

# Management of the energy harvesting for MEMS/NEMS via newmark current method

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**Abstract.** The free and forced vibration in addition to electric energy harvesting of a piezoelectric disk resting on two-parameter foundation modeled by modified couple stress as well as Kirchhoff plate theory is probed. The governing equations and boundary conditions are obtained using Hamilton's principle. Then, the free and forced vibration are solved using numerical solutions, generalized differential quadrature method (GDQM) and Newmark-beta method. The forced vibration is resulted from a base excitation load. Also, the possible voltage which can be harvested from this system is obtained using generalized integral quadrature method. The validity of the formulation and solution procedure is confirmed using a comparison study. The impact of parameters such as length effect, inner to outer radius ratio, and foundations parameters on the free and forced vibration as well as energy harvesting is investigated in detail. This paper can be a basis for future studies in the area of piezoelectric harvesters in small scales.

**Keywords:** energy harvesting; free and forced vibration; GDQM; micro/nano disk; modified couple stress theory; piezoelectric

## 1. Introduction

Piezoelectric materials, as one of the most important types of smart material, has brought a great deal of attention due to their usage in various engineering areas—such as mechanical, chemical, civil, and so on (Habibi *et al.* 2016, 2018b, Ebrahimi *et al.* 2019a, Esmailpoor Hajilak *et al.* 2019). These materials can be used in such assorted devices and structures as sensors (Tzou and Tseng 1990), actuators (Adriaens *et al.* 2000), energy harvesters (Yang *et al.* 2018), resonators (Casadei *et al.* 2012), etc. Nowadays, with advent of new wireless technologies and microelectron-mechanical systems (MEMS) (Habibi *et al.* 2018a, 2019b, d, e, Pourjabari *et al.* 2019, Safarpour *et al.* 2019a), the self-chargeable devices are inseparable part of these advanced technologies. In this regard, the mechanical vibration energy harvesters are of interest, since they not only can turn the waste energy to usable source of energy but also can damp the vibration and stabilize the system (Habibi *et al.* 2019a, Safarpour *et al.* 2019b, Alipour *et al.* 2020, Ebrahimi *et al.* 2020a).

In the recent years, it was attempted to model the microstructures in a fashion that the size dependent effects can be considered (Habibi *et al.* 2017, 2019c, Safarpour *et al.* 2018, 2020, Ghazanfari *et al.* 2020). To do so, scholars have used nonlocal elasticity (Eringen and Edelen 1972,

Eringen 2002), strain gradient theory (Lam *et al.* 2003, Zhou *et al.* 2016), and modified couple stress (Toupin 1962). Regarding this, Park and Gao (Park and Gao 2006) developed a model for Euler-Bernoulli beam based on modified couple stress. They studied the bending of cantilever using their model and considering the length and size effects. By incorporating Hamilton principle as well as modified couple stress, the model for Timoshenko beam which is able to capture the impacts related to the scale and size of the structure was introduced (Ma *et al.* 2008). Next, employing the modified couple stress, a size dependent model was developed (Ma *et al.* 2011) for Mindlin plate. The results in this paper indicated that the rotations together with deflections are less than the one extracted using classic theory. The size dependent nonlinear formulation for bending corresponded to axisymmetric disks utilizing modified couple stress was probed (Reddy and Berry 2012). In addition, Li and Pan (2015) examine the vibration as well as bending of FG piezoelectric plate incorporating modified couple stress theory as the size dependent elasticity. As the piezoelectric material usage has been grown in different structures over the recent years, the modeling of which is of importance. For example, incorporating Euler-Bernoulli beam theory and surface effects, the vibration investigation on the nonlocal nanowires was carried out (Gheshlaghi and Hasheminejad 2012). Also, they studied the critical voltage at which the buckling occurs. The vibration characteristics of Euler as well as Timoshenko beams are examined utilizing modified couple stress elasticity by Ansari *et al.* (2014). The static bending related to the FG piezoelectric

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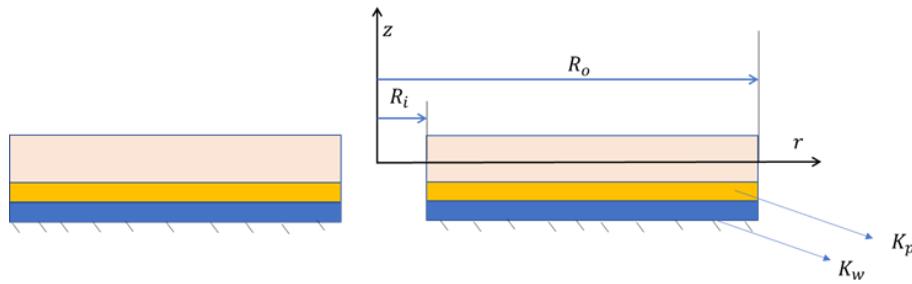


Fig. 1 Schematic of a piezoelectric disk resting on a Wrinkle-Pasternak medium

micro-plate which was placed on a two-parameter elastic foundation and was subjected to thermal, electrical, and mechanical load was probed (Abazid and Sobhy 2018). They considered all of the external load uniformly. Using modified couple stress, Malikan (2017) explored the shear buckling corresponded to piezoelectric nanoplate. In this paper, it was shown that, by intensifying the length scale, the critical load increases. On the basis of Kirchhoff plate theory, the vibration of a sandwich annular plate made of a core and two piezoelectric layers was analyzed (Wang *et al.* 2001). The credibility of the results was verified by a finite element model. In the recent years, with emergence of such devices as sensors, actuator, micro and nanoelectromechanical systems (MEMS/NEMS), the importance of having an energy source with higher durability has been increased (Granstrom *et al.* 2007). Given this, as the mechanical vibration energy is one of the most reachable energies in different mechanical systems, quite a bit of effort has been devoted in order to turn this energy to a usable form—electrical energy. Also, many type of materials which are able to do so such as electromagnetic materials (Davino *et al.* 2011), electrostatic (Lallart *et al.* 2011), and piezoelectric (Naderi *et al.* 2021) has been employed. The point which can be stated about piezoelectric material is that these type of material can be utilized in various scales—Micro, nano, or macro (Abdelkefi 2016, Madinei *et al.* 2016). There are many research in which the piezoelectric harvesters has been analyzed. Among these papers, Yang *et al.* (2009) investigated a two couple piezoelectric beam which are used as harvester of the mechanical energy near to resonant frequency into electrical energy. Also, by incorporating a finite element model and using Kirchhoff plate theory, an piezoelectric plate used as a mechanical vibration energy harvester was studied (Junior *et al.* 2009).

In this paper, the energy harvesting together with the vibration of a piezoelectric annular microplate modeled using Kirchhoff plate theory resting on a Winkler-Pasternak foundation, for the very first time, is investigated. To capture the effects of size, the modified couple stress elasticity is incorporated. The Hamilton's principle was employed in order to extract the governing equations in addition to the boundary conditions. In order to solve the problem, GDQM and Newmark beta is utilized. The credibility of solution procedure along with the equations are confirmed using some other studies. The influence of various parameters affecting the harvested energy and vibration of the disk is probed in detail. This study can be a

basis for future studies, theoretical and experimental, in which the piezoelectric disks are the aim.

## 2. Problem formulation

In this segment, the problem formulation corresponded to a piezoelectric micro-disk modeled using modified couple stress is presented. In Fig. 1 the schematic of the piezoelectric disk is shown.

Now, the displacement field related to a disk which is thin—the thickness to length ratio is more than ten—based on Kirchhoff plate theory are as follow (Ebrahimi *et al.* 2019b, Ebrahimi *et al.* 2019c, 2020b, Mohammadgholiha *et al.* 2019, Mohammadi *et al.* 2019, Habibi *et al.* 2020, Shariati *et al.* 2020a, Shokrgozar *et al.* 2020).

$$\begin{aligned} u_1(r, \theta, t) &= w(r, \theta, t) \\ u_2(r, \theta, t) &= -z \frac{\partial w(r, \theta, t)}{\partial r} \\ u_3(r, \theta, t) &= -z \frac{\partial w(r, \theta, t)}{r \partial \theta} \end{aligned} \quad (1)$$

Then, the corresponding nonzero strains can be written as (Hashemi *et al.* 2019, Moayedi *et al.* 2019, 2020a, b, Oyarhossein *et al.* 2020, Shariati *et al.* 2020b):

$$\begin{aligned} \varepsilon_{rr} &= \frac{\partial u_2(r, \theta, t)}{\partial r} = -z \frac{\partial^2 w(r, \theta, t)}{\partial r^2}, \\ \varepsilon_{\theta\theta} &= \frac{\partial u_3(r, \theta, t)}{r \partial \theta} + \frac{u_2(r, \theta, t)}{r} = \\ &= -z \left( \frac{\partial^2 w(r, \theta, t)}{r^2 \partial \theta^2} + \frac{\partial w(r, \theta, t)}{r \partial r} \right), \\ \varepsilon_{r\theta} &= \frac{1}{2} \left( \frac{\partial u_2(r, \theta, t)}{r \partial \theta} + \frac{\partial u_3(r, \theta, t)}{\partial r} - \frac{u_3(r, \theta, t)}{r} \right) = \\ &= z \left( -\frac{\partial^2 w(r, \theta, t)}{r \partial \theta \partial r} + \frac{\partial w(r, \theta, t)}{r^2 \partial \theta} \right) \end{aligned} \quad (2)$$

The stress equations related to a piezoelectric disk are (Hashemi *et al.* 2019, Al-Furjan *et al.* 2020q, Cheshmeh *et al.* 2020, Lori *et al.* 2020, Najaafi *et al.* 2020, Shariati *et al.* 2020c):

$$\begin{aligned} \sigma_{rr} &= \bar{Q}_{11} \varepsilon_{rr} + \bar{Q}_{12} \varepsilon_{\theta\theta} - \bar{e}_{31} E_z \\ \sigma_{\theta\theta} &= \bar{Q}_{12} \varepsilon_{rr} + \bar{Q}_{11} \varepsilon_{\theta\theta} - \bar{e}_{31} E_z \\ \tau_{r\theta} &= (-\bar{Q}_{12} + \bar{Q}_{11}) \varepsilon_{r\theta} \end{aligned} \quad (3)$$

In which (Al-Furjan *et al.* 2020e, o, s, Bai *et al.* 2020, Li *et al.* 2020a, Xiong *et al.* 2020, Guo *et al.* 2021b, Liu *et al.* 2021a):

$$\begin{aligned}\bar{Q}_{11} &= Q_{11} - \frac{(Q_{13})^2}{Q_{33}} \\ \bar{Q}_{12} &= Q_{12} - \frac{(Q_{13})^2}{Q_{33}} \\ \bar{e}_{31} &= e_{31} - \frac{Q_{13}e_{33}}{Q_{33}}\end{aligned}\quad (4)$$

Now, the electric fields and related electric displacements can be introduced respectively

$$\begin{aligned}E_r &= -\frac{\partial\Phi}{\partial r} \\ E_\theta &= -\frac{\partial\Phi}{\partial\theta} \\ E_z &= -\frac{\partial\Phi}{\partial z} \\ D_r &= \bar{\epsilon}_{11} E_r \\ D_\theta &= \bar{\epsilon}_{11} E_\theta \\ D_z &= \bar{\epsilon}_{33} E_r + \bar{e}_{31}(\epsilon_{rr} + \epsilon_{\theta\theta})\end{aligned}\quad (5)$$

where

$$\bar{\epsilon}_{11} = \epsilon_{11}, \bar{\epsilon}_{33} = \epsilon_{33} + \left(\frac{e_{33}^2}{Q_{33}}\right)\quad (6)$$

The electric potential distribution can be written as

$$\Phi = \left(1 - \left(\frac{z-h/2}{h/2}\right)^2\right)\phi(r, \theta)\quad (7)$$

Employing Eqs. (5) and (7) the electric fields and displacement can be rewritten

$$\begin{aligned}E_r &= -\left(1 - \left(\frac{z-h/2}{h/2}\right)^2\right)\frac{\partial\phi(r, \theta)}{\partial r} \\ E_\theta &= -\left(1 - \left(\frac{z-h/2}{h/2}\right)^2\right)\frac{\partial\phi(r, \theta)}{\partial\theta} \\ E_z &= \frac{8(z-h/2)}{h^2}\phi(r, \theta) \\ D_r &= -\bar{\epsilon}_{11}\left(1 - \left(\frac{z-h/2}{h/2}\right)^2\right)\frac{\partial\phi(r, \theta)}{\partial r} \\ D_\theta &= -\bar{\epsilon}_{11}\left(1 - \left(\frac{z-h/2}{h/2}\right)^2\right)\frac{\partial\phi(r, \theta)}{\partial\theta} \\ D_z &= \bar{\epsilon}_{33}\frac{8(z-h/2)}{h^2}\phi(r, \theta) + \bar{e}_{31}z\Delta w(r, \theta, t)\end{aligned}\quad (8)$$

In which the Laplacian operator in polar direction is defined as follow

$$\Delta^2 = \frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} + \frac{\partial^2}{r^2\partial\theta^2}\quad (9)$$

Now, the strain energy corresponded to a piezoelectric micro-disk modeled by modified couple stress can be written as (Adamian *et al.* 2020, Al-Furjan *et al.* 2020c, d, Li *et al.* 2020b, Zare *et al.* 2020, Dai *et al.* 2021b):

$$U = \frac{1}{2} \iint (\epsilon_{rr}\sigma_{rr} + \epsilon_{\theta\theta}\sigma_{\theta\theta} + \epsilon_{r\theta}\tau_{r\theta} + 2m_{r\theta}\chi_{r\theta}) r dz dA \quad (10)$$

where the couple stress tensor is (Liu *et al.* 2020b, Habibi *et al.* 2021, He *et al.* 2021, Huang *et al.* 2021a, Liu *et al.* 2021b, Zhang *et al.* 2021):

$$\begin{aligned}m_{r\theta} &= 2G_f l_0^2 \chi_{r\theta} \\ m_{\theta r} &= 2G_f l_0^2 \chi_{\theta r}\end{aligned}\quad (11)$$

and the tension tensor for circular plate can be defined as

$$\chi_{r\theta} = \chi_{\theta r} = \frac{1}{2} \left( -\frac{\partial^2 w(r, \theta, t)}{\partial r^2} + \frac{\partial w(r, \theta, t)}{r\partial r} \right)\quad (12)$$

Also, the kinetic energy of the plate can be introduced as follow (Liu *et al.* 2020a, Wang *et al.* 2020, Zhou *et al.* 2020, Dai *et al.* 2021a, Guo *et al.* 2021a, Shao *et al.* 2021, Wu and Habibi 2021):

$$\Pi = \frac{1}{2} \iint \left( \rho \frac{\partial^2 w(r, \theta, t)}{\partial t^2} \right) dz dA \quad (13)$$

Here, the variation of strain and kinetic energy can be rewritten as

$$\begin{aligned}U &= \frac{1}{2} \int \left( \begin{aligned} & -\delta \frac{\partial^2 w(r, \theta, t)}{\partial r^2} M_{rr} \\ & -\delta \left( \frac{\partial^2 w(r, \theta, t)}{r^2 \partial \theta^2} + \frac{\partial w(r, \theta, t)}{r \partial r} \right) M_{\theta\theta} + \\ & \delta \left( -\frac{\partial^2 w(r, \theta, t)}{r \partial \theta \partial r} + \frac{\partial w(r, \theta, t)}{r^2 \partial \theta} \right) M_{r\theta} \\ & + \left( -\frac{\partial^2 w(r, \theta, t)}{\partial r^2} + \frac{\partial w(r, \theta, t)}{r \partial r} \right) 2P \\ & + D_r \delta \frac{\partial \Phi}{\partial r} + D_\theta \delta \frac{\partial \Phi}{\partial \theta} + D_z \delta \frac{\partial \Phi}{\partial z} \end{aligned} \right) r dA \\ \delta \Pi &= \frac{1}{2} \int \rho \left( \frac{\partial w(r, \theta, t)}{\partial t} \delta \frac{\partial w(r, \theta, t)}{\partial t} \right) dA\end{aligned}\quad (14)$$

where the resultant forces are defined as follow (Zhao *et al.*, Huang *et al.* 2021b, Jiao *et al.* 2021, Moradi *et al.* 2021, Xu *et al.* 2021)

$$\begin{aligned}M_{rr} &= \int_{-h/2}^{h/2} z \sigma_{rr} dz \\ M_{\theta\theta} &= \int_{-h/2}^{h/2} z \sigma_{\theta\theta} dz \\ M_{r\theta} &= \int_{-h/2}^{h/2} z \sigma_{r\theta} dz \\ P &= 2 \int_{-h/2}^{h/2} m_{r\theta} dz \quad \text{or} \quad 2 \int_{-h/2}^{h/2} m_{\theta r} dz\end{aligned}\quad (15)$$

Additionally, the external work associated with the Winkler-Pasternak foundation is

$$W_{ext} = -\int (K_p \Delta^2 w - K_w w) w dA \tag{16}$$

Now using Hamilton’s principle (Ma et al. 2021, Hou et al. 2021, Huang et al. 2021c, Liu et al. 2021c, Chen et al. 2022, Yu et al. 2022):

$$\int_{t_0}^{t_1} \delta(U - \Pi + W_{ext}) dt = 0 \tag{17}$$

The governing equation corresponded to the motion of the piezoelectric disk resting on a two-parameter foundation can be obtained through the following equation

$$\begin{aligned} &-\frac{\partial^2 M_{rr}}{\partial r^2} - 2\frac{\partial M_{rr}}{r\partial r} + \frac{\partial M_{\theta\theta}}{r\partial r} - \frac{\partial^2 M_{\theta\theta}}{r^2\partial\theta^2} \\ &- 2\frac{\partial^2 M_{r\theta}}{r\partial r\partial\theta} - 2\frac{\partial M_{r\theta}}{r^2\partial\theta} - \frac{\partial^2 P}{\partial r^2} - 3\frac{\partial P}{r\partial r} \\ &+ hpw(r, \theta, t) - K_p \Delta^2 w + K_w w = 0 \end{aligned} \tag{18}$$

Now, for the electric equation the Maxwell’s equation should be satisfied in polar direction as follow (Duan et al. 2005)

$$\int_{-h/2}^{h/2} \left( \frac{\partial(rD_r)}{r\partial r} + \frac{\partial D_\theta}{r\partial\theta} + \frac{\partial D_z}{\partial z} \right) dz = 0 \tag{19}$$

Lastly, the corresponding end conditions can be obtained using Eq. (14)

$$\begin{aligned} \text{Simply supported: } &\begin{cases} M_{rr} + P = 0 \\ w = 0 \\ \phi' = 0 \end{cases} \\ \text{Clamped: } &\begin{cases} w' = 0 \\ w = 0 \\ \phi' = 0 \end{cases} \end{aligned} \tag{20}$$

### 3. Solution

Here, GDQM solution procedure as the numerical solution in order to discretize the formulation is employed (Al-Furjan et al. 2020f, g, j, k, l, n, r, u, v). Also, in order to solve the forced vibration related to a circular microplate Newmark beta method is utilized.

#### 3.1 Analytical solution

Fourth-order GDQM explains that the r-th order derivative of a function like  $\psi(x_i)$  can be defined as (Wu and Liu 2001).

$$\begin{aligned} \psi^{(r)}(x_i) &= \sum_{j=1}^{ns} h_{j0}^{(r)}(x_i) \psi_j + h_{11}^{(r)}(x_i) \psi_1^{(1)} \\ &+ h_{ns1}^{(r)}(x_i) \psi_{ns}^{(1)} = \sum_{j=1}^{ns+2} \beta_{ij}^{(r)} V_j \end{aligned} \tag{21}$$

$(i = 1, 2, \dots, ns)$

where

$$\beta_{ijl}^{(r)} = h_{jl}^{(r)}(x_i) = \frac{d^r h_{jl}(x_i)}{dx^r} \tag{22}$$

$$h_{jl}^{(r)}(x_i) = \begin{cases} 1 & \text{if } i = j \text{ \& } l = r \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

$$\begin{aligned} h_{pi}(x) &= (a_{pi}x^2 + b_{pi}x + c_{pi})l_p(x) \\ &(\text{p}=1, ns \text{ and } i = 0, 1) \\ h_{j0}(x) &= \frac{(x - x_1)(x - x_{ns})}{(x_j - x_1)(x_j - x_{ns})} l_j(x) \\ &(j = 2, 3, \dots, ns - 1) \end{aligned} \tag{24}$$

Also, the constants in the above equation are expressed in appendix-I. Now, the first order derivative corresponded to Lagrange interpolation are introduced as follow

$$\begin{aligned} l_j^{(1)}(x_i) &= \begin{cases} \frac{R^{(1)}(x_i)}{(x_i - x_j)R^{(1)}(x_j)} & (\text{for } i, j = 1, 2, \dots, ns; i \neq j) \\ -\sum_{j=1, j \neq i}^{ns} l_j^{(1)}(x_i) & (\text{for } i, j = 1, 2, \dots, ns) \end{cases} \\ R^{(1)}(x_i) &= \prod_{m=1, m \neq i}^{ns} (x_i - x_m) \end{aligned} \tag{25}$$

In addition, the other higher derivative for the Lagrange interpolation are (Shariati et al. 2012, 2016a, b, 2019, 2020g, 2021a, b):

$$l_j^{(r)}(x_i) = \begin{cases} r \left( l_j^{(r-1)}(x_i) l_j^{(1)}(x_i) - \frac{l_j^{(r-1)}(x_i)}{(x_i - x_j)} \right) \\ \quad (\text{for } i, j = 1, 2, \dots, ns) \\ -\sum_{j=1, j \neq i}^{ns} l_j^{(r)}(x_i) & (\text{for } i, j = 1, 2, \dots, ns) \end{cases} \tag{26}$$

Now, the discretized form of the function can be defined as follow

$$\begin{aligned} \{V\}^T &= \{\psi_1^{(0)}, \psi_1^{(1)}, \psi_2, \psi_3, \dots, \psi_{ns-1}, \psi_{ns}^{(0)}, \psi_{ns}^{(1)}\} \\ &= \{V_1, V_2, \dots, V_{ns+2}\} \end{aligned} \tag{27}$$

Chebyshev Gauss Lobatto is utilized to obtain the sample points.

$$x_i = \frac{L}{2} \left[ 1 - \cos \left( \frac{(i-1)}{(ns-1)} \pi \right) \right] \quad (i = 1, 2, \dots, ns) \tag{28}$$

In order to use GDQM, firstly, the variables are rewritten using separating of variables as follow (Al-Furjan et al. 2020a, b, h, i, m, p, t, 2021a, b)

$$\begin{cases} w(x, t) = W(x) \text{Cos}(n\theta) e^{i\lambda t} \\ \phi(x, t) = \varphi(x) \text{Cos}(n\theta) e^{i\lambda t} \end{cases} \tag{29}$$

Table 1 The geometry as well as material properties of the disk in the current study

$Q_{11}$ (Nm <sup>-2</sup> )	$Q_{12}$ (Nm <sup>-2</sup> )	$Q_{33}$ (Nm <sup>-2</sup> )	$Q_{13}$ (Nm <sup>-2</sup> )	$\rho$ (Kgm <sup>-3</sup> )	$e_{13}$ (Cm <sup>-2</sup> )	$e_{33}$ (Cm <sup>-2</sup> )	$\gamma_{11}$ (Fm <sup>-1</sup> )	$\gamma_{33}$ (Fm <sup>-1</sup> )	$R_o$ (mm)
$132 \times 10^9$	$71 \times 10^9$	$115 \times 10^9$	$73 \times 10^9$	$7.5 \times 10^3$	-4.1	14.1	$7.124 \times 10^{-9}$	$5.841 \times 10^{-9}$	6

 Table 2 The first two non-dimensional frequencies of CC Disk with two different outer to inner radius ratio and various  $n$ 

$n$		R <sub>o</sub> /R <sub>i</sub>			
		10		2	
		1 <sup>st</sup> mode	2 <sup>nd</sup> mode	1 <sup>st</sup> mode	2 <sup>nd</sup> mode
0	Present	27.28	75.36	89.25	246.33
	Ref. (Leissa and Qatu 2011)	27.3	75.3	89.2	246
1	Present	28.91	78.63	90.23	247.7
	Ref. (Leissa and Qatu 2011)	28.4	78.6	90.2	248
2	Present	36.61	90.44	93.32	251.9
	Ref. (Leissa and Qatu 2011)	36.7	90.5	93.3	253
3	Present	51.21	112.08	98.92	259.1
	Ref. (Leissa and Qatu 2011)	51.2	112	99	259

In the piezoelectric micro disk, the discretized form of displacement and the electric potential can be expressed as follow.

$$\{V\}^T = \{\{W_1, \dots, W_{ns+2}\}, \{\varphi_1, \dots, \varphi_{ns}\}\}^T \quad (30)$$

Now, by utilizing Eqs. (29) and (30) the governing equations Eqs. (18) and (19) and boundary conditions can be discretized. At last, the discretized equations of motion along with the boundary contentions can be rewritten in the following form.

$$\begin{bmatrix} [K_{bb}]_{12 \times 12} & [K_{bd}]_{12 \times (3ns-6)} \\ [K_{db}]_{(3ns-6) \times 12} & [K_{dd}]_{(3ns-6) \times (3ns-6)} \end{bmatrix} \cdot \begin{Bmatrix} \{V_b\} \\ \{V_d\} \end{Bmatrix} = 0 \quad (31)$$

$$-\lambda^2 \begin{bmatrix} [M_{bb}]_{12 \times 12} & [M_{bd}]_{12 \times (3ns-6)} \\ [M_{db}]_{(3ns-6) \times 12} & [M_{dd}]_{(3ns-6) \times (3ns-6)} \end{bmatrix} \cdot \begin{Bmatrix} \{V_b\} \\ \{V_d\} \end{Bmatrix} = 0$$

By solving the above eigenvalue problem, the eigenvalues and eigenvectors can be attained. Also, these matrixes are used in the Newmark beta method which is explained in the next section.

### 3.2 Newmark- $\beta$

In order to solve the forced vibration and energy harvesting of piezoelectric microdisk resting on a Wrinkle-Pasternak medium, the Newmark beta method is incorporated (Shariati *et al.* 2020d, e, f, h, i, j). Now, using the stiffness and mass matrix, obtained in the previous section, and using damping matrix which can be defined as follow, the effective stiffness matrix can be produced in Eq. (33). In addition, the displacement vector in each step can be extracted as in Eq. (35).

$$C = \beta[K] + \alpha[M]$$

$$\alpha = 2\xi_i \left( \frac{\omega_i \omega_j}{\omega_i + \omega_j} \right) \quad (32)$$

$$\beta = 2\xi_i \left( \frac{1}{\omega_i + \omega_j} \right)$$

$$[\bar{K}] = [K] + a_0[M] + a_1[C] \quad (33)$$

$$\begin{aligned} \{\bar{F}_{t+\Delta t}\} &= \{F_{t+\Delta t}\} + [M] \left( a_0 \{U_t^w\} + a_2 \{\dot{U}_t^w\} + a_3 \{\ddot{U}_t^w\} \right) \\ &+ [C] \left( a_1 \{U_t^w\} + a_4 \{\dot{U}_t^w\} + a_5 \{\ddot{U}_t^w\} \right) \end{aligned} \quad (34)$$

$$\{U_{t+\Delta t}^w\} = [\bar{K}]^{-1} \{\bar{F}_{t+\Delta t}\} \quad (35)$$

Also, the velocity vector, in each time step, can be attained as

$$\{\dot{U}_{t+\Delta t}^w\} = a_1 \left( \{U_{t+\Delta t}^w\} - \{U_t^w\} \right) - a_4 \{\dot{U}_t^w\} - a_5 \{\ddot{U}_t^w\} \quad (36)$$

Lastly, the vector associated with accelerating in each time step is expressed as

$$\{\ddot{U}_{t+\Delta t}^w\} = a_0 \left( \{U_{t+\Delta t}^w\} - \{U_t^w\} \right) - a_2 \{\dot{U}_t^w\} - a_3 \{\ddot{U}_t^w\} \quad (37)$$

In which time interval as well as constants are

$$\begin{aligned} a_0 &= \frac{1}{\beta(\Delta t)^2} & a_1 &= \frac{\alpha}{\beta \Delta t} & a_2 &= \frac{1}{\beta \Delta t} & a_3 &= \frac{1}{2\beta} - 1 \\ a_4 &= \frac{\alpha}{\beta} - 1 & a_5 &= \frac{\Delta t}{2} \left( \frac{\alpha}{\beta} - 2 \right) \\ \alpha &= \frac{1}{2} & \beta &= \frac{1}{4} \end{aligned} \quad (38)$$

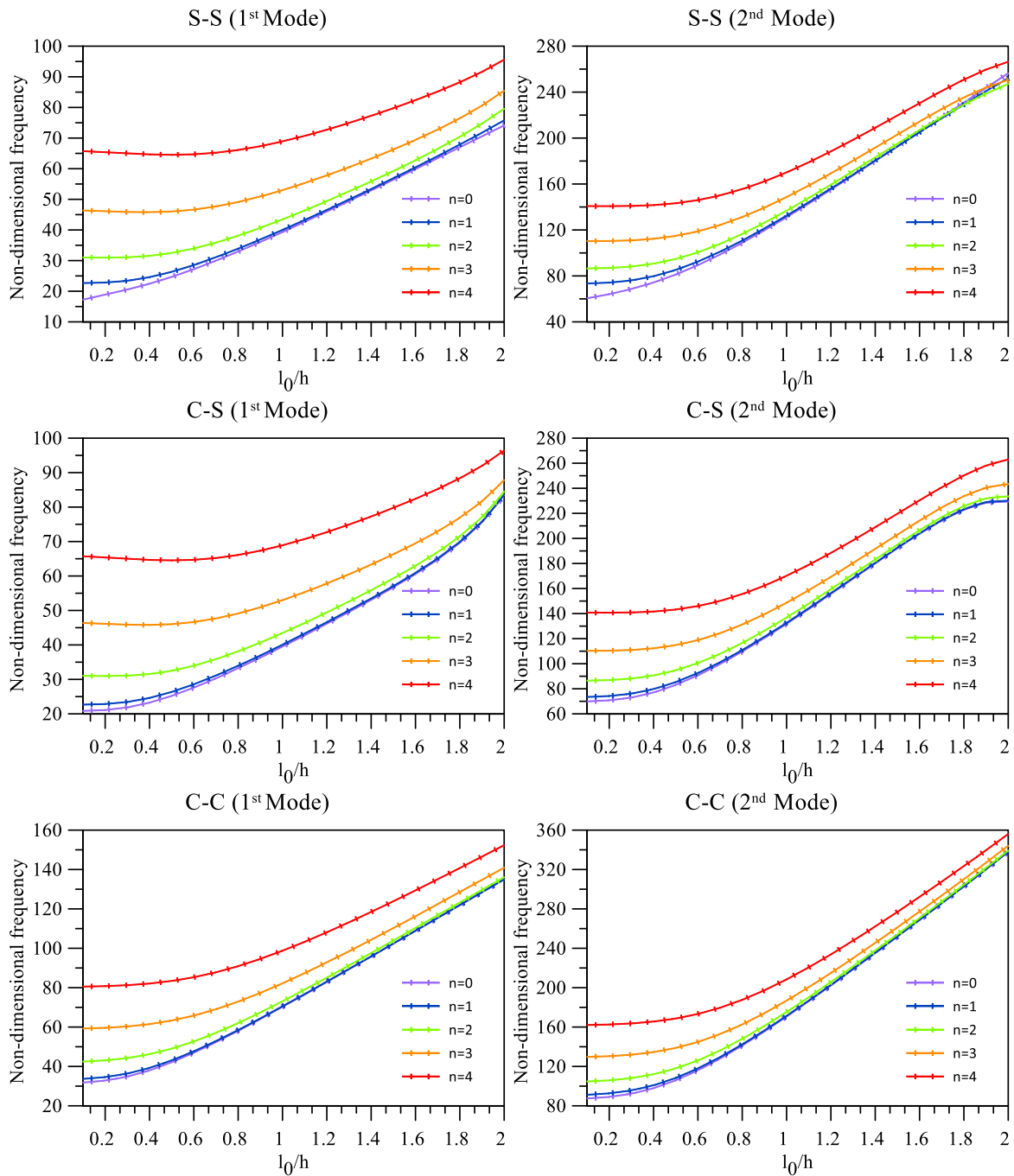


Fig. 2 First two nondimensional natural frequency of piezoelectric vs.  $l_0/h$

It should be mentioned that in order to obtain the harvested energy in the whole disk, the harvested voltages are integrated over the whole domain using GIQM procedure as explained in Ref. (Naderi *et al.* 2020)

#### 4. Result

In this section, first, using other papers' results, the formulation as well as the solution procedure are verified. Then, various parameters affecting the vibration along with energy harvesting of an annular piezoelectric microplate modeled by modified couple stress are examined in detail.

It is worthy to mention that the geometry and material properties related to the piezoelectric disk are shown in Table 2. Additionally, it should be mentioned that the damping ratio is  $\xi_i = 0.05$ .

Firstly, in Table 2, the first and second nondimensional  $\omega R_o \sqrt{\rho/C_{11}h^2}$  vibration frequency associated with piezoelectric microplate in which the piezoelectric and length effects are neglected are obtained for various  $P$  and clamped-clamped end conditions, and then the results are compared with those in Ref. (Leissa and Qatu 2011)

The results in this table indicate the very good agreement between the obtained results and those from

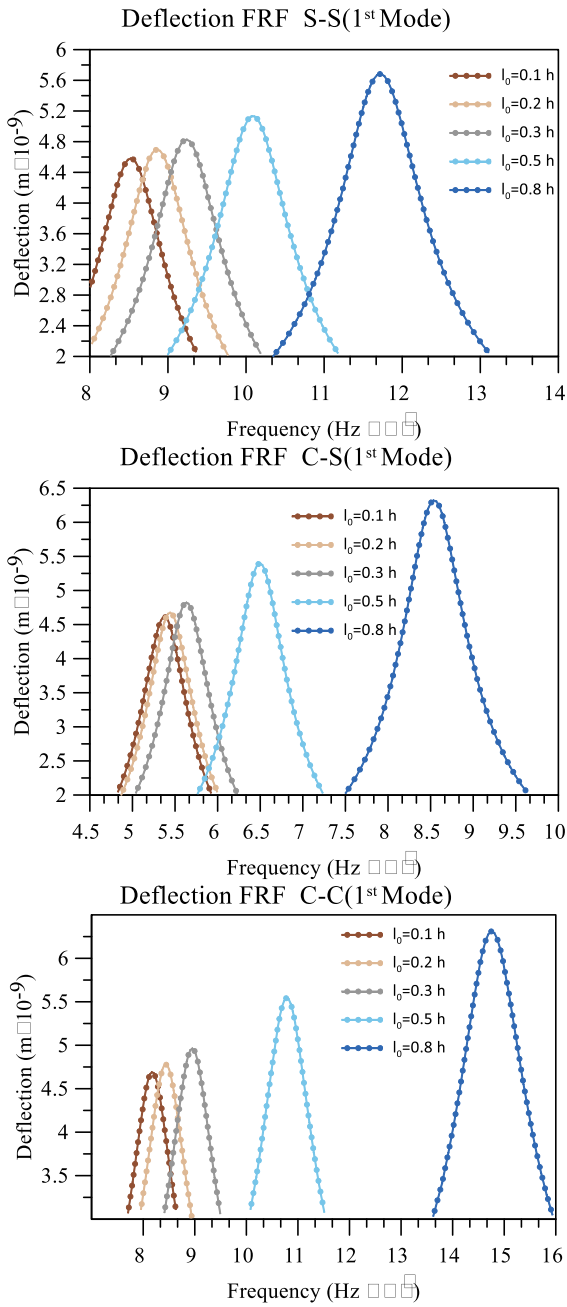


Fig. 3 Deflection FRFs of middle point of a piezoelectric micro disk subjected to base excitation for various  $l_0/h$  value

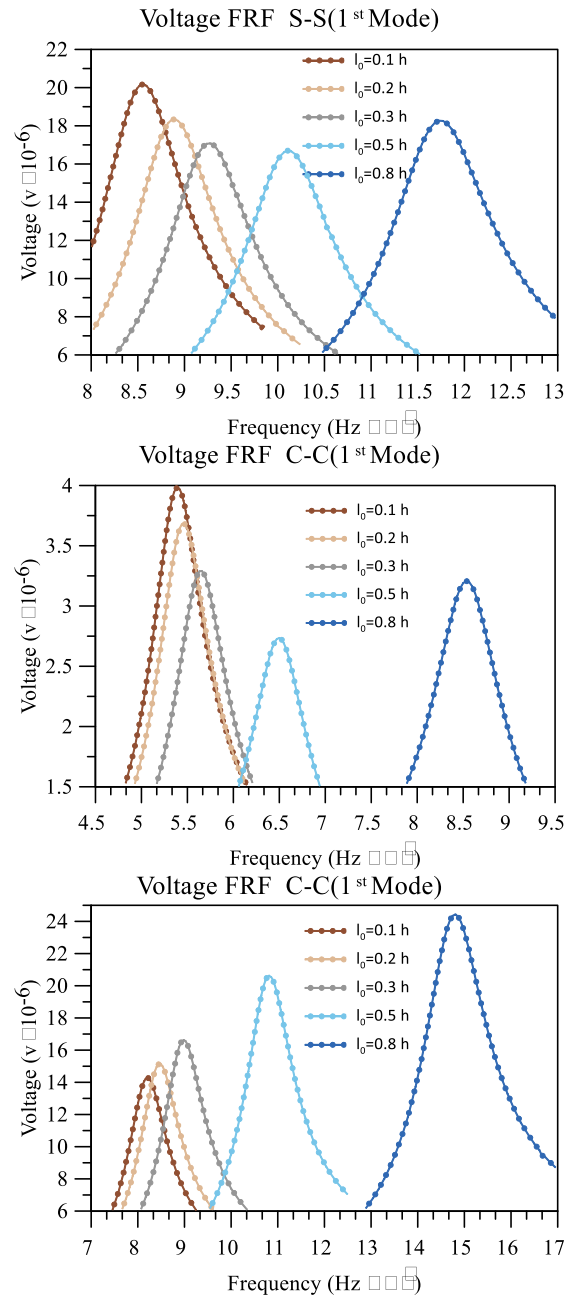


Fig. 4 Voltage FRFs for various  $l_0/h$  value of a piezoelectric microdisk subjected to base excitation

the reference, confirming the credibility of the formulation together with the solution procedure.

Now, the parameters, which play a significant role, in determining the vibration frequency and harvested energy for a piezoelectric micro disk modeled by modified couple stress are studied in detail. First, in order to investigate the length effect associated with the modified couple stress theory, the variation of the first and second vibrational frequency of a piezoelectric micro disk modeled by modified couple stress  $l_0/h$  against is shown in the Fig. 2 for various boundary conditions and  $n$ . It should be mentioned that the effects of the foundation is eliminated in these results.

As it is exhibited in Fig. 2, by intensifying the length scale of the disk, the vibration frequency in both of the modes increases. In addition, choosing higher mode number of vibration in  $\theta$ -direction leads in higher frequency, which is more observable in lower amount of  $l_0/h$ . Now, to examine the impact of  $l_0/h$  on the forced vibration as well as harvested energy of piezoelectric microdisks, Figs. 3 and 4 is presented. In these figures, respectively, the displacement FRF corresponded to the deflection of the middle of the disk and voltage FRF are presented for different amount of  $l_0/h$  and end conditions. It should be stated that the excitation frequency is around the first natural frequency of the disk in each state, and the foundations parameters are equal to zero.

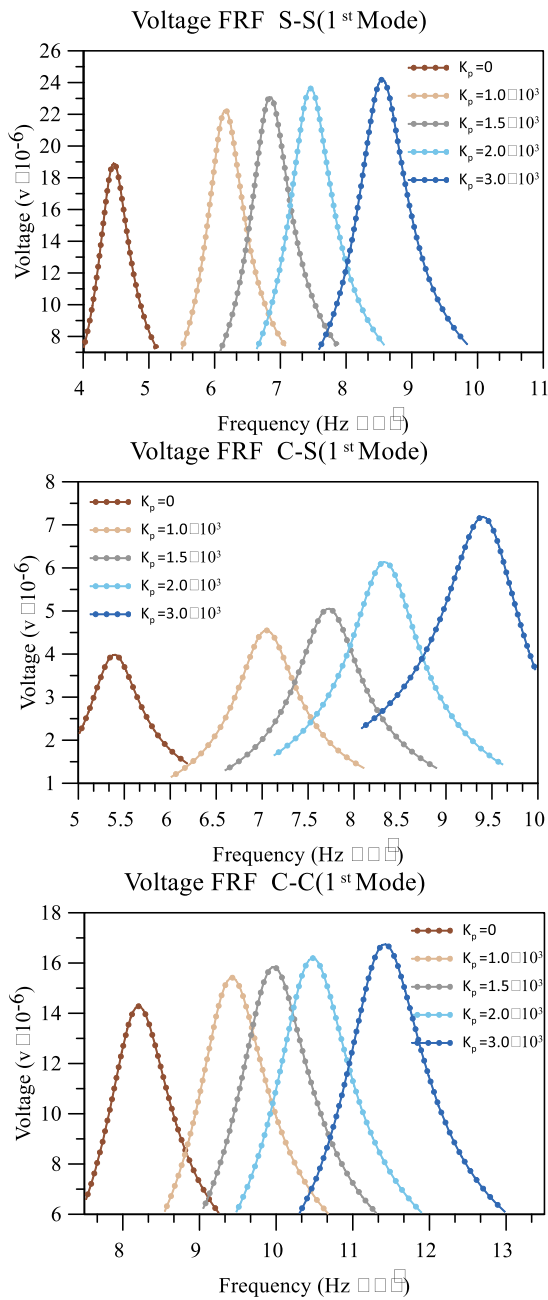


Fig. 6 Deflection FRF of middle point of a piezoelectric disk under base excitation for various  $K_p$

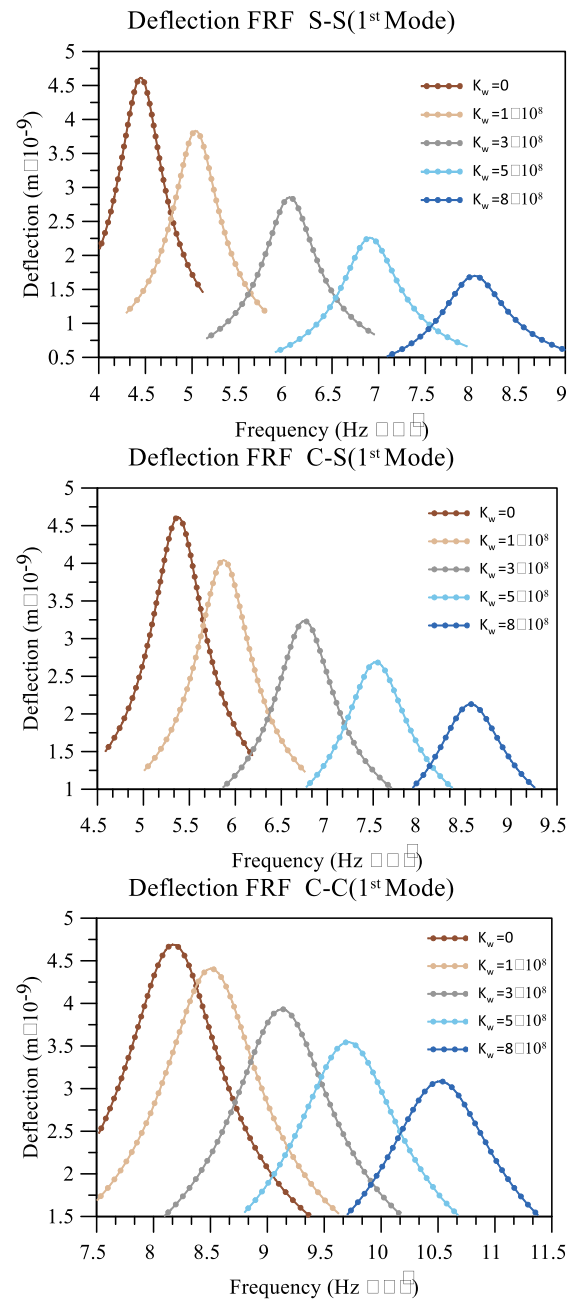


Fig. 7 Deflection FRF of middle point of a piezoelectric disk under base excitation for various  $K_w$

Fig. 3 reveals that intensifying the length scale not only move the peaks of deflection FRF to the right, which means that the vibrational frequency is increased, but also heighten up the value associated with the peaks, regardless of the boundary conditions.

This figure indicates that the higher the  $K_p$  is, the higher the peak values are, and also the peaks occur in higher excitation frequencies, in each of the boundary conditions.

In another word, the cases without the foundation effect has the lowest deflection peaks.

It can be comprehended from Fig. 4 that, despite the clamped end conditions, escalating  $l_0/h$  cause the peak of FRF voltage to have a decreasing- increasing effect, while in the clamped boundary condition, the values of the

voltage peaks increase with higher values of length scale. Now, the influence of parameters related to the foundations are investigated. Firstly, deflection FRF along with voltage FRF of piezoelectric micro disk subjected to a base excitation load is investigated for various  $K_p$  in Figs. 5 and 6, respectively. The frequency of the load is around the first natural vibration of the disk. Also, the other constant presenters in these figures are  $l_0/h = 0.1$ ,  $h = 0.01R_o$ ,  $n = 0$  and  $K_w = 0.0$ .

The conclusion which can be drawn from Fig. 7 is that stiffer Winkler foundation can cause the peaks of deflection FRF to reduces so much so that the case without the foundation effect have the highest peaks, regardless of the boundary conditions. Also, similar to the other foundation

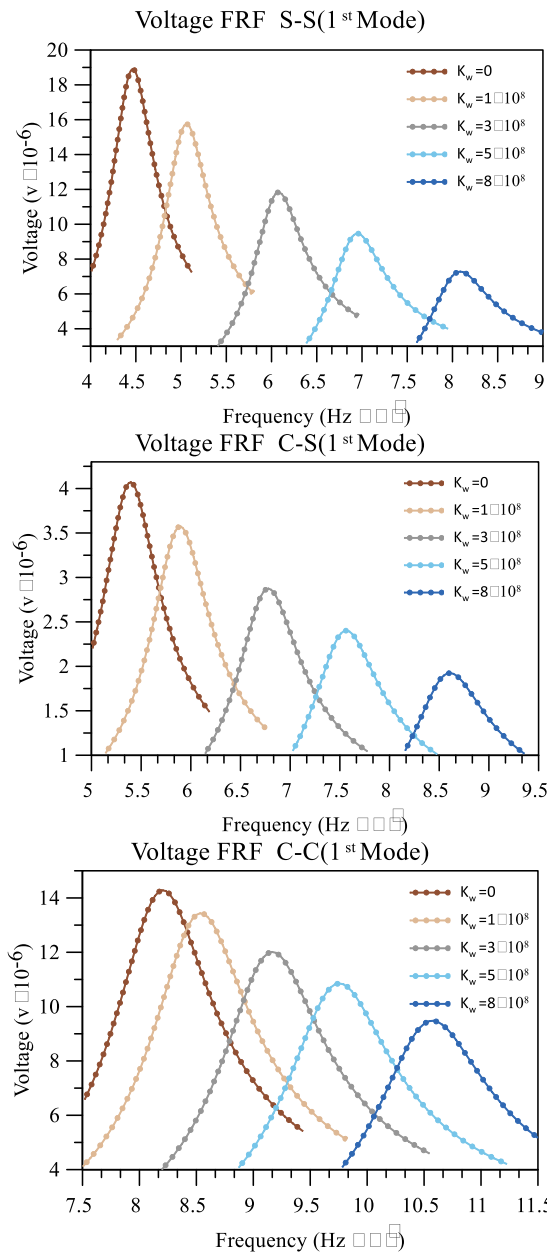


Fig. 8 Voltage FRF of a piezoelectric disk under base excitation for various  $K_w$

parameter, increasing this parameter leads in intensifying the peak frequency.

The highest voltage value, also, reduces as  $K_w$  increases based on Fig. 8. Similarly, the peaks excitation frequency is higher in cases with higher values for  $K_w$ . Therefore, it can be concluded that the highest voltage values occur when the Winkler foundation is at the lowest amount, and Pasternak foundation have higher values.

Lastly, the effects corresponded to inner to outer radius ratio of piezoelectric disk on its forced vibration and possibility of energy harvesting is investigated. To do so, displacement FRF along with the voltage FRF of the disk is plotted for four values of  $R_i/R_o$ . In addition, the effect related to foundations are neglected, and also the other constants are  $n = 0$ ,  $h = 0.01R_o$  and  $l_0/h = 0.1$ .

This figure indicates that the piezoelectric disks with

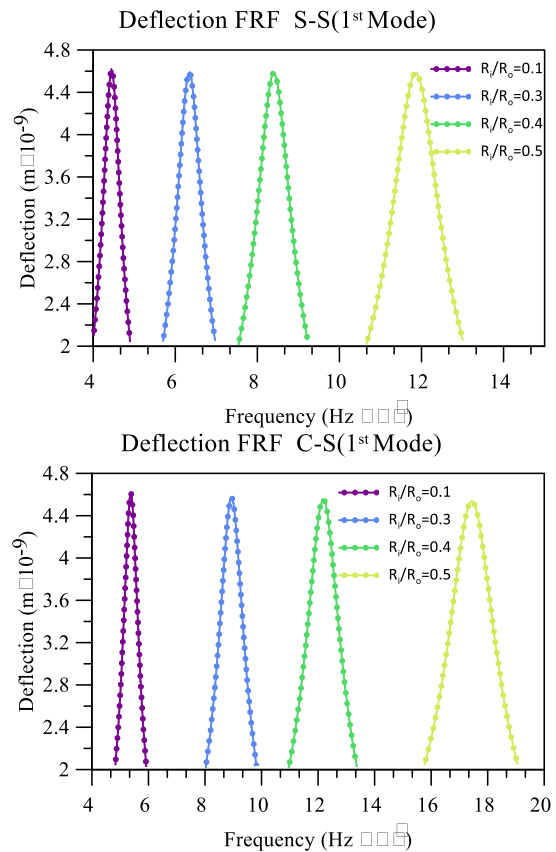


Fig. 9 Deflection FRF of middle point of a piezoelectric disk under base excitation for various  $R_i/R_o$

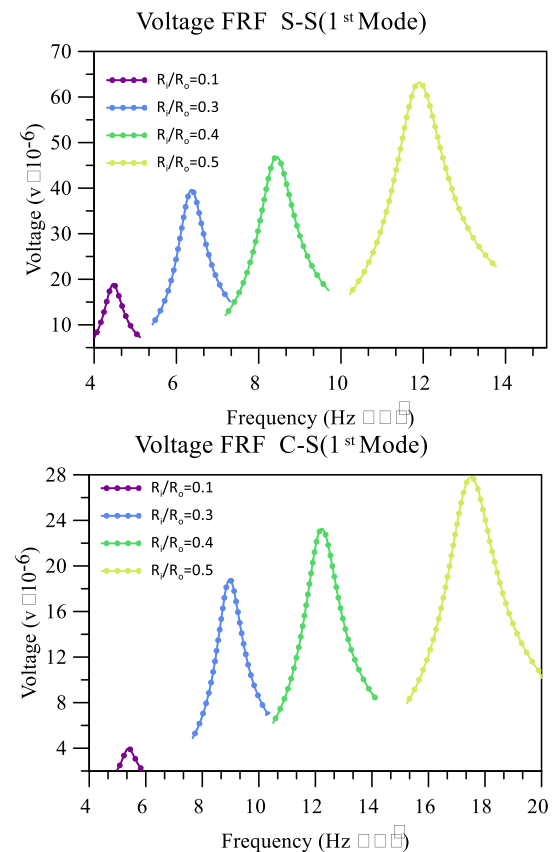


Fig. 10 Voltage FRF of a piezoelectric disk under base excitation for various  $R_i/R_o$

bigger inner radius possess higher excitation frequency, and also the highest value of deflection for the middle point of disks in cases with smaller inner radius is slightly higher.

Against the deflection FRF, the voltage FRFs demonstrate a compelling trend. Fig. 10 shows that by increasing the inner to outer radius ratio the possibility of energy harvesting increases so much so that the highest peak occurs in disks with highest value of  $R_i/R_o$ .

## 5. Conclusions

On the basis of modified couple stress and Kirchhoff plate theory, the free and forced vibration as well as possibility of electrical energy harvesting corresponded to a piezoelectric disk subjected to a base excitation and placed on a Winkler-Pasternak foundation is explored. Associated governing equations together with boundary conditions is extracted utilizing Hamilton's principle. Then, using GDQM in addition to Newmark-beta method, free and forced vibration results are obtained. Here are some of the conclusions can be drawn from this paper:

- Increasing length effect have an increasing impact on the vibration frequency, regardless of the boundary conditions.
- Higher values of  $l_0/h$  leads in higher peaks in deflection FRFs.
- Intensifying  $l_0/h$  cause Voltage FRFs to have decreasing and then increasing trend in cases with S-S and C-S boundary conditions, while, in C-C end condition, it cause an increasing effect only.
- Except C-S case for voltage FRF, intensifying  $K_p$  leads in escalating the peak in voltage and displacement FRFs.
- Displacement and voltage peaks decrease by increasing  $K_w$ .
- The highest amount of voltage can be harvested for disks with bigger inner radius.

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