

# Simulate of edge and an internal crack problem and estimation of stress intensity factor through finite element method

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**Abstract.** In this study, the elastic plane problem of a layered composite containing an internal or edge crack perpendicular to its boundaries in its lower layer is examined using numerical analysis. The layered composite consists of two elastic layers having different elastic constants and heights. Two bonded layers rest on a homogeneous elastic half plane and are pressed by a rigid cylindrical stamp. In this context, the Finite Element Method (FEM) based software called ANSYS is used for numerical solutions. The problem is solved under the assumptions that the contacts are frictionless, and the effect of gravity force is neglected. A comparison is made with analytical results in the literature to verify the model created and the results obtained. It was found that the results obtained from analytical formulation were in perfect agreements with the FEM study. The numerical results for the stress-intensity factor (SIF) are obtained for various dimensionless quantities related to the geometric and material parameters. Consequently, the effects of these parameters on the stress-intensity factor are discussed. If the FEM analysis is used correctly, it can be an efficient alternative method to the analytical solutions that need time.

**Keywords:** crack analysis; finite element method; fracture mechanics; stress-intensity factor

## 1. Introduction

Useful mechanical parts and materials, which progress with the changing and developing technology, is one of the main subjects of engineers. For this reason, analytical, numerical and experimental studies on the different behavior of mechanical components made of different types of materials have attracted the attention of researchers (Hajmohammad *et al.* 2018, Javani *et al.* 2019, Tahir *et al.* 2021, Mudhaffar *et al.* 2021, Zaitoun *et al.* 2021, Bouafia *et al.* 2021, Huang *et al.* 2021, Vinh and Tounsi 2021, Merazka *et al.* 2021, Zerrouki *et al.* 2021, Al-Osta *et al.* 2021, Bakoura *et al.* 2021, Guellil *et al.* 2021, Bellifa *et al.* 2021). Structure and machine elements are exposed to stress, deformation, and displacement under the influence of external load and environmental conditions. One of the most important steps in the dimensioning of engineering structures is to decide on the final shape and dimensions of the building to perform its duty under these external load and environmental conditions. For this, a critical load value and a parameter that defines the material are compared. The critical load factor depends on the external load applied, environmental conditions and geometry of the object. The material parameter is a measure of the strength of the material. To make this comparison, it is necessary to know under which conditions the strength of the object is exceeded. The most important factors affecting the exceeding of the strength of the object are mechanical behavior of the material, environmental conditions (temperature, humidity rate), character of external loads

(especially their dependence on time), and geometry of the object.

The loss of the strength of the structure under the influence of one or more of these factors causes the following situations;

- Fatigue
- Yielding, creep and excessive deformation
- Chemical structure change, corrosion
- Deterioration of stability (Buckling)
- Fracture (Knott 1973).

One of the factors that cause the strength of the structure to be destroyed is a fracture, as mentioned above. Fracture is generally the creation of new surfaces in the material under extreme loads. Fracture mechanics is the investigation of the strength of a member containing a crack. For investigation with fracture mechanics, structures are assumed to have initial cracks. The progression of these cracks will cause major problems for the structural members in the future.

The imperfections in the material's internal structure grow due to the factors mentioned above and the strength of the material decreases significantly. Therefore, crack problems have become an important issue in fracture mechanics recently. The SIF is the mostly widely used value in the practice of fracture criteria. The SIF defines the stress area near the tip of the crack and provides information about the behavior of the crack. For this reason, a lot of research has been done on this subject in recent years. Using different methods such as integral transform technique, finite element method, and other computational methods, many crack problems have been handled. A number of these are mentioned below as well.

The studies on the mechanical behaviors of structural and mechanical elements with different solution techniques

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have been the focus of interest of scientists (Alimirzaei *et al.* 2019, Maleki *et al.* 2019, Farokhian and Kolahchi 2020, Shariati *et al.* 2020, Azizkhani *et al.* 2020, Kumar *et al.* 2021, Huang *et al.* 2021, Al-Furjan *et al.* 2020a, b, c, 2021a, b, Bekkaye *et al.* 2021, Hachemi *et al.* 2021). Nowadays, there have been great developments in the fields of computer and numerical computing. So, the results have been obtained more precisely and faster. Due to these developments, the crack problem has been solved through different methods. Previous studies on crack problems have mostly focused on finding the biggest load that the structural element containing cracks can bear. The most important studies on this subject were made by Griffith (1920, 1924). According to Griffith, defects in the material play an important role in the loss of strength. Barenblatt (1962) stated in his study that in the solutions made with the theory of elasticity, there will be no endless stresses obtained at the crack ends. He suggested taking into account the cohesion zone at the ends and removing the singularity. Krenk (1973) dealt with the problem of an elastic layer with an inclined crack in terms of symmetry axes. The problem was reduced to a singular integral equation and the SIF was calculated by solving this integral equation numerically. Erdol and Erdogan (1976) calculated the SIF in the crack problem in the elastic layer that sits on simple supports and has perpendicular symmetrical cracks. The finite element model was developed by Vander and Grootenboer (1986) to determine the behavior of cracks in materials with different elastic properties. Determination of elastic behavior of plates containing cracks by the finite element method was discussed by Minguez (1993). The elastostatic plane problems of an elastic layer with crack were solved by using theory of elasticity and Fourier transform technique by researchers (Birinci and Cakiroglu 2003, Birinci and Erdol 2004, Birinci *et al.* 2010). Fatigue crack growth analyses of functionally graded materials using the extended finite element method (XFEM) were discussed by Bhattacharya *et al.* For simulation purposes, a variety of cases with an edge and a central crack, as well as many discontinuities and inhomogeneities, are examined. In their study, they concluded that the presence of inhomogeneities and discontinuities significantly affects the fatigue life of the components (Bhattacharya *et al.* 2013). The detailed examination of the crack problem, which causes a major problem in gears, using the finite element method was made by Zhao (2015). Yaylacı (2016) investigates a comparative study of the finite element method (FEM) and an analytical method for a layered composite consisting of two elastic layers containing crack. External load is applied to the upper elastic layer by means of a rigid punch, and the lower elastic layer rests on two simple supports. The researchers determined the SIF and the crack opening displacements for various dimensionless quantities. The natural frequency of a beam with a crack is investigated numerically with software system ANSYS which is based on the finite element method by Tiwari *et al.* (2017). Ismail *et al.* (2017) obtained J-integral with the ANSYS finite element software in examining the surface cracks in round bar. Nguyen *et al.* (2018) examined a numerical and analytical approaches of topologically optimal design of carbon fiber reinforced

plastics with crack. Pajand and Moghaddam (2018) studied on the finite elements for the crack problems. The tensile behavior and fracturing process of the Brazilian disc samples containing non-persistent joints were examined using experimental and numerical methods by Sarfarazi *et al.* (Sarfarazi *et al.* 2018, Sarfarazi and Haeri 2018). The chemo-mechanical formulation for describing the damage propagation in concrete affected by alkali-silica-reaction was investigated with numerical examples by Moallemia and Pietruszczak (2018). Figueira *et al.* (2018) investigated nonlinear FE modeling for crack problems. They examined types of applications and analyses of crack problems and compared them other works. Belinha *et al.* (2018) combined a without mesh method with a crack growth algorithm, considering a linear elastic model, to simulate the propagation of cracks in brittle materials. In numerical applications, the solutions obtained by considering the meshless approach and the crack opening paths estimated by other numerical approaches are compared. Wang *et al.* (2019a) studied a new method to predict the crack growth and flexural capacity of large-scale SCRC slabs. They obtained crack growth and flexural capacity of SCRC slabs from XFEM models and experimental methods. Saberi *et al.* (2019) investigated the effects of various boundary conditions, aspect ratios, and crack lengths on the buckling stability of thin plates under compression by numerical analysis. A theoretical and finite element (FE) study on the stress intensity factors of double-edged cracked steel beams strengthened with carbon fiber reinforced polymer plates were investigated by Wang *et al.* (2019b). The effects of various parameters such as crack length, crack location, and plate aspect ratio on the buckling stability performance of the cracked plates under tensile loading were examined through utilized analytical and numerical methods by Memarzadeh *et al.* (2020). Banh *et al.* (2020) solved the crack problem by using a novel numerical approach of multi-material optimal topology design based on extended isogeometric Analysis. The fracture behavior of linear elastic isotropic functionally graded materials, that is the variation of elastic and thermal properties, was investigated using finite element method by Nojumi and Wang (2020). Jiang and Xiang (2020) solved crack problems using the extended finite element method (XFEM) and the forward Euler approach. Malekan *et al.* (2021) performed fatigue analyses using commercial software and investigated fatigue crack growth. In their study, they considered five different 2-dimensional fatigue crack growth models and compared the obtained results with analytical and experimental results. A crack identification using artificial neural network and finite element method were studied to predict the crack depth in pipeline structure by Seguini *et al.* (2021). Considering the above literature, the finite element method is widely used to determine SIF in different crack problems. The software system ANSYS based on the FEM method is an important solution tool in order to obtain short-term and accurate results, especially in solving problems related to fracture mechanics. The comparison of numerical and analytical solutions of mechanical problems has been taken many times by researchers (Öner *et al.* 2015, Adiyaman *et al.* 2015, Yaylacı *et al.* 2019, 2020, 2021a, b,

Yaylacı and Avcar 2020, Uzun Yaylacı *et al.* 2020, Öner *et al.* 2022).

The problem of layer with internal and edge cracks is an intense subject of studies in fracture mechanics. The reason for this can be explained with the fact that many structural elements and standard test elements are close to the layer geometry. The most important criteria used to determine whether the crack has begun to advance in the material is the SIF. For this purpose, the crack problem of a composite layer with a vertical crack in its lower layer and loaded by a rigid block from the top was investigated. In this study, the problem of cracks in a composite layer, which sits on a half-plane, consists of two layers with different elastic properties and heights, and it is loaded by a rigid block from the top, which is investigated by the finite element method. It has a vertical crack in the lower layer. In the crack problem discussed, the SIF at the crack ends was calculated. Based on the literature review, a study on the crack mechanics of two elastic layers resting on a homogeneous elastic half-plane has not yet been investigated. In addition, this study has a novelty as the finite element method is used to analyze the effect of different parameters on the stress intensity factor.

### 2. Theoretical approach

Physical dimension and the material properties of structure play a vital role in the determination of characteristics of the crack structure such as the position, depth, orientation, and the variety of cracks.

Physical parameters such as the position, depth and direction of the crack play a vital role in determining the crack response. There can be cracks due to several reasons such as material defect and design law. After the crack occurs in the structure, the behavior of the crack is determined by the direction of the forces or moments. There are three types of modes (Fig. 1).

\*Mode I is an opening mode where the force is perpendicular to the crack faces,

\*Mode II is a sliding mode where crack faces slide on each other and the forces are parallel to the crack faces

\*Mode III is a shearing mode where this is mainly due to torsion moments.

SIF is widely used in analyzing the crack problems. If there is plastic deformation around the crack tip, obtaining SIF values becomes important for the analysis of the crack problem. For describing the state of stress around a crack, an infinite layer with a central crack is shown in Fig. 2. The connection between stresses and SIF around the crack tip can be given by the expression as Eq. (1):

$$\sigma_{ij} = -\frac{k}{\sqrt{r}} f_{ij}(\theta) \tag{1}$$

where the stress near the crack tip is rational to  $1/\sqrt{r}$ . This expression gives rise to the stress density factor ( $K = k\sqrt{2\pi}$ ). There are three modes of fracture ( $K_I$ ,  $K_{II}$  and  $K_{III}$ ) in each mode of loading, the  $1/\sqrt{r}$  singularity at the crack tip will be available, but the SIF and the function  $f_{ij}(\theta)$  will vary for the different modes.

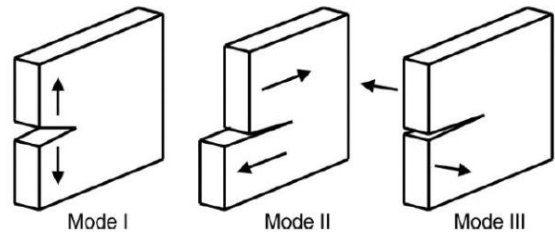


Fig. 1 Types of failure modes

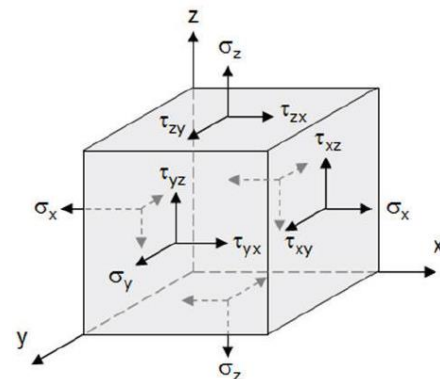
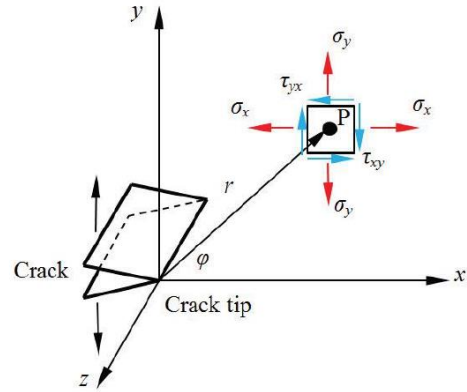


Fig. 2 Infinite plate with a central crack stress tensor

### 3. Result and discussion

In fracture mechanics, there are some equations for calculating the SIF. These equations are effective for general cracks, but there is no explicit formula for complex conditions. Therefore, a new approach is needed to calculate these SIF values. FEM is a numerical method which is used to solve engineering problems. If the problem is not solved analytically, or it takes too much time to solve, FEM is a very effective method. It is convenient for linear or nonlinear problems. Main rules for FEM are quite simple. In this solution method, complicated geometry is divided into smaller elements. All of these elements are interconnected. Connection points of the elements are known as nodes.

In this study, the finite element analysis software ANSYS Mechanical APDL (2013) will be used to calculate SIF of various cases. A crack in a plain strain layer is used and modeled using finite element program. Figs. 3 and 4 show two types of cracks such as edge and internal in the layer. The problem consists of a composite layer, which

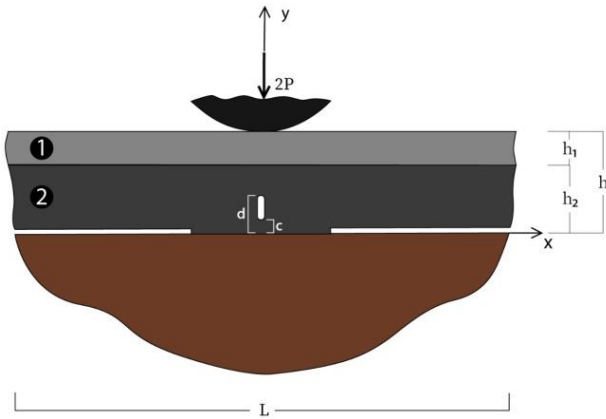


Fig. 3 Geometry and loading of the internal crack problem

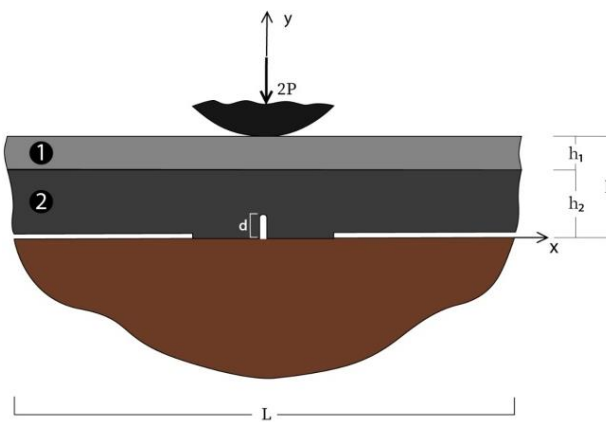


Fig. 4 Geometry and loading of the edge crack problem

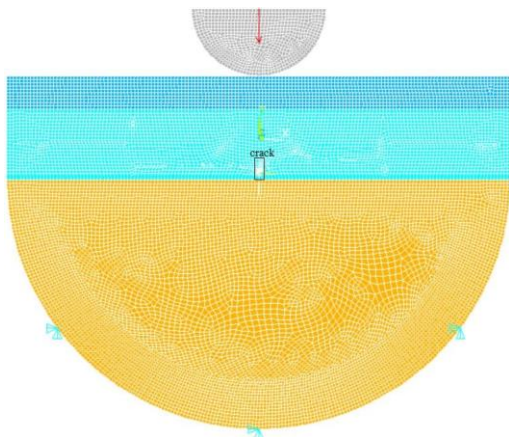


Fig. 5 Finite element model for crack analysis

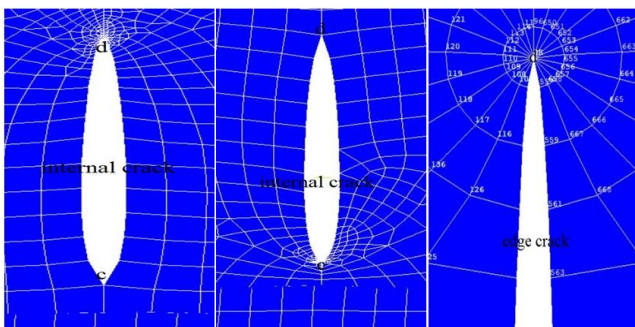


Fig. 6 Internal and edge crack models

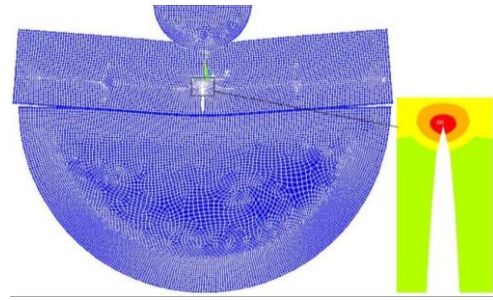


Fig. 7 Finite element model for edge crack analysis for point *d*

rests on the elastic half plane, and two layers of different elastic constants and heights, and it is loaded with a single load of  $2P$  magnitude by means of a rigid block from the top. A crack is a discontinuity in the material, where tensile stress cannot be transferred. The simplest method to model this is by making a small gap in the geometry where the crack should be, and mesh around it. If the gap is set to be very small, it will not have an important effect on the results of the analysis.

The first layer (upper layer) has a thickness  $h_1$ , and a rigid block with a contact surface is in continuous contact. The second layer (bottom layer) has a vertical crack along the  $y$  symmetry axis, its thickness is  $h_2$ , and it is in discontinuous contact with the half-plane. In the problem, all surfaces are frictionless, and the mass forces are neglected.

The problem is considered as a two-dimensional contact problem and the material of the layers are assumed to be elastic and isotropic. In the analyses, geometric properties are taken as  $L = 1m$  (length of the layer in  $x$  direction),  $h_1 = 10cm$  (thickness of the upper layer in  $y$  direction),  $P = 1000N$  load. Material properties are taken as  $E_2 = 30000 MPa$ ,  $\nu_1 = 0.25$ . Other parameters are chosen such that  $h_1/h_2$ ,  $G_2/G_1$ ,  $G_3/G_2$ ,  $R/h_1$  and  $G_1/(P/h_1)$  ratios which are compatible with dimensionless values obtained from analytical solution.

For this modeling, each step consists of:

1. Creating the initial geometry with the initial crack
2. Input material properties data of the problem.
3. Describing the FE model by elements PLANE183.
4. Mesh definition.
5. Identification of contact elements Targe169 and Conta172
6. FE analysis.
7. FE evaluation of SIF

In FEA, mesh convergence study is a very critical phase because it must be ensured that mesh size is small enough to find the correct result. In this study, firstly a bigger element size is used, then it is decreased by slow degrees. After a certain point, the change in mesh size did not significantly affect the result. This point was accepted as the optimum mesh size for the analysis. The model is meshed with a suitable two-dimensional plain strain element for an example of Quadrilateral element or PLANE183. The mesh discretization consists of 14137 elements and 42270 nodes (Fig. 5). Detailed images of the internal and external crack models defined in the program are given in Fig. 6.

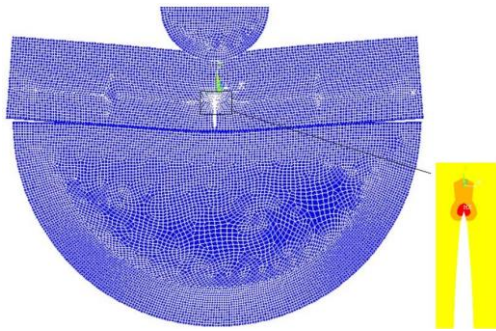


Fig. 8 Finite element model for internal crack analysis for point *d*

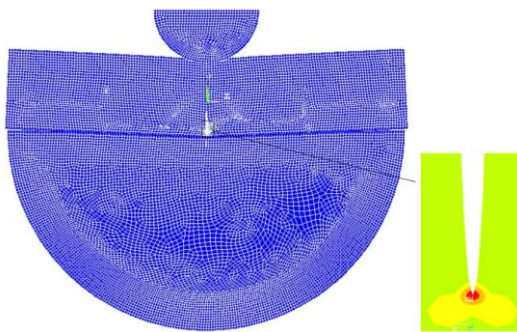


Fig. 9 Finite element model for internal crack analysis for point *c*

The most necessary thing during a crack model is the fact that the region rounds the fringe of the crack. There are many ways to construct the layer with a crack. One of the simplest methods is to identify the important coordinates. Once the coordinates are formed, then create the line and the area based on such coordinates. Figs. 7-9 illustrate three steps for the extension of the initial crack in the layer.

#### 4. Results

The SIF is a necessary factor to calculate the increasing stresses as it approaches the crack ends and provides information about the progress and danger of the crack. The SIF was examined according to the internal and edge cracks in the problem, which consists of two adherent layers that fit the elastic half plane and have a vertical crack in the lower layer. The relevant results are presented with graphics and tables. The expressions given in the numeric results are non-dimensional and defined as follows.

$R/h_1$ : The ratio of the radius of the rigid block to the height of the upper layer

$G_1/(P/h_1)$ : Load rate

$h_2/h_1$ : The ratio of the heights of the layers

$G_2/G_1$ : The ratio of shear modulus of layers

$G_3/G_2$ : The ratio of the half-plane to the shear modulus of the lower layer

##### 4.1 The internal crack case

In case of internal cracks, the graphics related to the SIF are given in Figs. 10-14.  $h_2/h_1 = 2$ ,  $G_3/G_2 = 0.5$ ,  $G_2/G_1 = 0.5$ ,

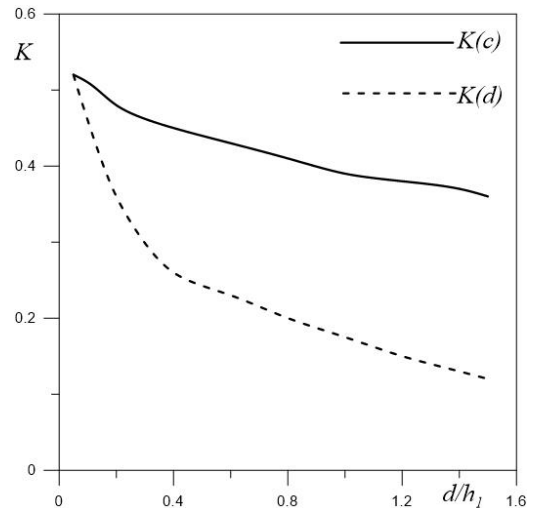


Fig. 10 The variation of the SIF with  $d/h_1$  for the internal crack ( $c/h_1 = 0.05$ )

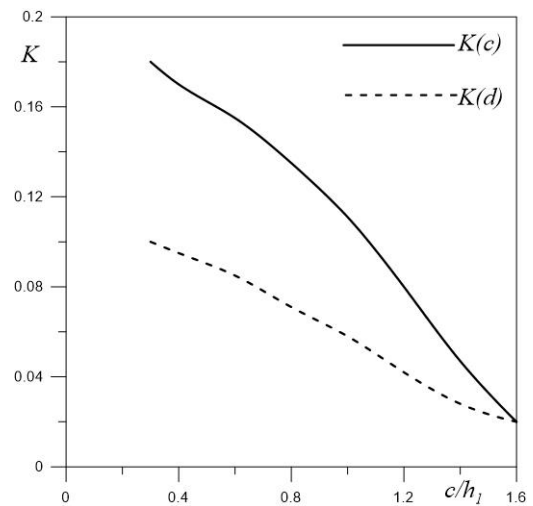


Fig. 11 The variation of the SIF with  $c/h_1$  for the internal crack ( $d/h_1 = 1.6$ )

$G_1/(P/h_1) = 100$ ,  $R/h_1 = 500$ ,  $\kappa_1 = \kappa_2 = \kappa_3 = 2$  values are used in Figs. (10-14).

$$K(c) = \frac{k(c)}{h_1 \sqrt{\frac{d-c}{2}}}$$

The stress-intensity factor at the *c* end of the crack

$$K(d) = \frac{k(d)}{h_1 \sqrt{\frac{d-c}{2}}}$$

The stress-intensity factor at the *d* end of the crack

Figs. 10-12 show the variations of the normalized SIF with the crack length for internal crack case. In the case of internal cracks in the elastic layer, the change of the SIF was investigated by keeping the *c* value constant and increasing the *d* value, and then increasing the *c* value by keeping the *d* value constant. In numerical applications, SIF was obtained for various crack lengths by keeping one end of the crack constant and changing its length. In Fig. 10, the crack tip *c* is fixed at  $c = 0.05h_1$ , and *d* varies until the SIF at *d*,  $K(d)$ , becomes approaches zero. The point at which  $K(d)$  approaches zero is approximately  $d = 1.6h_1$ . For fixed *c*, the normalized SIF ( $K(c)$ ) remains approximately constant.

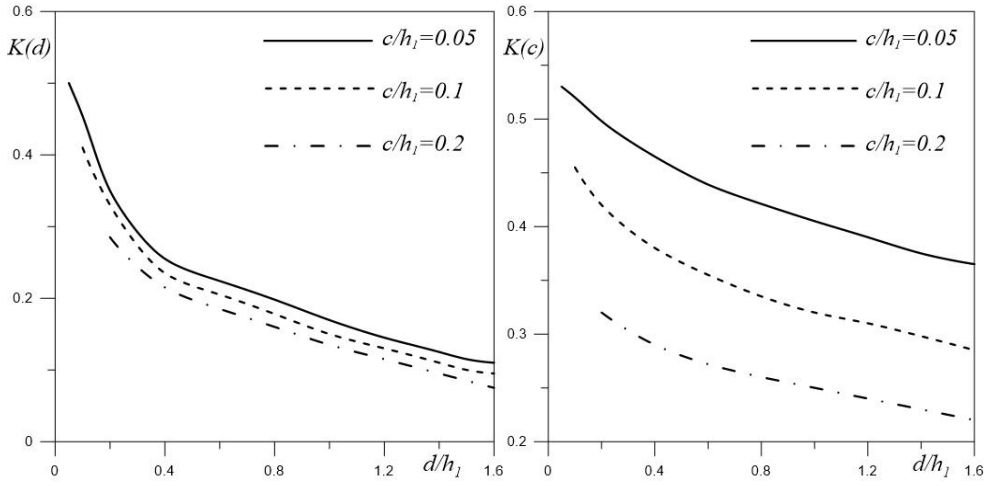


Fig. 12 The variation of the SIF with  $d/h_1$  for various values  $c/h_1$  for the internal crack

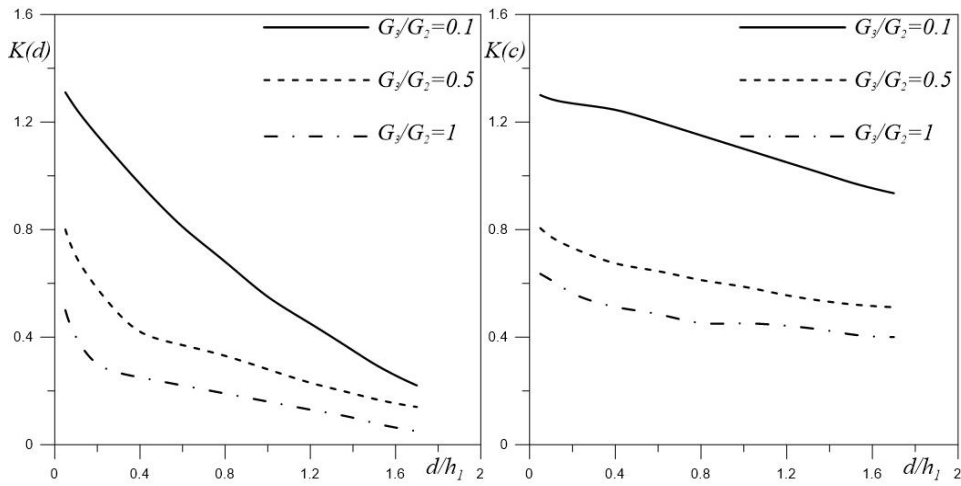


Fig. 13 The variation of the SIF with  $d/h_1$  for various values  $G_3/G_2$  for the internal crack ( $c/h_1 = 0.05$ )

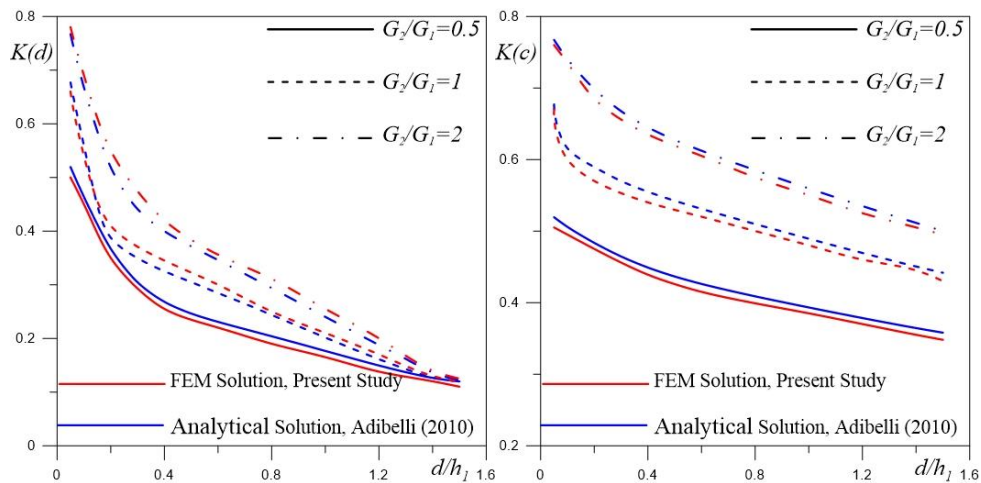


Fig. 14 Comparisons of analytical and FEM SIF results for various values  $G_2/G_1$  and  $d/h_1$  ( $c/h_1 = 0.05$ )

Note that  $K(d)$  is reduced rapidly and approaches zero as the crack tip  $d$  moves towards. As can be seen from the figures,  $K(d)$  values decrease faster than  $K(c)$  values.  $K(d) = K(c)$  is for a very small crack (i.e.,  $c = 0.05h_1$ ,  $d = 0.0501h_1$ ). In Fig. 11, the crack tip  $d$  is fixed at  $d = 1.6h_1$ , and  $c$  varies

between  $d$  and 0.  $K(c)$  goes to infinity as  $c$  approaches 0 and for a very small crack (i.e.,  $c = 1.5999h_1$ ,  $d = 1.6h_1$ ). It may be observed in Figs. 10 and 11 that  $K(c)$  is always greater than  $K(d)$ . In Fig. 12, the crack tip  $d$  varies until  $K(d)$  approaches zero, but  $c$  is fixed for various values of  $c/h_1$

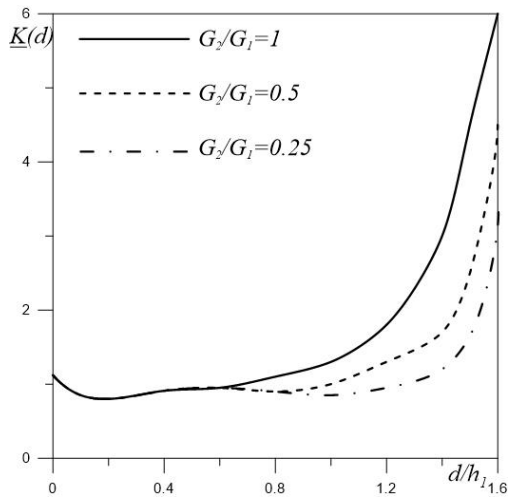


Fig. 15 The variation of the SIF with  $d/h_1$  for various values  $G_2/G_1$  for the edge crack

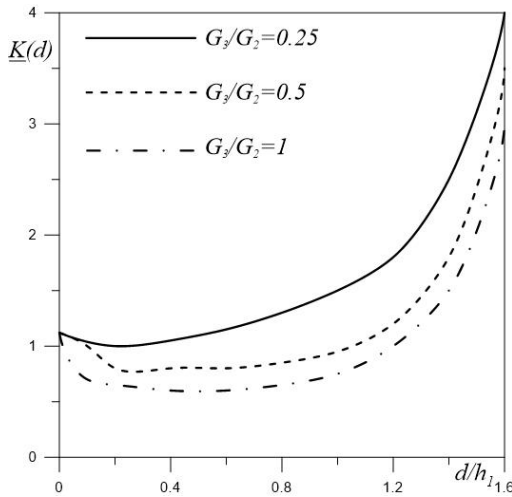


Fig. 16 The variation of the SIF with  $d/h_1$  for various values  $G_3/G_2$  for the edge crack

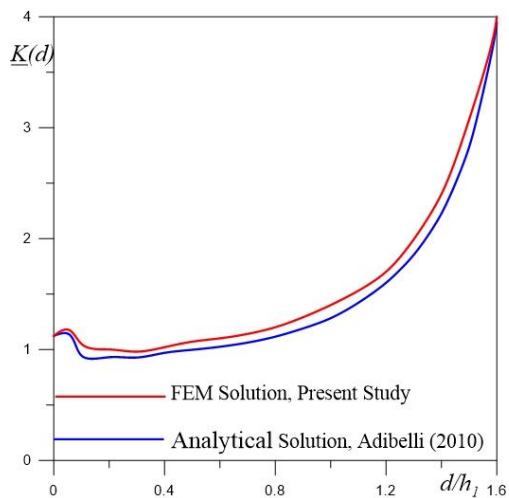


Fig. 17 Comparisons of analytical and FEM SIF results with  $d/h_1$

( $c/h_1 = 0.05$ ,  $c/h_1 = 0.1$ ,  $c/h_1 = 0.2$ ). It can be seen that the SIF at the crack tip  $d$ ,  $K(d)$ , decreases and approaches zero

more rapidly as the crack length decreases. That is, as the crack length decreases, the crack is closed earlier. When Figs. 10-12 are examined, the following expressions are obtained. The stress-intensity factors at the crack ends vary depending on  $\sigma_x(0, y)$  normal stresses. The reasons that increase these stress values also increase the stress-intensity factors at the crack ends. If the size of the crack decreases, the stress intensity-factors at the ends of  $c$  and  $d$  converge. On the contrary if the crack size increases, the stress-intensity factors diverge and their values increase.

The effect of the shear modulus of the layers ( $G_3/G_2$  and  $G_2/G_1$ ) on the stress-intensity factors ( $K(c)$  and  $K(d)$ ) are given in Figs. 13 and 14. As can be seen in Fig. 13, if the  $G_3/G_2$  ratio increases, the stress intensity factors decrease.  $K(c)$  and  $K(d)$  comparison in a study in the literature is given in Fig. 14. When the figure is examined, it is seen that the SIF increases as the  $G_2/G_1$  increases. It is observed that the values obtained in this study are compatible with the literature.

#### 4.2 The edge crack case

In case of edge cracks, the graphics related to the SIF are given in Figs. 15-17.  $h_2/h_1 = 2$ ,  $G_1/(P/h_1) = 100$ ,  $R/h_1 = 500$ ,  $\kappa_1 = \kappa_2 = \kappa_3 = 2$  values are used in Figs. 15-16 and  $G_3/G_2 = 0.5$ ,  $G_2/G_1 = 0.5$  in Fig. 17.

$\underline{K}(d) = \frac{k(d)}{\sigma_0 \sqrt{d}}$ : The stress-intensity factor at the  $d$  end of the crack

In the case of edge cracks, while the crack length decreases, the SIF decreases and the stress intensity factor goes to a certain limit value (Figs. 15-17). This limit for SIF is 1.12 in case of a very small crack ( $d/h_1 = 0.0001$ ). The variation of SIF for various non-dimensional material constants ( $G_2/G_1$  and  $G_3/G_2$ ) values is given in Figs. 15 and 16. The SIF decreases with the increase of the  $G_2/G_1$  ratio, on the contrary, with the increase of the  $G_3/G_2$  ratio, the SIF decreases.  $\underline{K}(d)$  comparison with a study in the literature is given in Fig. 17. When the figure is examined, it is observed that if the crack length increases, the SIF increases. It is seen that the values obtained in this study are compatible with the literature.

#### 5. Conclusions

The objective of this study is to assess the accuracy achieved with FEM by numerically estimating the stress-intensity factor. The results show that modeling a crack using the finite element method, with the help of ANSYS software, is a very accurate method. Results obtained are summarized below.

- Numerical technique may be used for determination of the stress-intensity factor of cracks. Based on the numerical simulation using the ANSYS finite element program, analysis of successful fracture mechanics is conducted to study the plain strain layer contained crack.

- It is shown that the SIF changes considerably as a result of the changes relying upon the placement and size of cracks.

- If the crack size is very small, the SIF at the ends of  $c$

and  $d$  is equal. As the ratio of the shear modulus of the lower layer to the upper layer ( $G_2/G_1$ ) increased, the SIF ( $K(d)$  and  $K(c)$ ) increased in case of internal cracks, on the contrary, it ( $\underline{K}(d)$ ) decreased in the case of edge cracks. As the ratio of the shear modulus of the elastic half plane to the lower layer increased, the SIF ( $K(d)$ ,  $K(c)$  and  $\underline{K}(d)$ ) decreased.

- In the case of internal crack in the lower layer of problem, it is seen that the SIF  $K(d)$  decreases faster than the SIF  $K(c)$ . In cases where the crack length decreases, the stiffness of the upper layer increases compared to the lower layer, and the stiffness of the half-plane increases compared to the lower layer, the crack closes earlier. As a result, the SIF at the crack ends decreases.

- In case of edge cracks, the SIF decreases as the crack length decreases, the stiffness of the layer 2 keeps constant and the stiffness of the half-plane and layer 1 increases.

- Vertical cracks which have an analytical solution were tested, and the results obtained numerically were compared to the analytical solutions throughout. As it was obtained that the analysis results of the FEM are in agreement with the theoretical results, the FEM models can be efficiently used for SIF detection at crack problems.

- When the results of SIF obtained by using numerical solution method (FEM) were compared with analytical results, the error rates were found to be very low. This work shows that the FEM is able to obtain SIF for the crack problems, saving significant time. We observed that all the results were compatible, thus proving that numerical approaches can be used for solving crack mechanics.

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