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Impacts of surface irregularity on vibration analysis of single-walled carbon nanotubes based on Donnell thin shell theory

Mahmoud M. Selim*[1,2], Saad Althobaiti[3], I.S. Yahia[4,5,6], Ibtisam M.O. Mohammed[7], Amira M. Hussin[1] and Abdel-Baset A. Mohamed[1]

[1]*Department of Mathematics, Al-Aflaj College of Sciences and Humanities, Prince Sattam bin Abdulaziz University, Al-Aflaj, 11912, KSA*
[2]*Department of Mathematics, Suez Faculty of Science, Suez University, Egypt*
[3]*Department of Sciences and Technology, Ranyah University Collage, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia*
[4]*Laboratory of Nano-Smart Materials for Science and Technology (LNSMST), Department of Physics, Faculty of Science, King Khalid University, P.O. Box 9004, Abha 61413, Saudi Arabia*
[5]*Research Center for Advanced Materials Science (RCAMS), King Khalid University, P.O. Box 9004, Abha 61413, Saudi Arabia*
[6]*Nanoscience Laboratory for Environmental and Biomedical Applications (NLEBA), Semiconductor Lab., Department of Physics, Faculty of Education, Ain Shams University, Roxy, Cairo 11757, Egypt*
[7]*Department of Mathematics, Al_ukhwa College of science and Art, Al-Baha University, Al_mukhwa, Saudi Arabia*

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Abstract. The present work is an attempt to study the vibration analysis of the single-walled carbon nanotubes (SWCNTs) under the effect of the surface irregularity using Donnell`s model. The surface irregularity represented by the parabolic form. According to Donnell`s model and three-dimensional elasticity theory, a novel governing equations and its solution are derived and matched with the case of no irregularity effects. To understand the reaction of the nanotube to the irregularity effects in terms of natural frequency, the numerical calculations are done. The results obtained could provide a better representation of the vibration behavior of an irregular single-walled carbon nanotube, where the aspect ratio (L/d) and surface irregularity all have a significant impact on the natural frequency of vibrating SWCNTs. Furthermore, the findings of surface irregularity effects on vibration SWCNT can be utilized to forecast and prevent the phenomena of resonance of single-walled carbon nanotubes.
Keywords: Donnell thin shell theory; irregularity; single-walled carbon nanotubes; vibration analysis

1. Introduction

In 1991, Iijima (Japanese scientist) discovered fullerene-related carbon nanotubes (CNTs) (Iijima 1991). Since Iijima discovery, numerus publications in this filed have grown quickly, such as (Qian *et al*. 2002) and (Thostenson *et al*. 2001), fabrication (Dai 2002) and (Liu e*t al*. 2004) and applications of such materials (Collins and Avouris 2000, Baughman *et al*. 2002, Qian *et al*. 2002, Avouris *et al*. 2003, Choi *et al*. 2004, Selim 2006, 2007, 2009, 2010).

Nowadays, many researchers are increasingly interested in carbon nanotubes, due to their unique properties. SWCNTs have a stiffness and superior thermal conductivity due to their structure, which is manufactured from a single layer graphene sheet rolled into a cylindrical tube with a perfect hexagonal lattice. SWCNTs are ideal candidates for a wide range of applications in nanoelectromechanical technologies, biomedical equipment, fuel cells, and a diversity of other components due to its unique features (Ghavanloo and Fazelzadeh 2012). As a result, the problem of assembling CNTs into nanodevices with distinctive structure and function, as well as maximizing CNTs' superior performance, has arisen (Abuhimd *et al*. 2013, Wang *et al*. 2013, Chuen 2017, Uallah *et al*. 2018, 2019 a, b, 2019, 2020, Rajwali *et al*. 2022). Vibrations of CNTs occur during some manufacturing processes and there are numerous publications investigated the vibrational properties of CNTs (Selim 2011, 2020a, b, 2021). The free vibration of SWCNT has been studied extensively (Xu *et al*. 2003, Chowdhury *et al*. 2010, Arghavan *et al*. 2011, Strozzi *et al*. 2014, Liu *et al*. 2015, Rakrak *et al*. 2016, Belhadj *et al*. 2017, Ajri *et al*. 2018, Preethi *et al*. 2018, Rajasekaran *et al*. 2018, Zargaripoor *et al*. 2018, Ebrahimi *et al*. 2018, Yi *et al*. 2019, Dehshahri *et al*. 2020), using continuum shell models during the last decade. In recent year, under external pressure, nonlinear vibration hygrothermal analysis of imperfect functionally graded carbon nanotube-reinforced composite has been examined (Foroutana *et al*. 2019, 2020). Due of technical restrictions, these models could not be used to predict the vibrational behavior of irregular SWCNT. The nature of the vibration analysis of single-walled carbon nanotubes is obviously influenced by changes in the nanotube's surface structures. Irregularities in carbon nanotube construction might emerge as a result of manufacturing flaws, careless servicing, environmental damage, and so on. As a result, interacting with various constructions to study the vibrational analysis of single-walled carbon nanotubes is essential. The effect of surface irregularity on natural frequencies of SWCNTs has not yet been investigated, to the best of the author's knowledge, and the present work is an attempt to show the effects of surface irregularity on vibration of SWCNTs. The Donnell`s

*Corresponding author, Professor,
E-mail: m.selim@psau.edu.sa

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model is applied to the governing equation to derive the stress and displacement fields due to its efficiency. Carbon nanotubes have structures that are comparable to those of cylindrical shells. This method can be used to investigate the vibrations of SWCNTs. MATLAB software is used to calculate the frequency of SWCNTs. It is noted that the current data can be used to analyze the influence of surface irregularity on SWCNTs with small modifications. It is concluded that, the irregularity parameter has a substantial impact on the vibration frequencies of SWCNTs, according to the findings, which could be useful for future research and accurate nanomachine design.

2. Theory and formulation

The cylindrical coordinates system, which is defined in Fig. 1, is used to describe the effect of surface irregularity on vibration analysis of the single-walled carbon nanotube. h, R and L thickness, radius and length of the considered SWCNT. The irregularity is considered as (Selim 2020 a, b):

$$z = \left(R + h + \varepsilon\, \delta(x)\right),$$
$$\delta(x) = \begin{cases} s(1 - \frac{4x^2}{s^2}) & \text{for} \quad |x| < \frac{s}{2}, \\ 0 & \text{for} \quad |x| \geq \frac{s}{2}, \end{cases} \tag{1}$$

where the maximum amplitude $\varepsilon = \frac{H'}{s} << 1$ represent the irregular boundary.

According to the three-dimensional elasticity theory (Shaban and Alibeigloo 2014), the governing equation of free harmonic vibration of SWCNTs with simply supported edges is given by

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{1}{R}\frac{\partial \sigma_{x\phi}}{\partial \phi} + \frac{\partial \sigma_{xz}}{\partial z} + \frac{\sigma_{xx} - \sigma_{\phi\phi}}{R} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{2}$$

$$\frac{\partial \sigma_{x\phi}}{\partial x} + \frac{1}{R}\frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{\partial \sigma_{\phi z}}{\partial z} + \frac{2\sigma_{x\phi}}{R} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{3}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{1}{R}\frac{\partial \sigma_{\phi z}}{\partial \phi} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{xz}}{R} = \rho \frac{\partial^2 w}{\partial t^2}, \tag{4}$$

where ρ is the density, $\sigma_{xx}, \sigma_{\phi\phi}, \sigma_{zz}, \sigma_{xz}, \sigma_{x\phi}$ and $\sigma_{\phi z}$ are the stress components and the displacement components u, v and $w$are represent the components in the radial, circumferential and axial directions.

For the present problem the following edges conditions will be satisfied:

$$\sigma_{xx} = \sigma_{\phi\phi} = \sigma_{zz} = 0. \tag{5}$$

2.1 Donnell thin shell theory for the vibration of SWCNT

Using the Donnell's model (He *et al*. 2006), the governing Eqs. (2)-(4) can be obtained as:

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{6}$$

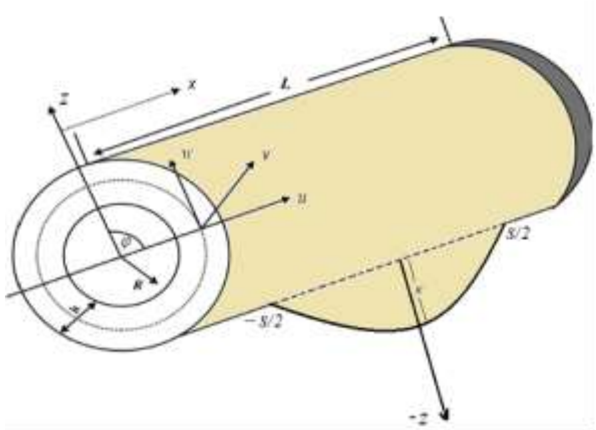

Fig. 1 Geometry of SWCNT with parabolic surface irregularity

where $D_{ij}(i, j = 1,2,3)$ are given by:

$$D_{11} = \frac{\partial^2}{\partial x^2} + \frac{1 - v}{2R^2}\frac{\partial^2}{\partial \phi^2} - \beta\rho\ h\frac{\partial^2}{\partial t^2}, \tag{7}$$

$$D_{12} = D_{21} = \frac{1 + v}{2R}\frac{\partial^2}{\partial x \partial \phi}, \tag{8}$$

$$D_{13} = \frac{-v}{R}\frac{\partial}{\partial x} = -D_{31}, \tag{9}$$

$$D_{22} = \frac{1 - v}{2}\frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \phi^2} - \beta\rho\, h\frac{\partial^2}{\partial t^2}, \tag{10}$$

$$D_{23} = \frac{-1}{R^2}\frac{\partial}{\partial \phi} = -D_{32}, \tag{11}$$

$$D_{33} = -\left[\frac{1}{R^2} + \frac{(1 - v^2)\alpha}{Eh}\left(\frac{\partial^4}{\partial x^4} + \frac{2}{R^2}\frac{\partial^4}{\partial x^2 \partial \phi^2} + \frac{1}{R^4}\frac{\partial^4}{\partial \phi^4}\right) + \beta\rho h\frac{\partial^2}{\partial t^2}\right], \tag{12}$$

where $\alpha = \frac{Eh^3}{12(1-v^2)}$ represents the effective bending stiffness, and $\beta = \frac{(1-v^2)}{Eh}$, E (Elastic modulus), ν (Poisson's ratio), E (rigidity) and t (Time).

3. Solution procedure

The displacements of the deformation can be written in the following form (Hussain *et al*. 2017):

$$u = u^0(x, \theta, t) - z\frac{\partial w(x, \theta, t)}{\partial x}, \tag{13}$$

$$u^0(x, \theta, t) = U_m e^{-i(k_m x - \omega t)} sin(n\theta) \tag{14}$$

$$v(x, \theta, t) = V_m e^{-i(k_m x - \omega t)} cos(n\theta), \tag{15}$$

$$w(x, \theta, t) = W_m e^{-i(k_m x - \omega t)} sin(n\theta), \tag{16}$$

where the vibration amplitude coefficients in the axial, circumferential, and radial directions are denoted by U_m, V_m and W_m respectively, u^0 is stretching the midline and k_m is the axial wave number.

The *m* and *n* are the axial and circumferential half wave numbers, respectively. The relation between angular frequency (ω) and the natural frequency is given from the following formula:

$$f = \omega/2\pi. \tag{17}$$

Using the operators in Eqs. (7)-(12) and expressions for u, v and $w$given in Eqs. (13)-(16), The vibration frequency equation of single-walled carbon nanotubes with surface irregularity based on Donnell`s model is designated by converting the Eq. (6) into matrix notation and solving it using the following Eigen-value problem approach:

$$\begin{bmatrix} L_{11}+\eta\omega^2 & L_{12} & \hat{L}_{13} \\ L_{21} & L_{22}+\eta\omega^2 & \hat{L}_{23} \\ L_{31} & L_{32} & \hat{L}_{33}+\eta\omega^2 \end{bmatrix} \begin{bmatrix} U_m \\ V_m \\ W_m \end{bmatrix} = 0 \tag{18}$$

Using the Appendix, the elements of the above matrix can be written as follows:

$$L_{11} = -{k_m}^2 - \left(\frac{1-v}{2R^2}\right) n^2, \tag{19}$$

$$L_{12} = ik_m \left(\frac{1+v}{2R}\right) n = -L_{21}, \tag{20}$$

$$\hat{L}_{13} = ik_m \left[\frac{v}{R} + \left[\left({k_m}^2 (R+h+\varepsilon\,\delta(x)) - 16ik_m\varepsilon\frac{x}{s} + \frac{8\varepsilon}{s}\right) + (R+h+\varepsilon\,\delta(x))\left(\frac{1-v}{2R^2}n^2 + \eta\,\omega^2\right)\right]\right] \tag{21}$$

$$L_{22} = -{k_m}^2\left(\frac{1-v}{2}\right) - \frac{n^2}{R^2}, \tag{22}$$

$$\hat{L}_{23} = \frac{-n}{R^2} + n\left(\frac{1+v}{2R}\right)\left(-{k_m}^2(R+h+\varepsilon\,\delta(x)) + ik_m\frac{8\varepsilon x}{s}\right), \tag{23}$$

$$L_{31} = -ik_m\left(\frac{v}{R}\right), \tag{24}$$

$$L_{32} = -\frac{n}{R^2}, \tag{25}$$

$$\hat{L}_{33} = \frac{-1}{R^2} - \frac{(1-v^2)\alpha}{Eh}\left({k_m}^4 + \frac{2}{R^2}{k_m}^2 n^2 + \frac{n^4}{R^4}\right) + \left(\frac{v}{R}\right)\left(-{k_m}^2(R+h+\varepsilon\,\delta(x)) + ik_m\frac{8\varepsilon x}{s}\right) \tag{26}$$

$$\eta = \frac{(1-v^2)\rho}{\mathrm{E}} \tag{27}$$

The non-zero solution of (U_m, V_m, W_m), which gives the vibration natural frequency and the shape modes of the irregular single-wall carbon nanotube, can be obtained from vanishing of the determinant of matrix (18) as follows:

$$det\begin{bmatrix} L_{11}+\eta\omega^2 & L_{12} & \hat{L}_{13} \\ -L_{21} & L_{22}+\eta\omega^2 & \hat{L}_{23} \\ L_{31} & L_{32} & \hat{L}_{33}+\eta\omega^2 \end{bmatrix} = 0, \tag{28}$$

As seen from Eq. (28), the lowest root of the eigenvalues gives the natural frequencies of irregular nanotube and the related vector $(U_m, V_m, W_m)^T$ gives the mode shapes.

In the current problem, we assume the axial wave number take the form $k_m = \frac{\mathrm{m}\pi}{L}$ (Hussain *et al*. 2017).

4. Particular case

When the surface irregularity is absent$(\text{i.e.}, \ \varepsilon = 0)$, Eq. (17) becomes

$$\begin{bmatrix} L_{11}+\eta\omega^2 & L_{12} & L_{13} \\ L_{21} & L_{22}+\eta\omega^2 & L_{23} \\ L_{31} & L_{32} & L_{33}+\eta\omega^2 \end{bmatrix} \begin{bmatrix} U_m \\ V_m \\ W_m \end{bmatrix} = 0 \tag{29}$$

where

$$L_{13} = ik_m\left[\frac{v}{R}\right],\ L_{23} = \frac{n}{R^2},\quad L_{33} = \frac{-1}{R^2} - \frac{(1-v^2)\alpha}{Eh}\left({k_m}^4 + \frac{2}{R^2}{k_m}^2 n^2 + \frac{n^4}{R^4}\right), \tag{30}$$

which is consistent with the finding of Hussain *et al*. (2017).

5. Numerical results and discussion

Using the parameters listed in Table 1 (Zhang *et al*. 2009), numerical findings for vibration of simply supported SWWCN with surface irregularity are reported in this section. As a function of time, the natural frequency (THz) for the simply supported SWCNTs at first vibration mode (*m* = *n* =*1*) have been investigated using the Donnell`s model (27) and the results are reported. The obtained results (variations in natural frequencies) are contrasted and explained in comparison to the uniform nanotube (i.e., $\varepsilon = 0.0$).

Figs. 2-4, show the effects of surface irregularity on the natural frequencies of the SWCNT. To show the effects of surface irregularity parameter, for varying values of the aspect ratio (L/d) of SWCNT, a series of simulations are performed for the natural frequencies of SWCNT. Figs. 2-4 show the natural frequency (THz) versus the L/d ratio for various values of the surface irregularity parameter ($\varepsilon = 0, 0.28, 0.56$).

Fig. 2 shows the variations of natural frequencies of uniform ($\varepsilon = 0.0$) single-walled carbon nanotube for different values of (*L/d* = 5-65). It is observed that, the range values of the natural frequencies for the uniform single-walled carbon nanotubes are quite different. The values change from (0.5-1.8) terahertzes for (*L/d* = 0-21) and (*L/d* = 27-67), but the values change from (0.5-4.0) terahertzes for the aspect ratio interval (22-26). The reasons for the discrepancy could be due to the diameter of carbon nanotubes increasing or decreasing.

Table 1 Simulation parameters of armchairs nanotube

h	E/ρ	υ	Eh	R
0.34 nm	3.6481×10^8 m^2 / s^2	0.2	278.25 Gpa·nm	0.343322 nm

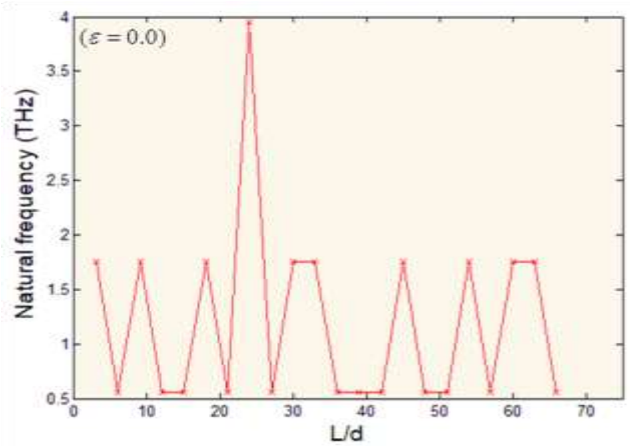

Fig. 2 Variations of natural frequencies of uniform SWCNT $(\varepsilon = 0.0)$ versus aspect ratio (L/d)

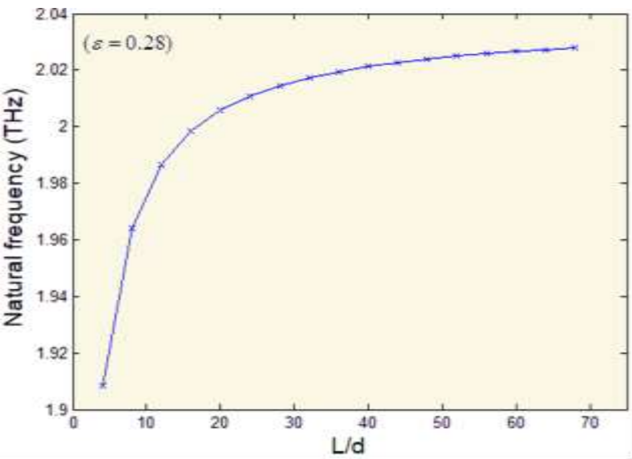

Fig. 3 Influences of the parabolic irregularity $(\varepsilon = 0.28)$ on the natural frequencies of SWCNT versus aspect ratio (L/d)

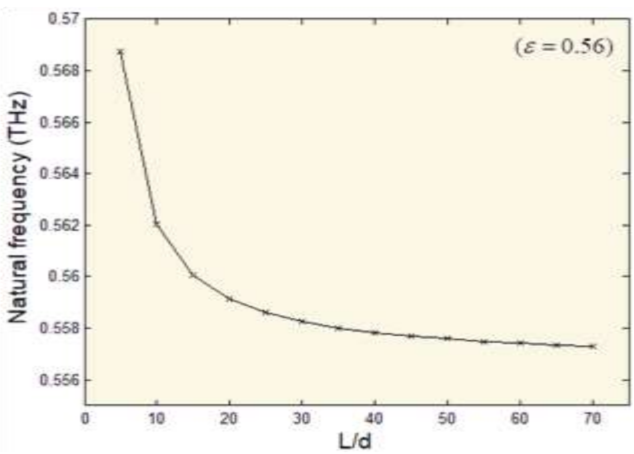

Fig. 4 Influences of the parabolic irregularity $(\varepsilon = 0.56)$ on the natural frequencies of SWCNT versus aspect ratio (L/d)

The impact of surface irregularity ($\varepsilon = 0.28$) on the natural frequencies for different values of (*L/d* = 5-65) of SWCNT is demonstrated in (Fig. 3). In this figure, it is observed that as the aspect ratio increase, the fundamental frequencies increase. It is clearly that the values of the fundamental frequencies are quite different, comparison with the case of the uniform single-walled carbon nanotubes (i.e., $\varepsilon = 0.0$). The values of the fundamental frequencies change from (1.9-2.02) terahertzes for (*L/d* = 5-65). The reason for the alteration is obvious: the carbon nanotube's surface irregularities are to cause.

The surface irregularity impact gets more noticeable when the surface irregularity parameter is increased ($\varepsilon = 0.56$) as shown in Fig. 4. The influence of the surface irregularity parameter is seen in this diagram ($\varepsilon = 0.56$) on the natural frequencies. The natural frequencies decrease, when the aspect ratio (L/d = 5-65) is increasing. When the nanotube is long, the fall in natural frequencies value is most significant. However, when compared to the vibration of nanotubes with a uniform surface ($\varepsilon = 0.0$), the surface irregularity parameter has a greater effects on the natural frequencies of the irregular nanotubes. From Figs. 2 and 4, one can observed that, the existence of surface irregularities in SWCNTs has an effect on the natural frequencies of SWCNTs, as shown in Figs. 2 and 4.

6. Conclusions

It is observed that, the effect of surface irregularity on vibration analysis of single-walled carbon nanotubes has not yet been investigated, the present work is an effort to show the effects of surface irregularity on natural frequencies of SWCNTs using the Donnell`s model. Surface irregularities and aspect ratio have an impact on the natural frequencies of single-walled carbon nanotubes. The numerical findings of this study revealed that the vibration characteristics are dependent on the nanotube length as well as the surface irregularity parameter. Furthermore, when the aspect ratio (L/d) is increased, the natural frequencies are observed to decrease. In addition, the decrease in natural frequency value is more evident when the nanotube is long. The main reason for these differences may be due to the increasing or decreasing of carbon nanotube diameter. However, it is observed, that the natural frequencies of SWCNT is more affected by the surface irregularity parameter ($\varepsilon = 0.28, 0.56$) compared with the case of uniform surface ($\varepsilon = 0.0$) of SWCNT. It is concluded that, the presence of surface irregularities in the single-walled carbon nanotubes, effects on the natural frequencies of the SWCNTs. Finally, the results of the present investigation may be provide a useful information for the next generation studies and accurate deigns of nanomachines, which irregular carbon nanotubes are the most common nano element.

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Appendix

$$\frac{\partial w(x,\varphi,t)}{\partial x} = -ik_m w(x,\varphi,t),$$

$$I_1 = e^{-i(k_m x-\omega t)}\,sin(n\varphi),$$
$$I_2 = e^{-i(k_m x-\omega t)}\,cos(n\varphi)$$

$$D_{11}u = \left(\frac{\partial^2}{\partial x^2} + \frac{1-v}{2R^2}\frac{\partial^2}{\partial \varphi^2} - \eta\frac{\partial^2}{\partial t^2}\right)u$$
$$= \frac{\partial^2 u}{\partial x^2} + \frac{1-v}{2R^2}\frac{\partial^2 u}{\partial \varphi^2} - \eta\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left(u^0(x,\varphi,t)\right)$$
$$+\frac{1-v}{2R^2}\frac{\partial^2}{\partial \varphi^2}\left(u^0(x,\varphi,t)\right) \quad - \eta\frac{\partial^2}{\partial t^2}\left(u^0(x,\varphi,t)\right) -$$
$$-ik_m\begin{bmatrix}\frac{\partial^2}{\partial x^2}\left(\left(R+\hbar+\varepsilon\,\delta(x)\right)w(x,\varphi,t)\right)\\ +\frac{1-v}{2R^2}\frac{\partial^2}{\partial \theta^2}\left(\left(R+\varepsilon\,\delta(x)\right)w(x,\varphi,t)\right)\end{bmatrix}+$$
$$+ik_m\eta\left(R+\hbar+\varepsilon\,\delta(x)\right)\frac{\partial^2}{\partial t^2}w(x,\varphi,t)$$
$$= -\left[\left({k_m}^2 + \frac{1-v}{2R^2}n^2\right) - \eta\omega^2\right]I_1\times U_m$$
$$-ik_m\begin{bmatrix}\begin{pmatrix}-{k_m}^2\left(R+\hbar+\varepsilon\,\delta(x)\right)\\ -16ik_m\frac{\varepsilon x}{s} - \frac{8\varepsilon}{s}\end{pmatrix}+\\ -\frac{1-v}{2R^2}n^2\left(R+\hbar+\varepsilon\,\delta(x)\right)\\ -\eta\left(R+\hbar+\varepsilon\,\delta(x)\right)\omega^2\end{bmatrix}I_1\times W_m,$$

$$D_{12}v = \frac{1+v}{2R}\frac{\partial^2 v}{\partial x\partial\varphi} = ik_m n\left(\frac{1+v}{2R}\right)I_1\times V_m$$

$$D_{13}w = \frac{-v}{R}\frac{\partial w}{\partial x} = ik_m\left(\frac{v}{R}\right)I_1\times W_m$$

$$D_{21}u = \left(\frac{1+v}{2R}\right)\frac{\partial^2 u}{\partial x\partial\theta} = \frac{1+v}{2R}\frac{\partial^2}{\partial x\partial\theta}\left(u^0(x,\varphi,t)\right)$$
$$-ik_m\left(\frac{1+v}{2R}\right)\frac{\partial^2}{\partial x\partial\varphi}\left(\left(R+\varepsilon\,\delta(x)\right)w(x,\varphi,t)\right)$$
$$= -ik_m n\left(\frac{1+v}{2R}\right)I\times U_m - ik_m\left(\frac{1+v}{2R}\right)\frac{\partial}{\partial\varphi}\left(\frac{\partial}{\partial x}\begin{pmatrix}\left(R+\hbar+\varepsilon\,\delta(x)\right)\\ w(x,\varphi,t)\end{pmatrix}\right)$$
$$= -ik_m n\left(\frac{1+v}{2R}\right)I\times U_m$$
$$-ik_m\left(\frac{1+v}{2R}\right)\frac{\partial}{\partial\varphi}\begin{pmatrix}-ik_m\left(R+\hbar+\varepsilon\,\delta(x)\right)w(x,\theta,t)\\ -\frac{8\varepsilon x}{s}w(x,\varphi,t)\end{pmatrix}$$
$$= -ik_m n\left(\frac{1+v}{2R}\right)I_2\times U_m$$
$$+ik_m n\left(\frac{1+v}{2R}\right)\left(ik_m\left(R+\hbar+\varepsilon\,\delta(x)\right) + \frac{8\varepsilon x}{s}\right)I_2\times W_m,$$

$$D_{22}v = \frac{1-v}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 v}{\partial\varphi^2} - \eta\frac{\partial^2 v}{\partial t^2}$$
$$= \left(-{k_m}^2\left(\frac{1-v}{2}\right) - \frac{n^2}{R^2} + \eta\omega^2\right)I_2\times V_m,$$

$$D_{23}w = \frac{-1}{R^2}\frac{\partial w}{\partial\varphi} = \left(\frac{-n}{R^2}\right)I_2\times W_m,$$

$$D_{31}u = \left(\frac{v}{R}\right)\frac{\partial u}{\partial x} = \frac{v}{R}\frac{\partial}{\partial x}\left(u^0(x,\varphi,t)\right)$$
$$-ik_m\left(\frac{v}{R}\right)\frac{\partial}{\partial x}\left(\left(R+h+\varepsilon\,\delta(x)\right)w(x,\varphi,t)\right)$$
$$= -ik_m\left(\frac{v}{R}\right)I_1\times U_m - ik_m\left(\frac{v}{R}\right)\frac{\partial}{\partial x}\left(\left(R+h+\varepsilon\,\delta(x)\right)w(x,\varphi,t)\right)$$
$$= -ik_m\left(\frac{v}{R}\right)I_1\times U_m + ik_m\left(\frac{v}{R}\right)\left(ik_m(R+h+\varepsilon\,\delta(x)) + \frac{8\varepsilon x}{s}\right)I_1\times W_m.$$

$$D_{32}v = \frac{1}{R^2}\frac{\partial v}{\partial\varphi} = \left(\frac{-n}{R^2}\right)I_1\times V_m,$$

$$D_{33}w = -\left[\frac{1}{R^2} + \frac{(1-v^2)\alpha}{Eh}\left(\frac{\partial^4}{\partial x^4} + \frac{2}{R^2}\frac{\partial^4}{\partial x^2\partial\varphi^2} + \frac{1}{R^4}\frac{\partial^4}{\partial\varphi^4}\right)\right]w - \eta\frac{\partial^2 w}{\partial t^2}$$
$$= -\left(\frac{1}{R^2} + \frac{(1-v^2)\alpha}{Eh}\left({k_m}^4 + \frac{2}{R^2}{k_m}^2 n^2 + \frac{n^4}{R^4}\right) + \eta\omega^2\right)I_1\times W_m$$