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Natural frequencies of FGM nanoplates embedded in an elastic medium

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Abstract. The small scale impact on the vibrational properties of "functionally graded" (FG) nanoplate embedded in an elastic medium is examined. The formulation is based on the four-unknown refined integral plate theory on aggregate with the nonlocal elasticity theory. Contrary to other theories, this one involves only four unknown variables. The solution procedure is obtained by employing the motion differential equations of physical phase that are converted into set of "linear algebraic equations". After, these are solved by a computer code. The influences of aspect ratio, material index, nonlocal parameter and elastic medium stiffness on the different modal vibrations of FG nanoplate are explored. The results demonstrate the significant impact of different physical and geometrical parameters on the vibration behavior of FG nanoplate.

Keywords: elastic medium; FG nanoplate; four-unknown refined integral plate theory; nonlocal theory; vibration

1. Introduction

The FGMs are novel generations of composite materials, these materials are generally composed of ceramic and metal which vary gradually in the direction of the thickness, this gradual change of the constituents leads to a gradual and continuous variation of the properties which eliminates the problems of the concentration of the stresses at the interfaces which is known in the traditional composites material. The advanced composites materials (FGM) have attracted the attention of several researchers (Kiani and Eslami 2010, Sedighi *et al*. 2015ab, Ebrahimi and Salari 2015, Kar and Panda 2015, Avcar 2016 and 2019, Kar and Panda 2016, Faleh *et al*. 2018, Karami and Janghorban 2019a, Safa *et al*. 2019, Zouatnia and Hadji 2019, Selmi 2020a, Yaylaci *et al*. 2020a and 2021ab). The micro/nano structures have been investigated for several times using the nonlocal theory of Eringen (Sedighi and Yaghootian 2016, Kolahchi 2017, Ebrahimi and Barati 2017a, Kolahchi *et al*. 2017a, Karami *et al*. 2018a, Al-Maliki *et al*. 2019, Fenjan *et al*. 2019, Abdulrazzaq *et al*. 2020ab, Asiri *et al*. 2020, Akbaş 2020a, Timesli 2020ab, Hadji and Avcar 2021, Bouhadra *et al*. 2021). Flexural behavior of P-FG nano-plates is examined by Kolahchi *et al*. (2015) employing the Eringen's nonlocal theory and a new SSDT. Barati and Shahverdi (2016) developed a four variable plate theory to examine thermal vibration behavior of FG-nanoplate with various boundary conditions. The effect of the longitudinal magnetic field on dynamic analysis of S-FGM nanobeams on elastic medium is analyzed by Ebrahimi and Barati (2017b) based on EBT model and Eringen nonlocal theory. The non-local thermal stability of sandwich piezoelectric nanoplates with FG core is investigated by Karami *et al*. (2018b) using second-order shear deformation theory. Attia and Abdel Rahman (2018) analyzed the dynamic behaviors of the FG viscoelastic nanobeams by employing the Bernoulli-Euler beam theory and Alembert's principle. Based on nonlocal elasticity theory, Mehar *et al*. (2018) studied the vibrational characteristics of nanoplate structure by developing a novel higher-order mathematical model and finite-element method. Ahmed *et al*. (2019) investigated the post-buckling response of FG porous nanobeam using the nonlocal theory and HSDT model. Recently, Attia *et al*. (2019) investigated the nonlinear vibrational characteristics of size-dependent FG nanobeams using the Timoshenko beam theory and DQM. Shanab *et al*. (2020) examined the Microstructure effect and Surface Energy on bending and vibrational Characteristics of FG- Nanobeam Embedded in an Elastic foundation using the Timoshenko theory. Based on the nonlocal continuum theory and the Timoshenko model, Bensattalah *et al*. (2020) analyzed the stability of TWCNTs under axial compression. The effect of the porosity distribution on the dynamic response of the FG nanobeam is examined by Ghandourah and Abdraboh (2020) based on

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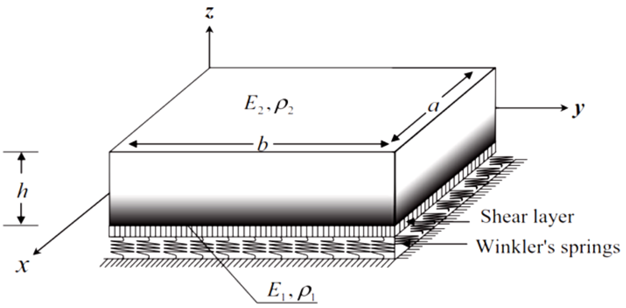

Fig. 1 geometry of FG nano-plate embedded in an elastic medium

Euler Bernoulli Beam theory and finite element method. Using a non-local shear deformation and energy principle, Gafour *et al*. (2020) examined the porosity-dependent free vibration analysis of FG nanobeam.

In the present investigation, the dynamic analysis of the FG nano-plate embedded in an elastic medium using a four-unknown refined integral plate and Eringen's nonlocal continuum theory. The developed model contains only four unknown-variables instead of five in conventional HSDTs and does not require any correction factors. The FG nano-plate effective's properties vary continuously through the direction of the thickness according to a new power law function. The non-local equations of motion are derived and solved via Hamilton's principle and analytical method of Navier, respectively. The accuracy of the computed results are checked out by the comparisons made with other models found in the literature. Several examples which show the effects of inhomogeneity parameter, elastic medium stiffness, scale effect parameter, and aspect ratio on the free vibration frequencies are presented and discussed in detail.

2. Problem formulation

2.1 Model of functionally graded materials (FGMs)

In this work, FG nano-plate is supposed to have an uniform thickness h, length "a" and width "b" in the coordinate system (see Fig. 1), with

$$0 \le x \le a; 0 \le y \le band - h/2 \le z \le h/2 \qquad (1)$$

The 2D structure studied is made of ceramic and metal, where his effective's properties (Young's modulus$E(z)$ and mass density$\rho(z)$) vary according to a new power law distribution which volume fraction is assumed to vary continuously through thickness as (Sobhy 2015):

$$E(z) = E_1\left(\frac{E_2}{E_1}\right)^{V_2}; \quad \rho(z) = \rho_1\left(\frac{\rho_2}{\rho_1}\right)^{V_2} \qquad (2a)$$

where E_1 and E_2 are the Young modulus of the top surface and the bottom one materials composed the FG-nano-plate. V_1 and V_2 are the volume fraction of materials 1 and 2 with $(V_1 + V_2 = 1)$. The FG volume fraction of the second material is given as (Kiani and Eslami 2013 and Avcar and Mohammed 2018):

$$V_2 = \left(\frac{z}{h} + \frac{1}{2}\right)^k \qquad (2b)$$

where k is the inhomogeneity parameter with $0 \le k \le \infty$

2.2 The generalized displacement field

In the present investigation, the displacement field is based to the HSDT assumptions with a reduced number of unknown variables. The displacements $(u, vandw)$ of current model are expressed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0(x,y)}{\partial x} \\ &\quad + k_1 f(z)\int \theta(x,y)\,dx \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0(x,y)}{\partial y} \\ &\quad + k_2 f(z)\int \theta(x,y)\,dy \\ w(x,y,z) &= w_0(x,y) \end{aligned} \qquad (3)$$

where, the terms "u_0, v_0, w_0 and θ " are the displacement functions of the median surface of the FG nano-plate. The constants " k_1 and k_2 " depend on the FG-plate's geometry. In this work, the shear warping function is given by

$$f(z) = \frac{z[\pi + 2\cos(\pi z/h)]}{2+\pi} \qquad (4)$$

2.3 Linear strain field

Based on kinematics of Eq. (3), the linear strain expressions are obtained as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \varepsilon_{xy}^1 \end{Bmatrix} + f(z)\begin{Bmatrix} \varepsilon_x^2 \\ \varepsilon_y^2 \\ \varepsilon_{xy}^2 \end{Bmatrix}, \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \qquad (5)$$

with

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \varepsilon_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \qquad (6a)$$

and

$$g(z) = \frac{df(z)}{dz} \qquad (6b)$$

By using the integrals considered in the above equations can be treated via the following expressed:

$$\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial x \partial y}, \qquad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial x \partial y}, \qquad (7)$$

$$\int \theta dx = A' \frac{\partial \theta}{\partial x}, \qquad \int \theta dy = B' \frac{\partial \theta}{\partial y}$$

where coefficients A', B', k_1 and k_2 are given by:

$$A' = \frac{1}{\lambda^2}, B' = \frac{1}{\beta^2}, k_1 = \lambda^2, k_2 = \beta^2 \tag{8}$$

with λ and μ are defined in Eq. (16).

2.4 Nonlocal constitutive relations

Based on nonlocal theory (Eringen 1972), the nonlocal constitutive relations of an FGM nanoplate can be written as:

$$\mathscr{L}\sigma_{xx} = \frac{E(z)}{1-\nu^2}\left[\varepsilon_{xx} + \nu\varepsilon_{yy}\right]$$
$$\mathscr{L}\sigma_{yy} = \frac{E(z)}{1-\nu^2}\left[\varepsilon_{yy} + \nu\varepsilon_{xx}\right]$$
$$\mathscr{L}\sigma_{xy} = \frac{E(z)}{1-\nu^2}\varepsilon_{xy}, \mathscr{L}\sigma_{xz} = \frac{E(z)}{2(1+\nu)}\varepsilon_{xz}, \mathscr{L}\sigma_{yz} = \frac{E(z)}{2(1+\nu)}\varepsilon_{yz} \tag{9}$$

where the nonlocal operator $\mathscr{L}$is defined as (Ghannadpour and Moradi 2019)

$$\mathscr{L} = 1 - \mu^2 \nabla^2 \tag{10}$$

2.5 Equations of motion

Using the Hamilton's principle (Karami and Janghorban 2016 and 2020, Barati 2017, Hamidi *et al*. 2018a, 2020a, Hadji *et al*. 2019, Attia and Mohamed 2020, Civalek *et al*. 2020, Hadji 2020, Hamed *et al*. 2020, Kar and Panda 2020, Eltaher and Mohamed 2020b) , the four equations of motion associated to the displacement field of Eq. (3) can be obtained as follows:

$$\delta u: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - I_1\left(\frac{\partial^2 u_0}{\partial t^2}\right) + I_2\left(\frac{\partial^3 w_0}{\partial x \partial t^2}\right) - I_3 K_1 B'\left(\frac{\partial^3 \theta}{\partial x \partial t^2}\right) + \mu^2 \begin{pmatrix} I_1\left(\left(\frac{\partial^4 u_0}{\partial x^2 \partial t^2}\right) + \left(\frac{\partial^4 u_0}{\partial y^2 \partial t^2}\right)\right) \\ -I_2\left(\left(\frac{\partial^5 w_0}{\partial x^3 \partial t^2}\right) + \left(\frac{\partial^5 w_0}{\partial x \partial y^2 \partial t^2}\right)\right) \\ +I_3 K_1 A'\left(\left(\frac{\partial^5 \theta}{\partial x \partial y^2 \partial t^2}\right) + \left(\frac{\partial^5 \theta}{\partial x^3 \partial t^2}\right)\right) \end{pmatrix} \tag{11a}$$

$$\delta v: \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} - I_1\left(\frac{\partial^2 v_0}{\partial t^2}\right) + I_2\left(\frac{\partial^3 w_0}{\partial y \partial t^2}\right) - I_3 K_2 B'\left(\frac{\partial^3 \theta}{\partial y \partial t^2}\right)\mu^2 \begin{pmatrix} I_1\left(\left(\frac{\partial^4 v_0}{\partial x^2 \partial t^2}\right) + \left(\frac{\partial^4 v_0}{\partial y^2 \partial t^2}\right)\right) \\ -I_2\left(\left(\frac{\partial^5 w_0}{\partial y \partial x^2 \partial t^2}\right) + \left(\frac{\partial^5 w_0}{\partial y^3 \partial t^2}\right)\right) \\ + \\ I_3 K_2 B'\left(\left(\frac{\partial^5 \theta}{\partial y \partial x^2 \partial t^2}\right) + \left(\frac{\partial^5 \theta}{\partial y^3 \partial t^2}\right)\right) \end{pmatrix} \tag{11b}$$

$$\begin{aligned} &\delta w: \left(\frac{\partial^2 M_x^b}{\partial x^2}\right) + \left(\frac{\partial^2 M_y^b}{\partial y^2}\right) + 2\left(\frac{\partial^2 M_{xy}^b}{\partial y \partial x}\right) - I_1\left(\frac{\partial^2 w_0}{\partial t^2}\right) \\ &-I_2\begin{pmatrix} \frac{\partial^3 u_0}{\partial x \partial t^2} + \frac{\partial^3 v_0}{\partial y \partial t^2} + I_4\left(\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + \frac{\partial^4 w_0}{\partial y^2 \partial t^2}\right) \\ -I_5\left(K_1 A'\left(\frac{\partial^4 \theta}{\partial x^2 \partial t^2}\right) + K_2 B'\left(\frac{\partial^4 \theta}{\partial y^2 \partial t^2}\right)\right) \end{pmatrix} \\ &-K_w w_0 + Ks\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2}\right) + \mu^2 \begin{pmatrix} I_1\left(\frac{\partial^4 w_0}{\partial x^2 \partial t^2}\right) \\ +I_2(\frac{\partial^5 u_0}{\partial x^3 \partial t^2} \end{pmatrix} \\ &+\frac{\partial^5 v_0}{\partial y \partial x^2 \partial t^2} - I_4\left(\frac{\partial^6 w_0}{\partial x^4 \partial t^2} + \frac{\partial^6 w_0}{\partial y^2 \partial x^2 \partial t^2}\right) \\ &+I_5\left(K_1 A'\left(\frac{\partial^6 \theta}{\partial x^4 \partial t^2}\right) + K_2 B'\left(\frac{\partial^6 \theta}{\partial y^2 \partial x^2 \partial t^2}\right)\right) \\ &+K_w\left(\frac{\partial^2 w_0}{\partial x^2}\right) - Ks\left(\frac{\partial^4 w_0}{\partial x^4} + \frac{\partial^4 w_0}{\partial y^4}\right) + I_1\left(\frac{\partial^4 w_0}{\partial y^2 \partial t^2}\right) \\ &+I_2\left(\frac{\partial^5 u_0}{\partial y^2 \partial x \partial t^2} + \frac{\partial^5 v_0}{\partial y^3 \partial t^2}\right) \\ &-I_4\left(\frac{\partial^6 w_0}{\partial x^4 \partial t^2} + \frac{\partial^6 w_0}{\partial y^2 \partial x^2 \partial t^2}\right) \begin{matrix} +I_5 K_1 A'\left(\frac{\partial^6 \theta}{\partial y^2 \partial x^2 \partial t^2}\right) \\ +K_2 B'\left(\frac{\partial^6 \theta}{\partial y^4 \partial t^2}\right. \end{matrix} \Bigg) \\ &+K_w\left(\frac{\partial^2 w_0}{\partial y^2}\right) - K_S\left(\frac{\partial^4 w_0}{\partial y^2 \partial x^2} + \frac{\partial^4 w_0}{\partial y^4}\right)\Big) \end{aligned} \tag{11c}$$

$$\begin{aligned} &\delta\theta: K_1 A'\left(\frac{\partial^2 M_x^S}{\partial x^2}\right) + K_2 B'\left(\frac{\partial^2 M_y^S}{\partial y^2}\right) + \left(K_1 A' + K_2 B'\right) \\ &\left(\frac{\partial^2 M_{xy}^S}{\partial y \partial x}\right) - K_1 A'\left(\frac{\partial Q_{xz}}{\partial x}\right) - K_2 B'\left(\frac{\partial Q_{yz}}{\partial y}\right) \\ &+I_3\begin{pmatrix} K_1 A'\left(\left(\frac{\partial^5 u_0}{\partial y^2 \partial x \partial t^2}\right) + \left(\frac{\partial^3 u_0}{\partial x \partial t^2}\right)\right) \\ +K_2 B'\left(\left(\frac{\partial^5 v_0}{\partial y^3 \partial t^2}\right) - I_5\left(\frac{\partial^3 v_0}{\partial y \partial t^2}\right)\right) \end{pmatrix} \\ &-I_5\begin{pmatrix} K_1 A'\left(\left(\frac{\partial^4 w_0}{\partial x^2 \partial t^2}\right) - \left(\frac{\partial^6 w_0}{\partial y^2 \partial x^2 \partial t^2}\right)\right) \\ +K_2 B'\left(\left(\frac{\partial^4 w_0}{\partial y^2 \partial t^2}\right) - \left(\frac{\partial^6 w_0}{\partial y^4 \partial t^2}\right)\right) \end{pmatrix} \\ &-I_6\left(K_1 A'\right)^2\left(\frac{\partial^4 \theta}{\partial x^2 \partial t^2}\right) + \left(K_2 B'\right)^2\left(\frac{\partial^4 \theta}{\partial y^2 \partial t^2}\right) \\ &-\mu^2 I_3\left(K_1 A'\left(\frac{\partial^5 u_0}{\partial x^3 \partial t^2}\right) + K_2 B'\left(\frac{\partial^5 v_0}{\partial y^3 \partial t^2}\right)\right) \\ &-I_5\left(K_1 A'\left(\frac{\partial^6 u_0}{\partial x^4 \partial t^2}\right) + K_2 B'\left(\frac{\partial^6 v_0}{\partial x^2 \partial y^2 \partial t^2}\right)\right) \end{aligned} \tag{11d}$$

$$+I_6\left(\begin{array}{l}(K_1A')^2\left(\left(\frac{\partial^6\theta}{\partial x^4\partial t^2}\right)-\left(\frac{\partial^6\theta}{\partial y^2\partial x^2\partial t^2}\right)\right)\\+(K_2B')^2\left(\frac{\partial^6\theta}{\partial x^2\partial y^2\partial t^2}\right)-\left(\frac{\partial^6\theta}{\partial y^4\partial t^2}\right)\end{array}\right)$$

where the stress resultants $(N_{ij}, M_{ij}^b, M_{ij}^s, Q_{iz})$ are given as:

$$\left\{N_{ij}, M_{ij}^b, M_{ij}^s\right\} = \int_{-h/2}^{h/2}\{1, z, f(z)\}\sigma_{ij}dz$$
$$Q_{iz} = \int_{-h/2}^{h/2} g(z)\sigma_{iz}dz, i = x, \mathrm{y} \quad (12)$$

and the inertias I_i are defined as

$$\begin{bmatrix} I_1 & I_2 & I_3 \\ I_2 & I_4 & I_5 \\ I_3 & I_5 & I_6 \end{bmatrix} = \int_{-h/2}^{h/2}\rho(z)\,[\bar{A}]dz, [\bar{A}] = \begin{bmatrix} 1 \\ z \\ f(z) \end{bmatrix}[1zf(z)] \quad (13)$$

Substituting the Eqs. (9) into Eq. (12), the stress resultants are given as:

$$\mathscr{L}\begin{Bmatrix} N_{xx} \\ M_{xx}^b \\ M_{xx}^s \end{Bmatrix} = [A]\begin{Bmatrix} \varepsilon_{xx}^0 \\ \kappa_{xx}^b \\ \kappa_{xx}^s \end{Bmatrix} + [B]\begin{Bmatrix} \varepsilon_{yy}^0 \\ \kappa_{yy}^b \\ \kappa_{yy}^s \end{Bmatrix} \quad (14a)$$

$$\mathscr{L}\begin{Bmatrix} N_{yy} \\ M_{yy}^b \\ M_{yy}^s \end{Bmatrix} = [A]\begin{Bmatrix} \varepsilon_{yy}^0 \\ \kappa_{yy}^b \\ \kappa_{yy}^s \end{Bmatrix} + [B]\begin{Bmatrix} \varepsilon_{xx}^0 \\ \kappa_{xx}^b \\ \kappa_{xx}^s \end{Bmatrix} \quad (14b)$$

$$\mathscr{L}\begin{Bmatrix} N_{xy} \\ M_{xy}^b \\ M_{xy}^s \end{Bmatrix} = [D]\begin{Bmatrix} \varepsilon_{xy}^0 \\ \kappa_{xy}^b \\ \kappa_{xy}^s \end{Bmatrix}, \begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = H\begin{Bmatrix} \kappa_{xz}^s \\ \kappa_{yz}^s \end{Bmatrix} \quad (14c)$$

where the stiffness's $(A, B, D\ and\ H)$ are defined as follows

$$[A] = \int_{\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{1-\nu^2}[\bar{A}]dz, [B] = \int_{\frac{h}{2}}^{\frac{h}{2}}\frac{\nu E(z)}{1-\nu^2}[\bar{A}]dz,$$
$$[D] = \int_{\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{2(1+\nu)}[\bar{A}]dz, H = \int_{\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{2(1+\nu)}[g(z)]^2dz, \quad (15)$$

3. Analytical solutions

The studied FG nanoplate is considered to be simply supported at all edges, for this purpose the Navier's solutions are used to ensure automatically the boundary condition cited above. The displacements expansions $(u_0, v_0, w_0\ and\ \theta)$ are given as (Yaghoobi and Taheri 2020)

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \begin{Bmatrix} U_0\, cos(\lambda\, x)\, sin(\beta\, y) \\ V_0\, sin(\lambda\, x)\, cos(\beta\, y) \\ W_0\, sin(\lambda\, x)\, sin(\beta\, y) \\ \theta_0\, sin(\lambda\, x)\, sin(\beta\, y) \end{Bmatrix} e^{i\omega t} \quad (16)$$

where U_0, V_0, W_0 and θ are the unknown coefficients, $\lambda = n\pi/a$, $\beta = m\pi/b$, $i = \sqrt{-1}$ and ω is the eigenfrequency.

By Substituting the Navier's procedure of Eq. (16) into above equation of motion. We obtain the following matrix system

$$\left(\begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} & \xi_{14} \\ \xi_{12} & \xi_{22} & \xi_{23} & \xi_{24} \\ \xi_{13} & \xi_{23} & \xi_{33} & \xi_{34} \\ \xi_{14} & \xi_{24} & \xi_{34} & \xi_{44} \end{bmatrix} - \omega^2\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{12} & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}\right)\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (17)$$

where ξ_{ij} and m_{ij} are the stiffness and mass components, respectively. with

$$\begin{array}{c}
\xi_{11} = -(A_{11}\lambda^2 + A_{66}\beta^2), \\
\xi_{12} = -\lambda\beta(A_{12} + A_{66}), \\
\xi_{13} = \lambda(B_{11}\lambda^2 + (B_{12} + 2B_{66})\beta^2), \\
\xi_{14} = k_1A'\lambda(B_{66}^s\beta^2 + B_{11}^s\lambda^2) - k_2B'\lambda\beta^2(B_{12}^s + B_{66}^s), \\
\xi_{22} = -(A_{66}\lambda^2 + A_{22}\beta^2), \\
\xi_{23} = \beta\lambda^2(B_{22}\beta^3 + (B_{12} + 2B_{66})), \\
\xi_{24} = k_1A'\beta\lambda^2(B_{12}^s + B_{66}^s) - k_2B'\beta(B_{22}^s\beta^2 + B_{66}^s\lambda^2) \\
\xi_{33} = -\lambda^2(D_{11}\lambda^2 + 2(D_{12} + 2D_{66})\beta^2) \\
-(k_w + k_s(\lambda^2 + \beta^2)) + \mu^2\begin{pmatrix} -k_w(\lambda^2 + \beta^2) \\ -k_s\lambda^2(\lambda^2 + \beta^2) \end{pmatrix}, \\
\xi_{34} = -k_1A'\lambda^2(D_{11}^s\lambda^2 + (D_{12}^s + 2D_{66}^s)\beta^2) \\
+k_2B'\beta^2((D_{12}^s + 2D_{66}^s)\lambda^2 + D_{22}^s\beta^2), \\
\xi_{44} = -k_1A'^{\lambda^2} + (k_1A'H_{11}^s\lambda^2 + k_2B'H_{12}^s\beta^2) \\
-k_2B'^{\beta^2} + (k_2B'H_{22}^s\beta^2 + k_1A'H_{12}^s\lambda^2) \\
-\lambda^2\beta^2(k_1A' + k_2B')^2H_{66}^s - (k_1A')^2A_{55}^s\lambda^2 - (k_2B')^2A_{44}^s\beta^2
\end{array} \quad (18)$$

and

$$\begin{array}{c}
m_{11} = I_1 + \mu^2(\lambda^2 + \beta^2)I_1 \\
m_{12} = 0 \\
m_{13} = -\lambda I_2 - \mu^2(\lambda^3 + \beta^2\lambda)I_2 \\
m_{14} = k_1A'\lambda I_3 + \mu^2k_1A'(\lambda^3 + \beta^2\lambda)I_3 \\
m_{22} = I_1 + \mu^2(\lambda^2 + \beta^2)I_1 \\
m_{23} = -\beta I_2 - \mu^2\beta(\lambda^2 + \beta^2)I_2 \\
m_{24} = k_2B'\beta I_3 + \mu^2k_1A'\beta(\lambda^2 + \beta^2)I_3 \\
m_{33} = I_1 - \mu^2(-\lambda^2(I_1 + I_4(\lambda^2 + \beta^2)) \\
-\beta^2(I_1 + I_4(\lambda^2 + \beta^2)) + I_4(\lambda^2 + \beta^2)) \\
m_{34} = -I_5(k_2B'\beta^2 + k_1A'\lambda^2) - \mu^2(-\lambda^2(k_1A'^{\lambda^2} \\
-k_2B'\beta^2) - \beta^2(k_1A'\lambda^2 - k_2B'\beta^2))I_5 \\
m_{44} = I_6((k_1A')^2\lambda^2 + (k_2B')^2\beta^2) - \mu^2(-((k_1A')^{2\lambda^2} \\
+(k_2B')^2\beta^2)\lambda^2 + ((k_1A')^2\lambda^2 + (k_2B')^2\beta^2)\beta^2)I_6
\end{array} \quad (19)$$

The eigenfrequency is extracted by putting $|[\xi] - \omega^2[M]| = 0$.

4. Analytical results and discussions

This section is composed of two parts, the first is devoted to the validation of the present model by comparing the computed results with those found in the literature, and the second part examines the different parameters influencing the free vibration of FG nano-plates.

The material properties constituting FG nanoplates are (Sobhy 2015):

$$E_m = E_1 = 70GPa, E_c = E_2 = 380GPa,$$
$$\rho_1 = 2707\frac{kg}{m^3}, \rho_2 = 3800\frac{kg}{m^3}$$

Table 1 Comparison of dimensionless frequency $\omega *= \omega h\sqrt{\frac{\rho_c}{G_c}}$ of a homogeneous nanoplate without elastic foundations

a/b	a/h	n,m	Source	$\mu^2(nm^2)$			
				0	1	2	3
0.5	10	1,1	Malekzadeh and Shojaee (2013)	0.058883	0.055556	0.052736	0.050305
			Sobhy (2015)	0.058883	0.055556	0.052736	0.050305
			Present	**0.058889**	**0.055561**	**0.052740**	**0.050310**
	20	1,1	Malekzadeh and Shojaee (2013)	0.014965	0.014119	0.013402	0.012785
			Sobhy (2015)	0.014965	0.014119	0.013402	0.012785
			Present	**0.014965**	**0.014740**	**0.014524**	**0.014317**
1	10	1,1	Malekzadeh and Shojaee (2013)	0.093029	0.085016	0.078771	0.073726
			Sobhy (2015)	0.093029	0.085016	0.078771	0.073726
			Present	**0.093042**	**0.085028**	**0.078782**	**0.073737**
		2,2	Malekzadeh and Shojaee (2013)	0.340640	0.254640	0.212120	0.167040
			Sobhy (2015)	0.340634	0.254633	0.212105	0.185591
			Present	**0.340817**	**0.254769**	**0.212219**	**0.185690**
		3,3	Malekzadeh and Shojaee (2013)	0.644000	0.410490	0.320550	0.271840
			Sobhy (2015)	0.683959	0.410467	0.320537	0.271858
			Present	**0.684731**	**0.410931**	**0.320899**	**0.272165**
	20	1,1	Malekzadeh and Shojaee (2013)	0.023864	0.021808	0.020206	0.018912
			Sobhy (2015)	0.023864	0.021808	0.020206	0.018912
			Present	**0.023865**	**0.023297**	**0.022768**	**0.022273**

The following dimensionless quantities are used in the present numerical results

$$\omega *= \omega h\sqrt{\frac{\rho_c}{G_c}},\quad \hat{\omega} = \omega h\sqrt{\rho_m/E_m}\ ,\quad \overline{\omega} = \omega\frac{b^2}{\pi^2}\sqrt{\frac{\rho_2 h}{D_2}},$$
$$\overline{K}_w = \frac{K_w a^4}{D_2}, \overline{K}_s = \frac{K_s a^2}{D_2}, D_2 = \frac{E_2 h^3}{12(1-\nu^2)} \tag{20}$$

Table 1 presents the comparisons of the dimensionless frequency $(\omega *)$ of a homogeneous simply supported nano-plate as function of vibrational mode (m,n), scale effect and aspect and geometry ratios (a/b and a/h).

The obtained results are compared with those given by Sobhy (2015) and Malekzadeh and Shojaee (2013).

From the table, it can be seen that the current results are almost the same as those of Sobhy (2015) and Malekzadeh and Shojaee (2013).

It is also remarkable that the dimensionless frequency$(\omega *)$ diminish with increasing of parameter "μ" and geometry ratio of a homogeneous nano-plate. But the increase of the vibration mode (m,n) and aspect ratio (a/b) increase the results of the dimensionless frequency.

Table 2 gives the non-local dimensionless frequency $(\hat{\omega})$ of a square FGM plate with and without elastic foundations for various values of ratio *h/a* and the inhomogeneity parameter *k*. the current results are compared with those obtained by Sobhy (2015). From the results, a good agreement is confirmed between current results and those given by Sobhy (2015). It can be seen that the nonlocal dimensionless frequency $(\hat{\omega})$ is in inverse relation with inhomogeneity parameter k. it can be also observed that the increase in the values of the ratio "*h/a*" and foundations parameters $(\overline{K}_w, \overline{K}_s)$.

The local and non-local dimensionless frequencies $\overline{\omega}$ of square FG nanoplate with and without elastic foundations for different values of inhomogeneity parameter and (a = h = 10) are presented in the table 3. It is clear from the results that present model gives almost the same values of the local and non-local dimensionless frequencies $\overline{\omega}$ as those computed by Sobhy (2015). The biggest values of frequency $\overline{\omega}$ are obtained for nanoplate on elastic foundation with ($\overline{K}_w$ = 100, $\overline{K}_s$ = 50).

It can be observed also that the dimensionless frequency $\overline{\omega}$ is inverse relation with inhomogeneity (k) and scale effect parameters (μ).

Fig. 2 plots Effect of the inhomogeneity parameter k and ratio *a/h* on the dimensionless frequency $\overline{\omega}$ of FG nanoplate reposed on elastic foundation with $(\overline{K}_1 = 100;\ \overline{K}_2 = 10; \mu = 1\ nm)$. From the obtained curves, it can be noted that the dimensionless frequency $\overline{\omega}$ is in direct correlation relation with geometry ratio a/h. it can be also observed that the increase in the values of the inhomogeneity parameter k leads to decrease the values of the frequency $\overline{\omega}$ when k < 1.5 but $\overline{\omega}$ increase with the increase of k in the case of k > 1.5.

The effects of the elastic foundation stiffnesses $(\overline{K}_w, \overline{K}_s)$ and ratio a/h on the nonlocal dimensionless the frequency $\overline{\omega}$ of simply supported P-FG nanoplates $(k = 1.5; \mu = 1\ nm)$are illustrated in the Fig. 3.

Table 2 Comparison of dimensionless frequency $\hat{\omega} = \omega h \sqrt{\frac{\rho_m}{E_m}}$ of a square FGM plate without or resting on elastic foundations

$\bar{K}_w$	$\bar{K}_s$	k	Sobhy (2015)				**Present**			
			h/a = 0.05	0.1	0.15	0.2	*h/a* = 0.05	0.1	0.15	0.2
0	0	0	0.0291	0.1134	0.2454	0.4154	**0.0291**	**0.1134**	**0.2453**	**0.4153**
		0.5	0.0247	0.0964	0.2091	0.3552	**0.0246**	**0.0963**	**0.2090**	**0.3552**
		1	0.0222	0.0869	0.1886	0.3206	**0.0222**	**0.0868**	**0.1886**	**0.3206**
		2	0.0202	0.0788	0.1706	0.2893	**0.0202**	**0.0788**	**0.1706**	**0.2893**
		5	0.0191	0.0740	0.1589	0.2667	**0.0191**	**0.0740**	**0.1588**	**0.2664**
	100	0	0.0406	0.1599	0.3515	0.6080	**0.0406**	**0.1597**	**0.3512**	**0.6076**
		0.5	0.0387	0.1527	0.3371	0.5861	**0.0386**	**0.1526**	**0.3370**	**0.5858**
		1	0.0378	0.1495	0.3305	0.5755	**0.0378**	**0.1494**	**0.3304**	**0.5753**
		2	0.0374	0.1479	0.3270	0.5696	**0.0374**	**0.1478**	**0.3269**	**0.5695**
		5	0.0377	0.1487	0.3286	0.5723	**0.0377**	**0.1487**	**0.3285**	**0.5721**
100	0	0	0.0298	0.1162	0.2518	0.4272	**0.0298**	**0.1162**	**0.2517**	**0.4271**
		0.5	0.0256	0.1000	0.2174	0.3704	**0.0255**	**0.0999**	**0.2173**	**0.3704**
		1	0.0233	0.0911	0.1982	0.3382	**0.0233**	**0.0910**	**0.1982**	**0.3382**
		2	0.0214	0.0837	0.1818	0.3097	**0.0214**	**0.0836**	**0.1818**	**0.3097**
		5	0.0205	0.0795	0.1716	0.2901	**0.0204**	**0.0795**	**0.1715**	**0.2899**
	100	0	0.0411	0.1619	0.3560	0.6161	**0.0411**	**0.1617**	**0.3558**	**0.6157**
		0.5	0.0392	0.1550	0.3423	0.5954	**0.0392**	**0.1549**	**0.3422**	**0.5951**
		1	0.0384	0.1520	0.3361	0.5855	**0.0384**	**0.1519**	**0.3360**	**0.5853**
		2	0.0381	0.1505	0.3329	0.5802	**0.0381**	**0.1505**	**0.3329**	**0.5800**
		5	0.0384	0.1515	0.3349	0.5834	**0.0384**	**0.1515**	**0.3348**	**0.5833**

Table 3 The frequency $\bar{\omega}$ of square FGM nanoplate without or resting on elastic foundations for different values of inhomogeneity parameter (a = h = 10)

$\bar{K}_w$	$\bar{K}_s$	k	Source	$\bar{\omega}$	
				$\mu = 0\ nm$	$\mu = 2\ nm$
0	0	0	Sobhy (2015)	1.9318	1.4441
			Present	**1.9320**	**1.4442**
		0.5	Sobhy (2015)	1.4969	1.1189
			Present	**1.4974**	**1.1194**
		2.5	Sobhy (2015)	1.2572	0.9397
			Present	**1.2579**	**0.9403**
		5.5	Sobhy (2015)	1.2087	0.9035
			Present	**1.2096**	**0.9042**
		10.50	Sobhy (2015)	1.1609	0.8678
			Present	**1.1621**	**0.8687**
100	0	0	Sobhy (2015)	2.1780	1.7598
			Present	**2.1781**	**1.7599**
		0.5	Sobhy (2015)	1.8354	1.5427
			Present	**1.8360**	**1.5432**
		2.5	Sobhy (2015)	1.6910	1.4704
			Present	**1.6921**	**1.4714**
		5.5	Sobhy (2015)	1.6738	1.4686
			Present	**1.6751**	**1.4698**

Table 3 Continued

$\overline{K}_w$	$\overline{K}_s$	k	Source	$\overline{\omega}$ $\mu = 0\ nm$	$\overline{\omega}$ $\mu = 2\ nm$
100	0	10.5	Sobhy (2015)	1.6499	1.4585
			Present	**1.6514**	**1.4599**
	50	0	Sobhy (2015)	3.8377	3.6167
			Present	**3.8378**	**3.6168**
		0.5	Sobhy (2015)	3.8077	3.6753
			Present	**3.8089**	**3.6765**
		2.5	Sobhy (2015)	3.9339	3.8432
			Present	**3.9364**	**3.8457**
		5.5	Sobhy (2015)	4.0031	3.9206
			Present	**4.0062**	**3.9236**
		10.5	Sobhy (2015)	4.0349	3.8553
			Present	**4.0383**	**3.8586**

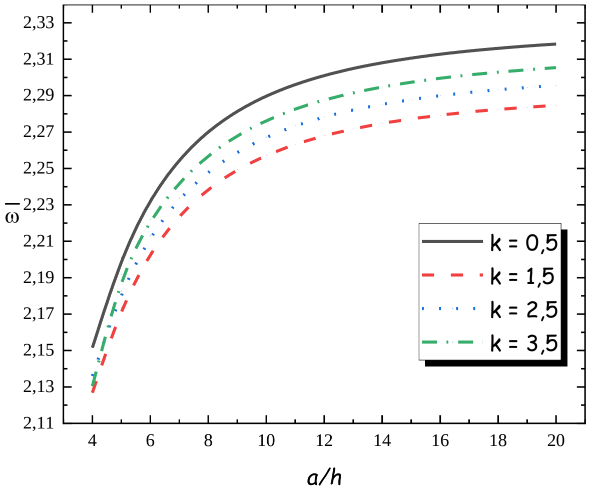

Fig. 2 Effect of the inhomogeneity parameter k on the frequency $\overline{\omega}$, of FGM nanoplates resting on elastic foundations $\left(\overline{K}_1 = 100; \overline{K}_2 = 10; \mu = 1nm\right)$

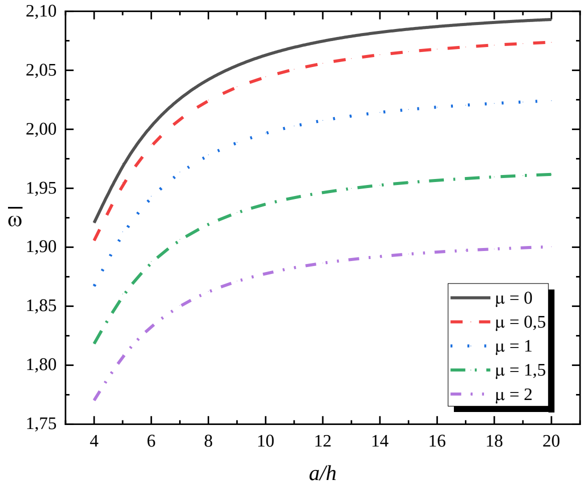

Fig. 4 Effect of the nonlocal coefficient (μ) on the frequency $\overline{\omega}$ of FGM nanoplates resting on elastic foundations $\left(\overline{K}_1 = 10; \overline{K}_2 = 10; k = 1.5\right)$

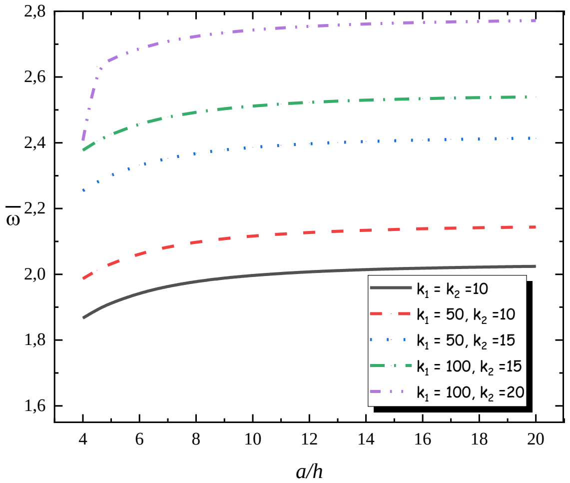

Fig. 3 Effect of the elastic foundation stiffnesses$\left(\overline{K}_w, \overline{K}_s\right)$ on the frequency$\overline{\omega}$ of FGM nanoplates$(k = 1.5; \mu = 1nm)$

From the plotted graphs, we can see that the nonlocal dimensionless the frequency $\overline{\omega}$ increase with increase of the geometry parameter *a/h* and foundation stiffness's $\left(\overline{K}_w, \overline{K}_s\right)$ because the FG-nanoplate becomes more rigid.

Fig. 4 illustrates the influence of the nonlocal coefficient (μ) on dimensionless frequency of the simply supported FG nanoplates resting on elastic foundations with$\left(\overline{K}_1 = 10; \overline{K}_2 = 10; k = 1.5\right)$. From the obtained curves, it is clear the increase in the scale parameter (μ) leads to reduce the values of the dimensionless frequency and this is confirmed for various values of geometry ratio *a/h*.

5. Conclusions

The nonlocal free vibrational behavior of simply supported P-FG nanoplates were investigated analytically using the a four-unknown refined integral plate and Eringen's formulation while including the effects of nonlocal material scale parameter, elastic medium properties, the inhomogeneity index, and the nanoplate's geometric ratios.

The main conclusions are:

• The dimensionless frequency $(\omega *)$ diminishes with increasing of parameter "μ" and geometry ratio.

• The nonlocal dimensionless frequency $(\hat{\omega})$ is in inverse relation with inhomogeneity parameter k.

• The nonlocal dimensionless the frequency $\overline{\omega}$ increases with increase of foundation stiffness's $\left(\overline{K}_w, \overline{K}_s\right)$.

Finally, we can be conclude that the current model is almost exact to predict the free vibrational parameters of homogeneous and FG nanoplates. Other works can be carried out in future by considering other types of materials and other models with shear deformation effect (Yaylaci and Birinci 2013, Oner *et al*. 2015, Adiyaman *et al*. 2015, Yaylaci 2016, Kolahchi *et al*. 2017b, Timesli *et al*. 2017, Timesli 2020c, Zghal *et al*. 2018, Eltaher *et al*. 2018b, 2019a, b, 2020b, Hamidi *et al*. 2018, Karami and Janghorban 2019b, Tounsi *et al*. 2019, Yaylaci *et al*. 2019, 2020b, Mehar and Panda 2020, Abed and Majeed 2020, Akbaş 2020b, Boulal *et al*. 2020, Chami *et al*. 2020, Karami *et al*. 2020, Selmi 2020b, Khazaei and Mohammadimehr 2020, Nejadi and Mohammadimehr 2020, Pandey *et al*. 2020, Patnaik *et al*. 2020, Eltaher and Mohamed 2020a, Rahmani and Asemani 2020, Taherifar *et al*. 2020, Tayeb *et al*. 2020, Yaylaci and Avcar 2020, Akbaş *et al*. 2020).

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