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Structural monitoring of layered FGM distribution ring support: Analysis with and without internal pressure

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Abstract. In this work, the vibrational frequency of two layered FGM cylindrical shell with and without the effects of internal pressure under ring support are discussed in detailed. The functionally graded materials of a cylindrical shell are designed for specific purpose and studied under various boundary conditions. The Love shell dynamical equations theory is utilized to find the relationship between the curvature displacement and strain displacement. Natural frequency vibrations are analyzed by using volume polynomial for bi-layered FGM shell under ring support both for with and without internal pressures.

Keywords: bi-layered FGM; functionally graded materials; internal pressure; natural frequency

1. Introduction

Functionally graded materials (FGM) are the most favorable materials which are widely used in broad terms, engineering many fields like mechanical, nuclear, chemical, civil and, naval (Markworth *et al*. 1995, Mahamood and Akinlabi, 2017, Mahamood *et al*. 2012, Gupta and Talha, 2015, Chandrasekaran 2020). Many existing cylindrical shells with different axis and different geometry have many uses in different industries (Bolomey 1989). Japanese scientists firstly used the idea of fabrication in 1984 (De Sousa *et al*. 2019, Niino *et al*. 1984). They are more beneficial at high temperature depends upon the environment and heat properties. The FGM materials can be made by using the technology of powder theory (Muller *et al*. 2013).

Bert and Malik (1996) analyzed the different techniques to obtain the equations of motion of the shell with different aspects of the functionally graded materials. Schöllhammer and Fries (2019) proposed a parameterization-free reformulation of the classical Kirchhoff-Love shell equations in terms of tangential differential calculus. An advantage of our approach is that the surface may be defined implicitly, and the resulting shell equations and stress resultants lead to a more compact and intuitive implementation. Numerical tests are performed and it is confirmed that the obtained approach is equivalent to the classical formulation based on local coordinates. Loy *et al*. (1999) analyzed the distribution of functionally graded material with different axial directions and also discuss about the vibrational frequency of FGM cylindrical shell. Cylindrical Shells and shell type structure are important structured components in different engineering fields such as marine, missile bodies, pressure vessels, naval construction and construction buildings in past few years. Zhang (2002) study the vibrational characteristics of the shell using wave propagation method. The vibrational characteristics of the shell are the systematic elements of various types of engineering structure.

Arshad *et al*. (2007) investigated that the trigonometric functions and volume exponential functions are depends on the natural logarithmic frequency of cylindrical shell. Naeem *et al*. (2009) studied in detailed the vibrational frequency of rotating and nonrotating cylindrical shell. They analyzed the cylindrical shell with a wave function of different materials. The wave propagation method is utilized to find out the problems faced in the dynamic equation of motion of the shell. Shah *et al*. (2009) analyzed the natural frequency of the different parameters and discuss about the Ritz-method for observing the vibrational frequency of FGM material of a cylindrical shell. The material properties are utilized to find out the equation of motion of the shell. Iqbal *et al*. (2011) generally FGM are fabricated by metal and ceramic. In the present study stainless steel, nickel and zirconia are utilized to structure the FGM cylindrical shell. In this way three types of shells are obtained. Naeem *et al*. (2012) in this study, the vibrational characteristics of FGM shell submerged in the

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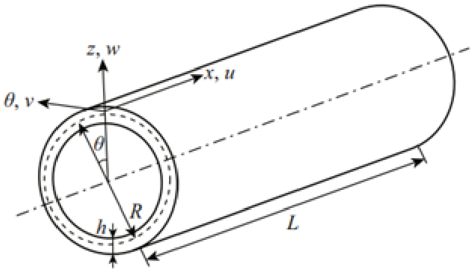

Fig. 1 Geometry of shell (Isvandzibaei *et al.* 2014, Shah *et al.* 2019)

thin FGM and the effect of fluid of acoustic wave equation are studied. Isvandzibaei *et al.* (2014) studied the FGM materials synthesis with different geometry like nickel and steel. They investigated the vibrational frequency of thin cylindrical shell and also studied the frequency of the material with internal pressure. Farahani and Barati (2015) in the present work, the vibration frequency of thick FGM conical, CS with arbitrary elastic restraints is presented. Golpayegani and Jafari (2017) study critical speed analysis of bi-layered rotating cylindrical shells made of functionally graded materials. Hussain and Naeem (2019) investigated the effects of ring supports on vibration of armchair and zigzag FGM rotating carbon nanotubes using Galerkin's method. Recently some researcher used different methods for nonlinear modeling (Eltaher *et al.* 2019, Ebrahimi *et al.* 2019, Safaei *et al.* 2019, Shahsavari *et al.* 2019, Benmansour *et al.* 2019, Kiani and Eslami 2013, Akbari *et al.* 2015).

The vibration of FGM cylindrical shell with ring support and internal pressure are discussed in detailed by the researchers. Behavior of vibration for cylindrical-shaped shell by applying Rayleigh-Ritz technique for clamped boundary conditions is analyzed. In the present study a bi layered cylindrical shell with ring support and internal pressure was investigated. Graphical representation shows the vibration of natural frequencies (Hz) for length of two layers CS against n under SS-SS boundary condition for Type-I for axial half wave parameter at ring support with internal pressure at FGM layers. Also the vibration of natural frequencies (Hz) for thickness to radius ratio against axial half wave parameter under SS-SS boundary condition for Type-I at ring support with internal pressure is investigated. The natural frequency falls, when thickness rises.

2. Mathematical formulation

A cylindrical shell of circular shape having thin walled with different parameters are studied in this work. The radius of shell is R, length L and thickness h shown in Fig. 1. The vibrations of natural frequency of circular cylindrical shell of FGMs having orthogonal coordinate system (t, θ, z), where t represent axial coordinate, z represent radial coordinate and θ shows circumferential coordinates. The Poisson ratio ϑ, Young's modulus E and mass density ρ are the materialistic properties of the shell expressed as

The equations for Love's thin shell theory are defined as

$$\begin{gathered}\frac{\partial N_t}{\partial t}+\frac{1}{R}\frac{\partial N_{t\theta}}{\partial\theta}=\rho h\frac{\partial^2 u}{\partial x^2}\\ \frac{\partial N_{t\theta}}{\partial t}+\frac{1}{R}\frac{\partial N_\theta}{\partial\theta}+\frac{2}{R}\frac{\partial M_{t\theta}}{\partial t}+\frac{1}{R^2}\frac{\partial M_\theta}{\partial\theta}=\rho h\frac{\partial^2 v}{\partial x^2}\\ \frac{\partial^2 M_t}{\partial t^2}+\frac{2}{R}\frac{\partial^2 M_{t\theta}}{\partial t\,\partial\theta}+\frac{1}{R^2}\frac{\partial^2 M_\theta}{\partial\theta^2}-\frac{N_\theta}{R}=\rho h\frac{\partial^2 w}{\partial x^2}\end{gathered}\tag{1}$$

where N_t, N_θ and $N_{t\theta}$ are force resultants and M_t, M_θ and $M_{t\theta}$ moment resultants along the shear direction and normal directions of the cylindrical shell. The stiffness for membrane, coupling and flexural X_{ij}, Y_{ij} and Z_{ij} where i,j = 1,2 and 6 are defined as

$$X_{ij},\quad Y_{ij},\ Z_{ij}\} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \Omega_{ij}\,(1, z, z^2)dz \tag{2}$$

Three materials are used in the constituents of FGMs as nickel, stainless steel and zirconia. These constituents are stainless steel, nickel and zirconia and the values that are used in present work describe the variation in natural frequencies affected by increasing the values of wave number n given below. For the study of a bi-layered FGM cylindrical shell, we consider constituent materials M_1 and M_2 for FGM inner layer whereas M_2 and M_3 for FGM outer layer.

2.1 Mathematical formulation of the FGM

The FGMs are advanced materials and more useful in engineering and technology. Functionally graded materials have many materialistic properties that are made from combination of two or more materials. These materials are very beneficial at high temperature environment that are defined by the following equations.

$$P = P_0[P_1T^1 + 1 + P_2T^2 + P_3T^3 + P_4T^4] \tag{3}$$

where P_0, P_1, P_2, P_3 and P_4 are constants at temperature T in Kelvin scale and fixed for any specific matter. Their combined material properties P can be written as a function of temperature and position of the respective material properties P_j and volume fractions V_{fj} for the j^{th} constituent material of the K constituent materials:

$$\grave{P}(T, z) = \sum_{j=1}^{k} \grave{P}_j(T)\, V_{fj}(z) \tag{4}$$

Moreover, summation of the entire FGM constituent is given by the following expression:

$$\sum_{j=1}^{k} V_{fj}(z) = 1 \tag{5}$$

The volume fraction law for bi-layered FGMs cylindrical shell can be defined as

$$V_1 = \left(\frac{z+\frac{h}{2}}{h}\right)^N,\ V_2 = 1 - \left(\frac{z+\frac{h}{2}}{h}\right)^N \tag{6}$$

The bi layered cylindrical shell having two layered which is made by FG materials are discussed. The cylindrical shell with two FGM layers with density ρ, Young's modulus E and Poisson ϑ are defined by following expressions.

$$\begin{aligned} \mathrm{E} &= \mathrm{E}_{\mathrm{FGM}}^{(1)} + \mathrm{E}_{\mathrm{FGM}}^{(2)} \\ \vartheta &= \vartheta_{\mathrm{FGM}}^{(1)} + \vartheta_{\mathrm{FGM}}^{(2)} \\ \rho &= \rho_{\mathrm{FGM}}^{(1)} + \rho_{\mathrm{FGM}}^{(2)} \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mathrm{E}_{\mathrm{fgm1}} &= (\mathrm{E}_1 - \mathrm{E}_2)\left(\frac{2z+h}{h}\right)^p + \mathrm{E}_1 \\ \nu_{\mathrm{fgm1}} &= (\nu_1 - \nu_2)\left(\frac{2z+h}{h}\right)^p + \nu_1 \\ \rho_{\mathrm{fgm1}} &= (\rho_1 - \rho_2)\left(\frac{2z+h}{h}\right)^p + \rho_1 \end{aligned} \tag{8}$$

$$\begin{aligned} \mathrm{E}_{\mathrm{fgm2}} &= (\mathrm{E}_2 - \mathrm{E}_1)\left(\frac{2z}{h}\right)^p + \mathrm{E}_2 \\ \nu_{\mathrm{fgm2}} &= (\nu_2 - \nu_1)\left(\frac{2z}{h}\right)^p + \nu_2 \\ \rho_{\mathrm{fgm2}} &= (\rho_2 - \rho_1)\left(\frac{2z}{h}\right)^p + \rho_2 \end{aligned} \tag{9}$$

MATLAB software obtains results of volume fraction laws regarding bi layered cylindrical shell with internal pressure at the temperature 300 K.

2.2 Axial function and displacement field

The displacement fields for vibrations of FGM cylindrical shell (CS) with inner pressure and ring support are assumed by following relations,

$$\begin{aligned} &\mathrm{w}_1(x,\theta,t) = \mathrm{X_m K_1}\ (x)(\cos(n\theta))(\cos(\omega t)) \\ &\{\mathrm{w}_2(x,\theta,t) = \mathrm{Y_m K_2}\ (x)(\sin(n\theta))(\sin(\omega t) \\ &\{\mathrm{w}_3(x,\theta,t) = \mathrm{Z_m K_3}(x)(\cos(n\theta))(\cos(\omega t)) \end{aligned} \tag{10}$$

where X_m, Y_m, Z_m represent the amplitude of vibrations in *x*, θ and *z* directions respectively, where *m* and *n* are represented by the axial and circumferential wave number of mode form respectively, angular vibrations frequency of shell wave is denoted by ω. K_1 *(x)*, K_2 *(x)* and K_3 *(x)* represents the axial model dependence in longitudinal, circumference and transverse direction respectively and we take here $K_1(x)=\frac{d\eta(x)}{dx}$

K_2 (x) = η(x) , K_3 (x) = η(x) $\prod_{i=1}^{k}(x - q_i)^{ur}$, where, (x) shows the axial functions which satisfy the geometric boundary condition, q_i is position of ring, k is the number of rings and r represent a parameter having one value with one ring and ω represents the natural frequency (Hz).

The axial function η(x) is chosen as the beam function in the following equation,

$$\begin{aligned} (x) = \alpha_1\cosh\left(\frac{\phi_m x}{L}\right) + \alpha_2\cos\left(\frac{\phi_m x}{L}\right) - \\ \mu_r\left(\alpha_3 \sinh\left(\frac{\phi_m x}{L}\right) + \alpha_4\sin\left(\frac{\phi_m x}{L}\right)\right) \end{aligned} \tag{11}$$

Here the values of α_i are varied with respect to boundary condition (i = 1, 2, 3, 4) μ_r are the parameters for bi-layered cylindrical shell (CS).

The end condition for SS-SS, Simply Support-Simply Support boundary condition

$$\eta(0) = \frac{\partial^2\eta(L)}{\partial x^2} = 0 \tag{12}$$

2.3 Theoretical considerations

For FGM Ω_{ij} (i,j = 1,2 and 6) are reduce stiffness and expressed as

$$\begin{aligned} \mathrm{Q}_{11} = \mathrm{Q}_{22} &= \frac{\mathrm{E}}{1-\vartheta^2} \\ \mathrm{Q}_{12} = \frac{\vartheta \mathrm{E}}{1-\vartheta^2}&, \mathrm{Q}_{66} = \frac{\mathrm{E}}{2(1+\vartheta)} \end{aligned} \tag{13}$$

$$\{\mathrm{X}_{ij},\quad \mathrm{Y}_{ij},\ \mathrm{Z}_{ij}\} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \Omega_{ij}\,(1,z,z^2)dz \tag{14}$$

The Love's thin shell theory for strain-curvature displacement are analyzed and these quantities are defined as

$$(\epsilon_1,\epsilon_2,\gamma) = \left\{\frac{\partial u}{\partial t},\frac{1}{R}\left(\frac{\partial v}{\partial\theta} + w\right),\frac{1}{R}\frac{\partial u}{\partial\theta}\right\} \tag{15}$$

$$\begin{aligned} &(\mathrm{k}_1,\mathrm{k}_2,\tau) \\ &= \left\{-\frac{\partial^2 u}{\partial t^2},\frac{1}{R}\left(\frac{\partial^2 w}{\partial\theta^2} - \frac{\partial v}{\partial\theta}\right),-\frac{1}{R}(\frac{\partial^2 w}{\partial t\,\partial\theta} - \frac{\partial v}{\partial t}\right\} \end{aligned} \tag{16}$$

2.4 Energy equations

The kinetic energy and potential energy of FGM cylindrical shell of different variation for desired theory, In this equations *u*, *v* and *w* are velocity components in three main directions. The kinetic energy of the shell is expressed by the following equation. The kinetic energy without rotating inertia of thin walled cylindrical shell are defined as

$$\Upsilon = \frac{1}{2}\int_0^L\int_0^{2\pi}\rho_i\left[\ \left|\begin{array}{c}\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 \\ + \left(\frac{\partial w}{\partial t}\right)^2\ \ R\,d\theta b\,dx\end{array}\right.\right] \tag{17}$$

$$\mathrm{P_i} = \int_{-h|2}^{h|2}\rho dz \tag{18}$$

For vibrating cylindrical shells, the strain energy is expressed by

$$S = \frac{1}{2}\int_0^L\int_0^{2\pi}\{J\}^T\,[C]\{J\}\ R\,d\theta\,dx \tag{19}$$

By substituting the values of Eqs. (10) and (11) in (12) we get the value of strain energy as

$$\begin{aligned} &S = \\ &\frac{1}{2}\int_0^L\int_0^{2\pi}\left\{\begin{array}{c} X_{11}{\epsilon_1}^2 + 2X_{12}\epsilon_1\epsilon_2 \\ +X_{22}\ {\epsilon_2}^2 + X_{66}{\epsilon_{12}}^2 + 2Y_{11}\epsilon_1 k_1 \\ +2Y_{12}\epsilon_1 k_2 + 2Y_{12}\epsilon_2 k_1 \\ +2Y_{22}\epsilon_2 k_2 + 4Y_{66}\epsilon_{12}k_{12} \\ +Z_{11}{k_1}^2 + 2Z_{12}k_1 k_2 \\ +Z_{22}{k_2}^2 + 4Z_{66}{k_{12}}^2 \end{array}\right\} Rd\theta\,dx \end{aligned} \tag{20}$$

So, $[S]$ can also be defined as,

$$S = \begin{bmatrix} X_{11} & X_{12} & 0 & Y_{11} & Y_{12} & 0 \\ X_{21} & X_{22} & 0 & Y_{21} & Y_{22} & 0 \\ 0 & 0 & X_{66} & 0 & 0 & Y_{66} \\ Y_{11} & Y_{12} & 0 & Z_{11} & Z_{12} & 0 \\ Y_{21} & Y_{22} & 0 & Z_{21} & Z_{22} & 0 \\ 0 & 0 & Y_{66} & 0 & 0 & Z_{66} \end{bmatrix} \tag{21}$$

Total energy of system is as follows

$$F^{\cdot} = \Upsilon - S - P \tag{22}$$

To determine the natural frequencies, the Rayleigh-Ritz method is used. The Lagrangian energy functional Π defined by the following equation

$$\Pi = \Upsilon_{max} - S_{max} \tag{23}$$

Here Υ_{max} and S_{max} are the maximum kinetic energy and strain energy, respectively.

The function is minimized with respect to unknowns X, Y and Z of the cylindrical shell as follows:

$$\frac{\partial\Pi}{\partial\alpha} = \frac{\partial\Pi}{\partial\beta} = \frac{\partial\Pi}{\partial\gamma} = 0 \tag{24}$$

In Eq. (23), Υ_{max} and S_{max} are the maximum kinetic energy and strain energy, respectively. Performing the minimization as in Eq. (16) yields a set of equations that can be expressed as follows:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \\ \gamma \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

where C_{ij} are some coefficients defined in the appendix Eq. (7) is solved by imposing the condition of non-trivial solutions and equating the characteristic determinant (C_{ij}) to zero. i.e.,

$$|C_{ij}| = 0 \ (i, j = 1,2,3) \tag{26}$$

Results are obtained by MATLAB programming software when diverse boundary conditions are applied. MATLAB software obtains results of volume fraction laws regarding bi-layered cylindrical shell with and without internal pressure.

The solution for Eq. (26) is acquired and a study of the functionally graded materials (FGM) cylindrical shell (CS) with ring support and internal pressure is also expressed in the power of ω such as:

$$b_0\omega^6 + b_1\omega^4 + b_2\omega^2 + b_3 = 0 \tag{27}$$

The solution of Eq. (27) consists of Eq. (6) roots where, three positive roots the natural frequencies and smallest positive roots is the frequency of significance in recent work.

3. Results and discussion

Table 1 Accuracy of the natural frequencies (Hz) for a soft simply supported FG cylindrical shell (h/R = 0.05, *L/R* = 20)

N	method	*n*		
		1	2	3
0	Malekzadeh and Heydarpour (2012)	12.898	31.543	87.977
	Present	11.262	29.756	86.182
1	Malekzadeh and Heydarpour (2012)	13.195	32.269	90.007
	Present	12.891	31.387	89.543

Table 2 Comparison of values of the frequency parameter *ω* for a cylindrical shell with S–S boundary conditions (*m* =1, *mR/L* = 0.05, *μ* = 0.3)

h/R	*m*	method	*n*				
			0	1	2	3	4
0.002	1	Lam and Loy (1995)	0.09292980	0.01610110	0.00545297	0.00504148	0.00853383
		Present	0.09292015	0.0161001	0.00545198	0.00504082	0.00853291
0.05		Lam and Loy (1995)	0.09296820	0.01610290	0.03927100	0.10981130	0.21027700
		Present	0.09296729	0.0161011	0.0392703	0.10981128	0.21027691

Table 3 Formation of constituents material

values	material properties		
materials	E (young's modulus)	ϑ (poisson ratio)	ρ (density)
stainless steel	2.07788E+11	0.317756	8166
nickel	2.05098E+11	0.310	8900
zirconia	1.68063E+11	0.298	5700

In this work vibration frequencies for a functionally graded cylindrical shells are presented and analysis. Before presenting results for cylindrical shells, some comparisons of results for them are performed to test the efficiency, validity, accuracy and robustness of the present numerical procedure. The calculated results are compared with those ones by Malekzadeh and Heydarpour (2012) and Lam and Loy (1995) as shown in Tables 1 and 2. The present calculated values show a good agreement between two sets of frequency parameters.

This work investigated the vibrational frequency of FGM material with simply supported boundary conditions. In our study, the cylindrical shell constituents of FGM material with two surfaces inner and outer layer surfaces are used, the inner surfaces are represents by M_1 and M_2 and the outer layers are M_3 and M_4 respectively. Three materials are used in the constituents of FGMs as nickl, steenless steel and zirconia. These constituents are stainless steel, nickel and zirconia and the values that are used in Table 3. The study variation in natural frequencies affected by increasing the values of wave number n from 1 to 10 and m is taken from 1 to 4. There are six types of variation called Type-I, Type-II, Type-III, Type-IV, Type-V and Type-VI obtaining by the variation in length *L* of the shell from 20 to 65, thickness h of the shell from 0.001 to 0.01 and q from 0.1 to 1.0. The first three types of tables for bi layered FGM shell with ring support and internal pressure and last three types of tables for bi layered FGM shell with ring

Table 4 Configuration of shell type with ring support

Type - I	Configuration of shell type with ring support	
n	*q* = 0.3	*q* = 0.4
1	1465.51779	1481.72884
2	2142.82263	2473.40273
3	2753.04151	3020.35555
4	4132.67451	4325.94464
5	6269.98739	6410.24218
6	7595.84398	7717.99418
7	1358.95225	1162.14157
8	2505.52588	2308.68139
9	6147.49749	6056.16141
10	7489.61562	7410.80179

Table 5 Variation of natural frequency against circumference wave number *n* for shell of Type-II (*m* = 1, *R* = 1, *h* = 0.03, *q* = 0.2)

Type - II	Length of shell with ring support and internal pressure	
n	*L* = 30	*L* = 35
1	2453.2342	2452.7954
2	3140.9681	3140.8461
3	3330.4868	3330.4459
4	3744.46397	3744.4041
5	4551.7940	4551.6981
6	5122.9478	5122.8355
7	3140.6449	3140.6231
8	3330.3786	3330.3713
9	3744.3054	3744.2947
10	5122.6504	5122.6303

Table 6 Variation of natural frequency against circumference wave number *n* for shell of Type-II (*m* = 1, *R* = 1, *L* = 18, *q* = 0.2)

Type - III	Thickness differences of shell with ring support and internal pressure	
n	*h* = 0.001	*h* = 0.002
1	14937.70622	10562.40718
2	20498.45498	14494.73477
3	20752.28168	14676.05131
4	20822.45908	14731.04656
5	20856.16609	14766.92488
6	20869.12538	14785.87017
7	8150.67601	7546.06283
8	8514.67378	7891.34917
9	8586.09343	7975.13992
10	8706.21597	8122.61056

support and without internal pressure. The radius of the shell *R* is taken as 1, young's modulus is taken as $E = 30\times10^6$and density of the material is $\rho = 7.34\times10^{-4}$ and $\vartheta = 0.25$. Table 4 represent the behavior frequency (HZ) with n for a Type-I FGM SS-SS cylindrical shell. The natural frequency obtained from different value of n rises. When *q* rises from 0.1 to 1.0 and also we take similar value without internal pressure. As the circumferential wave number n increases from 1 to 10 then it is observed that the natural frequency also increases as n increases from 1 to 5 and after that *q* from 6 to 1 the value of natural frequency decreases horizontally. The table shows the variation of natural frequency of bi layered FGM CS with ring support and without the effect internal pressure. As *n* increases from 1 to 5 then it is observed that the natural frequency also increases in the table as *q* increases from 0.1 to 0.5 and after that *q* from 0.6 to 1.0 the value of natural frequency decreases horizontally. The natural frequency of shell increases vertically when n rises from 1 to 10.

The table shows the variation for smaller values of *q* the value of the frequency tend to that of the SS conditions. Table 5 represent the behavior of frequency with n for a Type-II functionally graded SS-SS cylindrical shell. The natural frequency falls horizontally and rises vertically and also we take similar value without internal pressure. The natural frequency obtained from different value of L rises when n rises. The material property defined as the thickness is taken *h* = 0.03, radius *R* = 1, *q* = 0.2 and the axial wave number *m* = 1 to 4. It is also observed that the natural frequency increases vertically as *n* increases from 1 to 10. The table shows the variation of natural frequency of bi layered FGM CS with ring support and without the effect internal pressure. It is observed that the length of the CS *L* as increases from 20 to 40 the natural frequency of the material decreases from horizontally for *n* = 1 and similarly same results *n* from 1 to 10. It is also observed that the natural frequency increases from vertically as n increases from 1 to 10. Table 6 represents the behavior of frequency with n for a Type-III functionally graded simply supported – simply supported (SS - SS) for cylindrical shell (CS). The natural frequency falls when *h* rises. The natural frequency obtained from different value of h rises when n rises. The material property defined as the thickness is taken *L* = 18, *R* = 1, *q* = 0.3 and the axial wave number *m* = 1 to 4. It is concluded that the natural frequency of the shell with the effect of ring and pressure decreases horizontally as the thickness h of the shell increases from 0.001 to 0.005 and also we take similar value without internal pressure. It is also observed that the natural frequency of shell rises vertically when increases from 1 to 10. The table shows the variation of natural frequency of bi layered FGM CS with ring support and without the effect internal pressure. It is concluded that the natural frequency of the shell with ring pressure decreases horizontally as the thickness h of the shell increases from 0.001 to 0.005. It is also observed that the natural frequency of shell rises vertically when increases from 1 to 10. Graphical representation shows the vibration of natural frequencies (Hz) for length of two layers CS against *n* under SS-SS boundary condition for Type-I for axial half wave parameter (*m* = 1, 2 *R* = 1, *h* = 0.1) at ring support with internal pressure at FGM layers. The fig shows that vibration of natural frequencies (Hz) for thickness to radius ratio against n under SS-SS boundary condition for

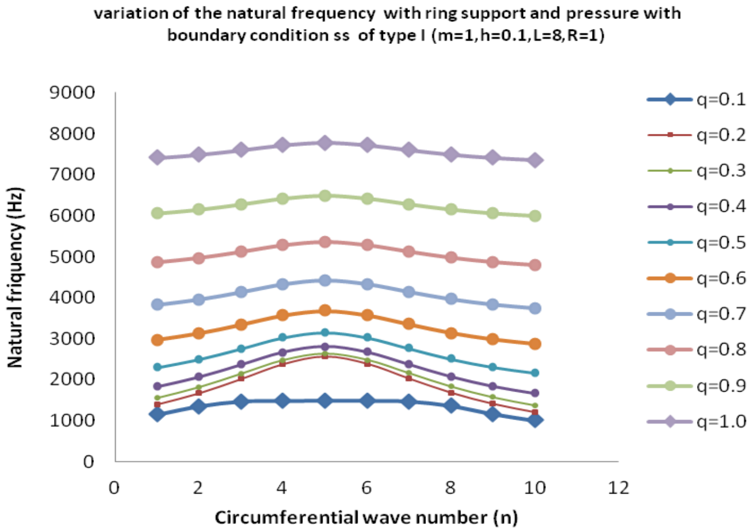

Fig. 2 Natural frequencies (Hz) for axial wave number of two layers CS against n under SS-SS boundary condition

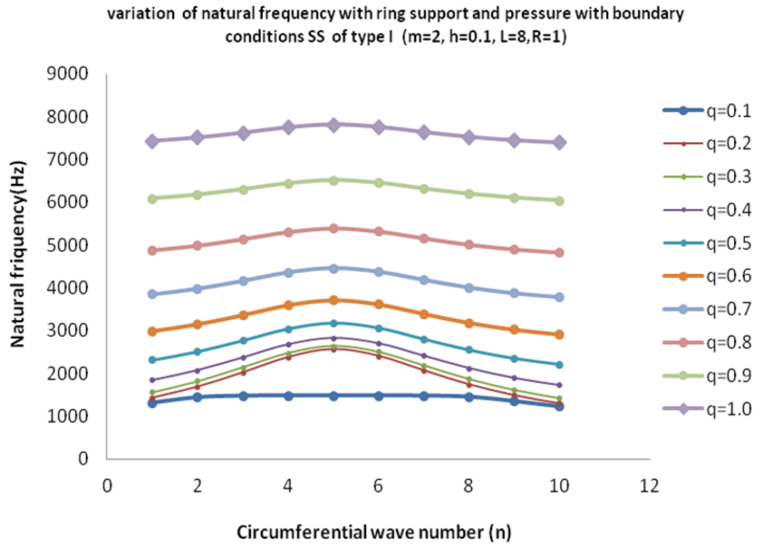

Fig. 3 Natural frequencies (Hz) for axial wave number of two layers CS against *n* under SS-SS boundary condition

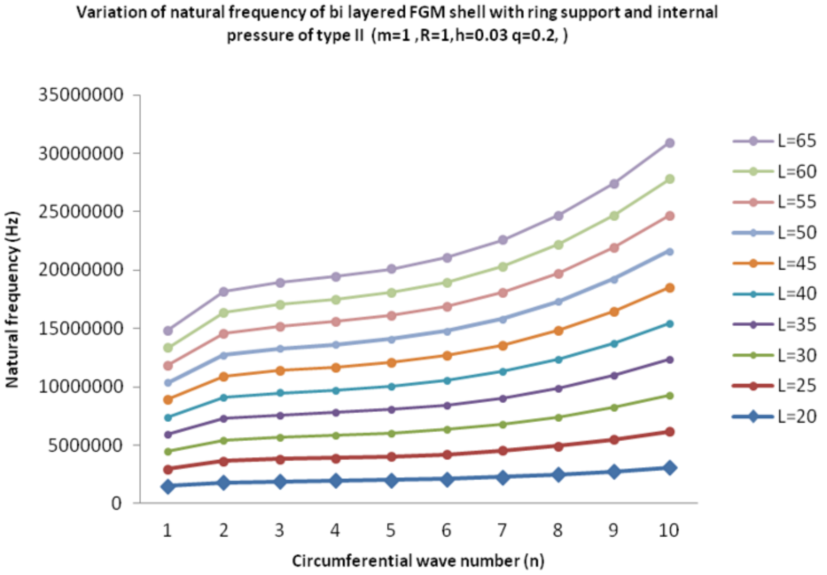

Fig. 4 Vibration of natural frequencies (Hz) for length of two layers CS against *n* under SS-SS boundary condition

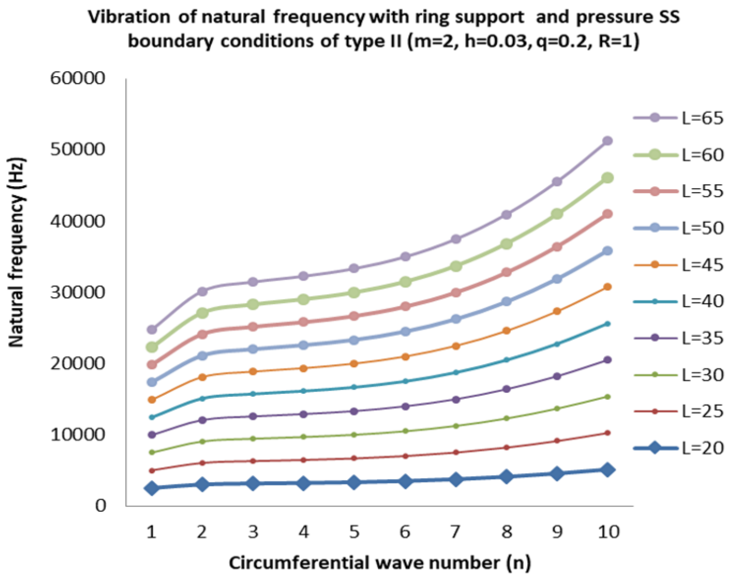

Fig. 5 Vibration of natural frequencies (Hz) for length of two layers CS against *n* under SS-SS boundary condition

Type-I for axial half wave parameter (*m* = 1, 2 *R* = 1, *L* = 20) at ring support with internal pressure. The natural frequency falls, when thickness *h* rises. The Figs. 2 and 3 shows the vibration of natural frequencies (Hz) for axial wave number of two layers CS against *n* under SS-SS boundary condition for Type-I for axial half wave parameter (*m* = 1, 2 *R* = 1, *h* = 0.1) at ring support of length with internal pressure at FGM layer. It is observed that the natural frequency also increases as *n* increases from 1 to 5 and after that *n* from 6 to 10 the value of natural frequency decreases horizontally. In Figs. 4 and 5 shows the vibration of natural frequencies (Hz) for length of two layers CS against *n* under SS-SS boundary condition for Type-II for axial half wave parameter (*m* = 1, 2 *R* = 1, *h* = 0.03) at ring support *q* = 0.3 with internal pressure at FGM layers. The natural frequency achieved from different value of length (*L* = 20 to 65) rises when circumferential wave number n rises at 1 to 10. It is observed that the length of the cylindrical shell *L* as increases the natural frequency of the material decreases from horizontally for circumferential wave number *n* = 1 and similarly same results as n from 1 to 10. It is also observed that the natural frequency increases vertically as *n* increases from 0 to 10.

4. Conclusions

The behavior of the natural frequencies (Hz) are assumed either increased or decreased by increasing the value of the ring support q with the placing of ring *q* of two layered functionally graded cylindrical shell (CS). From the table it is observed that the variation in vibrational characteristics through the circumferential wave number n gradually increases. The table shows the variation for smaller values of *q* the value of the frequency tends to that of the SS shell for circumferential. It is also observed from the table the natural frequency of shell rises vertically when increases from 1 to 10. The comparison is presented between the results obtained by the effect of internal pressure and without the effect of internal pressure. From the graphical representation and obtained the result that the vibrational frequency of the shell for internal pressure are different to each other as circumferential wave number n increases the vibrational frequency decreases by the effect of internal pressure. The natural frequency of the shell without the effect of internal pressure are decreases gradually by increasing the circumferential wave number n. graphical representation and obtained the result that the vibrational frequency of the shell for internal pressure are different to each other as circumferential wave number n increases the vibrational frequency decreases by the effect of internal pressure. The natural frequency of the shell without the effect of internal pressure are decreases gradually by increasing the circumferential wave number *n*. This model can be extended for the vibration plates with Galerkin's method.

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