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Structural detection of variation in Poisson's ratio: Monitoring system for zigzag double walled carbon nanotubes

Muzamal Hussain*[1], Sehar Asghar[1], Hamdi Ayed[2,3],
Mohamed A. Khadimallah[4,5], Adil Alshoaibi[6] and Abdelouahed Tounsi[7,8]

[1]Department of Mathematics, Govt. College University Faisalabad, 38040, Faisalabad, Pakistan
[2]Department of Civil Engineering, College of Engineering, King Khalid University, Abha, Kingdom of Saudi Arabia
[3]Higher Institute of Transport and Logistics of Sousse, University Sousse, Tunisia
[4]Prince Sattam Bin Abdulaziz University, College of Engineering, Civil Engineering Department, BP 655, Al-Kharj, 16273, Saudi Arabia
[5]Laboratory of Systems and Applied Mechanics, Polytechnic School of Tunisia, University of Carthage, Tunis, Tunisia
[6]Department of Physics, College of Science, King Faisal University, Al-Hassa, P.O Box, Hofuf, 31982, Saudi Arabia
[7]YFL (Yonsei Frontier Lab), Yonsei University, Seoul, Korea
[8]Department of Civil and Environmental Engineering, King Fahd University of Petroleum and Minerals,
31261 Dhahran, Eastern Province, Saudi Arabia

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Abstract. In this paper, natural frequency curves are presented for three specific end supports considering distinct values of nonlocal parameter. The vibrational behavior of zigzag double walled carbon nanotubes is investigated using wave propagation with nonlocal effect. Frequency spectra of zigzag (12, 0) double walled carbon nanotubes have been analyzed with proposed model. Effects of nonlocal parameters have been fully investigated on the natural frequency against against variation of Poisson's ratio. A slow increase in frequencies against variation of Poisson's ratio also indicates insensitivity of it for suggested nonlocal model. Moreover, decrease in frequencies with increase in nonlocal parameter authenticates the applicability of nonlocal Love shell model. Also the frequency curves for C-F are lower throughout the computation than that of C-C curves.

Keywords: double-walled CNTs; Love shell theory; nonlocal parameter; Poisson's ratio; vibration

1. Introduction

CNTs have a variety of uses and applications in potential looking fields, some of which are charge detectors, electronics, communication, composite materials, bio-technology, environment, energy storage, chemical, and optical (Tserpes and Papanikos 2005, Soltani *et al.* 2012, Das *et al.* 2013, Bocko and Lengvarský 2014, Reddy and Pang 2008). Therefore, in order to effectively use of CNTs in each of these fields, it is important that their vibration characteristics are examined. These cylindrical structures have many fascinating and valuable properties, well-bonding strength among carbon atoms (Soltani *et al.* 2012, Bocko and Lengvarský 2014, effective tensile efficiency and Young modulus (Das *et al.* 2013), high springiness (Reddy and Pang 2008) and several potential applications in different fields.

Vibrational characteristics of various nano-structures are widely investigated based on nonlocal beam model. Specifically, carbon nanotube as one of the most applicable miniature structure attracts many researchers in order to analytically and experimentally probes its dynamical properties using the nonlocal beam theory. The nonlocal theory mostly focused on the free vibrational analysis of the nano-structure, especially, carbon nanotubes. In addition, nano-structures can be mentioned as the important types of devices which have wide applications in a variety of technological and scientific fields (Tserpes and Papanikos 2005, Soltani *et al.* 2012). The nonlinear forced vibration of carbon nanotubes has seldom been observed (Das *et al.* 2013, Bocko and Lengvarský 2014, Reddy and Pang 2008). However, this issue is very crucial due to the widespread application of the forced nonlinear vibration carbon nanotubes in many practical instruments. Due to this, a new model is required to observe the nano-size structure. Some investigators studied the higher order elasticity theories (Murmu and Pradhan 2009, Civalek *et al.* 2009, Narendar and Gopalakrishnan 2011, Yayli 2013, Demir and Civalek 2016). Different non-classical elasticity theories have attracted the researcher's attention as: stress and strain theories (Mindlin and Tiersten, 1962 Toupin 1964) and nonlocal theory (Fleck and Hutchinson 1993, Eringen and Edelen 1972). For the interpretation of vibrational influence of single-walled carbon nanotubes (SWCNTs), the nonlocal elasticity theory and Timoshenko beam model are used (Mindlin and Tiersten 1962, Toupin 1964, Fleck and Hutchinson 1993, Muhammad *et al.* 2019) presented the large-amplitude thermal free vibration characteristics of nonlocal two-phase piezo-magnetic nano-size beams having geometric imperfections by considering piezoelectric reinforcement scheme. The piezoelectric reinforcement can cause an enhanced vibration behavior of smart nanobeams under magnetic field. Murmu and Pradhan (2009) investigated the vibrational frequencies with different

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*Corresponding author, Ph.D.,
E-mail: muzamal45@gmail.com

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modes along temperature change using nonlocal small scale effects. On the other side, for length scale coefficient and soft elastic medium with embedded carbon nanotube, the nonlocal frequencies were comparatively lower. It was also found that the frequencies of the nonlocal model at different stages of temperature are higher than the nonlocal with same temperature. Tayeb *et al*. (2020) studied the incorporation of carbon nanotubes in a polymer matrix makes it possible to obtain nanocomposite materials with exceptional properties. It's in this scientific background that this work was based. Sedighi (2020) performed the basis of finite element analysis to examine the vibrational characteristics of a hetero-nanotube made of carbon (C) and boron nitride (BN) nanotubes in magnetic and thermal environment. By incorporating the assumption of nonlocal elasticity theory, the size-dependent behavior of the considered structure is also taken into account. Eringen nonlocal theory and Von-Karman geometry were fully studied by Yang *et al*. (2010). Recently, some researchers have done their research in concrete construction (Thomée *et al*. 2006, Meftah *et al*. 2006, Yaman *et al*. 2006, Gasser, 2007).

Vibration frequencies of zigzag SWCNTs (5, 0), (8, 0), (9, 0) and (11, 0), with different boundary conditions were considered and calculated numerically through MD simulation. The influence of nonlocal parameter on height and radius was studied in detail. Fenjan *et al.* (2020c) studied the nonlinear stability behavior of a nanocrystalline silicon curved nanoshell considering strain gradient size-dependency. Nanocrystallines are composite materials with an interface phase and randomly distributed nano-size grains and pores. Imperfectness of the curved nanoshell has been defined based on an initial deflection. Das *et al*. (2013) studied the nonlocal theories for the in-extensional vibration of SWCNTs. The in-extensional mode frequency was treated by the positive strain gradient theory with circumferential wave number. Ansari and Arash (2013) investigated vibrations of DWCNTs based on NLT using differential quadrature method (DQM). The mechanical behavior of DWCNTs with geometrical parameters layer wise boundary conditions and small scale factors are fully investigated. Mirjavadi *et al*. (2020) presented the context of classic conical shell formulation, nonlinear forced vibration analysis of truncated conical shells and annular plates made of multi-scale epoxy/CNT/fiberglass composite. The composite material is reinforced by carbon nanotube (CNT) and also fiberglass for which the material properties are defined according to a 3D Mori-Tanaka micromechanical scheme.

Bocko and Lengvarský (2014) assessed the fundamental natural frequency (FNF) with different nonlocal parameters as well as two distinct diameter and continuously changed length was computed by the nonlocal theory. It has been represented by them the nonlocal parameters are highly influential on the bending vibration of a carbon nanotube. It was shown that the boundary conditions with nonlocal parameter are more effective on the nanotube vibration. Moreover, it has been observed that on enhancing the length of CNT, the frequencies decrease by increasing of the nonlocal parameter. Ouakad *et al*. (2020a) examined the fundamental vibrational characteristics of a spinning CNT-based nano-rotor assuming a nonlocal elasticity Euler-Bernoulli beam theory. The rotary inertia, gyroscopic, and rotor mass unbalance effects are all taken into consideration in the beam model. Assuming a nonlocal theory, two coupled 6th-order partial differential equations governing the vibration of the rotating SWCNT are first derived.

Fenjan *et al*. (2020b) investigated the nonlinear stability analysis of advanced microbeams reinforced by Graphene Platelets (GPLs) considering generic geometrical imperfections and thermal loading effect. Uniform, linear and nonlinear distributions of GPLs in transverse direction have been considered. Imperfection sensitivity of post-bucking behaviors of the microbeam to different kinds of geometric imperfections have been examined. Demir *et al*. (2020) investigated the effect of Chitosan (CS), Carbon Nanotube (CNT) and hybrid (CS-CNT) fillers on the natural frequency of drilled composite plate is investigated by experimentally in this study. The numerical validation is also made with a program based on Finite Element Method (SolidWorks). Soltani (2016) investigated the nonlinear vibrational characteristics of SWCNTs using the theory of nonlocal elasticity and Karman's geometric non-linearity theory. The controlling equation is derived from Donnell's shell theory and partial differential equations were converted into differential equations by invoking Galerkin's technique. The influence of aspect ratios, nonlocal parameters, nonlinear parameters and circumferential parameters was investigated. Recently, research on vibration of SWCNTs has been done by many material researchers (Wang *et al*. 2007, Gupta *et al*. 2010, Swain *et al*. 2013). Farokhian and Salmani-Tehrani (2020) investigated the dynamic buckling of a smart sandwich nanotube is studied. The nanostructure is composed of a carbon-nanotube with inner and outer surfaces coated with ZnO piezoelectric layers, which play the role of sensor and actuator. Nanotube is under magnetic field and ZnO layers are under electric field. Ouakad *et al*. (2020b) investigated the effects of material properties, nonlocal parameter, Lorentz and electric forces on maximum static deflections and natural frequencies of actuated hybrid carbon/boron-nitride nanotubes (CBNNT) subjected to thermal loads are studied for the first time. The displacement field of the nanotube satisfies assumptions of the Bernoulli–Euler beam theory. Recently some researcher used different methods for nonlinear modeling (Eltaher *et al*. 2019, Ebrahimi *et al*. 2019, Safaei *et al*. 2019, Shahsavari *et al*. 2019, Benmansour *et al*. 2019, Bouhlali *et al*. 2019, Ayat *et al*. 2018). Sedighi and Malikan (2020) applied the stress-driven nonlocal theory of elasticity, in its differential form to investigate the nonlinear vibrational characteristics of a hetero-nanotube in magneto-thermal environment with the help of finite element method. In order to more precisely deal with the dynamic behavior of size-dependent nanotubes, a two-node beam element with six degrees-of freedom including the nodal values of the deflection, slope and curvature is introduced. Fenjan *et al*. (2020a) studied the nonlinear dynamic characteristics of a nonlocal two-phase piezo-magnetic beam based on a refined higher-order beam formulation and piezoelectric reinforcement scheme. The piezoelectric reinforcement can cause an enhanced vibration behavior of smart nanobeams under magnetic field. Azizkhani *et al.* (2020) investigated the multi-Walled Carbon nanotubes (MWCNT) coupled with Silicone Rubber (SR) can represent applicable strain sensors with accessible materials, which result in good stretchability and great sensitivity. Sedighi *et al.* (2020) established the Eringen's strain-driven nonlocal differential model to exhibit inconsistencies when applied to bounded continua of applicative interest. The stress-driven nonlocal theory leads instead to well-posed nonlocal elastic formulations demonstration stiffening structural responses. Ahmed *et al.* (2019) concerned with post-buckling investigation of nano-scaled beams constructed from porous functionally graded (FG) materials taking into account geometrical imperfection shape. Hence, two types of nanobeams which are perfect and imperfect have been studied. Porous FG materials are classified based on even or uneven porosity distributions. Faleh *et al.* (2020) investigated the pulse load effects on forced transient vibrations of porous crystalline shells. A crystalline material contains many voids inside it and also there are nano-size grains which define the material character. The formulation for crystalline cylindrical shell is provided by first-order shell theory and a numerical approach is used to solve shell equations. Koochi and Goharimanes (2021) conducted the nonlinear oscillation of carbon nanotube manufactured nano-resonator. The governing equation of the nano-resonator is extracted in the context of the nonlocal elasticity. The impact of the Casimir force is also incorporated in the developed model. A closed-form solution based on the energy balance method is presented for investigating the oscillations of the nano-resonator. Fenjan *et al.* (2019b) studied the free vibrational characteristics of porous steel double-coupled nanoplate system in thermo-elastic medium. Different pore dispersions called uniform, symmetric and asymmetric have been defined. Nonlocal strain gradient theory (NSGT) containing two scale parameters has been adopted to stablish size-dependent modeling of the system. Sedighi and Daneshmand (2014) utilized the continuum model to extract the nonlinear governing equation for Carbon nanotube (CNT) probes near graphite sheets. The van der Waals (vdW) intermolecular force and electrostatic actuation are included in the equation of motion. Static and dynamic pull-in behavior of the system is investigated in this paper. Fenjan *et al.* (2019a) studied the dynamic stability of a porous metal foam nano-dimension plate on elastic substrate exposed to bi-axial time-dependent forces. Various pore contents based on uniform and non-uniform models have been introduced. Salah *et al.* (2019) employed a simple four-variable integral plate theory for examining the thermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. Abouelregal *et al.* (2021) examined the influence of thermal conductivity on the dynamics of a rotating nanobeam in the context of nonlocal thermo-elasticity theory. To this end, the governing equations are derived using generalized heat conduction including phase lags on the basis of the Euler–Bernoulli beam theory. Batou *et al.* (2019) studied the wave propagations in sigmoid functionally graded (S-FG) platesusing new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory.

In this paper, vibration characteristics of double-walled carbon nanotubes (CNTs) are studied based upon nonlocal elastic shell theory. The significance of small scale is being perceived by developing nonlocal Love shell model. The wave propagation approach has been utilized to frame the governing equations as eigen value system. The influence of nonlocal parameter subjected to diverse end supports clamped-clamped (C-C), simply supported-simply supported (S-S) and clamped-free (C-F) has been analyzed. The results generated furnish the evidence regarding applicability of nonlocal shell model.

2. Mathematical formulation

The fundamental equations from the Love shell theory (1952) are considered as

$$\begin{gathered}
\frac{\partial N_{xx}}{\partial x}+\frac{1}{R}\frac{\partial N_{x\theta}}{\partial\theta}-\frac{1}{2R^2}\frac{\partial M_{x\theta}}{\partial\theta}=\rho_t\frac{\partial^2 u}{\partial^2 t}\\
\frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial\theta}+\frac{\partial N_{x\theta}}{\partial x}+\frac{1}{R^2}\frac{\partial M_{\theta\theta}}{\partial\theta}+\frac{3}{2R}\frac{\partial M_{x\theta}}{\partial x}=\rho_t\frac{\partial^2 v}{\partial^2 t}\\
\frac{\partial^2 M_{xx}}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 M_{\theta\theta}}{\partial^2\theta}+\frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial\theta\partial x}-\frac{N_{\theta\theta}}{R}+p=\rho_t\frac{\partial^2 w}{\partial^2 t}
\end{gathered}\tag{1}$$

where *p* expresses the applied pressure on *i* tube through van der Waals (vdW) interaction forces. The vdW model explains the effects of interlayer relations among the tubes of double-walled CNTs.

The system of partial differential equations in form of three unknown field variables u^i, v^i, w^i (i = 1,2) for the *ith* tube of double-walled CNTs.

$$\begin{aligned}
&{y_{11}}^{(1)}u^1+{y_{12}}^{(1)}v^1+\\
&{y_{13}}^{(1)}w^1=\rho h\left(\ddot{u}^{(1)}-(e_o a)^2(\ddot{u}^{(1)}_{xx}+\frac{1}{{R_1}^2}\ddot{u}^{(1)}_{xx})\right)
\end{aligned}\tag{2}$$

$$\begin{aligned}
&{y_{21}}^{(1)}u^1+{y_{22}}^{(1)}v^1+\\
&{y_{23}}^{(1)}w^1=\rho h\left(\ddot{v}^{(1)}-(e_o a)^2(\ddot{v}^{(1)}_{xx}+\frac{1}{{R_1}^2}\ddot{v}^{(1)}_{xx})\right)
\end{aligned}\tag{3}$$

$$\begin{aligned}
&{y_{31}}^{(1)}u^1+{y_{32}}^{(1)}v^1+{y_{33}}^{(1)}w^1=\rho h\ddot{w}^{(1)}+w^{(1)}\sum_{\substack{j=1\\ j\neq i}}^{2}c_{1j}\\
&-\sum_{\substack{j=1\\ j\neq i}}^{2}c_{1j}w^{(j)}-(e_o a)^2\left[\begin{array}{l}
\rho h(\ddot{w}^{(1)}_{xx}+\frac{1}{{R_1}^2}\ddot{w}^{(1)}_{\theta\theta})\\
+(\ddot{w}^{(1)}_{xx}+\frac{1}{{R_1}^2}\ddot{w}^{(1)}_{\theta\theta})\sum_{\substack{j=1\\ j\neq i}}^{2}c_{1j}\\
-\sum_{\substack{j=1\\ j\neq i}}^{2}c_{1j}(\ddot{w}^{(j)}_{xx}+\frac{1}{{R_1}^2}\ddot{w}^{(j)}_{\theta\theta})
\end{array}\right]
\end{aligned}\tag{4}$$

$$y_{11}{}^{(2)}u^2 + y_{12}{}^{(2)}v^2 + y_{13}{}^{(2)}w^2 = \rho h\left(\ddot{u}^{(2)} - (e_o a)^2(\ddot{u}_{xx}^{(2)} + \frac{1}{R_2{}^2}\ddot{u}_{\theta\theta}{}^{(2)})\right) \tag{5}$$

$$y_{21}{}^{(2)}u^2 + y_{22}{}^{(2)}v^2 + y_{23}{}^{(2)}w^2 = \rho h\left(\ddot{v}^{(2)} - (e_o a)^2(\ddot{v}_{xx}^{(2)} + \frac{1}{R_2{}^2}\ddot{v}_{xx}^{(2)})\right) \tag{6}$$

$$\begin{aligned} & y_{31}{}^{(2)}u^2 + y_{32}{}^{(2)}v^2 + y_{33}{}^{(2)}w^2 \\ & = \rho h\ddot{w}^{(2)} + w^{(2)}\sum_{\substack{j=1\\ j\neq 2}}^{2} c_{2j} - \sum_{\substack{j=1\\ j\neq 2}}^{2} c_{2j}w^{(j)} - \\ & (e_o a)^2 \begin{bmatrix} \rho h(\ddot{w}_{xx}^{(2)} + \frac{1}{R_2{}^2}\ddot{w}_{\theta\theta}^{(2)}) + \\ (\ddot{w}_{xx}^{(2)} + \frac{1}{R_2{}^2}\ddot{w}_{\theta\theta}^{(2)})\sum\limits_{\substack{j=1\\ j\neq 2}}^{2} c_{2j} \\ -\sum\limits_{\substack{j=1\\ j\neq 2}}^{2} c_{2j}(\ddot{w}_{xx}^{(j)} + \frac{1}{R_2{}^2}\ddot{w}_{\theta\theta}^{(j)}) \end{bmatrix} \end{aligned} \tag{7}$$

here y_{pq} = (p,q = 1,2,3) are stated as partial operators can be seen in Appendix.

The three modal displacement functions of the shell for tube can be written as

$$u^{(i)}(x,\theta,t) = a_m \cos(n\theta)e^{(i\omega t - ik_m x)} \tag{8}$$

$$v^{(i)}(x,\theta,t) = b_m \sin(n\theta)e^{(i\omega t - ik_m x)} \tag{9}$$

$$w^{(i)}(x,\theta,t) = c_m \cos(n\theta)e^{(i\omega t - ik_m x)} \tag{10}$$

where a_m, b_m, c_m describe the displacement amplitude coefficients in x, θ and z directions correspondingly. The angular frequency is designated as ω, circumferential wave number by n and k_m referred to be axial wave number allied with end supports obligatory on double-walled CNTs. Replacing the functions and derivatives into the system of fundamental equations, henceforth derived a set of simultaneous as follows

$$Y_{11}{}^{(i)}a_m^i + Y_{12}{}^{(i)}b_m^i + Y_{13}{}^{(i)}c_m^i = -\omega^2\left(1-(e_o a)^2\nabla^2\right)\rho h a_m^i \tag{11}$$

$$Y_{21}{}^{(i)}a_m^i + Y_{22}{}^{(i)}b_m^i + Y_{23}{}^{(i)}c_m^i = -\omega^2\left(1-(e_o a)\nabla^2\right)\rho h b_m^i \tag{12}$$

$$\begin{aligned} & Y_{31}{}^{(i)}a_m^i + Y_{32}{}^{(i)}b_m^i + Y_{33}{}^{(i)}c_m^i + \left(1-(e_o a)^2\nabla^2\right) \\ & \left[\sum_{\substack{j=1\\ j\neq i}}^{2} c_{ij}c_m^i - \sum_{\substack{j=1\\ j\neq i}}^{2} c_{ij}c_m^i\right] = -\omega^2\left(1-(e_o a)^2\nabla^2\right)\rho h c_m^i \end{aligned} \tag{13}$$

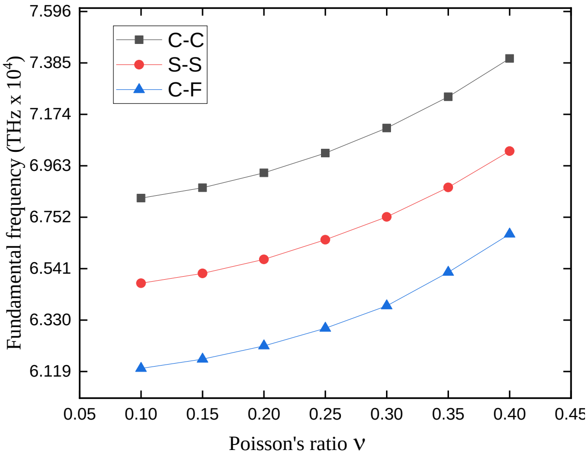

Fig. 1 Frequency variation of zigzag (12, 0) with $e_o = 0.2$ versus Poisson’s ratio

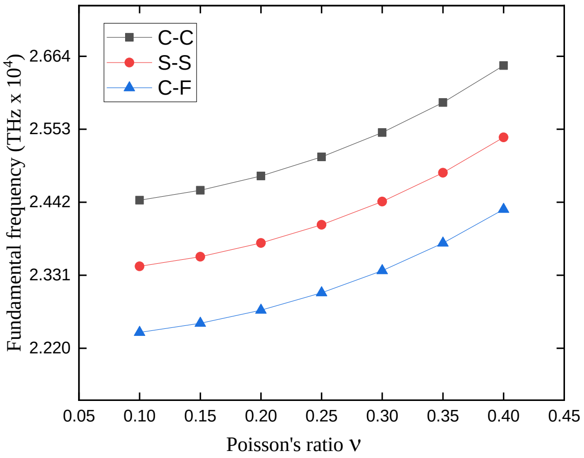

Fig. 2 Frequency variation of zigzag (12, 0) with e_o = 0.65 versus Poisson’s ratio

Since i = (1, 2) and the algebraic operators $Y_{pq}^{(i)}$ are acquired using Appendix with p,q = (1, 2, 3).

3. Result and discussion

Here, a methodology for constructing the nonlocal shell model based on wave propagation approach of double walled carbon nanotubes (DWCNTs) for vibration analysis has been presented The fundamental frequencies of the DWCNTS with clamped-clamped (C-C), simply supported-simply supported (S-S), clamped-free (C-F) against the variation of Poisson’s ratio. The frequency outcomes are validated with the existing results in the literature. The non-dimensional frequency parameters are also calculated to evaluate convergence rate of present results (Malekzadeh and Heydarpour 2012) as shown in the Table 1. There is once again comparison is made with Zhang *et al.* (2001) and Blevins (1979) as shown in Tables 2 and 3. Since the percentage of error is negligible, the model is concluded as valid. The proposed model based on WPA can incorporate in order to accurately predict the acquired results of material data point. The influence of nonlocal parameter

Table 1 Convergence of present results with Malekzadeh and Heydarpour (2012)

h/R	m	Method	*N*			
			7	9	11	13
0.05	1	Malekzadeh and Heydarpour (2012)	0.4392	0.4391	0.4392	0.4392
		Present	0.4217	0.4201	0.4217	0.4217
0.05	1	Malekzadeh and Heydarpour (2012)	0.4153	0.4147	0.4146	0.4147
		Present	0.4071	0.4010	0.4072	0.4039

Table 2 Convergence of present results with Zhang *et al.* (2001)

m	Method	*N*				
		1	2	3	4	5
1	Zhang *et al.* (2001)	0.034870	0.011760	0.007083	0.009016	0.013770
	Present	0.033710	0.010918	0.007019	0.009001	0.012345
1	Zhang *et al.* (2001)	0.087420	0.031550	0.015860	0.012240	0.014820
	Present	0.086992	0.030937	0.014327	0.012196	0.012387

Table 3 Convergence of present results with Blevins (1979)

m	Method	*N*				
		1	2	3	4	5
1	Blevins (1979)	0.034400	0.012040	0.007222	0.009048	0.013770
	Present	0.033654	0.011827	0.0065342	0.007865	0.012873
1	Blevins (1979)	0.084840	0.031620	0.016030	0.012330	0.014840
	Present	0.078653	0.030265	0.015263	0.011726	0.013287

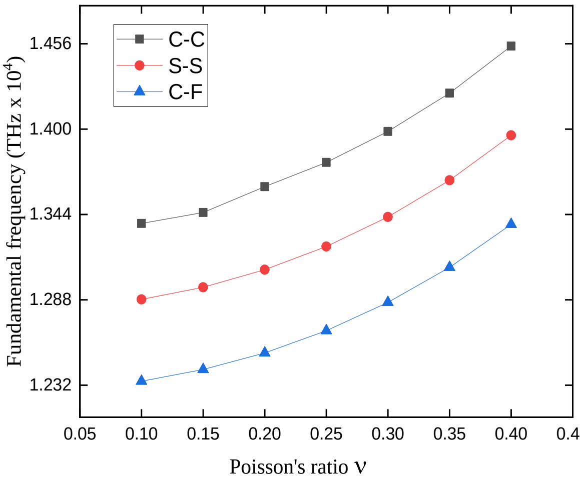

Fig. 3 Frequency variation of zigzag (12, 0) with $e_o = 1.2$ versus Poisson's ratio

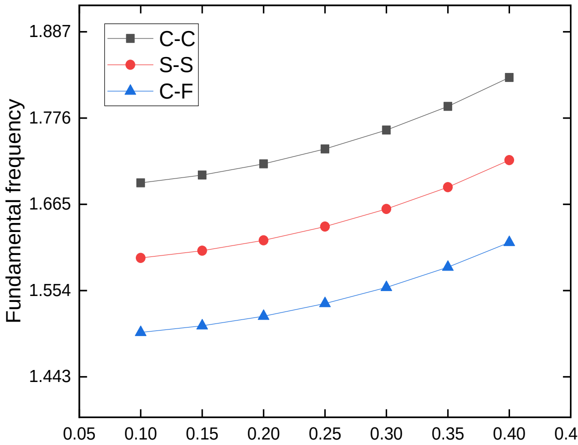

Fig. 4 Frequency variation of chiral (6, 4) with $e_o = 0.2$ versus Poisson's ratio

with variation of Poisson's ratio are investigated depending upon certain edge supports. The mass density is assumed to be 2300 kg/m^3, with Young's Modulus 1Tpa (Basirjafri *et al* 2012). In accordance of theoretical procedure, at first frequency of double-walled CNTs with mutation in values of Poisson's ratio is observed as shown in Fig. 1. As figure shows fundamental frequency rises by rising the Poisson's ratio. The results presented here are in good accordance with those established by Basirjafri *et al.* (2012). The fundamental frequencies are calculated of double-walled CNTs subjected to three distinct end supports C-C, S-S and C-F. The frequencies are obtained for the varying values of Poisson's ratio from 0.1 to 0.4. To inspect significance of nonlocal parameter on vibration of double-walled CNTs, two peculiar values of nonlocal parameter $e_o = 0.2$, 0.65 and 1.2 are considered. Figs. 1-3 illustrate the significance of scale effect on vibration of zigzag (12, 0) double-walled CNTs against Poisson ratio. The scale effect frequency curves are portrayed with $e_o = 0.2$, 0.65 and 1.2 depending upon three boundary conditions. The C-C frequencies of (12, 0) in Fig. 1 at $e_o = 0.2$ versus Poisson ratio are 6.8305,

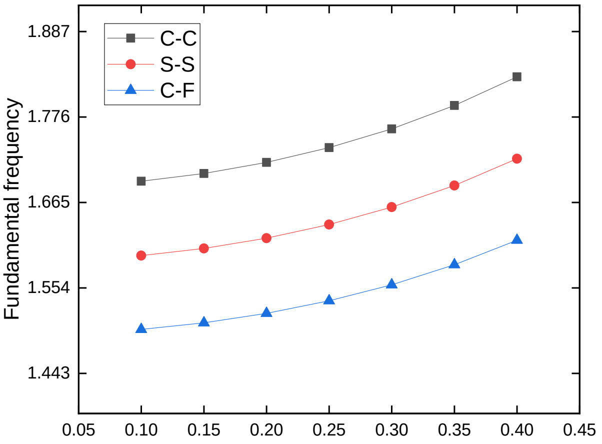

Fig. 5 Frequency variation of chiral (6, 4) with $e_o = 0.65$ versus Poisson's ratio

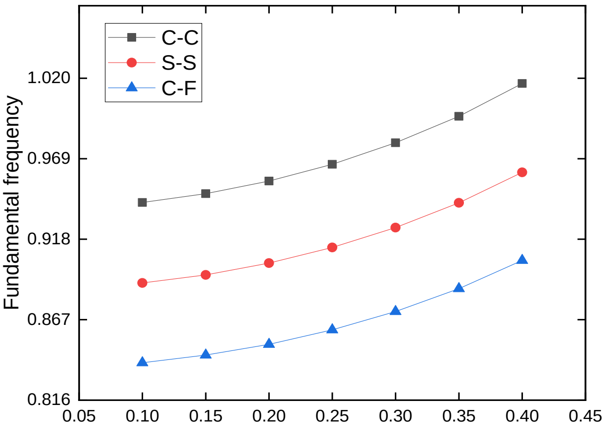

Fig. 6 Frequency variation of chiral (6, 4) with $e_o = 1.2$ versus Poisson's ratio

6.8730, 7.0150, 7.1180, 7.2460 and 7.4030, at $e_o = 0.65$ are 2.4450, 2.4602, 2.4820, 2.5110, 2.5479, 2.5937 and 2.6499 and in similar way at $e_o = 1.2$ are 1.3420, 1.3550, 1.3623, 1.3782, 1.3985, 1.4236, and 1.4545, respectively. Over again the trend presents a slow increase as ratio increases but at same time with increased nonlocal parameter, frequencies decrease for all boundary conditions. Another observation is clearly seen from these curves that zigzag possess higher frequencies as compared to armchair double-walled CNTs. It is explained as from basic carbon morphology zigzag owns numerous elements parallel to tube axis but armchair does not possess such feature. Consequently, zigzag CNTs are supposed to show strong bending and longitudinal permanence in comparison to armchair CNTs. Here in chiral case, Figs. 4-6 elucidate the influence of nonlocal parameters on the frequencies of chiral with indices (6, 4) based on nonlocal love shell model. These frequency curves confirmed the obvious higher values of C-C than those of S-S and C-F chiral double-walled CNTs. When $e_o = 0.2$ then the C-C chiral frequency peaks against ratio variation are 4.2260, 4.2510, 4.2870, 4.3347, 4.3955, 4.4713, and 4.5643, S-S 3.8971, 3.9198, 3.9957, 4.0509, 4.1196, 4.2041, are noticed as and C-F are 3.5737, 3.5941, 3.6233, 3.6621, 3.7116, 3.7734, and 3.8494, drawn in Fig. 4. In Fig. 6 for chiral (6, 4) with $e_o = 0.65$ C-C, 1.6926, 1.7027, 1.7170, 1.7609, and 1.8281, S-S, 1.5960, 1.6053, 1.6364, 1.6590, 1.6871 and 1.7218 and C-F 1.5002, 1.5088, 1.5210, 1.5373, 1.5581, 1.5841, and 1.6159, frequencies are displayed .It can be seen that frequencies are reducing as nonlocal parameter increasing .At the very moment, with higher Poisson's ratio it is noted that frequencies incline as well with respect to all considered end conditions.

4. Conclusions

In this paper, we developed a new model from the combination of the nonlocal shell model with wave propagation approach. The governing equation has been developed for the vibrations of double walled carbon nanotubes considering the nonlocal parameter with C-C, S-S and C-F boundary conditions. The results generated using computer software MATLAB to furnish the evidence regarding applicability of present model as following

- The fundamental frequency patterns for zigzag double-walled carbon nanotubes exhibit the resembling trends for varying values of ratio and considering distinct values of nonlocal parameter. Although we observe the phenomena for structural strength of double-walled CNTs.
- A slow increase in frequencies against variation of Poisson's ratio is observed that indicates the insensitivity for suggested nonlocal model. Because of the fact that greater the Poisson ratio, softer the material. An appropriate selection of material properties and nonlocal parameter has been considered.
- The influence of changing mechanical parameter Poisson's ratio has been investigated in detail. The dominance of boundary conditions via nonlocal parameter is shown graphically.

For future study, by considering present model based on the Winkler and Pasternak foundations, the nonlocal vibration characteristics of multi walled carbon nanotubes can be studied.

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