

## Propagation of waves with nonlocal effects for vibration response of armchair double-walled CNTs

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**Abstract.** In this paper, vibration characteristics of double-walled carbon nanotubes (CNTs) are studied based upon nonlocal elastic shell theory. The significance of small scale is being perceived by developing nonlocal Love shell model. The wave propagation approach has been utilized to frame the governing equations as eigen value system. The influence of nonlocal parameter subjected to diverse end supports has been overtly analyzed. An appropriate selection of material properties and nonlocal parameter has been considered. The influence of changing mechanical parameter Poisson's ratio has been investigated in detail. It is found that the frequencies decrease as nonlocal parameter increases and for the certain values of nonlocal parameter against range of Poisson ratio rise slowly with length double-walled CNTs. The dominance of boundary conditions via nonlocal parameter is shown graphically. The results generated furnish the evidence regarding applicability of nonlocal shell model and also verified by earlier published literature.

**Keywords:** double-walled CNTs; love shell theory; nonlocal parameter; poisson's ratio; vibration

### 1. Introduction

The rapid development of nano science and nano technology is phenomenal as echoed with an increase of its application in scientific research. Carbon nanotubes (CNTs) is such discovery by Iijima (1991), that may be used in a variety of fields like material reinforcement, aerospace, medicine, defense and microelectronic devices (Sosa *et al.* 2014, Soldano 2015, Fakhrabadi *et al.* 2015, Mouffoki *et al.* 2017, Bouadi *et al.* 2018). In particular, CNTs have been one of the leading minuscule structure appealed scientists and researchers to analyse experimentally and theoretically its potential aspects.

Major focus has been observed on the free vibrational response of CNTs especially among other nano structures. Additionally, in the age of technology nano sized structures have become utmost importance with large variety of applications in several fields. Since structure of double-walled CNTs consider as two concentric cylindrical shells. Owing the striking mechanical properties through the cylindrical mechanism CNT hold purposeful role in

conveying fluid and gas. With a vast area of potential innovation, however CNTs demands more understanding to investigate its mechanical properties. Free vibration analysis of CNTs have been influential aspect in dynamical science for the last one decade. Vibration characteristics are investigated using thin shell theory by Yakobson *et al.* (1996), beam theory by Wang *et al.* (2006) and nonlocal beam theory (Zermi *et al.* 2015, Youcef *et al.* 2018). An eminent study found in based upon ring theory by Vodenitcharova and Zhang (2003) whereas theories of continuum models developed by Li and chou (2003) in literature. Well known two main classes of models used to analyze the theoretical aspects of CNTs have been atomic model and other is continuum model. The classical molecular dynamics (MD) has shown to exceed those of other techniques such as tight-binding molecular dynamics and ab initio method included in class of atomic modeling (Iijima *et al.* 1996, Yakobson *et al.* 1997, Hernandez *et al.* 1998, Sanchez *et al.* 1999, Qian *et al.* 2002). The main reason continuum mechanics (Yoon *et al.* 2003, Fu *et al.* 2006, Kuang *et al.* 2009, Ansari *et al.* 2011) turned noticeable tool is its computational capability to generate results of large range system in nanometer range. The nonlocal elasticity introduced by Eringen (1983, 2002) becomes a turning point as small scale effect was inculcated in to fundamental equations as simply material parameter. Therefore, scientific community now propose to apply

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nonlocal continuum models to investigate nano-structured materials (Sudak 2003, Wang *et al.* 2006, Pradhan and Phadikar 2009, Ansari *et al.* 2010, Hao *et al.* 2010, Amara *et al.* 2010, Shen and Zhang, 2010). The first ever work presented on use of nonlocal elasticity was by Peddieson *et al.* (2003). Prominent computational competence and accuracy makes nonlocal models an attractive choice for further advancements in field. Akbaş (2016a, b) studied the forced vibration analysis of a simple supported viscoelastic nanobeam based on modified couple stress theory (MCST). The nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. The elastic medium is considered as Winkler-Pasternak elastic foundation. The damping effect is considered by using the Kelvin–Voigt viscoelastic model. The cracked beam is modelled using a proper modification of the classical cracked-beam theory consisting of two sub-beams connected through a massless elastic rotational spring. Donnell (1996) and Flügge (1962) have been two substantial shell theories practiced extensively in study of static and dynamic characteristics of CNTs. Flügge shell theory takes promising place to generate remarkably accurate developments to examine the CNTs. Akbaş (2017a, b) investigated the free vibration analysis of edge cracked cantilever microscale beams composed of functionally graded material (FGM) based on the modified couple stress theory (MCST). The material properties of the beam are assumed to change in the height direction according to the exponential distribution. The FG nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. Mechanical properties of FG beam depends on the position. The Kelvin–Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory. Sedighi *et al.* (2020) established the Eringen's strain-driven nonlocal differential model to exhibit inconsistencies when applied to bounded continua of applicative interest. The stress-driven nonlocal theory leads instead to well-posed nonlocal elastic formulations demonstrating stiffening structural responses. In the present article, using the stress-driven nonlocal model, a comprehensive analysis is conducted to explore the vibrational characteristics and critical divergence velocity of a hybrid-nanotube constructed by carbon. Akbaş (2018) presented the forced vibration responses of a cantilever nanobeam with crack using modified couple stress theory with damping effect. The crack is modeled with a rotational spring. The Kelvin–Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory in the time domain. Influences of the geometry, crack and material parameters on forced vibration responses of cracked nanobeams are examined and discussed.

Natuski and Qing *et al.* (2007) investigated single and double-walled CNTs filled with fluids by adopting wave propagation approach. Flügge shell theory was proposed to form governing equations of motion for CNTs. Rouhi and Ansari (2012) executed the axial buckling of double-walled CNT subject to various layer-wise conditions by using Rayleigh-Ritz based upon nonlocal Flügge shell theory.

Their study showed that the number of different layer-wise boundary conditions dominates the choice of values for nonlocal parameter. Akbaş (2019) presented axially forced vibration of a cracked nanorod under harmonic external dynamically load. In constitutive equation of problem, the nonlocal elasticity theory is used. The Crack is modelled as an axial spring in the crack section. In the axial spring model, the nonrod separates two sub-nanorods and the flexibility of the axial spring represents the effect of the crack. Boundary condition of the nanorod is selected as fixed-free and a harmonic load is subjected at the free end of the nanorod. In another paper, Natuski *et al.* (2006) carried out the vibration analysis of nested CNTs in elastic matrix. Flügge shell theory again had been engaged to establish administrative shell equations while proposed method was wave propagation. Akbaş (2017a, b) investigated the forced vibration analysis of a cracked functionally graded microbeam using modified couple stress theory with damping effect. Mechanical properties of the functionally graded beam change vary along the thickness direction. The crack is modelled with a rotational spring. The Kelvin–Voigt model is considered in the damping effect. static bending of an edge cracked cantilever nanobeam composed of functionally graded material (FGM) subjected to transversal point load at the free end of the beam is investigated based on modified couple stress theory. Material properties of the beam change in the height direction according to exponential distributions. Koochi and Goharimanesh (2021) studied the investigating for nonlinear oscillation of carbon nanotube manufactured nano-resonator. The governing equation of the nano-resonator is extracted in the context of the nonlocal elasticity. The impact of the Casimir force is also incorporated in the developed model. A closed-form solution based on the energy balance method is presented for investigating the oscillations of the nano-resonator. Akbaş (2020) investigated the axially damped forced vibration responses of viscoelastic nanorods within the frame of the modal analysis. The nonlocal elasticity theory is used in the constitutive relation of the nanorod with the Kelvin–Voigt viscoelastic model. In the forced vibration problem, a cantilever nanorod subjected to a harmonic load at the free end of the nanorod is considered in the numerical examples. Usuki and Yogo (2009) formed beam equations again based on Flugge shell theory, they concluded that if nonlocality and refined model are ignored then the generalized Beam theory and Flügge theory produce alike results. Further Wang and Zhang (2007) examined the bending and torsional stiffness of single-walled CNT applying the Flügge shell equations. They presented three-dimensional model of single-walled CNT in their work with effect of thickness. Ouakad and Sedighi (2016) have shown several nonlinear phenomena for significant effect on the electromechanical performance of single-walled carbon nanotube (SWCNT) based nanoelectromechanical (NEMS) devices. To name few: the van der Waals forces, the Casimir forces, the tip charge concentration and the rippling phenomenon. Civalek (2020) presented the free vibration characteristics of thick skew plates reinforced by functionally graded carbon nanotubes (CNTs) reinforced

composite. Discrete singular convolution (DSC) method is used for the numerical solution of vibration problems via geometric mapping technique. Using the geometric transformation via a four-node element, the straight-sided quadrilateral physical domain is mapped into a square domain in the computational space. Ansari and Rouhi (2013) summarized the effect of small scale, geometrical parameter and layer-wise end conditions of double-walled CNT by adopting Flügge shell model (FSM). They depicted that the continuum model considering the nonlocal effect compels the short double-walled CNT more flexible. Further Rouhi *et al.* (2015) investigated the vibration analysis of the multi-walled CNT by developing nonlocal FSM and presented the frequency spectrum against layer wise boundary conditions. Recently Hussain and Naeem (2019a, b) performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method. In recent studies double-walled CNT have been intensively attracted as that of single-walled CNT due to its effectively applicable thermal, mechanical and electronic features. Hu *et al.* (2008) reported a study on the transverse and torsion waves based on nonlocal shell model for single-walled and double-walled CNTs. Zare *et al.* (2020) study the performance of nanotube-based resonators is significantly affected by the variation of their natural frequencies. In reality, nanotubes may have some levels of initial imperfection, which definitely changes their vibrational characteristics. In the present article, the vibrational behavior of initially curved nanotubes is investigated on the basis of thin shell theory incorporating the nonlocal strain gradient theory.

Xu *et al.* (2008) modeled the nested tubes of double-walled CNT as separate elastic beam. Their work revealed that double-walled CNT had no change for a particular invariable frequency subject to distinct edge conditions. Using nonlocal Timoshenko beam theory, Ke *et al.* (2009) investigated free nonlinear vibrations of double-walled CNT and applied differential quadrature technique to derive frequency equations. After wards Khosrozadeh and Hajabasi (2012) carried out vibration analysis of double-walled CNT subject to nonlinear van der Waals forces. Aimed focus on values of nonlocal parameter, length of tube and surrounding elastic medium. Rouhi and Ansari (2013) adapted new numerical approach with nonlocal Donnell shell theory to inquire the small-scale effect on double walled-CNT depending on boundary conditions. Rysaeva *et al.* (2020) focused on close packed carbon nanotube bundles materials with highly deformable elements, for which unusual deformation mechanisms. Structural evolution of the zigzag carbon nanotube bundle subjected to biaxial lateral compression with the subsequent shear straining is studied under plane strain conditions using the chain model with a reduced number of degrees of freedom. Moreover, Benguidiab *et al.* (2014) explored the mechanical buckling features of zigzag double-walled CNT. A comprehensive research presented by Salvatore Brischetto (2015) to analyze the vibration characteristic of double-walled CNT by considering shell continuum model. The findings of article were evolved around effects of van der Waals interaction in terms of frequency ratio. Al-

Shamma demonstrated the experimental study of transient response for different load ratio (0.3, 0.4, 0.5, and 1.0)wt% of Multi-Walled Carbon Nano-Tubes (MWCNTs) reinforced matrix of composite in addition to woven carbon fiber. Thermal Low Velocity Impact (TLVI) apparatus have been designed to meet the different testing conditions. The effect of transient vibration by impact loading on simply supported curved composite shell with and without thermal condition has been studied experimentally. An improvement occurred in settling time about 62.0% at increased of weight ratio reinforcement up to 1.0wt% of MWCNTs, and under thermal condition the settling time reduced about 44.0%. Sedighi and Malikan (2020) investigated the stress-driven nonlocal theory of elasticity, in its differential form for nonlinear vibrational characteristics of a hetero-nanotube in magneto-thermal environment with the help of finite element method. In order to more precisely deal with the dynamic behavior of size-dependent nanotubes, a two-node beam element with six degrees-of freedom including the nodal values of the deflection, slope and curvature is introduced.

Vibration analysis of armchair double-walled CNTs are rarely done in recent past. A limited number of researchers performed analysis first time to investigate the vibration of double-walled CNTs (Wang *et al.* 2006, Natuski *et al.* 2007, Kuang *et al.* 2009, Shen and Zhang 2010, Ansari and Rouhi 2012, Ansari and Arash 2013). So far as reviewed from the literature, vibration response of armchair double-walled CNT using wave propagation approach based on nonlocal FSM has not been investigated/assumed. Many material researchers calculated the frequency of CNTs using different techniques, for example, structural mechanics approach (Li and Chou 2003, Tahouneh 2017, Moradiand Payganeh 2017, Shafiei and Setoodeh 2017), shear deformation theory (Arefi *et al.* 2018, Lei and Zhang 2018), nonlocal continuum models (Sudak 2003, Wang *et al.* 2006, Pradhan and Phadikar 2009, Ansari *et al.* 2010, Hao *et al.* 2010, Amara *et al.* 2010, Shen and Zhang 2010, She *et al.* 2019), stress and strain theory (Karami *et al.* 2018), shell theory (Yakobson *et al.* 1996), beam theory (Wang *et al.* 2006), atomic modeling (Iijima *et al.* 1996, Yakobson *et al.* 1997, Hernandez *et al.* 1998, Sanchez *et al.* 1999, Qian *et al.* 2002), Rayleigh-Ritz (Ansari and Rouhi 2012), Galerkin method (Do *et al.* 2019) and axially loaded double beam system (Xiaobin *et al.* 2014). Moreover, the existing novel theoretical model contributes inventive computational outputs for the vibration of CNTs as compare to prior models presented (Iijima *et al.* 1996, Qian *et al.* 2002, Peddison *et al.* 2003, Sudak 2003, Natuski *et al.* 2006, Shen and Zhang 2010, Ansari and Rouhi 2012). Recently some researcher used different methods for nonlinear modeling (Eltaher *et al.* 2019, Ebrahimi *et al.* 2019, Safaei *et al.* 2019, Shahsavari *et al.* 2019, Benmansour *et al.* 2019).

The foremost intension of this paper to investigate vibrations characteristics of double-walled CNTs by means of nonlocal elasticity shell model. The nonlocal shell model is established by inferring the nonlocal elasticity equations in to Love shell theory, which is our particular motivation. The suggested method to investigate the solution of fundamental eigen relations is wave propagation, which is a

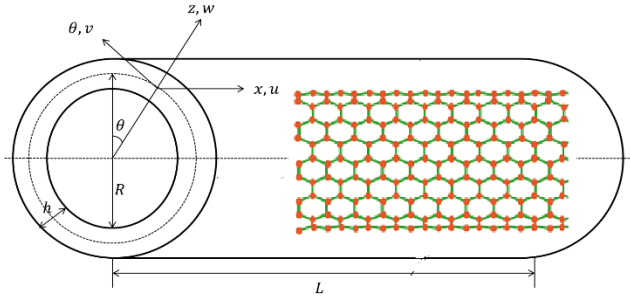


Fig. 1 A schematic representation of armchair CNTs structure

well-known and efficient technique to develop the fundamental frequency equations. It is carefully observed from the literature, no information is seen regarding present established model where such problem has been considered so it became an incentive to conduct current study. The specific influence of four different end supports based on nonlocal model such as clamped-clamped (C-C), clamped-simply supported (C-F) and simply supported-simply supported (S-S) with assorted values of nonlocal parameter are examined in detail.

## 2. Mathematical Formulation

The cylindrical shell is assumed to have length  $L$ , thickness  $h$  and the radius  $R$  for each tube of DWCNT with its coordinate system  $(x, \theta, z)$  as shown in Fig. 1.

The motion of double-walled CNTs predominates the nonlocal resultant forces and moments so expression for these forces are written as:

$$\begin{aligned} \{N_{xx}, N_{\theta\theta}, N_{x\theta}\} - (e_o a)^2 \nabla^2 \{N_{xx}, N_{\theta\theta}, N_{x\theta}\} \\ = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\tilde{\sigma}_{xx}, \tilde{\sigma}_{\theta\theta}, \tilde{\sigma}_{x\theta}) dz \end{aligned} \quad (1)$$

$$\begin{aligned} \{M_{xx}, M_{\theta\theta}, M_{x\theta}\} - (e_o a)^2 \nabla^2 \{M_{xx}, M_{\theta\theta}, M_{x\theta}\} \\ = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\tilde{\sigma}_{xx}, \tilde{\sigma}_{\theta\theta}, \tilde{\sigma}_{x\theta}) z dz \end{aligned} \quad (2)$$

here  $\tilde{\sigma}_{xx}$  and  $\tilde{\sigma}_{\theta\theta}$  stands for stress factors along the axial and tangential directions respectively and  $\tilde{\sigma}_{x\theta}$  indicates the shear stress in  $x\theta$ -plane.

The two-dimensional Hooke's law describes the elements of stress vector in Eqs. (1) and (2) as

$$\begin{pmatrix} \tilde{\sigma}_{xx} \\ \tilde{\sigma}_{\theta\theta} \\ \tilde{\sigma}_{x\theta} \end{pmatrix} - (e_o a)^2 \nabla^2 \begin{pmatrix} \tilde{\sigma}_{xx} \\ \tilde{\sigma}_{\theta\theta} \\ \tilde{\sigma}_{x\theta} \end{pmatrix} = \begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} & 0 \\ \hat{Q}_{12} & \hat{Q}_{22} & 0 \\ 0 & 0 & \hat{Q}_{66} \end{bmatrix} \begin{pmatrix} e_{xx} \\ e_{\theta\theta} \\ e_{x\theta} \end{pmatrix} \quad (3)$$

In same manner  $e_{xx}$  and  $e_{\theta\theta}$  exhibit the strain in  $x$ - and  $\theta$ -directions and  $e_{x\theta}$  presents the shear strain in the  $x\theta$ -plane.

For CNTs,  $\hat{Q}_{kl}$  ( $k, l = 1, 2, \dots, 6$ ) symbolizes stiffness as functions of Young's modulus and Poisson's ratio written as:

$$\begin{aligned} \hat{Q}_{11} = \frac{E}{1 - \nu^2} = \hat{Q}_{22}, \hat{Q}_{12} = \\ \frac{\nu E}{1 - \nu^2} = \hat{Q}_{21}, \hat{Q}_{66} = \frac{E}{2(1 + \nu)} \end{aligned} \quad (4)$$

here  $E, \nu$  and  $\hat{Q}$  are Young's modulus, Poisson's ratio and shear modulus respectively.

Love (1952) submitted the first thin shell theory on base of Kirchhoff's conception. Moreover, an additional modified form of thin shell theory (1963) is established. The diverse range of analytical assessment and comparisons have been examined of these shell theories by Markûs (1988).

The components of the strain vector ( $e$ ) in Eq. (3), that are considered by Love (1952) can be expressed as linear combinations:

$$\begin{aligned} e_{xx} = e_{11} + z\kappa_{11}, e_{\theta\theta} = e_{22} + z\kappa_{22}, \\ e_{x\theta} = e_{12} + 2z\kappa_{12} \end{aligned} \quad (5)$$

$\kappa_{11}$ ,  $\kappa_{22}$  and  $\kappa_{12}$  are known as surface curvatures whereas  $e_{11}$ ,  $e_{22}$  and  $e_{12}$  signify the reference surface strains

Since from Love's theory, the expressions of relation between strain and curvature displacement functions are considered as:

$$\begin{aligned} [e_{11}, e_{22}, e_{12}] = \left[ \frac{\partial u}{\partial x}, \frac{1}{R} \left( \frac{\partial v}{\partial \theta} + w \right), \left( \frac{\partial v}{\partial x} + \frac{1}{R} \frac{\partial u}{\partial \theta} \right) \right] \\ [\kappa_{11}, \kappa_{22}, \kappa_{12}] = \left[ -\frac{\partial^2 w}{\partial x^2}, -\frac{1}{R^2} \left( \frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta} \right), \right. \\ \left. -\frac{2}{R} \left( \frac{\partial^2 w}{\partial x \partial \theta} - \frac{3}{4} \frac{\partial v}{\partial x} + \frac{1}{4R} \frac{\partial u}{\partial \theta} \right) \right] \end{aligned} \quad (6)$$

By substituting Eqs. (3) to (6), nonlocal resultant forces and moments takes the following form.

$$\begin{aligned} N_{xx} - (e_o a)^2 \nabla^2 N_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\sigma}_{xx} dz \\ = \frac{Eh}{1 - \nu^2} \frac{\partial u}{\partial x} + \frac{\nu Eh}{1 - \nu^2} \frac{1}{R} \left( \frac{\partial v}{\partial \theta} + w \right) \end{aligned} \quad (7a)$$

$$\begin{aligned} N_{\theta\theta} - (e_o a)^2 \nabla^2 N_{\theta\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\sigma}_{\theta\theta} dz \\ = \frac{\nu Eh}{1 - \nu^2} \frac{\partial u}{\partial x} + \frac{Eh}{1 - \nu^2} \frac{1}{R} \left( \frac{\partial v}{\partial \theta} + w \right) \end{aligned} \quad (7b)$$

$$\begin{aligned} N_{x\theta} - (e_o a)^2 \nabla^2 N_{x\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\sigma}_{x\theta} dz \\ = \frac{Eh}{2(1 + \nu)} \left( \frac{1}{R} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial \theta} \right) \end{aligned} \quad (7c)$$

$$\begin{aligned} M_{xx} - (e_o a)^2 \nabla^2 M_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\sigma}_{xx} z dz \\ = \frac{\nu D}{R^2} \frac{\partial v}{\partial \theta} - D \left( \frac{\nu}{R^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w}{\partial x^2} \right) \end{aligned} \quad (7d)$$

$$M_{\theta\theta} - (e_o a)^2 \nabla^2 M_{\theta\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\sigma}_{\theta\theta} dz \quad (7e)$$

$$= \frac{D}{R^2} \frac{\partial v}{\partial \theta} - D \left( \frac{1}{R^2} \frac{\partial^2 w}{\partial \theta^2} + v \frac{\partial^2 w}{\partial x^2} \right)$$

$$M_{x\theta} - (e_o a)^2 \nabla^2 M_{x\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\sigma}_{x\theta} z dz \quad (7f)$$

$$= D \frac{(1-\nu)}{R} \left( \frac{\partial u}{\partial \theta} + \frac{3}{4} \frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial \theta \partial x} \right)$$

Meanwhile  $D = \frac{Eh^3}{12(1-\nu^2)}$  refers as bending rigidity of shell.

The mass density per unit length  $\rho_t$  is defined as:

$$\rho_t = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho dz \quad (8)$$

while  $\rho$  entitles as mass density.

The fundamental equations from the Love shell theory are considered as

$$\frac{\partial N_{xx}}{\partial x} + \frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} - \frac{1}{2R^2} \frac{\partial M_{x\theta}}{\partial \theta}$$

$$= \rho_t \frac{\partial^2 u}{\partial t^2} + \frac{1}{R} \frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R^2} \frac{\partial M_{\theta\theta}}{\partial \theta} + \frac{3}{2R} \frac{\partial M_{x\theta}}{\partial x} \quad (9)$$

$$= \rho_t \frac{\partial^2 v}{\partial t^2} + \frac{1}{R^2} \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2 M_{\theta\theta}}{\partial \theta^2} + \frac{2}{R} \frac{\partial^2 M_{x\theta}}{\partial \theta \partial x} - \frac{N_{\theta\theta}}{R} + p$$

$$= \rho_t \frac{\partial^2 w}{\partial t^2}$$

where  $p$  expresses the applied pressure on  $i$  tube through van der Waals (vdW) interaction forces. The vdW model explains the effects of interlayer relations among the tubes of double-walled CNTs.

By combining Eqs. (7) and (8) into (9), formulated the system of partial differential equations in form of three unknown field variables  $u^i, v^i, w^i$  ( $i = 1, 2$ ) for the  $i$ th tube of double-walled CNTs.

$$y_{11}^{(1)} u^1 + y_{12}^{(1)} v^1 + y_{13}^{(1)} w^1 = \rho h \left( \ddot{u}^{(1)} - (e_o a)^2 (\ddot{u}^{(1)}_{xx} + \frac{1}{R_1^2} \ddot{u}^{(1)}_{xx}) \right) \quad (10a)$$

$$y_{21}^{(1)} u^1 + y_{22}^{(1)} v^1 + y_{23}^{(1)} w^1 = \rho h \left( \ddot{v}^{(1)} - (e_o a)^2 (\ddot{v}^{(1)}_{xx} + \frac{1}{R_1^2} \ddot{v}^{(1)}_{xx}) \right) \quad (10b)$$

$$y_{31}^{(1)} u^1 + y_{32}^{(1)} v^1 + y_{33}^{(1)} w^1 = \rho h \ddot{w}^{(1)} + w^{(1)} \sum_{j=1}^2 c_{1j} - \sum_{j=1}^2 c_{1j} w^{(j)} - \left[ \rho h (\ddot{w}^{(1)}_{xx} + \frac{1}{R_1^2} \ddot{w}^{(1)}_{\theta\theta}) + (\ddot{w}^{(1)}_{xx} + \frac{1}{R_1^2} \ddot{w}^{(1)}_{\theta\theta}) \sum_{j=1}^2 c_{1j} - \sum_{j=1}^2 c_{1j} (\ddot{w}^{(j)}_{xx} + \frac{1}{R_1^2} \ddot{w}^{(j)}_{\theta\theta}) \right] \quad (10c)$$

$$y_{11}^{(2)} u^2 + y_{12}^{(2)} v^2 + y_{13}^{(2)} w^2 = \rho h \left( \ddot{u}^{(2)} - (e_o a)^2 (\ddot{u}^{(2)}_{xx} + \frac{1}{R_2^2} \ddot{u}^{(2)}_{xx}) \right) \quad (10d)$$

$$y_{21}^{(2)} u^2 + y_{22}^{(2)} v^2 + y_{23}^{(2)} w^2 = \rho h \left( \ddot{v}^{(2)} - (e_o a)^2 (\ddot{v}^{(2)}_{xx} + \frac{1}{R_2^2} \ddot{v}^{(2)}_{xx}) \right) \quad (10e)$$

$$y_{31}^{(2)} u^2 + y_{32}^{(2)} v^2 + y_{33}^{(2)} w^2 = \rho h \ddot{w}^{(2)} + w^{(2)} \sum_{j=1}^2 c_{2j} - \sum_{j=1}^2 c_{2j} w^{(j)} - \left[ \rho h (\ddot{w}^{(2)}_{xx} + \frac{1}{R_2^2} \ddot{w}^{(2)}_{\theta\theta}) + (\ddot{w}^{(2)}_{xx} + \frac{1}{R_2^2} \ddot{w}^{(2)}_{\theta\theta}) \sum_{j=1}^2 c_{2j} - \sum_{j=1}^2 c_{2j} (\ddot{w}^{(j)}_{xx} + \frac{1}{R_2^2} \ddot{w}^{(j)}_{\theta\theta}) \right] \quad (10f)$$

here  $y_{pq} = (p, q = 1, 2, 3)$  are stated as partial operators can be seen in Appendix.

## 2.1 Solution using the wave propagation approach

During the past few years, numerous theories have been extensively debated for vibration of nanotube, shell and plate morphologies of several conformations depending upon certain edge conditions. Wave propagation approach is among the most significant and successfully used numerical technique by researchers to investigate the free vibrations of cylinder-shaped shell, plates and nanotubes. The three modal displacement functions of the shell for  $i$ th tube can be written as (Flügge 1962, Forsberg 1964, Warburton 1965).

$$u^{(i)}(x, \theta, t) = a_m \cos(n\theta) e^{(i\omega t - ik_m x)} \quad (11a)$$

$$v^{(i)}(x, \theta, t) = b_m \sin(n\theta) e^{(i\omega t - ik_m x)} \quad (11b)$$

$$w^{(i)}(x, \theta, t) = c_m \cos(n\theta) e^{(i\omega t - ik_m x)} \quad (11c)$$

where  $a_m, b_m, c_m$  describe the displacement amplitude coefficients in  $x, \theta$  and  $z$  directions correspondingly. The angular frequency is designated as  $\omega$ , circumferential wave number by  $n$  and  $k_m$  referred to be axial wave number allied with end supports obligatory on double-walled CNTs. Replacing the functions and derivatives into the system of fundamental equations, henceforth derived a set of simultaneous as follows

$$Y_{11}^{(i)} a_m^i + Y_{12}^{(i)} b_m^i + Y_{13}^{(i)} c_m^i = -\omega^2 (1 - (e_o a)^2 \nabla^2) \rho h a_m^i \quad (12a)$$

$$Y_{21}^{(i)} a_m^i + Y_{22}^{(i)} b_m^i + Y_{23}^{(i)} c_m^i = -\omega^2 (1 - (e_o a)^2 \nabla^2) \rho h b_m^i \quad (12b)$$

$$Y_{31}^{(i)} a_m^i + Y_{32}^{(i)} b_m^i + Y_{33}^{(i)} c_m^i = -\omega^2 (1 - (e_o a)^2 \nabla^2) \rho h c_m^i \quad (12c)$$

$$\begin{aligned}
 & + (1 - (e_o a)^2 \nabla^2) \left[ \sum_{j=1}^2 c_{ij} c_m^i - \sum_{j \neq i}^2 c_{ij} c_m^i \right] \\
 & = -\omega^2 (1 - (e_o a)^2 \nabla^2) \rho h c_m^i
 \end{aligned}$$

Since  $i = (1,2)$  and the algebraic operators  $Y_{pq}^{(i)}$  are acquired using Appendix with  $p, q = (1,2,3)$ .

### 3. Result and discussion

An inventive approach to fabricate nonlocal Love shell model based on wave propagation technique of double-walled CNTs for vibrational response has been demonstrated and verified with results presented in the literature. Considering the negligible percentage of error, thus it confirms the validation of suggested nonlocal shell model. An innovational nonlocal model to examine the scale effect on vibrational behavior of armchair of double-walled CNTs. The influence of nonlocal parameter with variation of Poisson's ratio are investigated depending upon certain edge supports. The mass density is assumed to be  $2300 \text{ kg/m}^3$ , with Young's Modulus  $1 \text{ Tpa}$  (Basirjafri *et al.* 2012). In accordance of theoretical procedure, at first frequency of double-walled CNTs with mutation in values of Poisson's ratio is observed as shown in Fig. 2. As figure shows fundamental frequency rises by rising the Poisson's ratio. The results presented here are in good accordance with those established by Basirjafri *et al.* (2012). The fundamental frequencies are calculated of double-walled CNTs subjected to three distinct end supports C-C, S-S and C-F. The frequencies are obtained for the varying values of Poisson's ratio from 0.1 to 0.4. To inspect significance of nonlocal parameter on vibration of double-walled CNTs, three peculiar values of nonlocal parameter  $e_o = 0.2, 0.65$  and  $1.2$  are considered. When Poisson's ratio varies from  $\nu = 0.1$  to  $0.4$ , the frequencies of C-C armchair (7, 7) with  $r_1 = 1.5$  against  $e_o = 0.2$  are 4.3479, 4.3738, 4.4110, 4.4604, 4.5233, 4.6015 and 4.6811 and S-S frequencies are 4.0646, 4.0886, 4.1229, 4.1685, 4.2266, 4.2989 and 4.3711 respectively. Similarly, C-F frequencies are 3.7905, 3.8125, 3.840, 3.8859, 3.9394, 4.0058 and 4.0071. The corresponding frequencies have been sketched in Fig. 3. It is noticed that clamped-clamped frequencies are higher followed by simply supported - simply supported and clamped-free and gap between frequency curves are evident. As ratio increases, the frequencies also show gradually slow increasing pattern. Fig. 2 displays the frequency curves of armchair (7, 7) against  $e_o = 0.65$  subjected to aforementioned end supports. The curves affirm the gap between the end supports as shown in Fig. 1. For the first value of Poisson's ratio  $\nu = 0.1$  C-C armchair (7, 7) frequency is observed as 1.7283, S-S as 1.6453 and C-F 1.5645 respectively. Similarly, at the last value  $\nu = 0.4$ , C-C frequency is 1.8672, S-S and C-F are 1.7761, 1.6871. It is noticed that frequencies decline for all end supports. Furthermore, one important observation is clearly seen that with increased nonlocal parameter value, the frequency tends to

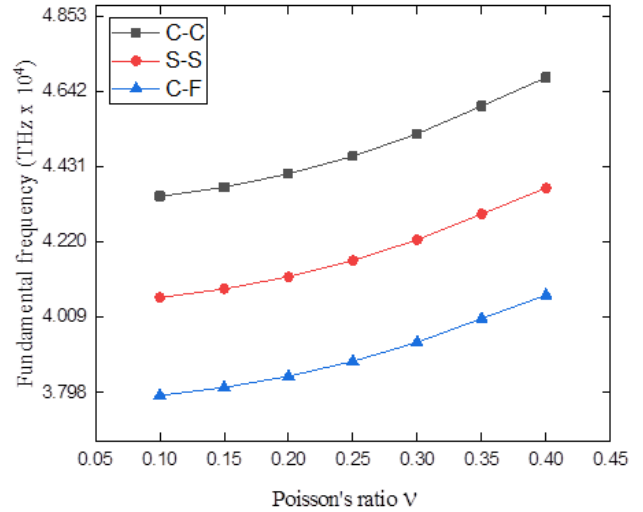


Fig. 2 Frequency variation of arm chair (7,7) with  $e_o = 0.2$  versus Poisson's ratio

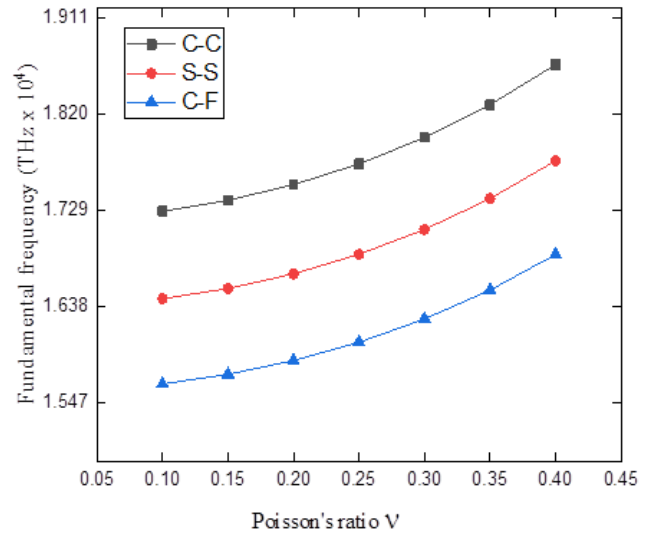


Fig. 3 Frequency variation of arm chair (7,7) with  $e_o = 0.65$  versus Poisson's ratio

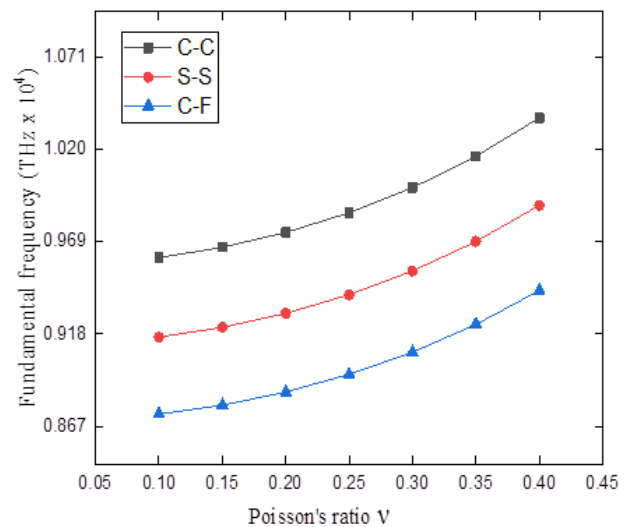


Fig. 4 Frequency variation of arm chair (7,7) with  $e_o = 1.2$  versus Poisson's ratio

decrease for all end conditions. The frequency variation with ratio of armchair (7, 7) double-walled CNTs against  $e_0 = 1.2$  can be viewed in Fig.4. The clamped-clamped frequencies are 0.9602, 0.9659, 0.9741, 0.9850, 0.9989, 1.0162 and 1.0371 whereas, clamped-free are calculated as 0.8737, 0.8787, 0.8860, 0.9080, 0.9223, and 0.9421. It is observed once again that frequencies decrease as nonlocal parameter increases. The frequency curve maintains the regular gap between end supports as seen before. It is also concluded that frequencies for these certain values of nonlocal parameter against range of Poisson ratio rise slowly with same parameters and length double-walled CNTs.

#### 4. Concluding remarks

The frequency patterns have been discussed for armchair double-walled CNTs to inspect the influence of Poisson's ratio based on nonlocal Love shell model. Natural frequency curves are presented for three specific clamped-clamped, simply supported and clamped free end supports considering distinct values of nonlocal parameter. The fundamental frequency patterns for armchair double-walled CNTs exhibit the resembling trends for varying values of ratio. Although we observe the phenomena for structural strength of double-walled CNTs by comparing the respective frequencies for armchair structure. Because of the fact that greater the Poisson ratio, softer the material. A slow increase in frequencies against variation of Poisson's ratio also indicates insensitivity of it for suggested nonlocal model. In addition, decrease in frequencies with increase in nonlocal parameter authenticates the applicability of nonlocal Love shell model. This model can be extendable for the vibration of multi-walled carbon nanotubes.

#### Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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## Appendix A

$$\begin{aligned}
y_{11}^{(i)} &= \frac{Eh}{1-\nu^2} \frac{\partial^2}{\partial x^2} + \left( \frac{1}{R_i^2} \frac{Eh}{2(1+\nu)} - \frac{1}{R_i^4} \frac{D(1-\nu)}{8} \right) \frac{\partial^2}{\partial \theta^2} \\
y_{12}^{(i)} &= \frac{1}{R_i} \left( \frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)} - \frac{3D(1-\nu)}{8} \frac{1}{R_i^2} \right) \frac{\partial^2}{\partial \theta \partial x} \\
y_{13}^{(i)} &= \frac{1}{R_i} \frac{Eh\nu}{1-\nu^2} \frac{\partial}{\partial x} + \left( \frac{D(1-\nu)}{2R_i^3} \frac{\partial^2}{\partial x^2} \right) \frac{\partial}{\partial \theta} \\
y_{21}^{(i)} &= y_{12}^{(i)} \\
y_{22}^{(i)} &= \left( \frac{Eh}{2(1+\nu)} + \frac{9D(1-\nu)}{8R_i^2} \right) \frac{\partial^2}{\partial x^2} \\
&\quad + \frac{1}{R_i^2} \left( \frac{Eh}{(1-\nu^2)} + \frac{D}{R_i^2} \right) \frac{\partial^2}{\partial \theta^2} \\
y_{23}^{(i)} &= - \left( \frac{\nu D}{R_i^2} + \frac{3D(1-\nu)}{2} \frac{1}{R_i^2} \right) \frac{\partial^3}{\partial x^2 \partial \theta} - \frac{D}{R_i^4} \frac{\partial^3}{\partial \theta^3} \\
&\quad + \frac{1}{R_i^2} \frac{Eh}{1-\nu^2} \frac{\partial}{\partial \theta} \\
y_{31}^{(i)} &= -y_{13}^{(i)} \\
y_{32}^{(i)} &= \left( \frac{D}{R_i^2} (2-\nu) + \frac{3D(1-\nu)}{4R_i^2} \right) \frac{\partial^3}{\partial x^2 \partial \theta} + \frac{D}{R_i^4} \frac{\partial^3}{\partial \theta^3} - \frac{1}{R_i^2} \frac{Eh}{1-\nu^2} \frac{\partial}{\partial \theta} \\
y_{33}^{(i)} &= -D \frac{\partial^4}{\partial x^4} - \left( \frac{2D}{R_i^2} + \frac{2D(1-\nu)}{R_i^2} \right) \frac{\partial^4}{\partial x^2 \partial \theta^2} - \frac{D}{R_i^4} \frac{\partial^4}{\partial \theta^4} \\
&\quad - \frac{1}{R_i^2} \frac{Eh}{1-\nu^2}
\end{aligned}$$

## Appendix B

$$\begin{aligned}
Y_{11}^{(i)} &= \frac{Eh}{1-\nu^2} (-k_m^2) + \left( \frac{1}{R_i^2} \frac{Eh}{2(1+\nu)} - \frac{1}{R_i^4} \frac{D(1-\nu)}{8} \right) (-n^2) \\
Y_{12}^{(i)} &= \frac{1}{R_i} \left( \frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)} - \frac{3D(1-\nu)}{8} \frac{1}{R_i^2} \right) (-nik_m) \\
Y_{13}^{(i)} &= \frac{1}{R_i} \frac{Eh\nu}{1-\nu^2} (-nik_m) + \left( \frac{D(1-\nu)}{2R_i^3} \frac{\partial^2}{\partial x^2} \right) (-n) \\
Y_{21}^{(i)} &= Y_{12}^{(i)} \\
Y_{22}^{(i)} &= \left( \frac{Eh}{2(1+\nu)} + \frac{9D(1-\nu)}{8R_i^2} \right) (-k_m^2) \\
&\quad + \frac{1}{R_i^2} \left( \frac{Eh}{(1-\nu^2)} + \frac{D}{R_i^2} \right) (-n^2) \\
Y_{23}^{(i)} &= \left( \frac{\nu D}{R_i^2} + \frac{3D(1-\nu)}{2} \frac{1}{R_i^2} \right) (nk_m^2) - \frac{D}{R_i^4} n^3 + \frac{1}{R_i^2} \frac{Eh}{1-\nu^2} (-n) \\
Y_{31}^{(i)} &= -Y_{13}^{(i)} \\
Y_{32}^{(i)} &= \left( \frac{D}{R_i^2} (2-\nu) + \frac{3D(1-\nu)}{4R_i^2} \right) (-nk_m^2) + \frac{D}{R_i^4} n^3 - \frac{1}{R_i^2} \frac{Eh}{1-\nu^2} n \\
Y_{33}^{(i)} &= -Dk_m^4 - \left( \frac{2D}{R_i^2} + \frac{2D(1-\nu)}{R_i^2} \right) n^2 k_m^2 - \frac{D}{R_i^4} n^4 \\
&\quad - \frac{1}{R_i^2} \frac{Eh}{1-\nu^2}
\end{aligned}$$