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Optimization dynamic responses of laminated multiphase shell in thermo-electro-mechanical conditions

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Abstract. The optimization for dynamic response associated with a cylindrical shell which is made of laminated composites embedded in a piezoelectric layer which is subjected to temperature rises and is resting on an elastic foundation is investigated for the first time. The first shear order theory (FSDT) is utilized in order to obtain the strain relations of the shell. Then, using the energy method, the equations of motions as well as boundary condition of the problem are attained. The formulation of this study together with the solution procedure which is a numerical solution method, differential quadrature method (DQM) is validated using other researches. This paper presents a thorough study on the parameters which impacts the vibration frequency of the laminated shell. The results of this paper shows that any type of laminated composite shell can reduce the vibration frequency providing that the angle related to layer are higher than 85 degrees. Also, in order to reduce the effect of temperature rises, the laminated composites instead of orthotropic one can be used.
Keywords: dynamic responses; laminated multiphase shell; optimization; thermo-electro-mechanical conditions

1. Introduction

Because of their extensive mechanical features in enhancing the material properties, laminated composites were the topic of various theoretical along with experimental researches in the past. Accordingly, these materials were employed in different geometries such as beams, plates, panels, and shells (Habibi *et al*. 2016, 2017, 2018a, b, 2019a, c, d, e, f, Safarpour *et al*. 2018a, 2019a, b, 2020, Ebrahimi *et al*. 2019a, 2020a, Esmailpoor Hajilak *et al*. 2019, Pourjabari *et al*. 2019, Alipour *et al*. 2020, Ghazanfari *et al*. 2020, Chen *et al*. 2022). Also, it should be mentioned that these composites have the capability to tolerate more loads in various direction in compassion with simple structures (Ebrahimi *et al*. 2019b, c, 2020b, Hashemi *et al*. 2019, Moayedi *et al*. 2019, Mohammadgholiha *et al*. 2019, Mohammadi *et al*. 2019, Al-Furjan *et al*. 2020c, d, e, f, Bai *et al*. 2020, Cheshmeh *et al*. 2020, Habibi *et al*. 2020, Li *et al*. 2020b, Lori *et al*. 2020, Moayedi *et al*. 2020a, b, Najaafi *et al*. 2020, Oyarhossein *et al*. 2020, Shariati *et al*. 2020a, b, c, Shokrgozar *et al*. 2020, Xiong *et al*. 2020, Guo *et al*. 2021b, Liu *et al*. 2021b).

Regarding this, among vast application of laminated composites in researches (Shariati *et al*. 2012, 2016a, b, 2019, 2020d, e, f, g, h, i, j, 2021a, b), it can refer to the works of Moayedi *et al*. (2021) in which they studied the vibration and buckling due to thermal loads related to a cylindrical shell made of laminated composites. Reddy and Khdeir (1989), explored the vibration as well as buckling related to plates which are made of laminated composites. In this article, they used first and third order plated theory to model the plate and analytical in addition to numerical solution to solve the formulation. The buckling and vibration of a laminated composite plate which is modeled by using the higher shear deformable theory was studied (Matsunaga 2000). By using a finite element model, the impact of heat along with the moistures on the vibration of plates which is made of laminated composites was probed by Ram and Sinha (1992). Also, in the frame work of laminated composites, the vibrational behavior of a micro disk which is placed in a viscoelastic medium, based on modified couple stress was examined (Al-Furjan *et al*. 2021). Additionally, using Donnell's theory, the nonlinear vibration associated with a rotating cylindrical shell whose material is laminated composites was investigated (Li *et al*. 2020a). Zhang (2001), by employing the wave propagation method, managed to present a research on the vibration of laminated composite shell. By employing the higher shear order theory for shells, the nonlinear post-buckling behavior together with vibration analysis corresponded to a panel

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whose material is laminated composites was presented (Panda and Singh 2013). Next, Shua and Du (1997), by utilizing the generalized differential quadrature as the numerical solution, investigated the vibration cylindrical shells which are made of laminated composites. The free vibrational characteristics related to a shell which is curved in two direction was investigated by means of finite element method by Chakravorty *et al*. (1996). By utilizing the Halpin-Tsaia model, the vibrational behavior of a shell made of laminated reinforced composite shell with the aid of graphene was investigated (Shen *et al*. 2018). In the mentioned paper, the shell is subjected to thermal loads. Shen and Yang (2014) presented a research on the nonlinear and linear vibration related to a shell which is made of laminated composites with piezoelectric fibers which are distributed uniformly and in graded fashion. The post buckling vibrational behavior of a laminated composite plates which are under thermal loading and coupled with a piezoelectric layer was studied with the aid of finite element model (Xia and Shen 2008). The vibration characteristics of a panel which made of laminated composites and curved in two direction and subjected to quick temperature gradient was explored by Huang and Tauchert (1992). Additionally, the nonlinear vibration of double-curved panel which is made of reinforced composite with functionally graded dispersed graphene was examined (Shen *et al*. 2019). Panda and Katariya (2015) studied the vibration as well as buckling which is resulted from mechanical and thermal loading of a laminated composite panel by using a finite element model.

The usage of piezoelectric materials, due to their specific characteristic by which they are able to transmit the electrical energy to the mechanical form and vice versa, has raised. Accordingly, these materials—as actuator, sensor, or energy harvester—have been employed in smart structures such as beams, plates (Tiersten 2013), and shells (Tzou 1993). In this regard, Habibi *et al*. (2019b) carried out a study on the vibrational behavior of cylindrical shells which is made of reinforced composites and coupled with a piezoelectric layer. They considered centrifugal forces resulted from spinning of the shell. Also, the vibrational behavior of a shell which is embedded in a piezoelectric layer was investigated based on a refined three-dimensional theory (Safarpour *et al*. 2019b). In the mentioned paper, the shell is rotating and carrying a mass at its end. Additionally, the three-layered plates and shell consisting a core and two piezoelectric layers was molded in order to explore the vibrational control of these systems (Balamurugan and Narayanan 2001). Yue *et al*. (2017) carried out a experimental research on the vibration control of shells which are coupled with eighth piezoelectric patches, four as actuator and four as sensor. The nonlinear vibrational characteristics of a plate made of functionally graded material which is covered by two piezoelectric layers and subjected to the temperature gradient was studied (Huang and Shen 2006). Also, by employing the higher order theory of plates, the nonlinear free and forced vibration related to the sandwich plates including two layers of piezoelectric was probed by Fakhari *et al*. (2011). By incorporation a finite element method, the post buckling in addition to the vibrational behavior associated with a laminated plated coupled with piezoelectric was investigated with considering thermal effects (Oh *et al*. 2000). Buckling, dynamic stability, as well as vibration analysis related to a beam made of three layer, two of which is made of piezoelectric material was carried out by using Euler Bernoulli beam theory and considering the temperature rises (Fu *et al*. 2012). Raja *et al*. (2004), by utilizing a finite element method, presented a paper on the vibrational control of shells along with plates made of composite material which is coupled with piezoelectric material and subjected to thermal loads. A shell which is coupled with two piezoelectric layers as sensor in addition to actuator was modeled by using finite element model in order to investigated its active vibrational control (Kumar *et al*. 2008). The vibrational behavior related to a cylindrical shell made of reinforced composites with help of carbon nanotubes which is coupled with piezoelectric material and laced in a thermal environment was solved with DQM (Safarpour *et al*. 2018b). By employing Donnell nonlinear theory, Liu *et al*. (2021d) managed to present an article on the forced vibration of FG shells made of piezoelectric materials which is under thermal, mechanical, and electric loading.

In this paper, the thermoelectrical vibrational behavior of laminated composite cylindrical shells which is coved by a piezoelectric layer is optimized. The formulation of the system is attained by energy method, aka Hamilton's principle. The formulation is solved by GDQM numerically. The solution method in addition to the formulation is validated using other literatures. Then, the impact of various parameters on the vibrational characteristic of the hell is investigated.

2. Problem formulation

Now, in the current segment of the paper the mathematical modeling of a cylindrical shell which is made of laminated composites and covered by a layer of piezoelectric material is presented. The schematic of the shell can be seen in Fig. 1.

Based on first shear order theory, the displacement field related to the shell is as follow.

$$\begin{aligned} u_1(x,\theta,z,t) &= U(x,\theta,t) + z\psi_x(x,\theta,t) \\ u_2(x,\theta,z,t) &= V(x,\theta,t) + z\psi_\theta(x,\theta,t) \\ u_3(x,\theta,z,t) &= W(x,\theta,t) \end{aligned} \tag{1}$$

And based on these displacement fields (Bai *et al*. 2021, Jiang *et al*. 2021, Li *et al*. 2021a, b, Peng *et al*. 2021, Xu *et al*. 2021b), the nonzero strains in angular, axial, and lateral direction can be written as follow.

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial U(x,\theta,t)}{\partial x} + z\frac{\partial \psi_x(x,\theta,t)}{\partial x} \\ \varepsilon_{\theta\theta} &= \frac{1}{R}\left(\frac{\partial V(x,\theta,t)}{\partial \theta} + W(x,\theta,t) + z\frac{\partial \psi_\theta(x,\theta,t)}{\partial \theta}\right) \\ \varepsilon_{x\theta} &= \frac{1}{2}\left(\frac{\partial U(x,\theta,t)}{R\partial \theta} + \frac{\partial V(x,\theta,t)}{\partial x} + \frac{z}{R}\frac{\partial \psi_x(x,\theta,t)}{\partial \theta} \right. \\ &\qquad \left. + z\frac{\partial \psi_\theta(x,\theta,t)}{\partial x}\right) \end{aligned} \tag{2}$$

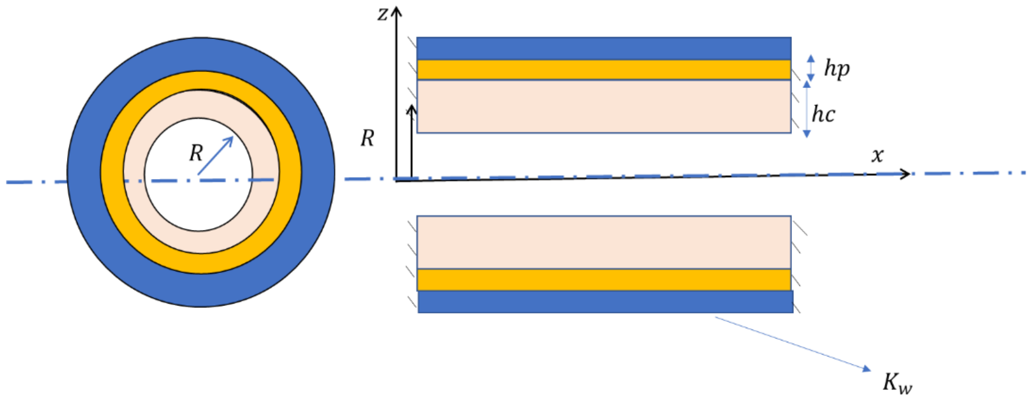

Fig. 1 Schematic of a cylindrical shell made of laminated composites which is embedded in a piezoelectric layer and resting on a elastic foundation

$$\varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial W(x,\theta,t)}{\partial x}+\psi_x(x,\theta,t)\right)$$
$$\varepsilon_{z\theta}=\frac{1}{2}\left(\frac{\partial W(x,\theta,t)}{R\partial\theta}-\frac{V(x,\theta,t)}{R}+\psi_\theta(x,\theta,t)\right) \tag{2}$$

Here, the stress forces of the laminated composite core under thermal load can be attained.

$$\begin{Bmatrix}\sigma^c{}_{xx}\\ \sigma^c{}_{\theta\theta}\\ \sigma^c{}_{z\theta}\\ \sigma^c{}_{xz}\\ \sigma^c{}_{x\theta}\end{Bmatrix}=\begin{pmatrix}\bar{Q}^k{}_{11} & \bar{Q}^k{}_{12} & 0 & 0 & \bar{Q}^k{}_{16}\\ \bar{Q}^k{}_{21} & \bar{Q}^k{}_{22} & 0 & 0 & \bar{Q}^k{}_{26}\\ 0 & 0 & \bar{Q}^k{}_{44} & \bar{Q}^k{}_{45} & 0\\ 0 & 0 & \bar{Q}^k{}_{54} & \bar{Q}^k{}_{55} & 0\\ \bar{Q}^k{}_{61} & \bar{Q}^k{}_{62} & 0 & 0 & \bar{Q}^k{}_{66}\end{pmatrix}$$
$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{z\theta}\\ \varepsilon_{xz}\\ \varepsilon_{x\theta}\end{Bmatrix}-\begin{Bmatrix}\alpha^c{}_{xx}\\ a^c{}_{\theta\theta}\\ 0\\ 0\\ 0\end{Bmatrix}\Delta T \tag{3}$$

In which the transformation matrix related to the stiffness of the shell is

$$\begin{pmatrix}\bar{Q}^k{}_{11} & \bar{Q}^k{}_{12} & 0 & 0 & \bar{Q}^k{}_{16}\\ \bar{Q}^k{}_{21} & \bar{Q}^k{}_{22} & 0 & 0 & \bar{Q}^k{}_{26}\\ 0 & 0 & \bar{Q}^k{}_{44} & \bar{Q}^k{}_{45} & 0\\ 0 & 0 & \bar{Q}^k{}_{54} & \bar{Q}^k{}_{55} & 0\\ \bar{Q}^k{}_{61} & \bar{Q}^k{}_{62} & 0 & 0 & \bar{Q}^k{}_{66}\end{pmatrix}$$
$$=T\begin{pmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{21} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{66}\end{pmatrix}T^T \tag{4}$$

where, T is the transformation matrix and can be written as

$$T=\begin{pmatrix}m^2 & n^2 & 0 & 0 & -2mn\\ n^2 & m^2 & 0 & 0 & 2mn\\ 0 & 0 & m & n & 0\\ 0 & 0 & -n & m & 0\\ mn & -mn & 0 & 0 & m^2-n^2\end{pmatrix} \tag{5}$$

In which $n=Sin(\beta)$ and $m=Cos(\beta)$. Also, the stiffness modules of an orthotropic material are (Adamian *et al*. 2020, Al-Furjan *et al*. 2020a, b, Li *et al*. 2020c, Liu *et al*. 2020a, b, 2021c, Wang *et al*. 2020, Zare *et al*. 2020, Zhou *et al*. 2020, Dai *et al*. 2021a, b, Guo *et al*. 2021a, Habibi *et al*. 2021, He *et al*. 2021b, Huang *et al*. 2021a, Shao *et al*. 2021, Wu and Habibi 2021, Zhang *et al*. 2021):

$$Q_{11}=\frac{E_1}{1-\mu_{12}\mu_{21}},Q_{12}=Q_{21}=\frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}},$$
$$Q_{22}=\frac{E_2}{1-\mu_{12}\mu_{21}},Q_{44}=G_{23},$$
$$Q_{55}=G_{13},Q_{66}=G_{12} \tag{6}$$

Additionally, the stress forces (He *et al*. 2021a, Liu *et al*. 2021a, Liu *et al*. 2022, Wang *et al*. 2022, Zhong *et al*. 2022) related to the piezoelectric layer can be written as follow.

$$\begin{Bmatrix}\sigma^p{}_{xx}\\ \sigma^p{}_{\theta\theta}\\ \sigma^p{}_{x\theta}\\ \sigma^p{}_{z\theta}\\ \sigma^p{}_{xz}\end{Bmatrix}=\begin{pmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{21} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{66} & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & C_{55}\end{pmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{x\theta}\\ \varepsilon_{z\theta}\\ \varepsilon_{xz}\end{Bmatrix}$$
$$-\begin{Bmatrix}\alpha^p{}_{xx}\\ a^p{}_{\theta\theta}\\ 0\\ 0\\ 0\end{Bmatrix}\Delta T-\begin{pmatrix}0 & 0 & e_{31}\\ 0 & 0 & e_{32}\\ 0 & 0 & 0\\ e_{15} & 0 & 0\\ 0 & e_{15} & 0\end{pmatrix}\begin{Bmatrix}E_x\\ E_\theta\\ E_z\end{Bmatrix} \tag{7}$$

Here, the electrical displacement in three directions is

$$\begin{Bmatrix}D_{xx}\\ D_{\theta\theta}\\ D_{zz}\end{Bmatrix}=\begin{pmatrix}0 & 0 & 0 & e_{15} & 0\\ 0 & 0 & 0 & 0 & e_{15}\\ e_{31} & e_{32} & 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{x\theta}\\ \varepsilon_{xz}\\ \varepsilon_{z\theta}\end{Bmatrix}-\begin{Bmatrix}P_1\\ P_2\\ P_3\end{Bmatrix}\Delta T$$
$$+\begin{pmatrix}\kappa_{11} & 0 & 0\\ 0 & \kappa_{22} & 0\\ 0 & 0 & \kappa_{33}\end{pmatrix}\begin{Bmatrix}E_x\\ E_\theta\\ E_z\end{Bmatrix} \tag{8}$$

In which the coefficients for the cylindrical shell made of piezoelectric material are as follow

$$C_{11}=\bar{C}_{11}-\frac{(\bar{C}_{13})^2}{\bar{C}_{33}},C_{12}=\bar{C}_{12}-\frac{\bar{C}_{13}\bar{C}_{23}}{\bar{C}_{33}}, \tag{9}$$

$$
\begin{aligned}
&C_{22} = \bar{C}_{22} - \frac{(\bar{C}_{23})^2}{\bar{C}_{33}} C_{44} = \bar{C}_{44},\\
&C_{55} = \bar{C}_{55}, C_{66} = \bar{C}_{66}\\
&e_{31} = \bar{e}_{31} - \frac{\bar{C}_{13} e_{33}}{\bar{C}_{33}},\\
&e_{32} = \bar{e}_{32} - \frac{\bar{C}_{23} e_{33}}{\bar{C}_{33}}, e_{15} = \bar{e}_{15}\\
&\kappa_{11} = \bar{\kappa}_{11}, \kappa_{22} = \bar{\kappa}_{22},\\
&\kappa_{33} = \bar{\kappa}_{33} + \frac{{e_{33}}^2}{\bar{C}_{33}}
\end{aligned}
$$

Also, the electrical potential can be introduced as follow

$$
\Phi(x,\theta,z,t) = -\cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\phi(x,\theta,t) + \frac{2V_0\left(z-\frac{h_c}{2}\right)}{h_p} \tag{10}
$$

And now based on the electrical potential, the electric field in axial, lateral, as well as angular related to a piezoelectric cylindrical shell can be written as follow:

$$
E_x = -\frac{\partial\Phi}{\partial x}, \qquad E_z = -\frac{\partial\Phi}{\partial z}, \qquad E_\theta = -\frac{1}{R+z}\frac{\partial\Phi}{\partial\theta} \tag{11}
$$

Now, the strain energy (Ma *et al*. 2020, Zhao *et al*. 2020, Hou *et al*. 2021, Huang *et al*. 2021b, c, Jiao *et al*. 2021, Liu *et al*. 2021e, Moradi *et al*. 2021, Xu *et al*. 2021a, Dong *et al*. 2022, Luo *et al*. 2022, Yang *et al*. 2022, Yu *et al*. 2022) associated with the cylindrical shell made of laminated composites and covered by a piezoelectric layer is as follow.

$$
\begin{aligned}
U &= \frac{1}{2}\int\int\int_{\frac{-h_c}{2}}^{\frac{h_c}{2}}\begin{pmatrix}\varepsilon_{xx}{\sigma^c}_{xx} + \varepsilon_{\theta\theta}{\sigma^c}_{\theta\theta}\\ +\varepsilon_{x\theta}{\sigma^c}_{x\theta}\\ +\varepsilon_{z\theta}{\sigma^c}_{z\theta} + \varepsilon_{xz}{\sigma^c}_{xz}\end{pmatrix} R\,d\theta dxdz\\
&+ \frac{1}{2}\int\int\int_{\frac{h_c}{2}}^{\frac{h_c}{2+h_p}}\begin{pmatrix}\varepsilon_{xx}{\sigma^p}_{xx} + \varepsilon_{\theta\theta}{\sigma^p}_{\theta\theta}\\ +\varepsilon_{x\theta}{\sigma^p}_{x\theta}\\ +\varepsilon_{z\theta}{\sigma^p}_{z\theta}\\ +\varepsilon_{xz}{\sigma^p}_{xz}\\ -D_{zz}E_z\\ -D_{xx}E_x - D_{\theta\theta}E_\theta\end{pmatrix} R\,d\theta dxdz
\end{aligned} \tag{12}
$$

Also, the kinetic energy (Lu *et al*. 2021, Wang *et al*. 2021, Zhou *et al*. 2021, Huang *et al*. 2022, Zhang *et al*. 2022) of the system is

$$
\Pi = \frac{1}{2}\iiint \rho\left(\frac{\partial u_1(x,\theta,z,t)}{\partial t} + \frac{\partial u_2(x,\theta,z,t)}{\partial t} + \frac{\partial u_3(x,\theta,z,t)}{\partial t}\right)^2 Rdzd\theta dx \tag{13}
$$

Here, the variation of the two strains as well as kinetic energy of the system can be written as

$$
\begin{aligned}
U &= \frac{1}{2}\int\int\int_{\frac{-h_c}{2}}^{\frac{h_c}{2}}\begin{pmatrix}
N_{xx}\delta\frac{\partial U(x,\theta,t)}{\partial x} + M_{xx}\delta\frac{\partial\psi_x(x,\theta,t)}{\partial x} +\\
\frac{1}{R}\begin{pmatrix}N_{\theta\theta}\begin{pmatrix}\delta\frac{\partial V(x,\theta,t)}{\partial x}\\ +\delta W(x,\theta,t)\end{pmatrix}\\ +M_{\theta\theta}\delta\frac{\partial\psi_\theta(x,\theta,t)}{\partial x}\end{pmatrix}\\
+\frac{1}{2}\begin{pmatrix}N_{x\theta}\begin{pmatrix}\delta\frac{\partial V(x,\theta,t)}{\partial x}\\ +\delta\frac{\partial U(x,\theta,t)}{R\partial\theta}\end{pmatrix}\\ +M_{x\theta}\begin{pmatrix}\delta\frac{1}{R}\frac{\partial\psi_x(x,\theta,t)}{\partial\theta}\\ +\delta\frac{\partial\psi_\theta(x,\theta,t)}{\partial x}\end{pmatrix}\end{pmatrix}\\
+\frac{1}{2}Q_{z\theta}\begin{pmatrix}\delta\frac{\partial W(x,\theta,t)}{R\partial\theta}\\ -\delta\frac{V(x,\theta,t)}{R}\\ +\delta\psi_\theta(x,\theta,t)\end{pmatrix}\\
+\frac{1}{2}Q_{xz}\begin{pmatrix}\delta\frac{\partial W(x,\theta,t)}{\partial x}\\ +\delta\psi_x(x,\theta,t)\end{pmatrix}
\end{pmatrix} R\,d\theta dxdz\\
&+\frac{1}{2}\int\int\int_{\frac{h_c}{2}}^{\frac{h_c}{2+h_p}}\begin{pmatrix}
+D_{zz}\frac{\pi}{h_p}Sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\delta\phi(x,\theta,t)\\
-D_{xx}\cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\delta\frac{\partial\phi(x,\theta,t)}{\partial x}\\
-D_{\theta\theta}\cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\delta\frac{\partial\phi(x,\theta,t)}{\partial\theta}
\end{pmatrix} R\,d\theta dxdz
\end{aligned} \tag{14}
$$

$$
\delta\Pi = \frac{1}{2}\iint\begin{pmatrix}
\begin{pmatrix}I_0\frac{\partial^2 U(x,\theta,t)}{\partial t^2}\\ +I_1\frac{\partial^2\psi_x(x,\theta,t)}{\partial t^2}\end{pmatrix}\delta U(x,\theta,t) +\\
\begin{pmatrix}I_0\frac{\partial^2 V(x,\theta,t)}{\partial t^2}\\ +I_1\frac{\partial^2\psi_\theta(x,\theta,t)}{\partial t^2}\end{pmatrix}\delta V(x,\theta,t) +\\
\begin{pmatrix}I_1\frac{\partial^2 U(x,\theta,t)}{\partial t^2}\\ +I_2\frac{\partial^2\psi_x(x,\theta,t)}{\partial t^2}\end{pmatrix}\delta\psi_x(x,\theta,t) +\\
\begin{pmatrix}I_1\frac{\partial^2 V(x,\theta,t)}{\partial t^2}\\ +I_2\frac{\partial^2\psi_\theta(x,\theta,t)}{\partial t^2}\end{pmatrix}\delta\psi_\theta(x,\theta,t) +\\
I_0\frac{\partial^2 W(x,\theta,t)}{\partial t^2}\delta W(x,\theta,t)
\end{pmatrix} Rd\theta dx \tag{15}
$$

In which

$$
\{I_0, I_1, I_2\} = \int_{-h_c/2}^{h_c/2}\rho_c\{1,z,z^2\}dz + \int_{h_c/2}^{h_c/2+h_p}\rho_p\{1,z,z^2\}dz \tag{16}
$$

Also, the external work done by the thermal loads as well as foundation can be written.

$$
\begin{aligned}
\delta W_{ext} &= (\bar{N}^x)\frac{\partial W(x,\theta,t)}{\partial x}\delta\frac{\partial W(x,\theta,t)}{\partial x}\\
&+(\bar{N}^\theta)\frac{\partial W(x,\theta,t)}{\partial\theta}\delta\frac{\partial W(x,\theta,t)}{\partial\theta}\\
&- K_w W(x,\theta,t)\delta W(x,\theta,t)
\end{aligned} \tag{17}
$$

In which

$$\begin{aligned}\bar{N}^x &= -\left(\alpha^p{}_{xx}h_p + \sum_{k=1}^{nl}\alpha^k{}_{xx}(z_{k+1}-z_k)\right)\Delta T + 2e_{31}V \\ \bar{N}^\theta &= -\left(\alpha^p{}_{\theta\theta}h_p + \sum_{k=1}^{nl}\alpha^k{}_{\theta\theta}(z_{k+1}-z_k)\right)\Delta T + 2e_{31}V\end{aligned} \tag{18}$$

where

$$\begin{pmatrix}\alpha^k{}_{xx} & 0\\ 0 & \alpha^k{}_{\theta\theta}\end{pmatrix} = \begin{pmatrix}m & n\\ -n & m\end{pmatrix}\begin{pmatrix}\alpha^c{}_{xx} & 0\\ 0 & \alpha^c{}_{\theta\theta}\end{pmatrix}\begin{pmatrix}m & n\\ -n & m\end{pmatrix}^T \tag{19}$$

Now, by employing the Hamilton's principle, Eq. (20)

$$\int_{t_0}^{t_1}\delta(U-\Pi+W_{ext})\,dt = 0 \tag{20}$$

Now, after mathematical manipulation and by setting the coefficient of the variation of the independent variables, the governing equations as well as boundary conditions related to a laminated composite shell covered by a piezoelectric layer can be obtained as follow.

$$\partial U{:}\ \frac{\partial N_{xx}}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial\theta} = I_0\frac{\partial^2 U(x,\theta,t)}{\partial t^2} + I_1\frac{\partial^2\psi_x(x,\theta,t)}{\partial t^2} \tag{21}$$

$$\partial V{:}\ \frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial\theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}Q_{z\theta} = I_0\frac{\partial^2 V(x,\theta,t)}{\partial t^2} + I_1\frac{\partial^2\psi_\theta(x,\theta,t)}{\partial t^2} \tag{22}$$

$$\partial W{:}\ -\frac{1}{R}N_{\theta\theta} + \frac{\partial Q_{xz}}{\partial x} + \frac{1}{R}\frac{\partial Q_{z\theta}}{\partial\theta} + \bar{N}^\theta\frac{\partial^2 W(x,\theta,t)}{\partial\theta^2} + \bar{N}^x\frac{\partial^2 W(x,\theta,t)}{\partial x^2} - K_w W(x,\theta,t) = I_0\frac{\partial^2 W(x,\theta,t)}{\partial t^2} \tag{23}$$

$$\partial\psi_x{:}\ \frac{\partial M_{xx}}{\partial x} + \frac{1}{R}\frac{\partial M_{x\theta}}{\partial\theta} - Q_{xz} = I_1\frac{\partial^2 U(x,\theta,t)}{\partial t^2} + I_2\frac{\partial^2\psi_x(x,\theta,t)}{\partial t^2} \tag{24}$$

$$\partial\psi_\theta{:}\ \frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial M_{\theta\theta}}{\partial\theta} - Q_{z\theta} = I_1\frac{\partial^2 V(x,\theta,t)}{\partial t^2} + I_2\frac{\partial^2\psi_\theta(x,\theta,t)}{\partial t^2} \tag{25}$$

$$\partial\phi{:}\ \frac{1}{2}\int\int\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p}\left(\begin{array}{c} +D_{zz}\frac{\pi}{h_p}Sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\\ -\frac{\partial D_{xx}}{\partial x}cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\\ -\frac{\partial D_{\theta\theta}}{\partial\theta}\frac{1}{R+z}cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\end{array}\right)R\,d\theta dxdz \tag{26}$$

In which

$$\begin{aligned}\{N_{xx},N_{\theta\theta},N_{x\theta}\} &= \left(\int_{-h_c/2}^{h_c/2}\{\sigma^c_{xx},\sigma^c_{\theta\theta},\sigma^c_{x\theta}\}dz + \int_{h_c/2}^{h_c/2+h_p}\{\sigma^p_{xx},\sigma^p_{\theta\theta},\sigma^p_{x\theta}\}dz\right)\\ \{M_{xx},M_{\theta\theta},M_{x\theta}\} &= \left(\int_{-h_c/2}^{h_c/2}z\{\sigma^c_{xx},\sigma^c_{\theta\theta},\sigma^c_{x\theta}\}dz + \int_{h_c/2}^{h_c/2+h_p}z\{\sigma^p_{xx},\sigma^p_{\theta\theta},\sigma^p_{x\theta}\}dz\right)\\ \{Q_{xz},Q_{z\theta}\} &= \left(\int_{-h_c/2}^{h_c/2}\{\sigma^c_{xz},\sigma^c_{z\theta}\}dz + \int_{h_c/2}^{h_c/2+h_p}\{\sigma^p_{xz},\sigma^p_{z\theta}\}dz\right)\end{aligned} \tag{27}$$

The resultant forces can be rewritten as follows

$$\begin{Bmatrix}N_{xx}\\N_{\theta\theta}\\N_{x\theta}\\M_{xx}\\M_{\theta\theta}\\M_{x\theta}\\Q_{z\theta}\\Q_{zx}\end{Bmatrix} = \begin{bmatrix}A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & 0 & 0\\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & 0 & 0\\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0\\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & 0 & 0\\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & 0 & 0\\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{55}\end{bmatrix}\begin{Bmatrix}\frac{\partial U}{\partial x}\\ \frac{1}{R}\left(\frac{\partial V}{\partial\theta}+W\right)\\ \frac{1}{R}\frac{\partial U}{\partial\theta}+\frac{\partial V}{\partial x}\\ \frac{\partial\psi_x}{\partial x}\\ \frac{1}{R}\frac{\partial\psi_\theta}{\partial\theta}\\ \frac{1}{R}\frac{\partial\psi_x}{\partial\theta}+\frac{\partial\psi_\theta}{\partial x}\\ \psi_\theta+\frac{1}{R}\frac{\partial W}{\partial\theta}-\frac{V}{R}\\ \psi_x+\frac{\partial W}{\partial x}\end{Bmatrix} + \begin{bmatrix}0 & 0 & F_{31}\\ 0 & 0 & F_{32}\\ 0 & 0 & 0\\ 0 & 0 & G_{31}\\ 0 & 0 & G_{32}\\ 0 & 0 & 0\\ 0 & -F_{25} & 0\\ -F_{15} & 0 & 0\end{bmatrix}\begin{Bmatrix}\phi\\ \phi\\ \frac{\partial\phi}{\partial z}\end{Bmatrix} \tag{28}$$

In which

$$\begin{aligned}\{A_{ij},B_{ij},D_{ij}\} &= \sum_{k=1}^{nl}\bar{Q}^k{}_{ij}\begin{Bmatrix}(z_{k+1}-z_k),\\ \frac{({z_{k+1}}^2-{z_k}^2)}{2},\frac{({z_{k+1}}^3-{z_k}^3)}{3}\end{Bmatrix} + \int_{h_c/2}^{h_c/2+h_p}C_{ij}\{1,z,z^2\}dz\\ \{F_{31},G_{31}\} &= \int_{h_c/2}^{h_c/2+h_p}e_{31}\frac{\pi}{h_p}Sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\{1,z\}dz\end{aligned} \tag{29}$$

$$F_{32}=\int_{h_c/2}^{h_c/2+h_p} e_{32}\frac{\pi}{h_p}Sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)dz$$
$$F_{15}=\int_{h_c/2}^{h_c/2+h_p} e_{15}\,cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)dz$$
$$F_{25}=\int_{h_c/2}^{h_c/2+h_p} \frac{1}{R+z}e_{15}\,cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)dz$$

Additionally, the electrical displacement using Eq. (29) can be written as

$$\begin{aligned}
&\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} D_{xx}\,cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)dz\\
&=F_{15}\left(\frac{\partial W(x,\theta,t)}{\partial x}+\psi_x(x,\theta,t)\right)-\bar{P}_1\Delta T+X_{11}\frac{\partial\phi(x,\theta,t)}{\partial x}\\
&\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} D_{\theta\theta}\frac{1}{R+z}cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)dz\\
&=F_{25}\left(\frac{\partial W(x,\theta,t)}{R\partial\theta}-\frac{V(x,\theta,t)}{R}+\psi_\theta(x,\theta,t)\right)\\
&-\bar{P}_2\Delta T+X_{22}\frac{\partial\phi(x,\theta,t)}{\partial\theta}\\
&\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} D_{zz}\frac{\pi}{h_p}Sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)dz\\
&=F_{31}\left(\frac{\partial U(x,\theta,t)}{\partial x}+\frac{1}{R}\left(\frac{\partial V(x,\theta,t)}{\partial\theta}+W(x,\theta,t)\right)\right)+\\
&G_{31}\left(\frac{\partial\psi_x(x,\theta,t)}{\partial x}+\frac{\partial\psi_\theta(x,\theta,t)}{R\partial\theta}\right)-\bar{P}_3\Delta T-X_{33}\phi(x,\theta,t)
\end{aligned}\tag{30}$$

In which

$$\begin{aligned}
&\bar{P}_1=\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} P_1\left(\frac{\pi}{h_p}Sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\right)dz\ \bar{P}_2\\
&=\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} P_2\,cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)dz\\
&\bar{P}_3=\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} P_3\left(\frac{1}{R+z}cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\right)dz\ X_{11}\\
&=\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} \kappa_{11}\left(\frac{\pi}{h_p}Sin\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\right)^2dz\\
&X_{22}=\int_{\frac{h_c}{2}}^{\frac{h_c}{2}+h_p} \kappa_{22}\left(cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\right)^2dz\ X_{33}\\
&=\int_{h_c/2}^{h_c/2+h_p} \kappa_{33}\left(\frac{1}{R+z}cos\left(\frac{\pi\left(z-\frac{h_c}{2}\right)}{h_p}\right)\right)^2dz
\end{aligned}\tag{31}$$

And the boundary conditions are

$$\begin{aligned}
&Simply-supported: N_{xx}=0, V=0, W=0, M_{xx}=0, \psi_x=0\\
&Clamped: U=0, V=0, W=0, \psi_\theta=0, \psi_x=0
\end{aligned}\tag{32}$$

Therefore, the equations of motions can be presented as follow.

$$\begin{aligned}
&\frac{\partial}{\partial x}\begin{pmatrix}A_{11}\frac{\partial U}{\partial x}+A_{12}\frac{1}{R}\left(\frac{\partial V}{\partial\theta}+W\right)\\+B_{11}\frac{\partial\psi_x}{\partial x}+B_{12}\frac{1}{R}\frac{\partial\psi_\theta}{\partial\theta}+F_{31}\frac{\partial\phi}{\partial z}\end{pmatrix}\\
&+\frac{1}{R}\frac{\partial}{\partial\theta}\left(A_{66}\left(\frac{1}{R}\frac{\partial U}{\partial\theta}+\frac{\partial V}{\partial x}\right)+B_{66}\left(\frac{1}{R}\frac{\partial\psi_x}{\partial\theta}+\frac{\partial\psi_\theta}{\partial x}\right)\right)\\
&=I_0\frac{\partial^2U(x,\theta,t)}{\partial t^2}+I_1\frac{\partial^2\psi_x(x,\theta,t)}{\partial t^2}
\end{aligned}\tag{33}$$

$$\begin{aligned}
&\frac{1}{R}\frac{\partial}{\partial\theta}\begin{pmatrix}\frac{A_{22}}{R}\left(\frac{\partial V}{\partial\theta}+W\right)+A_{12}\frac{\partial U}{\partial x}\\+B_{12}\frac{\partial\psi_x}{\partial x}+B_{22}\frac{1}{R}\frac{\partial\psi_\theta}{\partial\theta}+F_{32}\frac{\partial\phi}{\partial z}\end{pmatrix}\\
&+\frac{\partial}{\partial x}\left(A_{66}\left(\frac{1}{R}\frac{\partial U}{\partial\theta}+\frac{\partial V}{\partial x}\right)+B_{66}\left(\frac{1}{R}\frac{\partial\psi_x}{\partial\theta}+\frac{\partial\psi_\theta}{\partial x}\right)\right)\\
&+\frac{A_{44}}{R}\left(\frac{1}{R}\frac{\partial\psi_x}{\partial\theta}+\frac{\partial\psi_\theta}{\partial x}\right)-\frac{1}{R}F_{25}\phi\\
&=I_0\frac{\partial^2V(x,\theta,t)}{\partial t^2}+I_1\frac{\partial^2\psi_\theta(x,\theta,t)}{\partial t^2}
\end{aligned}\tag{34}$$

$$\begin{aligned}
&-\frac{1}{R}\begin{pmatrix}\frac{A_{22}}{R}\left(\frac{\partial V}{\partial\theta}+W\right)+A_{12}\frac{\partial U}{\partial x}\\+B_{12}\frac{\partial\psi_x}{\partial x}+B_{22}\frac{1}{R}\frac{\partial\psi_\theta}{\partial\theta}+F_{32}\frac{\partial\phi}{\partial z}\end{pmatrix}\\
&+\frac{\partial}{\partial x}\left(A_{55}\left(\psi_x+\frac{\partial W}{\partial x}\right)-F_{15}\phi\right)\\
&+\frac{1}{R}\frac{\partial}{\partial\theta}\left(\frac{A_{44}}{R}\frac{\partial\psi_x}{\partial\theta}+A_{44}\frac{\partial\psi_\theta}{\partial x}-F_{25}\phi\right)+\bar{N}^\theta\frac{\partial^2W(x,\theta,t)}{\partial\theta^2}\\
&+\bar{N}^x\frac{\partial^2W(x,\theta,t)}{\partial x^2}-K_wW(x,\theta,t)=I_0\frac{\partial^2W(x,\theta,t)}{\partial t^2}
\end{aligned}\tag{35}$$

$$\begin{aligned}
&\frac{\partial}{\partial x}\begin{pmatrix}B_{11}\frac{\partial U}{\partial x}+B_{12}\frac{1}{R}\left(\frac{\partial V}{\partial\theta}+W\right)\\+D_{11}\frac{\partial\psi_x}{\partial x}+D_{12}\frac{1}{R}\frac{\partial\psi_\theta}{\partial\theta}+G_{31}\frac{\partial\phi}{\partial z}\end{pmatrix}\\
&+\frac{1}{R}\frac{\partial}{\partial\theta}\left(B_{66}\left(\frac{1}{R}\frac{\partial U}{\partial\theta}+\frac{\partial V}{\partial x}\right)+D_{66}\left(\frac{1}{R}\frac{\partial\psi_x}{\partial\theta}+\frac{\partial\psi_\theta}{\partial x}\right)\right)\\
&-A_{55}\left(\psi_x+\frac{\partial W}{\partial x}\right)+F_{15}\phi\\
&=I_1\frac{\partial^2U(x,\theta,t)}{\partial t^2}+I_2\frac{\partial^2\psi_x(x,\theta,t)}{\partial t^2}
\end{aligned}\tag{36}$$

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(B_{66}\left(\frac{1}{R}\frac{\partial U}{\partial\theta}+\frac{\partial V}{\partial x}\right)+D_{66}\left(\frac{1}{R}\frac{\partial\psi_x}{\partial\theta}+\frac{\partial\psi_\theta}{\partial x}\right)\right)+\\
&\frac{1}{R}\frac{\partial}{\partial\theta}\left(\begin{pmatrix}\frac{B_{22}}{R}\left(\frac{\partial V}{\partial\theta}+W\right)+B_{12}\frac{\partial U}{\partial x}\\+D_{12}\frac{\partial\psi_x}{\partial x}+D_{22}\frac{1}{R}\frac{\partial\psi_\theta}{\partial\theta}\end{pmatrix}+G_{32}\frac{\partial\phi}{\partial z}\right)\\
&-A_{44}\left(\frac{1}{R}\frac{\partial\psi_x}{\partial\theta}+\frac{\partial\psi_\theta}{\partial x}\right)+F_{25}\phi\\
&=I_1\frac{\partial^2V(x,\theta,t)}{\partial t^2}+I_2\frac{\partial^2\psi_\theta(x,\theta,t)}{\partial t^2}
\end{aligned}\tag{37}$$

$$\begin{aligned}
&F_{31}\left(\frac{\partial U(x,\theta,t)}{\partial x}+\frac{1}{R}\left(\frac{\partial V(x,\theta,t)}{\partial\theta}+W(x,\theta,t)\right)\right)\\
&+G_{31}\left(\frac{\partial\psi_x(x,\theta,t)}{\partial x}+\frac{\partial\psi_\theta(x,\theta,t)}{R\partial\theta}\right)-\bar{P}_3\Delta T-X_{33}\phi(x,\theta,t)
\end{aligned}\tag{38}$$

$$-\frac{\partial}{\partial x}\left(F_{15}\left(\frac{\partial W(x,\theta,t)}{\partial x}+\psi_x(x,\theta,t)\right)+X_{11}\frac{\partial \phi(x,\theta,t)}{\partial x}\right)$$
$$\frac{\partial}{\partial \theta}\left(\begin{array}{c}F_{25}\left(\frac{\partial W(x,\theta,t)}{R\partial \theta}-\frac{V(x,\theta,t)}{R}+\psi_\theta(x,\theta,t)\right)\\ +X_{22}\frac{\partial \phi(x,\theta,t)}{\partial \theta}\end{array}\right)=0$$

3. Solution procedure

Now, the solution procedure in order to extract the vibrational frequencies related to a laminated composite shell covered by a piezoelectric layer is presented. In this regard, a numerical solution, DQM, is utilized. Based on DQM, the n-th order of a function can be rewritten in the following format.

$$\Psi^{(r)}(x_i)=\sum_{j=1}^{ns} l_{ij}^{(r)} V_j \; (i=1,2,\ldots,ns) \tag{39}$$

In which ns is reprehensive of the grid point numbers, and $l_{ij}^{(r)}$ is the Lagrange interpolation which is introduced in the following equation.

$$l_j^{(1)}(x_i)=\begin{cases}\dfrac{R^{(1)}(x_i)}{(x_i-x_j)R^{(1)}(x_j)} \text{(for } i,j=1,2,\ldots,ns, i\neq j)\\ -\displaystyle\sum_{j=1,i\neq j}^{ns} l_j^{(1)}(x_i) \text{ (for } i,j=1,2,\ldots,ns)\end{cases} \tag{40}$$
$$R^{(1)}(x_i)=\prod_{m=1,m\neq i}^{ns}(x_i-x_m)$$

Additionally, the Lagrange interpolation for higher order can be obtained as follow.

$$l_j^{(r)}(x_i)=\begin{cases} r\left(l_j^{(r-1)}(x_i)l_j^{(1)}(x_i)-\dfrac{l_j^{(r-1)}(x_i)}{(x_i-x_j)}\right)\\ \text{(for } i,j=1,2,\ldots,ns)\\ -\displaystyle\sum_{j=1,i\neq j}^{ns} l_j^{(r)}(x_i) \text{ (for } i,j=1,2,\ldots,ns)\end{cases} \tag{41}$$

It should be mentioned that the grid points through the domain is chosen based on Chebyshev Gauss Lobatto which is

$$x_i=\frac{L}{2}\left[1-cos\left(\frac{(i-1)}{(ns-1)}\pi\right)\right] (i=1,2,\ldots,ns) \tag{42}$$

Now, to utilize DQM, the independent variables should be rewritten using separation of variables as follow.

$$\begin{cases} W(x,\theta,t)=\bar{W}(x)\,Cos(n\theta)\,e^{i\lambda t}\\ U(x,\theta,t)=\bar{U}(x)\,Sin(n\theta)\,e^{i\lambda t}\\ V(x,\theta,t)=\bar{V}(x)\,Cos(n\theta)\,e^{i\lambda t}\\ \psi_x(x,\theta,t)=\bar{\psi}_x(x)\,Cos(n\theta)\,e^{i\lambda t}\\ \psi_\theta(x,\theta,t)=\bar{\psi}_\theta(x)\,Sin(n\theta)\,e^{i\lambda t}\\ \phi(x,\theta,t)=\bar{\phi}(x)\,Cos(n\theta)\,e^{i\lambda t}\end{cases} \tag{43}$$

Also, the discrete form of the displacement fields together with the electric potential can be written as follow.

$$\{V\}^T=\left\{\begin{array}{c}\{\bar{W}_1,\ldots,\bar{W}_{ns}\},\{\bar{U}_1,\ldots,\bar{U}_{ns}\},\{\bar{V}_1,\ldots,\bar{V}_{ns}\},\\ \{\bar{\psi}_{x1},\ldots,\bar{\psi}_{xns}\},\{\bar{\psi}_{\theta 1},\ldots,\bar{\psi}_{\theta ns}\},\{\bar{\phi}_1,\ldots,\bar{\phi}_{ns}\}\end{array}\right\}^T \tag{44}$$

Here, the governing equations as well as boundary conditions can be discretized using Eq. (43) and (44). Now, these equations can be rewritten in the matrix format containing a stiffness matrix as well as a mass matrix as follow.

$$\begin{bmatrix}[K_{bb}]_{12\times12} & [K_{bd}]_{12\times(3ns-6)}\\ [K_{db}]_{(3ns-6)\times12} & [K_{dd}]_{(3ns-6)\times(3ns-6)}\end{bmatrix}\cdot\begin{Bmatrix}\{V_b\}\\ \{V_d\}\end{Bmatrix}-\lambda^2\begin{bmatrix}[M_{bb}]_{12\times12} & [M_{bd}]_{12\times(3ns-6)}\\ [M_{db}]_{(3ns-6)\times12} & [M_{dd}]_{(3ns-6)\times(3ns-6)}\end{bmatrix}\cdot\begin{Bmatrix}\{V_b\}\\ \{V_d\}\end{Bmatrix}=0. \tag{45}$$

By solving this eigenvalue problem, the frequencies as well as mode shape of a laminated composite shell embedded in a piezoelectric layer which is subjected to thermal loads can be attained. Also, the nondimensional vibration frequency is $\bar{\lambda}=\lambda R\sqrt{\frac{\rho_c}{E_1}}$.

4. Parametric result

Now, firstly, the validity of the presented formulation as well as numerical solution procedure is confirmed, and then the parameter that can affect the vibrational response of a cylindrical shell which is made of laminated composites and embedded in a piezoelectric layer resting on an elastic foundation is studied in details. The mechanical properties of both the composite core together with the piezoelectric layer is presented in Table 1.

Now, in order to investigate the credibility of the formulation and solution method for studying the vibration of cylindrical shells, by eliminating the piezoelectric layer, the nondimensional vibration frequencies of an orthotropic cylindrical shell with various boundary conditions is obtained and is compared with those presented in Ref. (Loy *et al*. 1997). Also, other constants are presented in Table 1.

The results of Table 2 proves that the presented solution and formulation are credible in order to study the vibration of cylindrical shells. Now, the parameters which affect the thermoelectrical vibrational behavior of a laminated composite shell which is embedded in a piezoelectric layer as well as an elastic foundation is investigated. To begin with, the vibration frequency of a laminated composite shell with various foundry conditions and layer number—1, 2, 3, and 6—is plotted against the number of circumferential modes.

As it can be seen from Fig. 2, the vibrational frequencies for different shells with different core type are closer when the mode number is lower. Additionally, generally, the higher the laminated composite layers are, the higher the frequency is, in every mode number and end conditions.

Table 1 The geometry as well as material properties of the disk in the current study

Core	$E_1\ (GPa)$	$E_2\ (GPa)$	$G_{13} = G_{12}\ (GPa)$	$G_{23}(GPa)$	μ_{12}	$\alpha_{xx}\left(\frac{1}{K}\right)$	$\alpha_{xx}\left(\frac{1}{K}\right)$	$\rho_c\left(\frac{Kg}{m^3}\right)$
	137	8.96	7.1	3.477	0.3	26.08×10^{-9}	0.3×10^{-9}	1.603
Piezo	$\bar{C}_{11}(GPa)$	$\bar{C}_{12}(GPa)$	$\bar{C}_{13}(GPa)$	$\bar{C}_{22}(GPa)$	$\bar{C}_{23}(GPa)$	$\bar{C}_{33}(GPa)$	$\rho_p\left(\frac{Kg}{m^3}\right)$	
	137	71	73	132	74	115	7.5	
	$e_{13}(Cm^{-2})$	$e_{15}(Cm^{-2})$	$e_{33}(Cm^{-2})$	$\kappa_{11}(Fm^{-1})$	$\kappa_{33}(Fm^{-1})$		$\alpha_{xx} = \alpha_{\theta\theta}\left(\frac{1}{K}\right)$	
	-4.1	10.5	14.1	7.124×10^{-9}	5.841×10^{-9}		4.738×10^{5}	

Table 2 the nondimensional vibration frequency of orthotropic cylindrical shell

n	B.C					
	S-S		C-C		C-S	
	Present	Ref. (Loy *et al.* 1997)	Present	Ref. (Loy *et al.* 1997)	Present	Ref. (Loy *et al.* 1997)
1	0.016110	0.016101	0.032792	0.032885	0.023925	0.023974
2	0.0094314	0.009382	0.013906	0.013932	0.011242	0.011225
3	0.022127	0.022105	0.022668	0.022672	0.022326	0.022310
4	0.042098	0.042095	0.042199	0.042208	0.042138	0.042139
5	0.067989	0.068008	0.068020	0.068046	0.068003	0.068024

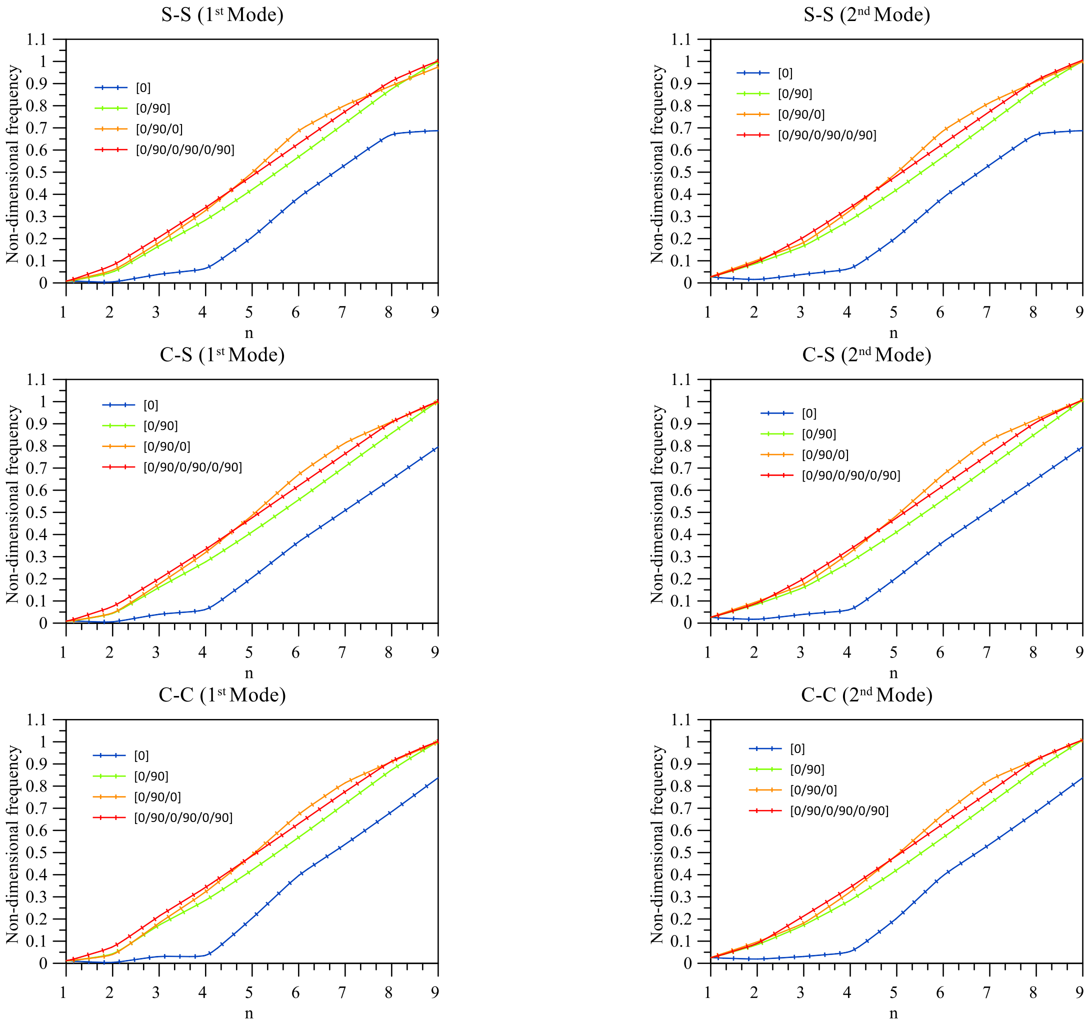

Fig. 2 Variation of nondimensional vibration frequency against n for various type of laminated composite

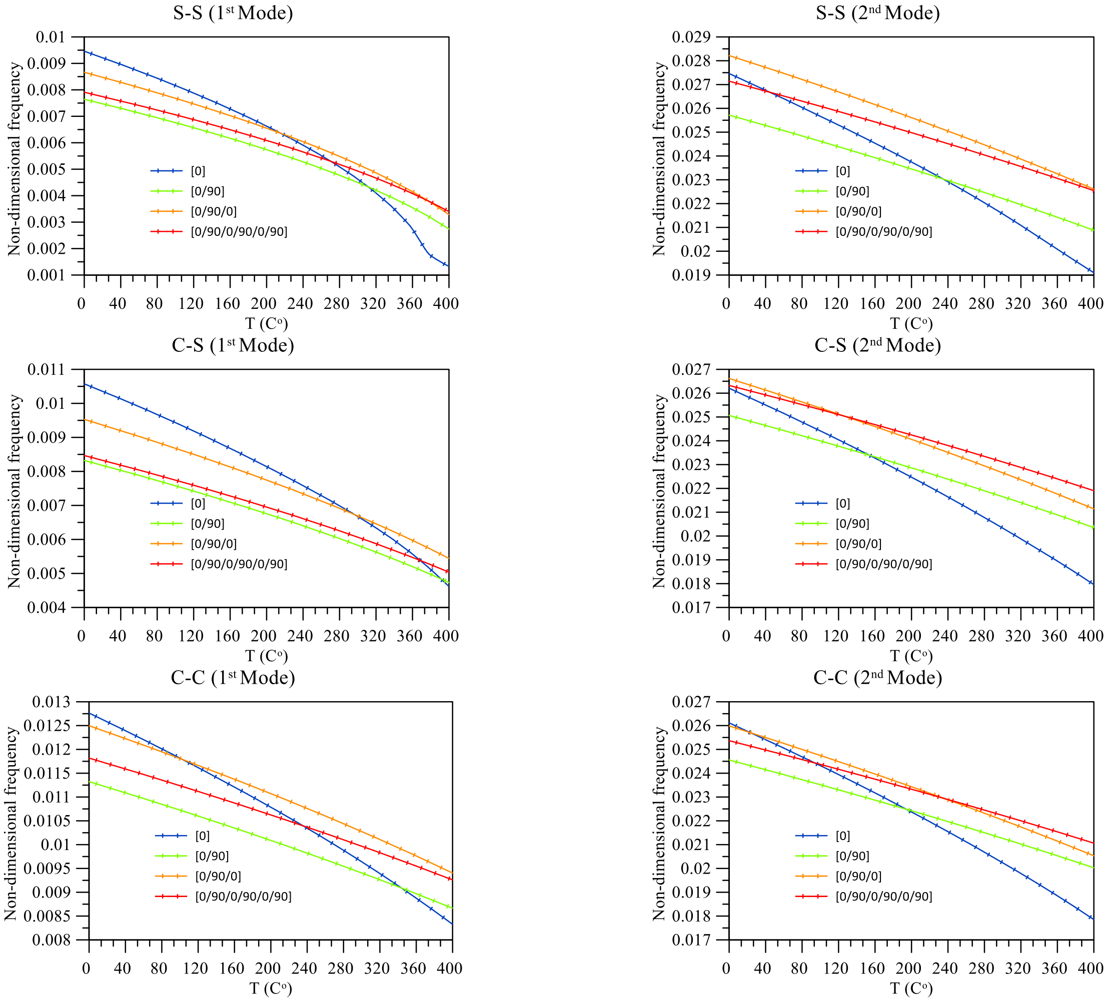

Fig. 3 Variation of nondimensional vibration frequency against temperature value for various type of laminated composite

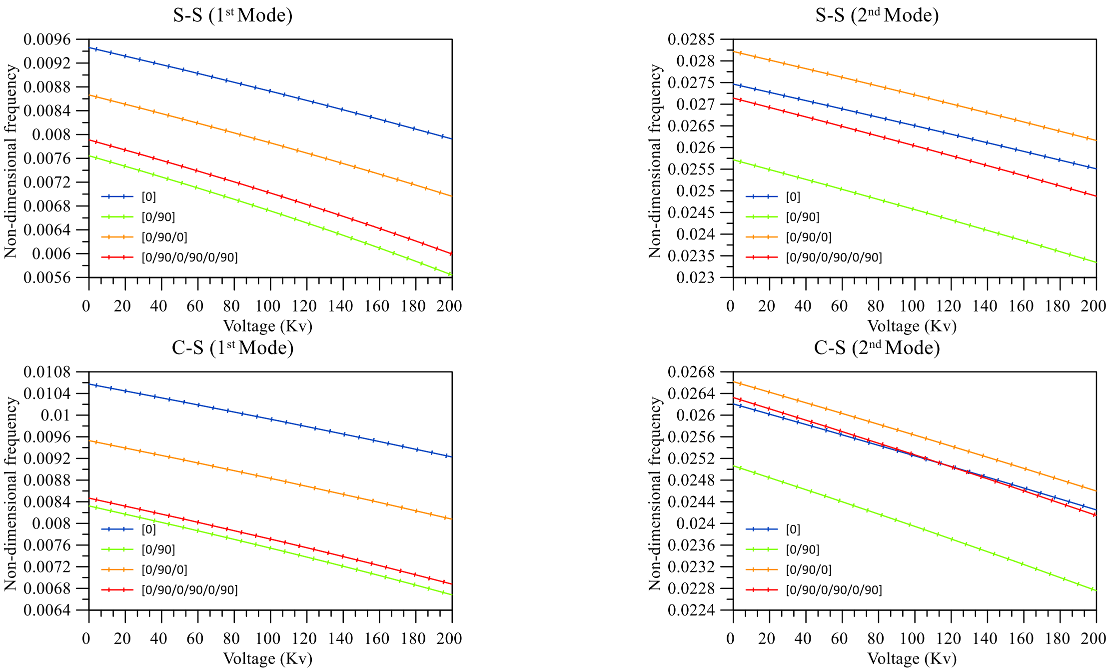

Fig. 4 Variation of nondimensional vibration frequency against external voltage for various type of laminated composite

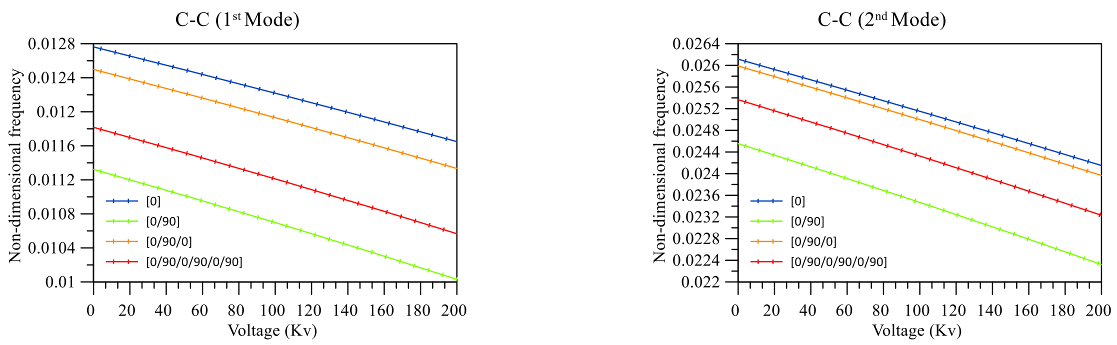

Fig. 4 Continued

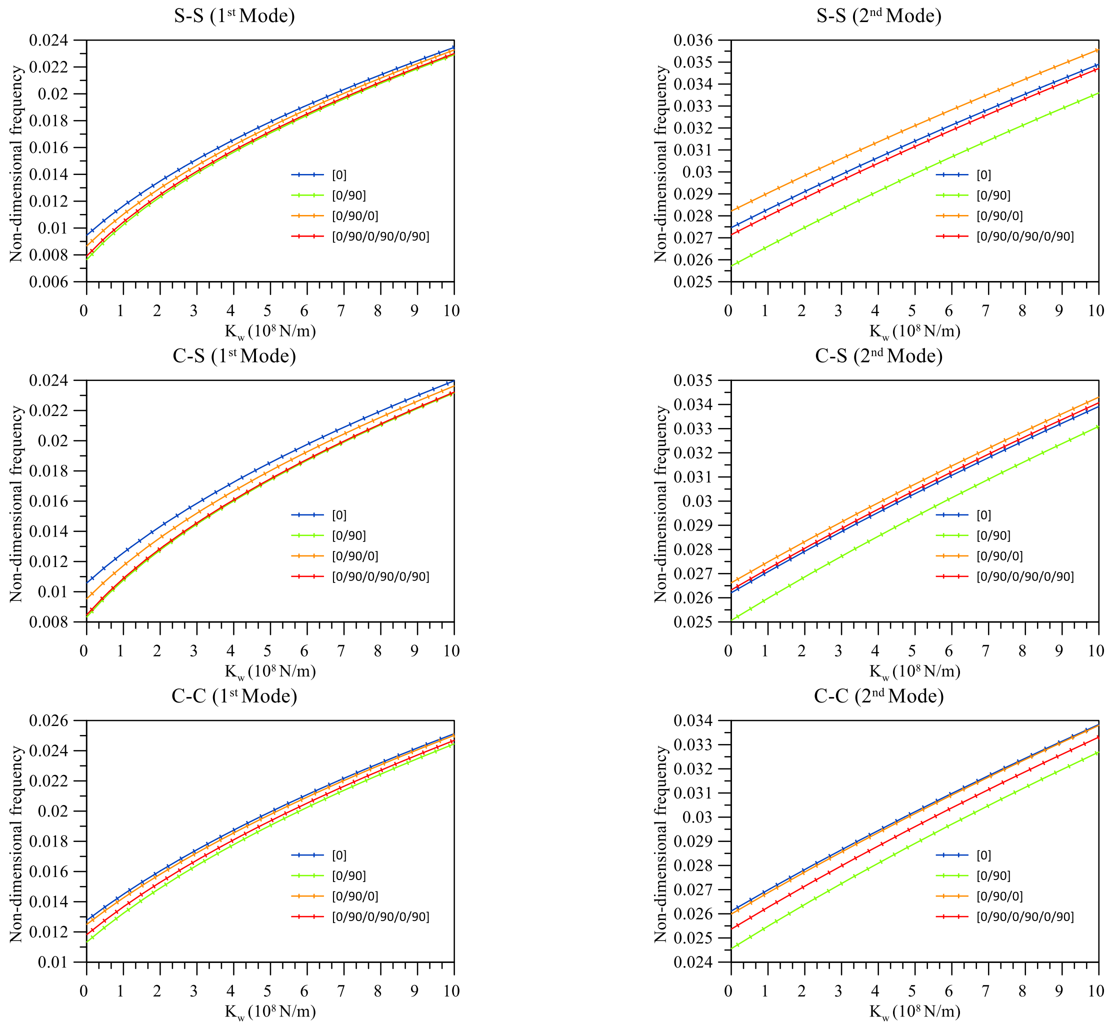

Fig. 5 Variation of nondimensional vibration frequency against foundation parameter value for various type of laminated composite

In Fig. 3, to study the impact of temperature rises on the dynamic response of the laminated composite shells, the variation of nondimensional vibration frequencies of the shell are exhibited against the value of temperature. In this figure, the various boundary conditions as well as various type of laminated composites are considered.

The interesting results in Fig. 3 indicates that the orthotropic shell is more dependent on the temperature in contrasts with the laminated composites shell as, by increasing the temperature, the gradient of vibrational frequency of orthotropic shell varies more quickly than other cases. In another word, the orthotropic shell, in all of the boundary types – except second mode of CS and SS – has the higher value of vibration frequency, and, by increasing the temperature, its value drops so much so that, at the end of the process, it has the lowest value among all

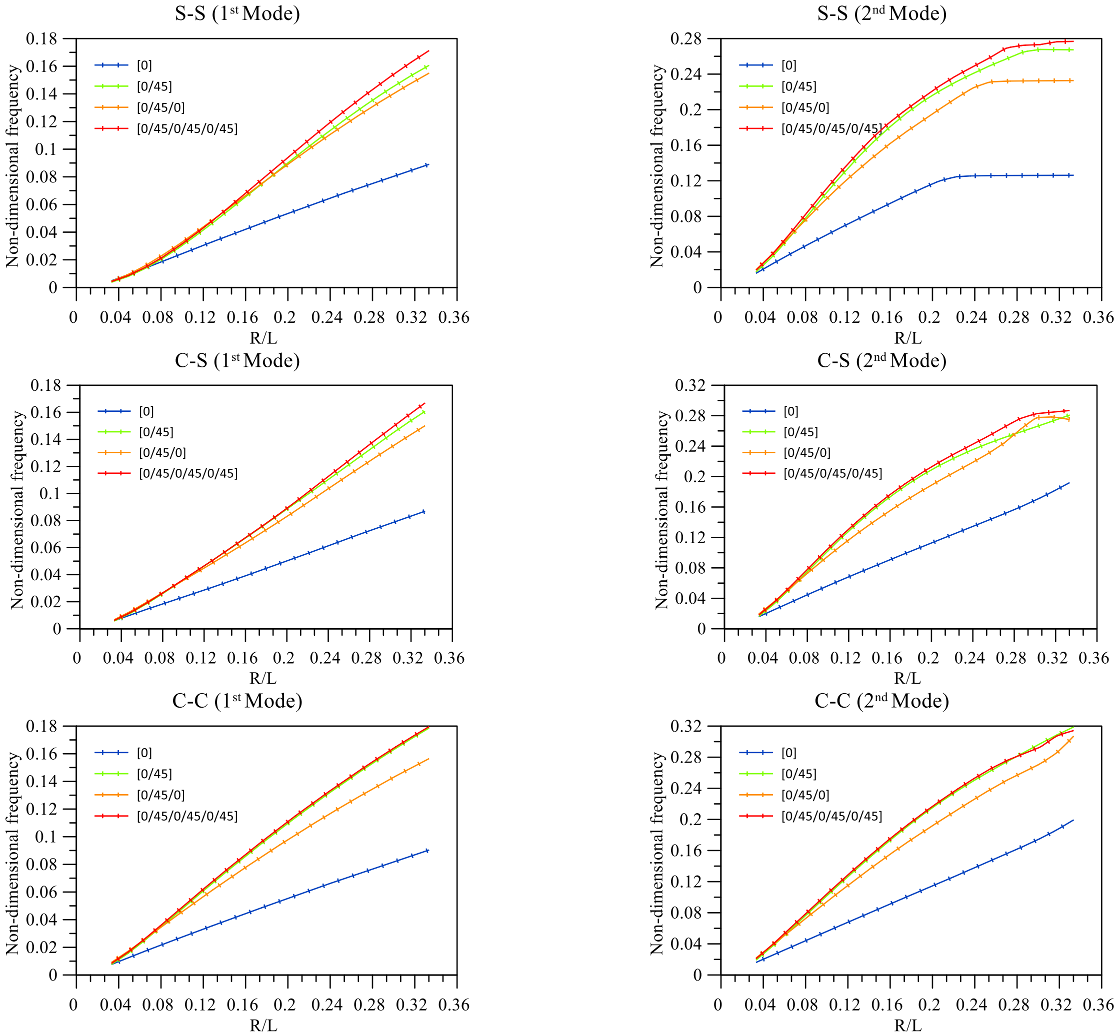

Fig. 6 Variation of nondimensional vibration frequency against *R/L* for various type of laminated composite.

the cases. Additionally, as a general conclusion, it is observable that the frequency of laminated composite with 2 layers has the lowest value among laminated shells, while the 3 layers has the highest one.Next, the electrical loading influence on the vibration of laminated cylindrical shell which is covered by a piezoelectric layer is explored in Fig. 4. in the mentioned figure, the nondimensional frequencies are plotted against the value of external voltage for different shell with different cores.

As expected, the results in this figure shows that the intensifying of externa voltage leads in reducing the value of natural frequencies, despite the boundary condition or shell type. Also, the trend which can see in the previous figures are observable in this figure as the orthotropic shell has the highest vibration frequency among all the cases.

In Fig. 5 the effect of various values of Winkler elastic foundation on the vibration of laminated composite cylindrical shell is investigated. In this figure the various type of laminated core together with different boundary conditions are presented.

The results of the figure show that, despite the type of the end conditions or the laminated core, escalating the value of the foundation cause the vibration frequencies to rise. In addition, the other interesting results of this figure is that the vibration frequencies of various laminated shells are getting closer as the foundation get stiffer.

Now, the impact one of the most important parameters in determining the value of natural frequency, the radius of the shell, is examined in Fig. 6. In this regard, the variation of vibrational frequency of laminated shell are plotted versus the value of radius for various boundary conditions.

The interesting results of Fig. 6 exhibit an increasing effect of higher values associated with radius on the natural vibration frequency of the cylindrical shell. Also, it can be observed that the effect of radius on the cases with laminated composites is higher than that on orthotropic shell.

5. Conclusions

The vibration frequency of a laminated composite cylindrical shell coved by a piezoelectric layer is optimized. The displacement field is chosen based on FSDT. The formulation of thermoelectrical vibration of is obtained through energy method and solved by DQM. The credibility of the current study is validated by employing another research. Here is the highlight of the study:

• The effect of temperature is more apparent on the shells with orthotropic cores.

• Increasing the circumferential mode number leads in higher vibrational frequencies.

• The value for initial frequency related to the shells with orthotropic shell are lower in cases that shell is not subjected to temperature loading.

• The higher the external voltage is, the lower the frequency of the cylindrical shell is.

• The laminated composites can have lower frequency than orthotropic shell if the angle of laminated layers is higher than 85.

• The vibration frequencies are rising when θ is around 45.

• The bigger radius of cylindrical shell leads in higher value of vibration frequencies.

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