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Performance of FGM bilayered cylindrical shell placed on cantilever edge

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Abstract. Functionally graded materials (FGMs) are designed for specific purpose and applications. Functionally graded materials for bi-layered cylindrical shell was discussed for different boundary conditions. Functionally graded materials (FGMs) are that kind of material in which function and formation may deviate continuously. Cylindrical shells are mainly significant in various fields of science as well as advanced technology of engineering like aerospace engineering, mechanical engineering and civil engineering. Wide applications of cylindrical shell in different fields like aircraft, aerospace and pressure vessels etc. Bi-layered cylindrical shells consist of two layers and in this work, one layer is of FGM material whose constituents are nickel (Ni) and zirconia (Zr) and other is of isotropic material whose constituent is stainless steel. In this work, effect of trigonometric volume fraction law on cantilever FGM bi-layered cylindrical shell with internal pressure has analyzed by using Rayleigh-Ritz technique and Love's shell theory. Present results of FGM bi-layered cylindrical shell are compared with FGM cylindrical shell. Validity of present technique has verified by way of comparisons with current conclusions and those obtained in the past studies.
Keywords: bi-layered; clamped boundary condition; FGMs; Rayleigh-Ritz technique; trigonometric volume fraction law

1. Introduction

Functionally graded materials (FGMs) are that kind of material in which function and formation may deviate continuously. FGMs are inhomogeneous that their properties changes continuously now a days, FGM are considered for use in environment with high temperature. Bryan (1890) is considered to be the primer research worker who examined studied vibrations of rotating cylindrical shells. The free vibrations of a rotating ring were related with those of these shells. Sharma *et al*. (1998) determined frequencies of composite cylindrical shells containing fluid. They estimated the axial modal deformations by trigonometric functions. Di Taranto and Lessen (1964) investigated the vibrations of thin isotropic and infinite long rotating cylindrical shells. Sharma (1974) analyzed vibration frequencies circular cylinder with using the Rayleigh-Ritz formulation and made comparisons of his results with some experimental ones. Srinivasan and Lauterbach (1971) conducted the research on isotropic long rotating cylindrical shells including influence of coriolis actions on their travelling modes. Chung *et al*. (1981) studied the frequency response of fluid-filled CSs and presented an analysis of experimental and analytical investigation. Penzes and Kraus (1972) applied generalized end conditions to analyze vibrations of rotating cylindrical shells. The analysis of rotating shells was confined to some special cases owing to need of approximate approach and calculation process. With powerful numerical methodologies, shell vibration analysis has completely revolutionized by advanced computers. Sewall and Naumann (1968) considered the vibration analysis of CSs based on analytical and experimental methods. The shells were strengthened with longitudinal stiffeners. Zohar and Aboudi (1973) studied vibrations of rotating cylindrical shells having finite length and matrix approach was used to derive the shell vibration. Najafizadeh and Isvandzibaei (2007) applied ring supports to CSs for vibration analysis of along the tangential direction and founded their research on angular deformation theory of higher order. The angular deformation was used for shell equations and determined the effects of constituent volume fractions and shell configurations on the shell vibrations. FG material parameters were changed step by step. Wang and Chen (1974) performed frequencies of rotating cylindrical shells based on energy variational approach. Ergin and Temarel (2002) did a vibration study of cylindrical shells. The shells lied in a horizontal direction

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and contained fluid and submerged in it. Padovan (1975) did analysis of pre-stress influence on buckling and vibration aspects of rotating cylindrical shells. Goncalves and Batista (1987) gave an analytical investigation of submerged CSs with fluid. Fox and Hardie (1985) examined vibrations of rotating cylindrical shells. They used shell theory due to Flugge for shell motion equations. Amabili *et al*. (1999) used Donnell's shallow-shell model with the quiescent, dense, inviscid and incompressible fluid. Also the dense fluid is studied for the influence of both the internal and external side of the shell. In the external side of the shell, the fluid was considered as an unbounded domain in the radial direction, while internally, the shell was considered as filled completely. The shell motion equations were used for rotating cylindrical shell by different researchers (Saito and Endo 1996, Wang *et al*. 1994, Chen *et al*. 1993). Lam and Loy (1994) investigated the vibrations of rotating composite and sandwich cylindrical shells. They performed comparisons of vibration frequencies of composited rotating cylindrical shells and evaluated the results applying different shell theories. Pankaj *et al*. (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Li and Lam (1998) studied influence of edge conditions vibration frequencies and modes of rotating composite CSs. Recently some researcher used different methods for nonlinear modeling (Avcar 2019, Karami *et al*. 2017, 2018, Madani *et al*. 2016, Simsek, 2011). Bellman and Casti (1971) studied computational solution of nonlinear differential integral equations and generalized linear functionals for approximation technique and Quadrature method for linear and non-linear functions. Bert and Malik (1996) studied differential quadrature (DQ) method in the field of structural mechanics and solved problem of eight-order coupled differential equation and compared results of (DQ) solution with natural frequencies of shell. Farahani and Barati (2015) studied vibration investigation of shell which is inundated in the incompressible fluid and by seeing axial and lateral hydrostatic pressure Flugge shell equation is used and the equation from which method it is recognized is wave propagation. Golpayegani and Ghorbani (2016) presented free vibrations of FGM thin cylindrical shell and consequence of different parameters on natural frequencies and free vibration influence of non-uniform internal pressure. They described vibration characteristics of rotating functionally graded cylindrical shell and fabricated by functionally graded material. Recently some researcher used different methods for nonlinear modeling (Eltaher *et al*. 2019, Ebrahimi *et al*. 2019, Safaei *et al*. 2019, Shahsavari *et al*. 2019, Benmansour *et al*. 2019).

In this study, functionally graded materials (FGMs) are that kind of material in which function and formation may deviate continuously. The FGM layer is composed of different materials like nickel (Ni), aluminum, stainless steel and zirconia (Zr). In this research, FGM bi-layered cylindrical shell consists of two layers, one of them is of FGM material whose constituents are nickel (Ni) and zirconia (Zr) and other is of isotropic layer whose constituent is stainless steel. effect of trigonometric volume fraction law on cantilever FGM bi-layered cylindrical shell with internal pressure has analyzed by using Rayleigh-Ritz technique and Love's shell theory. Effect of volume fraction law has been investigated for clamped boundary condition under non-uniform internal pressure, which is particular motive. The material properties of the cylindrical shells change continuously, rapidly and slowly, from the inner to the outer surface of the FGM layer over the shell thickness. At the inner surface of the FGM layer one substance is known to be pure, while the other has zero concentration.

2. Methodology

The general property q of FGM function temperature which is defined as follows

$$q = q_0(q_{-1}T^{-1} + 1 + q_1T + q_2T^2 + q_3T^3) \tag{1}$$

At temperature T in kelvin scale the constants are q_0, q_{-1}, q_1, q_2 and q_3.

$$z(x,y) = L\left(\frac{G}{L}\right)^{D-2}\left(\frac{In\,\gamma}{M}\right)^{1/2}\sum_{m=1}^{M}\sum_{n=n_1}^{n_{max}}\gamma^{(D-3)n} \tag{1}$$

Now volume ratio is defined as

$$q = \sum_{i=1}^{k} q_i\,V_{fi} \tag{2}$$

Composing material's volume ratio must be equal to one i.e.

$$\sum_{i=1}^{k} q_i\,V_{fi} = 1 \tag{3}$$

Velocity vector of the shell at each point is

$$\bar{v} = \dot{\bar{r}} \tag{4}$$

By putting value of position vector r then velocity vector v becomes

$$\bar{v} = \dot{u}\bar{\imath} + \dot{v}\bar{\jmath} + \dot{w}\bar{k} \tag{5}$$

For theoretical considerations, the first and utmost purpose is to reach the motion equations for FGM cylindrical shell as shown in Fig. 1 where different parameters are used like h indicates thickness, R indicates radius, ρ indicates mass density and L indicates length. Displacement of Longitudinal, circumferential and radial direction is shown by u, v and w respectively. In different direction like θ unit vectors are represented by $\bar{\imath}$, $\bar{\jmath}$ and $\bar{k}$. Stress values have range in a dissimilar material and this range is named as Proportionality limit.

The strain energy is written as follows

$$T = \frac{1}{2}\int_0^L\int_0^{2\pi}(\epsilon^T)\,[S](\epsilon)R\,d\theta\,dx \tag{6}$$

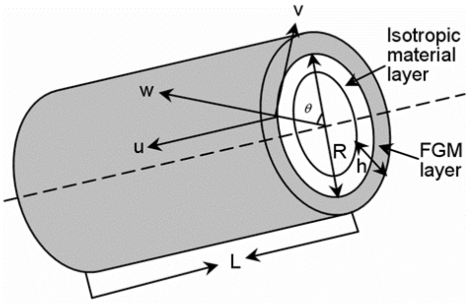

Fig. 1 Bilayered cylindrical shell

Stiffness and strain matrix ϵ can be written as

$$\epsilon^T = \{e_1, e_2, \gamma, k_1, k_2, 2\tau\} \quad (7)$$

Surface strains are represented e_1, e_2, γ and surface curvatures are $k_1, k_2, 2\tau$represented as respectively. These surface strains are written as

$$(e_1, \gamma, e_2) = \left[\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}, \frac{1}{R}\left(\frac{\partial v}{\partial x} + w\right)\right] \quad (8)$$

$$(k_1, \tau, k_2) = \begin{bmatrix} -\frac{\partial^2 w}{\partial x^2}, -\frac{1}{R}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right), \\ -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) \end{bmatrix} \quad (9)$$

Stiffness matrix for composites can be written as F_{11}

$$[S] = \begin{bmatrix} F_{11} & F_{12} & 0 & G_{11} & G_{12} & 0 \\ F_{12} & F_{22} & 0 & G_{12} & G_{22} & 0 \\ 0 & 0 & F_{66} & 0 & 0 & G_{66} \\ G_{11} & G_{12} & 0 & H_{11} & H_{12} & 0 \\ G_{12} & G_{22} & 0 & H_{12} & G_{22} & 0 \\ 0 & 0 & G_{66} & 0 & 0 & H_{66} \end{bmatrix} \quad (10)$$

On putting the the value of Eqs. 7 and 10 in Eq. 6, we get new form of strain energy

$$T = \frac{R}{2}\int_0^L \int_0^{2\pi} \begin{pmatrix} A_{11}e_1^2 + 2e_1e_2A_{12} \\ +2e_1k_1B_{11} \\ +2e_1k_2B_{12} + e_2^2A_{22} \\ +2e_2k_2B_{12} + 2e_2k_2B_{22} \\ +\gamma^2A_{66} + 4\tau\gamma B_{66} \\ +k_1^2D_{22} + 2k_1k_2D_{12} \\ +k_2^2D_{22} + 4\tau^2D_{66} \end{pmatrix} d\theta dx \quad (11)$$

The expression for kinetic energy is as follows

$$J = \frac{1}{2}\int_0^L \int_0^{2\pi} \rho\left\{\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right\} R d\theta dx \quad (12)$$

Results are gotten by MATLAB programming software when diverse boundary conditions are applied. MATLAB software obtains results of volume fraction laws regarding bilayered cylindrical shell. The tensile stiffness F_{ij}, flexural rigidity G_{ij} and torsional rigidity H_{ij} are obtained as

$$(F_{ij}, G_{ij}, H_{ij}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\,(1, Z, Z^2)dz \quad (13)$$

Reduced stiffness matrix Q can be written as

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, Q_{12} = \frac{\nu E}{1-\nu^2}, Q_{66} = \frac{E}{2(1+\nu)} \quad (14)$$

Total energy of system is as follows

$$\prod = T - J \quad (15)$$

The followings displacement functions u, v, w are selected to separate the variables spatially x, θ and t the time variable. The general solutions for modal vibration can be written as

$$\begin{aligned} u(x,\theta,t) &= x_m U(x)\cos(n\theta)\sin\omega t \\ v(x,\theta,t) &= y_m V(x)\sin(n\theta)\cos\omega t \\ w(x,\theta,t) &= z_m W(x)\cos(n\theta)\sin\omega t \end{aligned} \quad (16)$$

where x_m, y_m and z_m represent the amplitudes of vibration in the x, θ and z direction respectively, the axial and circumferential wave numbers of mode shapes are denoted by m and n respectively, ωsignifies the angular vibration frequency of the shell wave. U, V and Wdenotes the axial model dependence in the longitudinal, circumferential and transverse directions respectively. Here we take $U(x) = \frac{d\varphi}{dx}, V(x) = \varphi(x)$, $W(x) = \varphi(x)$, where $\varphi(x)$ represents the axial function which satisfies the geometric edge condition.

The axial function $\varphi(x)$ is taken as the beam function in the following form

$$\begin{aligned} \varphi(x) = {} & \beta_1\cosh(\mu_m x) + \beta_2\cos(\mu_m x) \\ & -\sigma_m\beta_3\sinh(\mu_m x) + \sigma_m\beta_4\sin(\mu_m x)a \end{aligned} \quad (17)$$

$$\frac{\partial \prod_{max}}{\partial x_m} = \frac{\partial \prod_{max}}{\partial y_m} = \frac{\partial \prod_{max}}{\partial z_m} \quad (18)$$

The obtained equation by arrangements of terms are written in matrix form as

$$\{[C] - \Omega^2[M]\}X = 0 \quad (19)$$

where

$$\Omega^2 = R^2\omega^2\rho_t \quad (20)$$

$[C]$ and $[M]$ are stiffness and mass matrices of the cylindrical shell respectively and

$$X = [x_m, y_m, z_m]$$

Cylindrical shell with two layers and both layers are composed of isotropic material then Poisson ratio ν, density ρ and Young modulus E are as follows

$$\begin{aligned} \nu &= \nu_1 + \nu_2 \\ \rho &= \rho_1 + \rho_2 \\ E &= E_1 + E_2 \end{aligned} \quad (21)$$

Volume fraction constructed from two constituents with the trignometric volume fraction law are as follows

$$\begin{aligned} V_{isotropic} &= sin^2\left[\left(\frac{z}{h} + 0.5\right)^N\right], 0 \le N \le \infty \\ V_{isotropic} &= cos^2\left[\left(\frac{z}{h} + 0.5\right)^N\right] \end{aligned} \quad (22)$$

Table 1 Behavior of natural frequency for bilayered cylindrical shell when (m = 2, L/R = 8.2, h/R = 0.4) for clamped boundary condition where one layer is of FGM material and other is of isotropic material

n	*N* = 1		*N* = 2		*N* = 3	
	Iqbal *et al*. (2009)	Present values	Iqbal *et al*. (2009)	Present values	Iqbal *et al*. (2009)	Present values
1	30.4758	30.4601	29.4152	29.3076	28.1622	28.3463
2	10.2744	10.2512	09.9152	09.5647	09.4947	09.5432
3	6.19196	6.17620	05.9631	4.7645	05.6989	04.4329
4	7.89979	7.81234	07.5927	06.6543	07.2290	07.6543
5	12.0787	12.0283	11.6063	11.9865	11.0386	11.6213

Table 2 Comparison of variation of natural frequencies (Hz) against circumferential wave number (n) for FGM cylindrical shell with clamped boundary condition when (m = 1, L/R = 20, h/R = 0.05)

n	*N* = 1	*N* = 2	*N* = 3
1	0184.8213	0171.1213	0163.2287
2	0190.5287	0173.7342	0164.5287
3	0345.8543	0313.1564	0296.3954
4	0593.8643	0537.3245	0509.8432
5	0912.6189	0826.3876	0978.9324
6	1292.8235	1171.8781	1117.5956
7	1726.1098	1566.7298	1498.5034
8	2202.3453	2002.5245	1921.6261
9	2628.7241	2242.2872	2346.7287
10	2925.9981	2613.7971	2508.6041

Table 3 Comparison of behavior of natural frequency (Hz) against circumferential wave number n for FGM cylindrical shell for simply supported (S-S) boundary condition when (m = 1, L/R = 20, h/R = 500)

n	*N* = 0.5		*N* = 2		*N* = 15	
	Naeem *et al*. (2009)	Present values	Naeem *et al*. (2009)	Present values	Naeem *et al*. (2009)	Present values
1	13.321	13.144	13.211	13.108	12.933	12.965
2	4.5162	4.2987	4.4794	4.4054	4.3829	4.4023
3	4.1903	4.1723	4.1562	4.1865	4.0646	4.3287
4	7.0967	7.0657	7.0379	7.0546	6.8851	6.4433
5	11.335	11.301	11.241	11.432	10.998	10.786
6	16.594	16.754	16.455	16.765	16.101	16.089
7	22.826	22.101	22.635	22.102	22.148	22.062
8	30.023	30.564	29.771	28.223	29.132	28.435
9	38.181	37.908	37.862	36.453	37.028	36.432
10	47.301	46.763	46.905	44.876	45.897	44.987

3. Result and discussion

Table 1 represents a comparison of natural frequencies (Hz) for FGM cylindrical shell (CS) configured according to those ones obtained in (Iqbal *et al*. 2009) for clamped boundary condition and power law exponents N = 0.3, 1, 10. Nickel and zirconium (Zr) are considered as the constituent materials of the FGM cylindrical shell. Nickel (Ni) is at the outer surface of the shell and zirconium (Zr) is at the inner surface. The natural frequencies (NFs) decreased by increasing the power law exponent in an expected sense. It is noted that, the respective frequency are same for the circumferential wave number n = 1 and n = 5 and the frequencies are slightly lower for the circumferential wave number n between 2 and 5. The minimum frequency occurs at the circumferential wave number n = 3. Table 2 shows behavior of natural frequency for bilayered cylindrical shell when (m = 2, L/R = 8.2, h/R = 0.1) with circumferential wave number n = 1 to 10 and clamped boundary condition is applied. Table 3 represents a

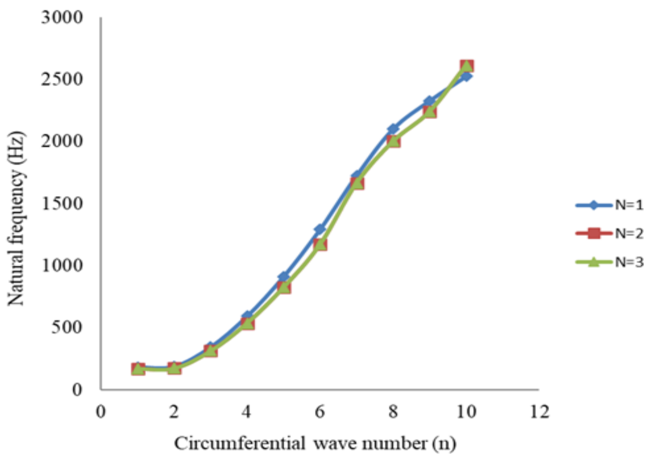

Fig. 2 Graphical representation of behavior of natural frequency for bilayered cylindrical shell when (m = 1, L/R = 8, h/R = 0.1) and (m = 3, L/R = 8.2, h/R = 0.2) for clamped boundary condition.

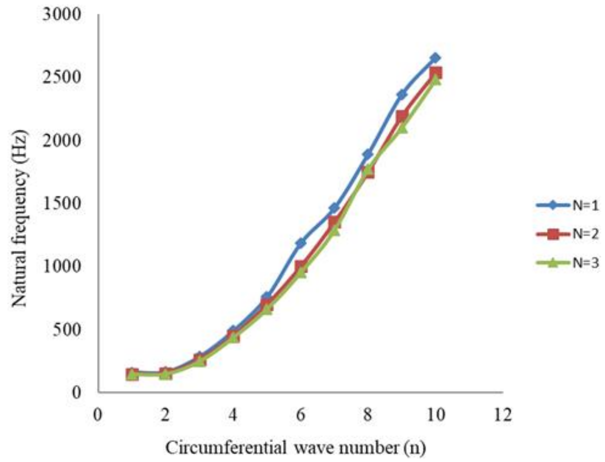

Fig. 5 Graphical representation of behavior of natural frequency for bilayered cylindrical shell when (m = 4, L/R = 8, h/R = 0.2) and (m = 2, L/R = 8.1, h/R = 0.1) for clamped boundary condition

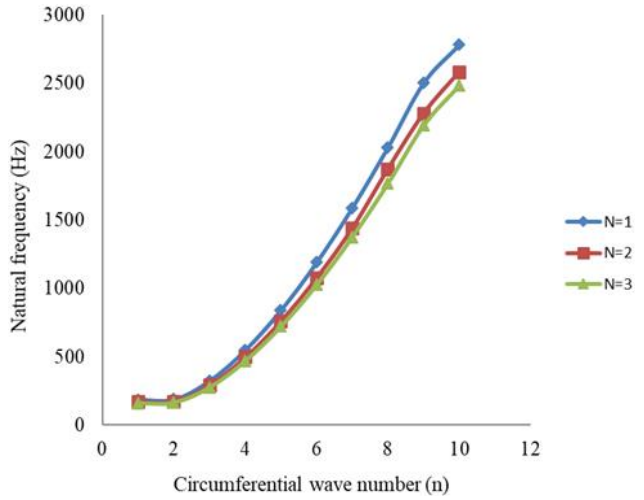

Fig. 3 Graphical representation of behavior of natural frequency for bilayered cylindrical shell when (m = 1, L/R = 8, h/R = 0.1) and (m = 3, L/R = 8.2, h/R = 0.2) for clamped boundary condition

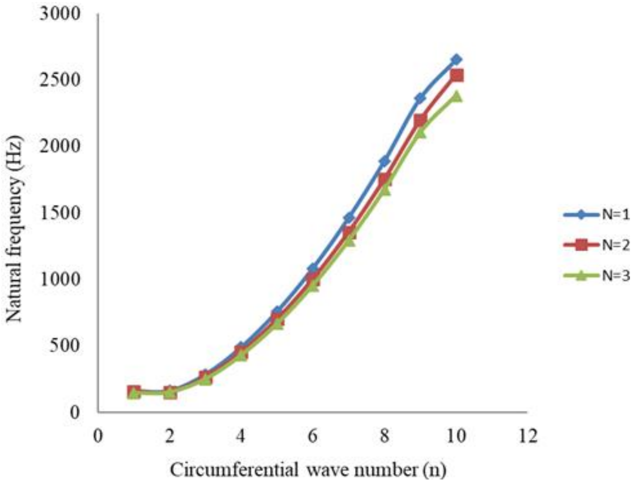

Fig. 6 Graphical representation of behavior of natural frequency for bilayered cylindrical shell when (m = 4, L/R = 8, h/R = 0.2) and (m = 2, L/R = 8.1, h/R = 0.1) for clamped boundary condition

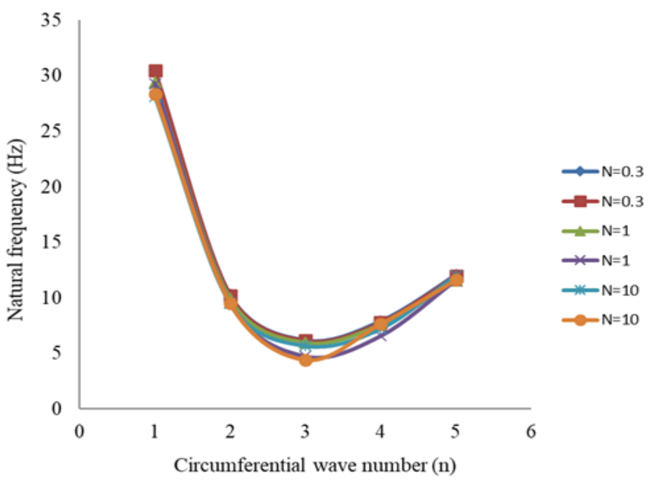

Fig. 4 Graphical representation of comparison of behavior of natural frequency (Hz) against circumferential wave number n for simply supported (S-S) boundary condition and clamped boundary condition

comparison of natural frequencies (NFs) for FGM cylindrical shell (CS) configured according to those ones obtained in (Arshad *et al*. 1997) for simply supported (S-S) boundary condition and power law exponents N = 0.3, 1, 5. Stainless steel and nickel (Ni) are considered as the constituent materials of the FG cylindrical shell. Stainless steel is at the outer surface of the shell and nickel is at the inner surface. The natural frequencies (NFs) decreased by increasing the power law exponent. It is evident that the normal frequencies (Hz) of these shells first increase to a certain circumferential wave number n.

Graphical representation of behavior of natural frequency for bilayered cylindrical shell when (m = 1, L/R = 8, h/R = 0.1) of clamped boundary condition. Figs. 2 and 3 show comparison of behavior of natural frequency of FGM cylindrical shell against circumferential wave number n for clamped and simply supported boundary condition respectively. First the frequencies increase then decrease at n = 2 and then increasing smoothly with rising values of n.

The shell parameters are L/R = 20, h/R = 0.05 and L/R = 20, h/R = 0.01 respectively. By increasing the values of N, the frequency decreases then there is increase in frequency with the values of N. Fig. 4 shows the behavior of natural frequency with circumferential wave number. The frequencies decrease on increasing the circumferential wave number. In Figs. 5 and 6 show graphical representation of behavior of natural frequencies (Hz) of FGM bilayered cylindrical shell with clamped boundary conditions versus circumferential wave number. It is observed that the frequencies increases on increasing the circumferential wave number. It can be seen that the frequencies decreases on increasing the power index N.

4. Conclusions

In this paper, effect of trigonometric volume fraction law on cantilever FGM bi-layered cylindrical shell with internal pressure were analyzed. Generally, the FGM layer is composed of different materials like nickel (Ni), aluminum, stainless steel and zirconia (Zr). In this research, FGM bi-layered cylindrical shell consist of two layers, one of them is of FGM material whose constituents are nickel (Ni) and zirconia (Zr) and other is of isotropic layer whose constituent is stainless steel. The material properties of the cylindrical shells change continuously, rapidly and slowly, from the inner to the outer surface of the FGM layer over the shell thickness. At the inner surface of the FGM layer one substance is known to be pure, while the other has zero concentration. A well-defined mathematical technique, the Rayleigh-Ritz approach is employed because it is broadly utilized in resolving the shell problems. The current approach is simple and reliable to get results by the efficient use of MATLAB software. These comparisons displayed a quick convergence of the process towards the accurate frequency values.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

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