

Mathematical modeling of concrete beams containing GO nanoparticles for vibration analysis and measuring their compressive strength using an experimental method

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Abstract. Due to the extensive use of concrete structures in various applications, the improvement of their strength and quality has become of great importance. A new way of achieving this purpose is to add different types of nanoparticles to concrete admixtures. In this work, a mathematical model has been employed to analyze the vibration of concrete beams reinforced by graphene oxide (GO) nanoparticles. To verify the accuracy of the presented model, an experimental study has been conducted to compare the compressive strengths of these beams. Since GO nanoparticles are not readily dissolved in water, before producing the concrete samples, the GO nanoparticles are dispersed in the mixture by using a shaker, magnetic striker, ultrasonic devices, and finally, by means of a mechanical mixer. The sinusoidal shear deformation beam theory (SSDBT) is employed to model the concrete beams. The Mori-Tanaka model is used to determine the effective properties of the structure, including the agglomeration influences. The motion equations are calculated by applying the energy method and Hamilton's principle. The vibration frequencies of the concrete beam samples are obtained by an analytical method. Three samples containing 0.02% GO nanoparticles are made and their compressive strengths are measured and compared. There is a good agreement between our results and those of the mathematical model and other papers, with a maximum difference of 1.29% between them. The aim of this work is to investigate the effects of nanoparticle volume fraction and agglomeration and the influences of beam length and thickness on the vibration frequency of concrete structures. The results show that by adding the GO nanoparticles, the vibration frequency of the beams is increased.

Keywords: analytical method; concrete beam; experimental method; GO nanoparticles; vibration

1. Introduction

Numerous empirical research works have been devoted to the modeling of various structures. These modeling approaches can be divided into two categories: atomic modeling, and continuum mechanics modeling (Solhjo and Vakis 2015). The atomic modeling techniques include the molecular dynamics, molecular dynamics strong bond, and the theory of density. This type of modeling involves complex calculations for the systems that contain a large number of atoms, and also its practical applications are very limited. However, the limitations of atomic modeling have been overcome with the development of continuum mechanics. Today, continuum mechanics models are widely used for modeling various structures. A comparison between the results of these two modeling approaches shows that the continuum mechanics method is more successful in predicting the dynamic and static behaviors of different systems. So, more researchers are relying on the continuum mechanics models to study the dynamic and static behaviors of structures.

In recent years, more theoretical and experimental

studies have been devoted to nano-composites. With the advancement of technology, nano-composites have gained a special reputation in the construction industry and their use in different structures is growing. The reason is that by using nanoparticles as reinforcement, excellent mechanical and thermal properties can be achieved and the static and dynamic behaviors can be improved.

Mirza *et al.* (1991) showed that the strength of a short composite steel-concrete beam is affected by variations in the strengths of concrete and steel, the cross-sectional dimensions of concrete and steel sections, the placement of steel sections and reinforcing bars, and the strength model used. To calculate the properties of composites, Tan and Tong (2001) presented a Micro-electromechanical model. The free vibration analysis of a composite cylindrical shell containing fluid was performed by Kadoli and Ganesan (2003). Wuite and Adali (2005) investigated stress analysis of beams reinforced with carbon nanotubes. They concluded that the presence of carbon nanotubes as reinforcement enhances the beam stability and rigidity. Matsuna (2007) analyzed the stability of composite cylindrical shells with the help of the third-order shear theory. Formica (2010) examined the vibrations of a plate reinforced with carbon nanotubes and used the Mori-Tanaka model for determining its composite properties. The axial failure of RC beams damaged by shear reversals was presented by Henkhaus *et al.* (2013). General information

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about the definitions of slenderness in the well-known standards and ten selected industrial reinforced concrete (RC) chimneys were given by Karaca and Türkeli (2014). Liew *et al.* (2014) studied the postbuckling of nano composite cylindrical panels under axial compression. In this study, the mixture law was used to obtain the properties of the nanocomposites and the meshless method was applied to analyze and calculate the buckling load the of nanocomposite structures. The dynamic behavior of composite cylindrical shells containing fluid flow was studied by Seo *et al.* (2015). In this work, an asymmetrical thermal load and a uniform electrical charge were considered. The buckling of polymeric plates reinforced with carbon nanotubes was studied by Kolahchi *et al.* (2015). They used the DQM to obtain the structural buckling load. In a work by Kolahchi *et al.* (2016a), the dynamic buckling of plates reinforced with functionally graded carbon nanotubes was investigated. In this work, the temperature-dependent properties were taken into account and the orthotropic Pasternak model was used for simulating the elastic foundation. Berghouti *et al.* (2019) studie Vibration analysis of nonlocal porous nanobeams made of functionally graded material. Hadji *et al.* (2021) analyzed the bending and free vibrations of FGM plates containing pores of various distributions and shapes. These studies are completed by Al-Furjan *et al.* (2020, 2021a, b, c), Fakhari and Kolahchi (2018), Hajmohammad *et al.* (2018, 2021) and Keshtegar *et al.* (2020, 2021a, b).

In the field of mathematical modeling of concrete structures, Jafarian and Kolahchi (2016) studied the buckling of concrete beams reinforced with carbon nanotubes by using the Euler - Bernoulli and Timoshenko beam models. The nonlinear buckling of an embedded straight concrete beam reinforced with silicon dioxide (SiO₂) nanoparticles was investigated by Zamanian *et al.* (2017). Taherifar *et al.* (2020) modeled a pad concrete foundation with a smart layer for earthquake analysis. Yang and Li (2020) experimentally studied the shear behaviors of partial precast steel reinforced concrete beams.

A survey of the scientific literature shows no investigation of the effects of GO nanoparticles on the vibration of concrete beams. So in this work, the vibration analysis of concrete beams reinforced with GO nanoparticles is presented. The Mori-Tanaka model is used for calculating the equivalent material properties of the beams. The concrete beam is simulated with SSDT mathematically and analytical model is used for obtaining the frequency of structure. To validate the presented model, the compressive strengths of the concrete beams are experimentally determined and compared. The effects of GO nanoparticles volume fraction and agglomeration and of beam length and thickness on the vibration frequency are also investigated.

2. Mathematical modeling

Fig. 1 shows a concrete beam reinforced with GO nanoparticles.

In addition, through the presence of GO nanoparticles, the structure is subjected to an electric field. Using SSDT, the displacement field can be written as (Thai and Vo 2012):

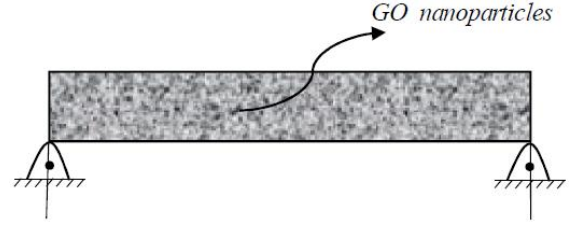


Fig. 1 Schematic of a concrete beam reinforced with GO nanoparticles

$$u_1(x, z, t) = u(x, t) - z \frac{\partial w(x, t)}{\partial x} + f\psi(x, t) \quad (1)$$

$$u_2(x, z, t) = 0 \quad (2)$$

$$u_3(x, z, t) = w(x, t) \quad (3)$$

where u_1 , u_2 and u_3 are the displacements of the beam's mid-plane in the axial, transverse and thickness directions, respectively, ψ represents the rotation of cross section about y axis, and $f = \frac{h}{\pi} \sin\left(\frac{\pi z}{h}\right)$. Using Eqs. (1) to (3), the nonlinear strain-displacement relations based on Von -Karman theory are as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + f \frac{\partial \psi}{\partial x} \quad (4)$$

$$\varepsilon_{xz} = \cos\left(\frac{\pi z}{h}\right) \psi \quad (5)$$

The stress equations of the beam are

$$\sigma_{xx} = Q_{11} \varepsilon_{xx} \quad (6)$$

$$\sigma_{xz} = Q_{55} \varepsilon_{xz} \quad (7)$$

2.1 Mori-Tanaka model

Here, E and ν are the Young's modulus and Poisson's ratio of the GO-reinforced concrete beam; which can be calculated by the Mori-Tanaka model as

$$E = \frac{9KG}{3K + G} \quad (8)$$

$$\nu = \frac{3K - 2G}{6K + 2G} \quad (9)$$

where the effective bulk modulus (K) and effective shear modulus (G) can be expressed as

$$K = K_{out} \left[1 + \frac{\xi \left(\frac{K_{in}}{K_{out}} - 1 \right)}{1 + \alpha(1 - \xi) \left(\frac{K_{in}}{K_{out}} - 1 \right)} \right] \quad (10)$$

$$G = G_{out} \left[1 + \frac{\xi \left(\frac{G_{in}}{G_{out}} - 1 \right)}{1 + \beta(1 - \xi) \left(\frac{G_{in}}{G_{out}} - 1 \right)} \right] \quad (11)$$

where

$$K_{in} = K_m + \frac{(\delta_r - 3K_m\chi_r)C_r\zeta}{3(\xi - C_r\zeta + C_r\zeta\chi_r)} \quad (12)$$

$$K_{out} = K_m + \frac{C_r(\delta_r - 3K_m\chi_r)(1 - \zeta)}{3[1 - \xi - C_r(1 - \zeta) + C_r\chi_r(1 - \zeta)]} \quad (13)$$

$$G_{in} = G_m + \frac{(\eta_r - 3G_m\beta_r)C_r\zeta}{2(\xi - C_r\zeta + C_r\zeta\beta_r)} \quad (14)$$

$$G_{out} = G_m + \frac{C_r(\eta_r - 3G_m\beta_r)(1 - \zeta)}{2[1 - \xi - C_r(1 - \zeta) + C_r\beta_r(1 - \zeta)]} \quad (15)$$

where two parameters ξ and ζ describe the agglomeration of nanoparticles and C_r relates to the SiO₂ volume fraction. In addition, $\chi_r, \beta_r, \delta_r, \eta_r$ are calculated as

$$\chi_r = \frac{3(K_m + G_m) + k_r - l_r}{3(k_r + G_m)} \quad (16)$$

$$\beta_r = \frac{1}{5} \left\{ \begin{array}{l} \frac{4G_m + 2k_r + l_r}{3(k_r + G_m)} + \frac{4G_m}{(p_r + G_m)} \\ \frac{2[G_m(3K_m + G_m) + G_m(3K_m + 7G_m)]}{G_m(3K_m + G_m) + m_r(3K_m + 7G_m)} \end{array} \right\} \quad (17)$$

$$\delta_r = \frac{1}{3} \left[n_r + 2l_r + \frac{(2k_r - l_r)(3K_m + 2G_m - l_r)}{k_r + G_m} \right] \quad (18)$$

$$\eta_r = \frac{1}{5} \left[\begin{array}{l} \frac{2}{3}(n_r - l_r) + \frac{4G_m p_r}{(p_r + G_m)} \\ \frac{8G_m m_r(3K_m + 4G_m)}{3K_m(m_r + G_m) + G_m(7m_r + G_m)} \\ \frac{2(k_r - l_r)(2G_m + l_r)}{3(k_r + G_m)} \end{array} \right] \quad (19)$$

where k_r, l_r, n_r, p_r and m_r are the Hill's elastic modulus for the nanoparticles (Mori and Tanaka 1973), and K_m and G_m are the bulk and shear moduli of the matrix; which can be written as

$$K_m = \frac{E_m}{3(1 - 2\nu_m)} \quad (20)$$

$$G_m = \frac{E_m}{2(1 + \nu_m)} \quad (21)$$

where E_m and ν_m are Young's modulus and the Poisson's ratio of the concrete beam, respectively. Furthermore, β, α can be obtained from

$$\alpha = \frac{(1 + \nu_{out})}{3(1 - \nu_{out})} \quad (22)$$

$$\beta = \frac{2(4 - 5\nu_{out})}{15(1 - \nu_{out})} \quad (23)$$

$$\nu_{out} = \frac{3K_{out} - 2G_{out}}{6K_{out} + 2G_{out}} \quad (24)$$

2.2 Governing equations

One way of deriving the governing equations is to use

the energy method and the Hamilton's principle. The potential energy of the structure can be written as follows:

$$U = \frac{1}{2} \int_V (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\varepsilon_{xz}) dV \quad (25)$$

By substituting Eqs. (4) and (5) into Eq. (25), the potential energy is obtained as

$$U = \frac{1}{2} \int_V \left(\sigma_{xx} \left(\frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + f \frac{\partial \psi}{\partial x} \right) + \sigma_{xz} \left(\cos \left(\frac{\pi z}{h} \right) \psi \right) \right) dV \quad (26)$$

The in-plane forces and moments are defined as

$$(N, M, P) = \int_A (1, z, f) \sigma_x dA \quad (27)$$

$$Q = \int_A \cos \left(\frac{\pi z}{h} \right) \sigma_{xz} dA \quad (28)$$

Using Eqs. (27) and (28), the potential energy can be simplified as follows:

$$U = \int_x \left(N \frac{\partial u}{\partial x} - M \frac{\partial^2 w}{\partial x^2} + P \frac{\partial \psi}{\partial x} + Q \psi \right) dx \quad (29)$$

The kinetic energy of the structure is expressed as

$$K = \frac{\rho}{2} \int_V (\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2) dV \quad (30)$$

By substituting Eqs. (1)-(3) into Eq. (30) we have

$$K = \frac{\rho}{2} \int \left(\left(\frac{\partial u}{\partial t} - z \frac{\partial^2 w}{\partial x \partial t} + f(z) \frac{\partial \psi}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right) dV \quad (31)$$

where ρ is the beam density. By defining the inertia moment terms as

$$\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{Bmatrix} = \int \begin{Bmatrix} \rho \\ \rho z \\ \rho z^2 \\ \rho f(z) \\ \rho z f(z) \\ \rho f(z)^2 \end{Bmatrix} dA \quad (32)$$

Eq. (31) can be rewritten as

$$K = 0.5 \int \left[I_0 \left(\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right) - 2I_1 \left(\frac{\partial u}{\partial t} \frac{\partial^2 w}{\partial x \partial t} \right) + I_2 \left(\frac{\partial^2 w}{\partial x \partial t} \right)^2 - 2I_3 \left(\frac{\partial u}{\partial t} \frac{\partial \psi}{\partial t} \right) - I_4 \left(\frac{\partial^2 w}{\partial x \partial t} \frac{\partial \psi}{\partial t} \right) + I_5 \left(\frac{\partial \psi}{\partial t} \right)^2 \right] dx \quad (33)$$

The Hamilton's principle is expressed as follows:

$$\int_0^t (\delta U - \delta K) dt = 0 \quad (34)$$

Now by applying the Hamilton's principle, the three governing equations can be obtained as follows:

$$\delta u: \frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 w}{\partial x \partial t^2} - I_3 \frac{\partial^2 \psi}{\partial t^2} \quad (35)$$

Table 1 The results of cubic samples compressive strength at different ages

Volume percent of GO	Without GO			With 0.02% GO		
	Xu <i>et al.</i> (2019)	Present work	Diff%	Xu <i>et al.</i> (2019)	Present work	Diff%
3 days	-	21.67	-	-	24.85	-
7 days	32	33.91	5	43	44.41	3
14 days	50	52.28	4	62	56.73	9
28 days	59	55.1	7	77	68.24	10

$$\delta w: \frac{\partial^2 M}{\partial x^2} - \frac{\partial^2 F}{\partial x^2} + \frac{\partial Q}{\partial x} = I_0 \frac{\partial^2 w}{\partial t^2} + I_1 \frac{\partial^3 u}{\partial x \partial t^2} - I_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} + I_4 \frac{\partial^3 \psi}{\partial x \partial t^2} \quad (36)$$

$$\delta \psi: Q_x - \frac{\partial F_x}{\partial x} = I_5 \frac{\partial^2 \psi}{\partial t^2} - I_3 \frac{\partial^2 u}{\partial t^2} + I_4 \frac{\partial^3 w}{\partial x \partial t^2} \quad (37)$$

By substituting Eqs. (6) and (7) into (27) and (28), the internal forces and moments can be calculated as follows:

$$N = hQ_{11} \frac{\partial u}{\partial x} \quad (38)$$

$$M = -Q_{11} I \frac{\partial^2 w}{\partial x^2} + \frac{24Q_{11} I}{\pi^3} \frac{\partial \psi}{\partial x} \quad (39)$$

$$P = -\frac{24Q_{11} I}{\pi^3} \frac{\partial^2 w}{\partial x^2} + \frac{6Q_{11} I}{\pi^2} \frac{\partial \psi}{\partial x} \quad (40)$$

$$Q = \frac{Q_{55} A}{2} \psi \quad (41)$$

where

$$(A, I) = \int_A (1, z^2) dA \quad (42)$$

3. Analytical solution

Assuming simply-supported boundary conditions, the three displacement mode shapes can be written as

$$d = \begin{Bmatrix} u \\ w \\ \psi \end{Bmatrix} = \sum_{m=0}^{\infty} \begin{Bmatrix} A_1 \cos\left(\frac{m\pi x}{L}\right) e^{i\omega t} \\ A_2 \sin\left(\frac{m\pi x}{L}\right) e^{i\omega t} \\ A_3 \sin\left(\frac{m\pi x}{L}\right) e^{i\omega t} \end{Bmatrix} \quad (43)$$

where ω represents the vibration frequency of the beam, and m is the axial wave number. Substituting Eq. (43) into the motion equations yields

$$[K][d] + [M][\ddot{d}] = 0 \quad (44)$$

where $[K]$ and $[M]$ are the stiffness and mass matrixes, respectively. The frequency of the structure can now be obtained by using the eigenvalue method.

4. Experimental analysis

Since the nanoparticles are not readily dissolved in

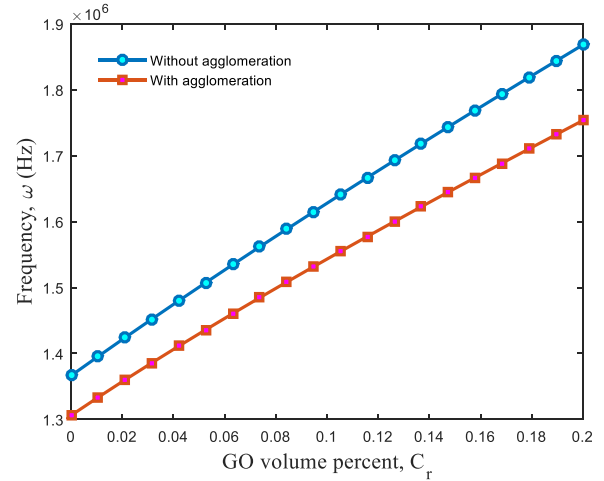


Fig. 2 GO volume percent effects on the frequency of the structure

water without any specific process, before producing the concrete samples, the GO nanoparticles are dispersed in the mixture by using shaker, magnetic stirrer, and ultrasonic devices and, finally, by a mechanical mixer based on the amount of nanoparticles used relative to cement at specific times. In order to determine the average compressive strength of concrete, cubic samples of 51*51*51 mm dimensions were produced and cured for 3, 7, 14, and 28 days before measuring their compressive strength. Table 1 shows the compressive strengths of these cubic samples and compares them with the results obtained by Xu *et al.* (2019).

As can be seen, by adding GO nanoparticles to the concrete samples, their compressive strength has been improved. In addition, the experimental data of this work are in a good agreement with the results of Xu *et al.* (2019). Based on the mathematical modeling, the compressive strength has been calculated with $f'_c = 4700\sqrt{E}$ utilizing Mori-Tanaka model which is 78 MPa. This shows a good validation of the analytical and experimental results.

5. Numerical results

In this chapter, using the analytical model, the beam vibration frequencies are calculated and the effects of various parameters such as volume percentage of GO nanoparticles, geometrical parameters and the agglomeration of GO nanoparticles on beam frequency are examined. For this purpose, a concrete beam with elastic modulus of

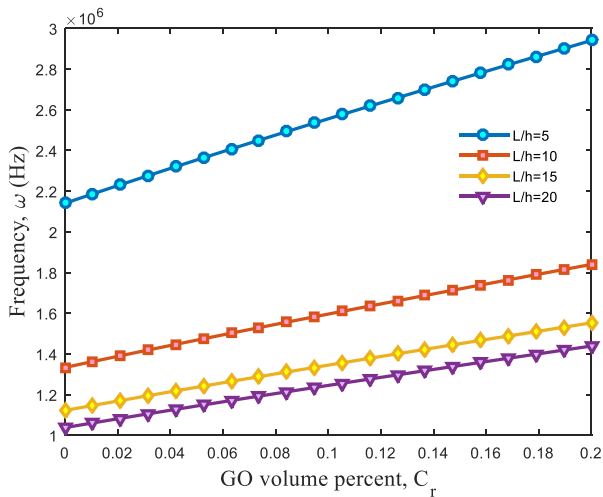


Fig. 3 Length to thickness ratio effect on the frequency of the structure

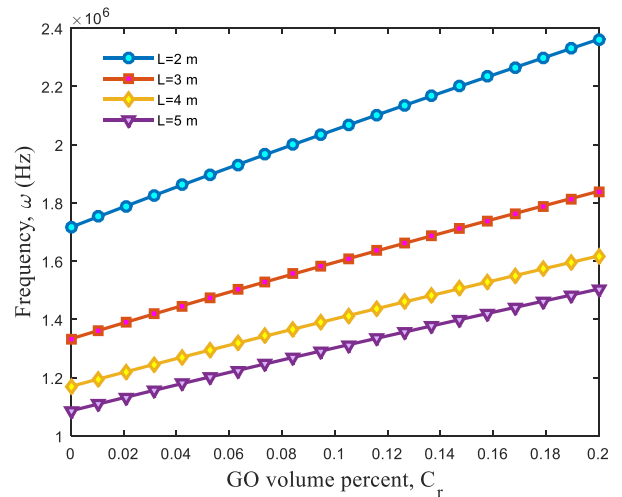


Fig. 5 Length effect on the frequency of the structure

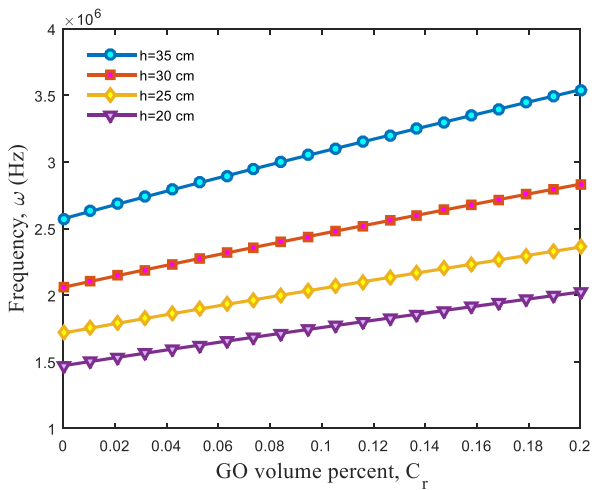


Fig. 4 Thickness effect on the frequency of the structure

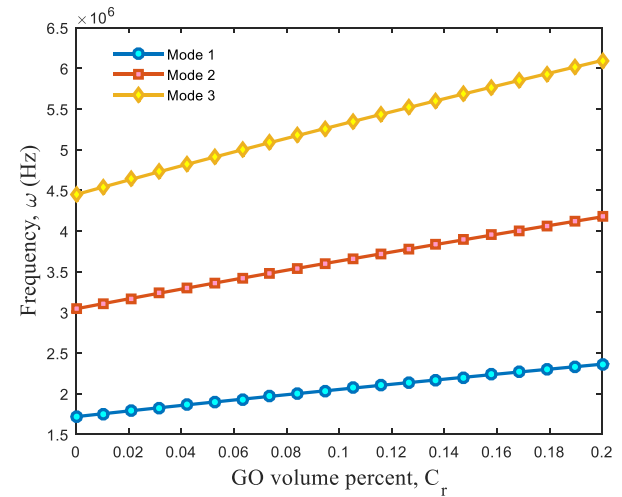


Fig. 6 Mode number effect on the frequency of the structure

$E_m = 20 \text{ GPa}$ reinforced with GO nanoparticles with elastic modulus of $E_r = 250 \text{ GPa}$ is considered.

The effects of agglomeration on the concrete beam's vibration frequency at different volume fractions of GO nanoparticles are demonstrated in Fig. 2. It can be found that considering agglomeration effects leads to lower frequency. It is due to the fact that considering agglomeration effect leads to lower stiffness in structure. However, the agglomeration of nanoparticles has a major effect on the vibration behaviour concrete beams. In addition, due to the high bending rigidity, the beams' vibration frequency goes up with the increase of C_r .

Fig. 3 displays the effect of beam length-to-thickness ratio on vibration frequency at different volume fractions of GO nanoparticles. As can be seen, with increasing the length to thickness ratio of the concrete beam, the frequency is increased since the stiffness of structure enhances.

Figs. 4 and 5 illustrate the influence of length and thickness on the frequency along the volume percent of GO nanoparticles. It can be concluded that the frequency is increased with enhancing the thickness and is decreased with increasing the length.

Fig. 6 presents the influence of mode number on vibration frequency at different volume fractions of GO nanoparticles. Obviously, the vibration frequency rises with the increase of mode number.

6. Conclusions

In this study, the vibration characteristics of concrete beams reinforced with GO nanoparticles were studied. The beams were mathematically modeled by applying the SSDT. Using the strain-displacement equations, the energy method, and the Hamilton's principle, the coupled governing equations were derived. Finally, using an analytical method, the vibration frequencies of the structures were calculated and the effects of various parameters such as the volume fraction of GO nanoparticles, agglomeration, and the beam's geometrical parameters on vibration frequency were investigated. It was found that the vibration frequency goes up with the increase in the volume fraction of GO nanoparticles. In general, the presence of agglomeration leads to a decline in the vibration

frequency of the beams. In addition, the mode shapes have a significant effect on the beam frequency. The results were validated by other published works. It is hoped that the findings of this work can be useful in improving the quality and strength of the concrete structures reinforced with nanoparticles.

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