tapered axially functionally graded Timoshenko and Euler–Bernoulli microbeams", *Physica E*, **83**, 74-87. https://doi.org/10.1016/j.physe.2016.04.011.

Shafiei, N., Kazemi, M. and Ghadiri, M. (2016b), "Nonlinear vibration behavior of a rotating nanobeam under thermal stress using Eringen's nonlocal elasticity and DQM", *Appl. Phys. A*. **122**(8), 728. https://doi.org/10.1007/s00339-016-0245-y.

Shafiei, N., Kazemi, M. and Ghadiri, M. (2016c), "Nonlinear vibration of axially functionally graded tapered microbeams", *Int. J. Eng. Sci.*, **102**, 12-26. https://doi.org/10.1016/j.ijengsci.2016.02.007.

Shafiei, N., Kazemi, M. and Ghadiri, M. (2016d), "On size-dependent vibration of rotary axially functionally graded microbeam", *Int. J. Eng. Sci.*, **101**, 29-44. https://doi.org/10.1016/j.ijengsci.2015.12.008.

Shafiei, N., Kazemi, M., Safi, M. and Ghadiri, M. (2016e), “Nonlinear vibration of axially functionally graded non-uniform nanobeams”, *Int. J. Eng. Sci.*, **106**, 77-94. https://doi.org/10.1016/j.ijengsci.2016.05.009.

Shafiei, N., Mirjavadi, S.S., Afshari, B.M., Rabby, S. and Hamouda, A.M.S. (2017c), “Nonlinear thermal buckling of axially functionally graded micro and nanobeams”, *Compos. Struct.*, **168**, 428-439. https://doi.org/10.1016/j.compstruct.2017.02.048.

Shafiei, N., Mirjavadi, S.S., MohaselAfshari, B., Rabby, S. and Kazemi, M. (2017d), “Vibration of two-dimensional imperfect functionally graded (2D-FG) porous nano-/micro-beams”, *Comput. Method Appl. Mech. Eng.*, **322**, 615-632. https://doi.org/10.1016/j.cma.2017.05.007.

Shafiei, N., Mousavi, A. and Ghadiri, M. (2016f), “On size-dependent nonlinear vibration of porous and imperfect functionally graded tapered microbeams”, *Int. J. Eng. Sci.*, **106**, 42-56. https://doi.org/10.1016/j.ijengsci.2016.05.007.

Shafiei, N., Mousavi, A. and Ghadiri, M. (2016g), “Vibration behavior of a rotating non-uniform FG microbeam based on the modified couple stress theory and GDQEM”, *Compos. Struct.*, **149**, 157-169. https://doi.org/10.1016/j.compstruct.2016.04.024.

Shafiei, N. and She, G.L. (2018), “On vibration of functionally graded nano-tubes in the thermal environment”, *Int. J. Eng. Sci.*, **133**, 84-98. https://doi.org/10.1016/j.ijengsci.2018.08.004.

Shanab, R.A. and Attia, M.A. (2021), “On bending, buckling and free vibration analysis of 2D-FG tapered Timoshenko nanobeams based on modified couple stress and surface energy theories”, *Wave Random Complex Med.*, 1-47. https://doi.org/10.1080/17455030.2021.1884770.

Shao, Y., Zhao, Y., Gao, J. and Habibi, M. (2021), “Energy absorption of the strengthened viscoelastic multi-curved composite panel under friction force”, *Arch. Civil Mech. Eng.*, **21**(4), 1-29. https://doi.org/10.1007/s43452-021-00279-3.

Shariati, A., Habibi, M., Tounsi, A., Safarpour, H. and Safa, M. (2020a), “Application of exact continuum size-dependent theory for stability and frequency analysis of a curved cantilevered microtubule by considering viscoelastic properties”, *Eng. Comput.*, 1-20. https://doi.org/10.1007/s00366-020-01024-9.

Shariati, A., Mohammad-Sedighi, H., Żur, K.K., Habibi, M. and Safa, M. (2020b), “On the vibrations and stability of moving viscoelastic axially functionally graded nanobeams”, *Materials*, **13**(7), 1707. https://doi.org/10.3390/ma13071707.

Shen, H.-S. and Xiang, Y. (2021), “Assessment of negative poisson’s ratio effect on thermal post-buckling of FG-GRMMC laminated cylindrical panels”, *Adv. Nano Res.*, **10**(5), 423-435. https://doi.org/10.12989/ANR.2021.10.5.423.

Shivanian, E., Ghadiri, M. and Shafiei, N. (2017), “Influence of size effect on flapwise vibration behavior of rotary microbeam and its analysis through spectral meshless radial point interpolation”, *Appl. Phys. A.* **123**(5), 329. https://doi.org/10.1007/s00339-017-0955-9.

Shokrgozar, A., Safarpour, H. and Habibi, M. (2020), “Influence of system parameters on buckling and frequency analysis of a spinning cantilever cylindrical 3D shell coupled with piezoelectric actuator”, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*. **234**(2), 512-529. https://doi.org/10.1177/0954406219883312.

Tang, Y., Liu, S., Deng, Y., Zhang, Y., Yin, L. and Zheng, W. (2021), “An improved method for soft tissue modeling”, *Biomed. Signal Pr.*, **65**, 102367. https://doi.org/10.1016/j.bspc.2020.102367.

Teng, Z., Wang, W. and Gu, C. (2022), “Free vibration and buckling characteristics of porous functionally graded materials (FGMs) micro-beams based on the modified couple stress theory”, *ZAMM Z. Angew. Math. Me.*, **102**(4), e202100219. https://doi.org/10.1002/zamm.202100219.

Wang, H., Zandi, Y., Gholizadeh, M. and Issakhov, A. (2021), “Buckling of porosity-dependent bi-directional FG nanotube using numerical method”, *Adv. Nano Res.*, **10**(5), 493-507. https://doi.org/10.12989/ANR.2021.10.5.493.

Wang, S., Guo, H., Zhang, S., Barton, D. and Brooks, P. (2022), “Analysis and prediction of double-carriage train wheel wear based on SIMPACK and neural networks”, *Adv. Mech. Eng.*, **14**(3), 16878132221078491. https://doi.org/10.1177/16878132221078491.

Wang, Z., Yu, S., Xiao, Z. and Habibi, M. (2020), “Frequency and buckling responses of a high-speed rotating fiber metal laminated cantilevered microdisk”, *Mech. Adv. Mater. Struct.*, 1-14. https://doi.org/10.1080/15376494.2020.1824284.

Wei, J., Xie, Z., Zhang, W., Luo, X., Yang, Y. and Chen, B. (2021), “Experimental study on circular steel tube-confined reinforced UHPC columns under axial loading”, *Eng. Struct.*, **230**, 111599. https://doi.org/10.1016/j.engstruct.2020.111599.

Wu, J. and Habibi, M. (2021), “Dynamic simulation of the ultra-fast-rotating sandwich cantilever disk via finite element and semi-numerical methods”, *Eng. Comput.*, 1-17. https://doi.org/10.1007/s00366-021-01396-6.

Xu, W., Pan, G., Moradi, Z. and Shafiei, N. (2021a), “Nonlinear forced vibration analysis of functionally graded non-uniform cylindrical microbeams applying the semi-analytical solution”, *Compos. Struct.*, 114395. https://doi.org/10.1016/j.compstruct.2021.114395.

Xu, Y., Zhang, H., Yang, F., Tong, L., Yan, D., Yang, Y., Wang, Y. and Wu, Y. (2021b), “Experimental investigation of pneumatic motor for transport application”, *Renew. Energ.*, **179**, 517-527. https://doi.org/10.1016/j.renene.2021.07.072.

Yu, J., Lu, L., Chen, Y., Zhu, Y. and Kong, L. (2021), “An Indirect Eavesdropping Attack of Keystrokes on Touch Screen through Acoustic Sensing”, *IEEE T. Mobile Comput.*, **20**(2), 337-351. https://doi.org/10.1109/TMC.2019.2947468.

Yu, X., Maalla, A. and Moradi, Z. (2022), “Electroelastic high-order computational continuum strategy for critical voltage and frequency of piezoelectric NEMS via modified multi-physical couple stress theory”, *Mech. Syst. Signal Pr.*, **165**, 108373. https://doi.org/10.1016/j.ymssp.2021.108373.

Zare, R., Najaafi, N., Habibi, M., Ebrahimi, F. and Safarpour, H. (2020), “Influence of imperfection on the smart control frequency characteristics of a cylindrical sensor-actuator GPLRC cylindrical shell using a proportional-derivative smart controller”, *Smart Struct. Syst.*, **26**(4), 469-480. https://doi.org/10.12989/sss.2020.26.4.469.

Zhang, R., Bai, H. and Chen, X. (2022), “The consistent couple stress theory-based vibration and post-buckling analysis of bi-directional functionally graded microbeam”, *Symmetry*, **14**(3). https://doi.org/10.3390/sym14030602.

Zhao, Y., Moradi, Z., Davoudi, M. and Zhuang, J. “Bending and stress responses of the hybrid axisymmetric system via state-space method and 3D-elasticity theory”, *Eng. Comput.*, 1-23. https://doi.org/10.1007/s00366-020-01242-1.

Zheng, C., An, Y., Wang, Z., Qin, X., Eynard, B., Bricogne, M., Le Duigou, J. and Zhang, Y. (2022a), “Knowledge-based engineering approach for defining robotic manufacturing system architectures”, *Int. J. Prod. Res.*, 1-19. https://doi.org/10.1080/00207543.2022.2037025.

Zheng, C., An, Y., Wang, Z., Wu, H., Qin, X., Eynard, B. and Zhang, Y. (2022b), “Hybrid offline programming method for robotic welding systems”, *Robot. Comput. Integrat. Manuf.*, **73**, 102238. https://doi.org/10.1016/j.rcim.2021.102238.

Zheng, W., Cheng, J., Wu, X., Sun, R., Wang, X. and Sun, X. (2022c), “Domain knowledge-based security bug reports prediction”, *Knowl. Based Syst.*, **241**, 108293. https://doi.org/10.1016/j.knosys.2022.108293.

Zheng, W., Shen, T., Chen, X. and Deng, P. (2022d), “Interpretability application of the Just-in-Time software defect prediction model”, *J. Syst. Software*, **188**, 111245. https://doi.org/10.1016/j.jss.2022.111245.

Zhong, C., Li, H., Zhou, Y., Lv, Y., Chen, J. and Li, Y. (2022), “Virtual synchronous generator of PV generation without energy storage for frequency support in autonomous microgrid”, *Int. J. Electr. Power*, **134**, 107343. https://doi.org/10.1016/j.ijepes.2021.107343.

Zhou, C., Zhao, Y., Zhang, J., Fang, Y. and Habibi, M. (2020), “Vibrational characteristics of multi-phase nanocomposite reinforced circular/annular system”, *Adv. Nano Res.*, **9**(4), 295-307. https://doi.org/10.12989/anr.2020.9.4.295.

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Appendix

$$
\begin{aligned}
\delta P = &-\int_0^L \frac{\partial}{\partial x}\left(A\frac{\partial u}{\partial x}\right)dx\,\delta(u) + A\frac{\partial u}{\partial x}\bigg|_0^L \delta(u) + \int_0^L \frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2}\right)dx\,\delta(w) + D\frac{\partial \varphi}{\partial x}\bigg|_0^L \delta\left(\frac{\partial w}{\partial x}\right) - \frac{\partial}{\partial x}\left(D\frac{\partial \varphi}{\partial x}\right)\bigg|_0^L \delta(w) \\
&-\int_0^L \frac{\partial}{\partial x}\left(D\frac{\partial \varphi}{\partial x}\right)dx\,\delta(\varphi) - 2\int_0^L \frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2}\right)dx\,\delta(w) + D\frac{\partial^2 w}{\partial x^2}\bigg|_0^L \delta\left(\frac{\partial w}{\partial x}\right) - \frac{\partial}{\partial x}\left(D\frac{\partial^2 w}{\partial x^2}\right)\bigg|_0^L \delta(w) \\
&-\int_0^L \frac{\partial}{\partial x}\left(D\frac{\partial^2 w}{\partial x^2}\right)dx\,\delta(\varphi) + D\frac{\partial \varphi}{\partial x}\bigg|_0^L \delta(\psi) + D\frac{\partial^2 w}{\partial x^2}\bigg|_0^L \delta(\psi) + \int_0^L \frac{\partial^2}{\partial x^2}\left(D\frac{\partial \varphi}{\partial x}\right)dx\,\delta(w) \\
&+\int_0^L \frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2}\right)dx\,\delta(w) - 2\,D\frac{\partial^2 w}{\partial x^2}\bigg|_0^L \delta\left(\frac{\partial w}{\partial x}\right) + D\frac{\partial^2 w}{\partial x^2}\bigg|_0^L \delta\left(\frac{\partial w}{\partial x}\right) - \frac{\partial}{\partial x}\left(D\frac{\partial^2 w}{\partial x^2}\right)\bigg|_0^L \delta(w) \\
&+\frac{\partial}{\partial x}\left(D\frac{\partial \varphi}{\partial x}\right)\bigg|_0^L \delta(w) + 2\frac{\partial}{\partial x}\left(D\frac{\partial^2 w}{\partial x^2}\right)\bigg|_0^L \delta(w) + \int_0^L \frac{\partial}{\partial x}\left(D\frac{\partial^2 w}{\partial x^2}\right)dx\,\delta(\psi) - D\frac{\partial^2 w}{\partial x^2}\bigg|_0^L \delta(\varphi) \\
&-\int_0^L \frac{\partial^2}{\partial x^2}\left(D\frac{\partial \varphi}{\partial x}\right)dx\,\delta(w) - D\frac{\partial \varphi}{\partial x}\bigg|_0^L \delta\left(\frac{\partial w}{\partial x}\right) - \int_0^L \frac{\partial}{\partial x}\left(C_{10}\frac{\partial w}{\partial x}\right)dx\,\delta(w) + C_{10}\frac{\partial w}{\partial x}\bigg|_0^L \delta(w) \\
&+\int_0^L C_{10}\varphi\delta(\varphi) + C_{10}\frac{\partial w}{\partial x}\delta(\varphi)\,dx - \int_0^L \frac{\partial}{\partial x}\big(C_{10}(\varphi)\big)dx\,\delta(w) + C_{10}\varphi|_0^L\delta(w) \\
&+l^2\left\{\begin{array}{c}
-\displaystyle\int_0^L \frac{\partial}{\partial x}\left[\frac{1}{4}A_{10}\left(\frac{\partial \varphi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)-\frac{1}{2}A_{10}\frac{\partial^2 w}{\partial x^2}\right]dx\,\delta(\varphi) + \left(\frac{1}{4}A_{10}\left(\frac{\partial \varphi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)\right)\bigg|_0^L \delta(\varphi) \\
+\left(A_{10}\left(\dfrac{\partial \varphi}{\partial x}+\dfrac{\partial^2 w}{\partial x^2}\right)+\dfrac{1}{2}A_{10}\dfrac{\partial^2 w}{\partial x^2}\right)\bigg|_0^L \delta\left(\dfrac{\partial w}{\partial x}\right) \\
-\dfrac{\partial}{\partial x}\left[-\dfrac{1}{4}A_{10}\left(\dfrac{\partial \varphi}{\partial x}+\dfrac{\partial^2 w}{\partial x^2}\right)+\left(A_{10}-\dfrac{1}{2}A_{10}\right)\dfrac{\partial^2 w}{\partial x^2}\right]\bigg|_0^L \delta(w) \\
+\displaystyle\int_0^L \frac{\partial^2}{\partial x^2}\left[-\frac{1}{4}A_{10}\left(\frac{\partial \varphi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+\frac{1}{2}A_{10}\frac{\partial^2 w}{\partial x^2}\right]dx\,\delta(w) - \left(\frac{1}{2}A_{10}\frac{\partial^2 w}{\partial x^2}\right)\bigg|_0^L \delta(\varphi)
\end{array}\right\}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&\begin{bmatrix}
\displaystyle\sum_{j=1}^{n} Q_{ij}^{(1)}\left(A\sum_{j=1}^{n} Q_{ij}^{(1)}\right) & 0 & 0 \\
0 & \displaystyle\frac{\partial}{\partial x}\left(C_{10}\sum_{j=1}^{n} Q_{ij}^{(1)}\right)-\frac{1}{4}l^2\frac{\partial^2}{\partial x^2}\left(A_{10}\sum_{j=1}^{n} Q_{ij}^{(2)}\right) & \displaystyle\frac{dC_{10}}{dx}+C_{10}\sum_{j=1}^{n} Q_{ij}^{(1)}+\frac{1}{4}l^2\frac{\partial^2}{\partial x^2}\left(A_{10}\sum_{j=1}^{n} Q_{ij}^{(1)}\right) \\
0 & \displaystyle C_{10}\sum_{j=1}^{n} Q_{ij}^{(1)}-\frac{1}{4}l^2A_{10}\sum_{j=1}^{n} Q_{ij}^{(2)} & \displaystyle C_{10}-\frac{1}{4}l^2A_{10}\sum_{j=1}^{n} Q_{ij}^{(1)}-\frac{\partial}{\partial x}\left(D\sum_{j=1}^{n} Q_{ij}^{(1)}\right)
\end{bmatrix}
\begin{Bmatrix} u \\ w \\ \varphi \end{Bmatrix} \\
&\qquad = F\begin{bmatrix} 0 \\ \displaystyle\sum_{j=1}^{n} Q_{ij}^{(2)} \\ 0 \end{bmatrix}
\end{aligned}
\tag{16}
$$