

# Temperature jump and concentration slip effects on bioconvection past a vertical porous plate in the existence of nanoparticles and gyrotactic microorganism with inclined MHD

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**Abstract.** The paper presents the effects of temperature jump and concentration slip on inclined MHD bioconvection past a vertical porous plate via porous media. The authors have examined how the presence of both nanoparticles and gyrotactic microorganism impact the whole procedure. It is researched that the numerical scheme, called Runge-Kutta fourth fifth order Fehlberg method (RKF45) has been used to solve the governing partial differential equations. The equations are reduced into ordinary differential equations by using suitable similarity transformation. The effects of pertinent parameter for variation in the velocity profile, velocity profile at far field, temperature profile, concentration profile and motile microorganism density profile have been obtained. The results obtained from current study in the concluding part of the paper match to the pre researched data which validate the authenticity of the study.

**Keywords:** concentration slip; gyrotactic microorganism; inclined MHD; nanoparticles; RKF-45; temperature jump

## 1. Introduction

Many scientist and researchers are working in the direction to increase the efficiency of heat transfer phenomenon of fluid dynamics, particularly the fluids used in daily life. For this purpose, bioconvection plays a vital role in fluid mechanism, modern engineering and environmental system. Some important applications of bioconvection in biological system and biotechnology are such as bio-microsystem, enzyme biosensors, mass transfer of biotechnology etc. This kind of convection is induced due to an unstable density stratification caused by up swimming microorganisms. Motile microorganism survives only in base fluid, hence the nanofluid must be water based nanofluid. Many researchers have investigated gyrotactic microorganism in fluid dynamics such as Childress *et al.* (1975) analyzed the theory of a suspension of swimming microorganism. In this study they concluded that the motility of the organism is defined by an average upward swimming speed  $U$  and a diffusivity tensor  $D$ . Hillesdon and Pedley (1996) studied bioconvection in suspensions of oxytactic bacteria. They focused on suspension of the bacterium *Bacillus subtilis*, is located in a chamber with its higher surface open to the atmosphere, complex bioconvection patterns form. Later, Hill and Pedley (2005) introduced the bioconvection term in fluid dynamics. The effects of bio thermal convection in a suspension of gyrotactic microorganism in boundary layer flows were

studied by Nield and Kuzentsov (2006). Alloui *et al.* (2007) did numerical study of thermo bioconvection in a suspension of gravitactic microorganisms. Kuzentsov (2010, 2011) described the nanofluid bioconvection in a suspension containing both gyrotactic and oxytactic microorganism with nanoparticles in a water-based fluid. The readers can further investigate heat transfer phenomena in different aspects such as Eltahir *et al.* (2019), Matouk *et al.* (2020), Hussain *et al.* (2020) and Fenjan *et al.* (2020).

Avramenko and Kuzentsov (2004) carried out an analysis of stability of a suspension of gyrotactic microorganism in the presence of porous media in boundary layers' flow. Aziz *et al.* (2012) studied the suction/injection effects of free convection boundary layer flow over a horizontal flat plate filled by nanofluid having gyrotactic microorganism. They described the influences of bioconvection constraints on the dimensionless velocity, temperature, nanoparticle concentration and density of motile microorganisms as well as on the local Nusselt, Sherwood and motile microorganism numbers. Xu and Pop (2014) investigated the mixed convection flow of a water based nanofluid with uniform free stream over a stretching surface in the presence of gyrotactic microorganism. Raees *et al.* (2015) extended the work of Xu and Pop and studied microorganism in gravity-driven nano-liquid film containing nanoparticles. Beg *et al.* (2015) presented the bioconvection effects for non-Newtonian nanofluid containing microorganism embedded in a flat plate in the presence of porosity. Jain and Bohra (2016) studied radiative flow through porous media past a rotating disk with variable fluid properties. Uddin *et al.* (2016), Tausif *et al.* (2016) and Siddiqua *et al.* (2016) were investigated the

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bioconvection suspension in presence of both nanoparticle and microorganism in several aspects. If we talk about graded porous plate, several researchers did wonderful work such as, Berghouti *et al.* (2019) described the vibration analysis of nanolocal porous graded material. Medani *et al.* (2019) studied the dynamic behavior of FG-CNT using energy principle. Kaddari *et al.* (2020) discussed about the structural behavior of graded porous plate of a new quasi-3D model. Zine *et al.* (2020) investigated the bending analysis of functionally graded porous plates.

MHD boundary layer flow past a vertical plate has an important significance due to its frequent occurrence in many industrial and technology aspects such as MHD pumps, drug delivery, micro-mixing of physiological samples and biological transportations. We know that the presence of nanoparticle increases the electrical conductivity of the nanofluids, hence making them more efficient we use external magnetic field in boundary layer flow. There are several applications of MHD including engineering, biomedical, magnetofluidic leakage free seals, MHD blood flow meters etc. Chauhan and Agarwal (2012) discussed influence of MHD couette flow in a channel partially embedded with porous medium. Khan *et al.* (2014) investigated the bioconvection MHD nanofluid due to microorganism over a stretching sheet in the presence of nanoparticles. Mutuku *et al.* (2014) focused on the study of magnetohydrodynamic bioconvection of nanofluid due to gyrotactic microorganism over a vertical plate with suction/injection. Khan *et al.* (2014) have extended the work of Mutuku *et al.* (2014) and investigated the gyrotactic microorganism over a heat stretching sheet in the presence of nanoparticles and convective boundary conditions. Jain and Choudhary (2015) described influences of MHD boundary layer flow over an exponentially stretching sheet with porous medium and slip. Makinde *et al.* (2016) have investigated the effects of non-linear thermal radiation of MHD boundary layer nanofluid with chemical reaction. Khan *et al.* (2016) introduced the effects of Joule heating and viscous dissipation on MHD bioconvection flow in the presence of nanoparticles and gyrotactic microorganism over a porous wedge. Makinde and Animasaun (2016) discussed the radiative, thermophoresis and Brownian motion effects on magnetohydrodynamic bioconvection nanofluid past a horizontal surface with chemical reaction. Recently, Chakraborty *et al.* (2016) Alsaedi *et al.* (2017) and Raju *et al.* (2017) studied gyrotactic microorganism with nanoparticles under different characteristics.

Having the meticulous and detailed study on the topic of the behavior of gyrotactic microorganism in the presence of inclined magnetic field, temperature slip and concentration slip, the authors found that no work has been carried out yet on it. Since microorganism is an imperative component of water-based nanofluid, so water based nanofluid has been included in this study for the desired outcomes.

## 2. Mathematical formulation

We have considered a water based nanofluid containing gyrotactic microorganisms in the presence of inclined

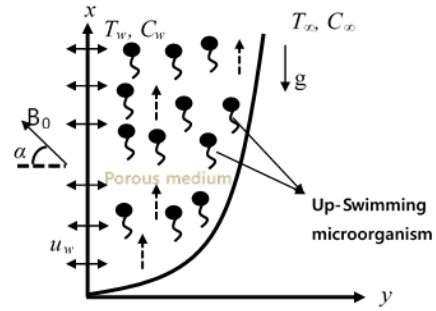


Fig. 1 Geometrical sketch of rotating CS

magnetic field with porous medium over a permeable vertical flat plate. We anticipated that the existence of nanoparticle does not possess any influence on the route of microorganism's swimming and on their velocity too. Also we assumed that the suspension of nanoparticles is stable and dilute, this hypothesis is only true, when the concentration of that nanoparticle is less than 1%. And this is well known or a logical assumption that only in a dilute suspension of nanoparticles, nanoparticle bioconvection is occurred. Inclined magnetic field is applied perpendicular to the plate. Where  $B_0$  is the magnetic field strength and  $\alpha$  is the inclination angle. The fluid flow is assumed to be in porous medium. The flow is started from the bottom of the plate, where  $x$ -axis remains upright and  $y$ -axis is perpendicular to the  $x$ -axis (see Fig. 1). The stretching velocity of the fluid is given as  $u_w = ax$ , where  $a$  is supposed to a positive constant.

The microorganism is swimming in the upward direction with the  $x$ -axis and gravitational force  $g$  applied on the fluid.

Thus, under the above assumption the governing equations of considered setup followed by Xu and Pop (2016) and Chakraborty *et al.* (2016) is given as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_f} \left[ \begin{array}{l} (1 - C_\infty) \rho_{f\infty} \beta g (T - T_\infty) \\ -(\rho_p - \rho_{f\infty}) g (C - C_\infty) \\ -(n - n_\infty) g \zeta \Delta P \end{array} \right] \quad (2)$$

$$\frac{\sigma B_0^2 \sin^2 \alpha}{\rho_f} (u - U_\infty) - \frac{\nu}{k_1} (u - U_\infty),$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha^* \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left[ \begin{array}{l} D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \left( \frac{D_T}{T_\infty} \right) \left[ \left( \frac{\partial T}{\partial x} \right)^2 \right. \\ \left. + \left( \frac{\partial T}{\partial y} \right)^2 \right] \end{array} \right] + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2 \sin^2 \alpha}{\rho_f c_p} (u - U_\infty)^2, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (4)$$

$$u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \frac{bW_c}{(C_w - C_\infty)} \left[ \frac{\partial}{\partial y} \left( n \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial x} \left( n \frac{\partial C}{\partial x} \right) \right] = D_m \left( \frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + 2 \frac{\partial^2 n}{\partial x \partial y} \right). \quad (5)$$

where  $u$  and  $v$  are the velocity components along the directions  $x$  and  $y$ , respectively.  $\rho_f$  is the density of the base fluid,  $\rho_p$  signifies the density of the nanoparticles,  $\rho_{f\infty}$  denotes the density of primary fluid at the far field,  $\beta$  is the volume growth coefficient of the primary fluid,  $\sigma$  is the electrical conductivity,  $\nu$  is the kinematic viscosity,  $k_l$  the permeability medium of the fluid,  $C$  denotes the volume fraction of nanoparticles and its ambient value is  $C_\infty$ ,  $U_\infty$  is the velocity of unrestricted flow,  $\alpha^*$  denotes the thermal diffusivity,  $c_p$  is the specific heat at constant pressure,  $\tau = (\rho c)_p / (\rho c)_f$  is referred the proportion between the effectual heat capability of the nanoparticles and base fluid,  $D_B$  and  $D_T$  denoted by Brownian diffusion coefficient and thermophoretic diffusion coefficient, respectively.  $T$  and  $T_\infty$  are the nanofluid temperatures near the wall and ambient temperature,  $\xi$  is the standard volume of a microorganism,  $\Delta P$  denotes the difference between a cell and primary fluid,  $n$  is the concentration of microorganism,  $b$  is the chemotaxis constant,  $W_c$  is referred maximum cell swimming speed,  $D_m$  represents the diffusivity of microorganism and  $g$  is the acceleration due to gravity.

The corresponding boundary conditions for the Eqs. (1)-(5) are

$$\begin{aligned} u_w &= ax, \quad v = V, \\ T &= T_w + l_1 \frac{\partial T}{\partial y}, \\ C &= C_w + l_2 \frac{\partial C}{\partial y}, \quad n = n_w \end{aligned} \quad \text{at } y = 0, \quad (6)$$

$$u \rightarrow U_\infty, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad n \rightarrow n_\infty \quad \text{at } y \rightarrow \infty.$$

Here  $l_1$  and  $l_2$  are the thermal and concentration slip factor, respectively.  $T_w, C_w, n_w$  are the temperature, nanoparticle volume fraction and density of the motile microorganism at the plate surface, correspondingly. While  $T_\infty, C_\infty, n_\infty$  are the ambient values of above corresponds. Also we supposed that the free stream velocity and the suction/injection velocity are as follows Mutuku and Makinde (2014)

$$u_w = ax \quad \text{and} \quad V = -(av)^{1/2} f_w$$

where  $f_w > 0$  denotes suction velocity, while  $f_w < 0$  represents injection velocity and  $f_w = 0$  is the situation for impermeable plate surface.

To convert the above partial differential equations (1-5) into ordinary differential equations, we have introducing similarity transformations, as

$$\eta = (a/\nu)^{1/2} y,$$

$$\psi(x, y) = (a\nu)^{1/2} x f(\eta)$$

$$+ (U_\infty \nu / a)^{1/2} s(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad (7)$$

$$w(\eta) = \frac{n - n_\infty}{n_w - n_\infty}.$$

Initially, we have to satisfy the equation of continuity Eq (1) using

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \quad (8)$$

By substituting Eqs. (7) and (8) into Eqs. (2)-(5), we attained the following ordinary differential equations

$$f''' + ff'' - f'^2 - M^2 \sin^2 \alpha f' - Kf' = 0, \quad (9)$$

$$\begin{aligned} s''' + fs'' - f's' + G_r(\theta - N_r \phi - R_b w) \\ - M^2 \sin^2 \alpha (s' - 1) - K(s' - 1) = 0, \end{aligned} \quad (10)$$

$$\begin{aligned} \theta'' + N_t \theta'^2 + Pr f \theta' + N_b \theta' \phi' \\ + Pr Ec (f'' + As'')^2 \\ + Pr Ec M^2 \sin^2 \alpha \{f' + A(s' - 1)\}^2 = 0, \end{aligned} \quad (11)$$

$$\phi'' + Le f \phi' + \frac{N_t}{N_b} \theta'' = 0, \quad (12)$$

$$\begin{aligned} w'' + Sc f w' \\ - Pe \{ \phi' w' + \phi'' (w + \Omega) \} = 0, \end{aligned} \quad (13)$$

Subject to boundary conditions;

$$\begin{aligned} f(0) &= f_w, \quad f'(0) = 1, \quad s(0) = 0, \\ s'(0) &= 0, \quad \theta(0) = 1 + \delta \theta'(0), \\ \phi(0) &= 1 + \delta_1 \phi'(0), \quad w(0) = 1 \quad \text{at } \eta = 0 \\ f'(\infty) &= 0, \quad s'(\infty) = 1, \quad \theta(\infty) = 0, \\ \phi(\infty) &= 0, \quad w(\infty) = 0 \quad \text{as } \eta \rightarrow \infty. \end{aligned} \quad (14)$$

Here  $f'(\eta), s'(\eta), \theta(\eta), \phi(\eta)$  and  $w(\eta)$  denotes, non-dimension velocity close to the surface, non-dimension velocity at far field, non-dimension temperature, non-dimension nanoparticle concentration, non-dimension

denseness of motile microorganism, respectively.

Where  $M^2 = \frac{\sigma B_0^2}{a\rho_f}$  denotes the magnetic field

parameter,  $K = \frac{\nu}{ak_1}$  refers permeability parameter,

$G_r = \frac{(1-C_\infty)\rho_{f\infty}\beta g(T_w-T_\infty)}{a\rho_f U_\infty}$  signify Grashof number,

$N_r = \frac{(\rho_f - \rho_{f\infty})(C_w - C_\infty)}{\rho_{f\infty}\beta(1-C_\infty)(T_w - T_\infty)}$  indicates buoyancy-ratio

parameter,  $R_b = \frac{(n_w - n_\infty)\zeta\Delta P}{\rho_{f\infty}\beta(1-C_\infty)(T_w - T_\infty)}$  conveys

bioconvection Rayleigh number,  $Pr = \frac{\nu}{\alpha}$  is the Prandtl

number,  $Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}$  is the Eckert number,

$N_b = \frac{\tau D_B(C_w - C_\infty)}{\alpha}$  denotes Brownian motion

parameter,  $N_t = \frac{\tau D_T(T_w - T_\infty)}{T_\infty\alpha}$  is the thermophoresis

parameter,  $A = \frac{U_\infty}{u_w}$  describes the ratio of the fluid

velocity at the far field and near the wall,  $Le = \frac{\nu}{D_B}$  is the

Lewis number,  $Sc = \frac{\nu}{D_m}$  represents the Schmidt number,

$Pe = \frac{bW_c}{D_m}$  is the bioconvection Peclet number,

$\Omega = \frac{n_\infty}{n_w - n_\infty}$  is the microorganism concentration

difference parameter,  $\delta, \delta_1$  are the thermal slip and concentration slip parameter, correspondingly and  $f_w$  stands for suction/injection parameter.

Here, skin friction  $C_f$ , Nusselt number  $Nu$ , Sherwood number  $Sh$  and density number of motile microorganism  $Nn$  followed by Xu and Pop (2014) is as follows

$$\begin{aligned} C_f &= \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu = \frac{xq_w}{k(T_w - T_\infty)}, \\ Sh &= \frac{xq_m}{D_B(C_w - C_\infty)}, \quad Nn = \frac{xq_n}{D_m(n_w - n_\infty)}. \end{aligned} \quad (16)$$

where  $\tau_w, q_w, q_m$  and  $q_n$  are the skin friction, surface heat flux, surface mass flux and surface motile flux denoted as

$$\begin{aligned} \tau_w &= \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}, \quad q_w = -k \left. \frac{\partial T}{\partial y} \right|_{y=0}, \\ q_m &= -D_B \left. \frac{\partial C}{\partial y} \right|_{y=0}, \quad q_n = -D_m \left. \frac{\partial n}{\partial y} \right|_{y=0}. \end{aligned} \quad (17)$$

Using Eqs. (16) and (17), we get

$$\begin{aligned} Re_x^{1/2} C_f &= \left(\frac{x}{L}\right)^2 f''(0) + \left(\frac{x}{L}\right) s''(0), \\ Re_x^{-1/2} Nu &= -\theta'(0), \\ Re_x^{-1/2} Sh &= \frac{-\phi'(0)}{\phi(0)}, \quad Re_x^{-1/2} Nn = -w'(0). \end{aligned} \quad (18)$$

where  $L = U_\infty / a$  is a reference length.

### 3. Numerical solution

Governing system of equations are coupled nonlinear boundary value problems (BVP), we used Runge-Kutta Fourth Fifth (RKF-45) order method with shooting technique to solve these system of equations. Using similarity transformation boundary value problems converted into initial value problems, this procedure is used to convert higher order nonlinear equations into first order ordinary differential equations. The new converted IVP as follows

$$f = f_1, f' = f_2, f'' = f_3, f''' = f_3', \quad (19)$$

$$s = f_4, s' = f_5, s'' = f_6, s''' = f_6', \quad (20)$$

$$\theta = f_7, \theta' = f_8, \theta'' = f_8', \quad (21)$$

$$\phi = f_9, \phi' = f_{10}, \phi'' = f_{10}', \quad (22)$$

$$w = f_{11}, w' = f_{12}, w'' = f_{12}'. \quad (23)$$

Using Eqs. (19)-(23), Eqs. (9)-(14) reduced into first order ordinary differential equations given as

$$f_3' = f_2^2 - f_1 f_3 + M^2 \sin^2 \alpha f_2 + K f_2, \quad (24)$$

$$\begin{aligned} f_6' &= f_2 f_5 - f_1 f_6 - G_r (f_7 - N_r f_9 - R_b f_{11}) \\ &+ M^2 \sin^2 \alpha (f_5 - 1) + K (f_5 - 1) = 0, \end{aligned} \quad (25)$$

$$\begin{aligned} f_8' &= -Pr f_1 f_8 - N_b f_8 f_{10} - N_t f_8^2 \\ &- Pr Ec \left[ (f_3 + A f_6)^2 + \right. \\ &\left. M^2 \sin^2 \alpha \{f_2 + A(f_5 - 1)\}^2 \right] \end{aligned} \quad (26)$$

$$f_{10}' = -Lef_1 f_{10} - \frac{N_t}{N_b} f_8', \quad (27)$$

$$\begin{aligned} f_{12}' &= Pe \left[ f_{10} f_{12} + f_{10}' \{f_{11} + \Omega\} \right] \\ &- Sc f_1 f_{12}. \end{aligned} \quad (28)$$

where  $f_1, f_2, f_3, f_4$  and  $f_5$  are the initial guesses for  $f''(0), s''(0), \theta'(0), \phi'(0)$  and  $w'(0)$ , respectively. To

find out these values, we apply Secant method with shooting techniques.

The Runge-Kutta-Fehlberg (RKF-45) method is a well-accurate method. To obtain the solution of initial value problem for computational procedure we need to choose the fix finite value for  $\eta_\infty$ . In current study, we have elect  $\eta_\infty$  as  $\eta_8 (0 \leq \eta \leq 8)$ . The step size is taken  $\Delta\eta = 0.001$ , the procedure is continuing till we obtained the desired convergence exactness of  $10^{-6}$ .

Algorithm of this method is as follows

$$y_{m+1} = y_m + h \left( \begin{array}{l} \frac{25}{216}k_0 + \frac{1408}{2565}k_2 \\ + \frac{2197}{4104}k_3 - \frac{1}{5}k_4 \end{array} \right), \quad (30)$$

$$y_{m+1} = y_m + h \left( \begin{array}{l} \frac{16}{135}k_0 + \frac{6656}{12825}k_2 \\ + \frac{28561}{56430}k_3 - \frac{9}{50}k_4 \\ + \frac{2}{25}k_5 \end{array} \right), \quad (31)$$

Eqs. (30) and (31) are fourth and fifth order Runge-Kutta scheme respectively

$$\begin{aligned} k_0 &= f(x_m, y_m), \\ k_1 &= f\left(x_m + \frac{h}{4}, y_m + \frac{hk_0}{4}\right), \\ k_2 &= f\left(x_m + \frac{3}{8}h, y_m + \left(\frac{3}{32}k_0 + \frac{9}{32}k_1\right)h\right), \\ k_3 &= f\left(x_m + \frac{12}{13}h, y_m + \left(\begin{array}{l} \frac{1932}{2197}k_0 \\ - \frac{7200}{2197}k_1 \\ + \frac{7296}{2197}k_2 \end{array}\right)h\right), \\ k_4 &= f\left(x_m + h, y_m + \left(\begin{array}{l} \frac{439}{216}k_0 - 8k_1 \\ + \frac{3680}{513}k_2' \\ - \frac{845}{4104}k_3 \end{array}\right)h\right), \\ k_5 &= f\left(x_m + \frac{h}{2}, y_m + \left(\begin{array}{l} -\frac{8}{27}k_0 + 2k_1 \\ - \frac{3544}{2565}k_2 \\ + \frac{1859}{4104}k_3 - \frac{11}{40}k_4 \end{array}\right)h\right). \end{aligned}$$

Table 1 Comparison with existing results for Nusselt number and density number of motile microorganism when  $K = M2 = \Omega = Ec = 0$

Nu (Nusselt Number)		Nn (Density number of motile microorganism)			
Xu and Pop (2014)	Chakraborty <i>et al.</i> (2016)	Present Result	Xu and Pop (2014)	Chakraborty <i>et al.</i> (2016)	Present Result
0.346872	0.34689118	0.345985648	0.319434	0.31941987	0.316544658
0.574283	0.57428288	0.579865155	0.147219	0.14721928	0.146569879
1.159426	1.15942580	1.165689745	-0.311003	-0.31099156	-0.315656495
1.563334	1.56331503	1.589565463	-0.643148	-0.64301399	-0.656545568

#### 4. Result and discussion

In this segment, we feature the impacts of some significant relevant parameters on the velocity profile near the wall, velocity profile at far field, temperature profile, nanoparticles concentration profile and motile microorganism density profiles graphically and in tabular form.

Table 1 shows the correlation between existing outcomes and present outcome. It is seen that current result is very well in concurrence with existing outcomes of Xu and Pop (2014) and Chakraborty *et al.* (2016), this table additionally confirmed our method of solution.

Figs. (2)-(5) show the effects of velocity profile near the wall for different parameters. Fig. 2 shows the variation caused by the suction/injection parameter on velocity profile for porous and free fluid region. It is clearly seen that for injection  $f_w < 0$  gives a rise in the fluid velocity, this is due to inner effort of the fluid due to injection, more fluid enter in the region of the plate, this in arrival impacts the velocity of the fluid quite meaningfully, whereas for suction  $f_w > 0$  fluid velocity decreasing significantly. The outward effort of the fluid due to suction causes reduction in velocity of the fluid flow. Also it is observed that free fluid region velocity is greater than with porous medium in both situations. Fig. 3 depicts the effects of magnetic field parameter on velocity profile near the wall when suction or injection applied. We know that in magnetic field there exists a force called Lorentz force, which works against the fluid flow, this causes decrease in the fluid velocity. It is also noted that injection velocity is more than suction velocity. Fig. 4 shows the effects of porous parameter on velocity profile with or without magnetic field. It is observed that boundary layer thickness decreases as we increase porous parameter. Fig. 5 reveals effects of inclination angle on velocity profile, when we increase the angle magnetic field parameter affect much on velocity profile, this causes reduction in momentum boundary layer thickness.

Figs. 6-16 demonstrate the effects of various parameters on velocity at far filed. Fig. 6 represents the suction/injection on velocity profile at far field. This figure shows that velocity profile increases for suction and decreases for injection, with or without porous medium. Fig. 7 exhibits the influence of magnetic field parameter

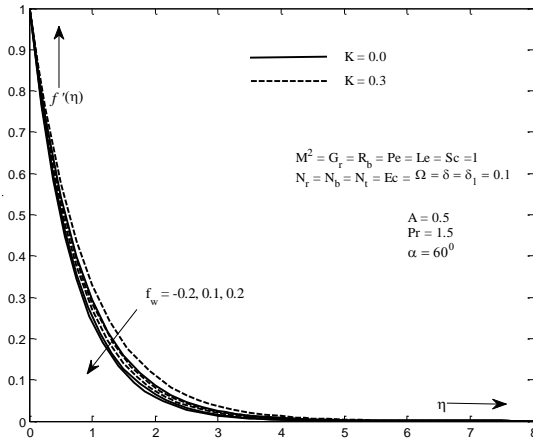


Fig. 2 Influence of  $f_w$  on velocity profile

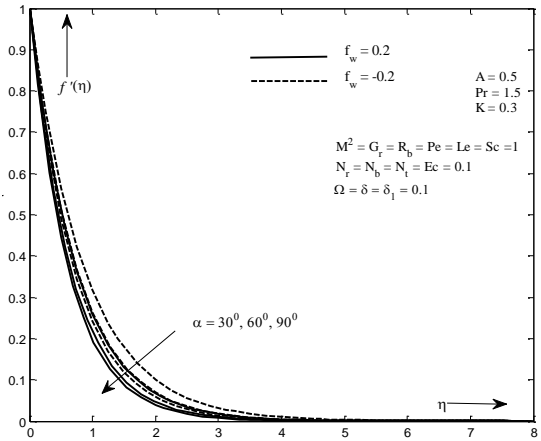


Fig. 5 Influence of  $\alpha$  on velocity profile

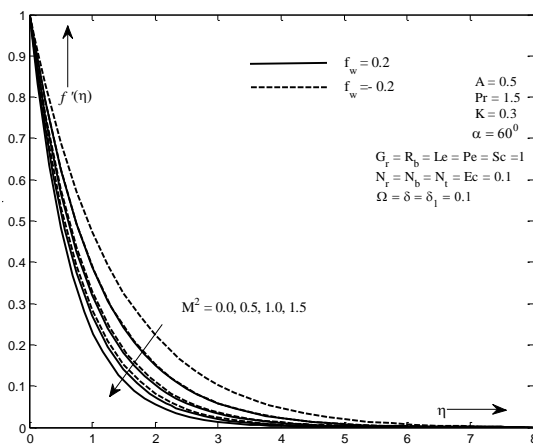


Fig. 3 Influence of  $M^2$  on velocity profile

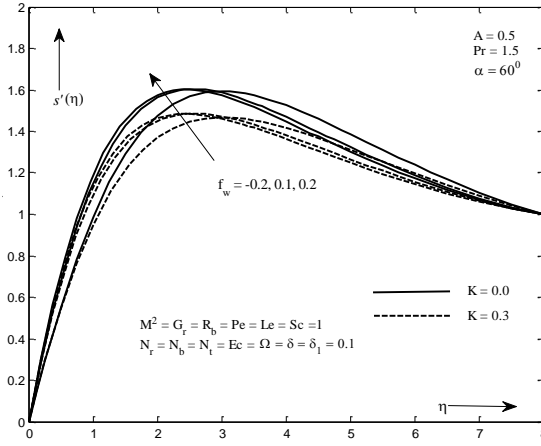


Fig. 6 Influence of  $f_w$  on velocity profile at far field

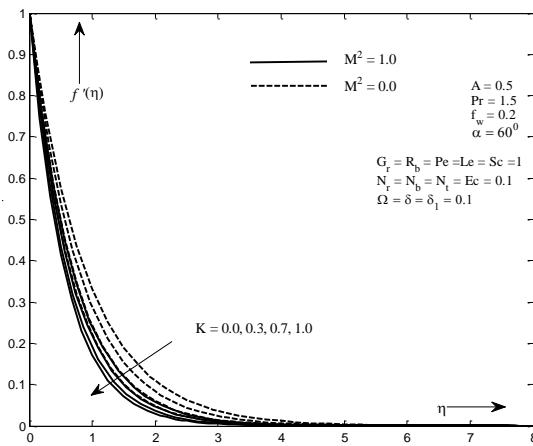


Fig. 4 Influence of  $K$  on velocity profile

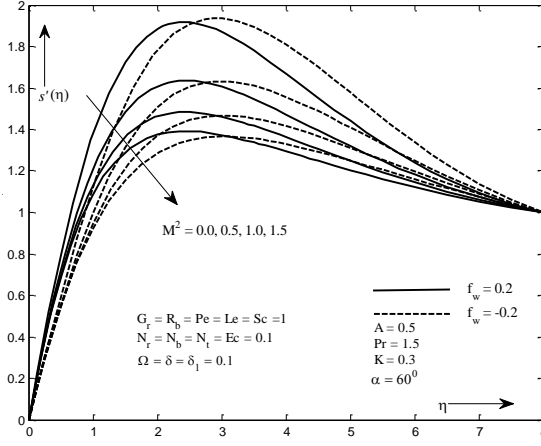


Fig. 7 Influence of  $M^2$  on velocity profile at far field

on velocity profile at far field when suction/injection applied. The velocity profile found decreasing function of magnetic field parameter. The reason behind it is that Lorentz force increases with magnetic field parameter, this force has a tendency to drag the fluid flow near the surface or wall. Similar effects shown for both suction and injection. Fig. 8 and 9 show the effects of porous parameter and inclined angel on velocity profile. For both parameter velocity profile is a decreasing function of  $K$  and  $\alpha$ .

Since the porosity of the plate enhances, then the permeability of the vertical porous plate works to draw back the velocity of the fluid. These results are arising for magnetic and non-magnetic field. Similar effects show for inclination angle.

Fig. 10 displays the influence of Brownian motion parameter and thermophoresis parameters at a far field on velocity profile. The increment of both parameters leads to enhance the fluid velocity at far field. As we know that in

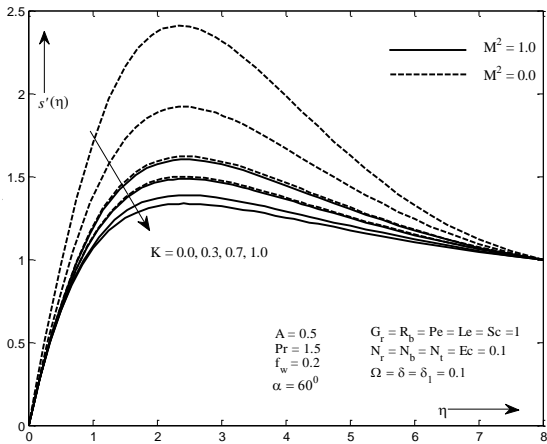


Fig. 8 Influence of  $K$  on velocity profile at far field

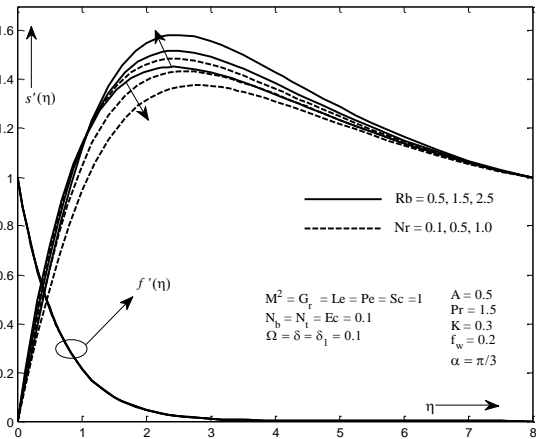


Fig. 11 Influence of  $R_b$  and  $N_r$  on velocity profile at far field

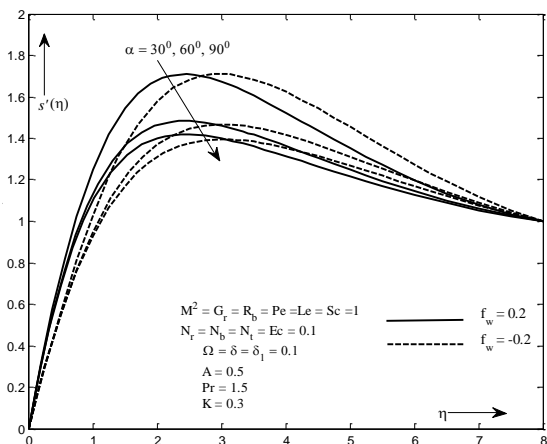


Fig. 9 Influence of  $\alpha$  on velocity profile at far field

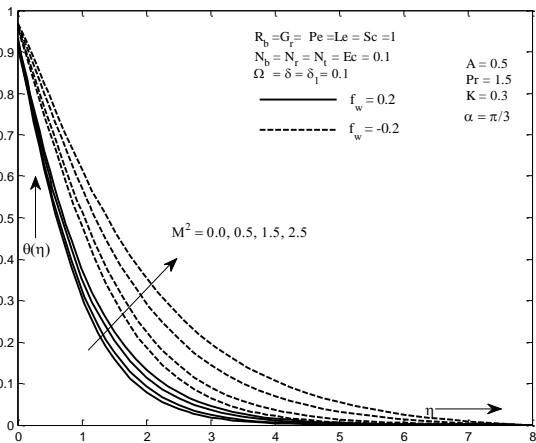


Fig. 12 Influence of  $M^2$  on temperature profile

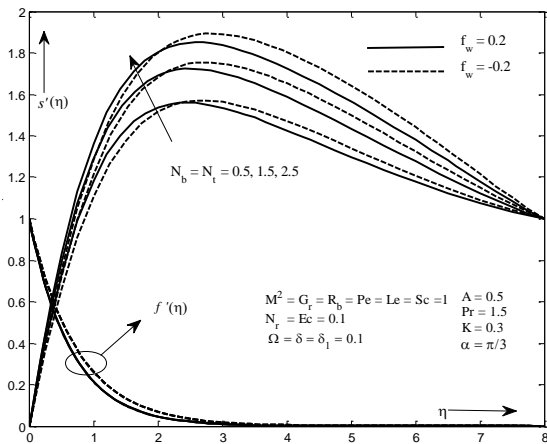


Fig. 10 Influence of  $N_b$  and  $N_t$  on velocity profile at far field

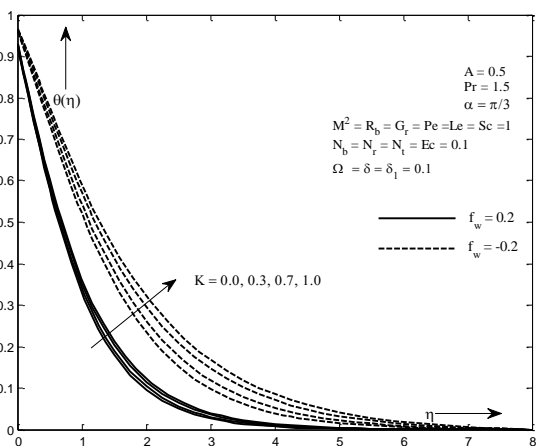


Fig. 13 Influence of  $K$  on temperature profile

nanofluid Brownian motion parameter and thermophoresis parameter works with the fluid velocity, hence for both conditions fluid velocity increases. It is noted that the effects of Brownian motion parameter and thermophoresis parameter is negligible for velocity profile near the wall. Fig. 11 presents the effects of bioconvection Rayleigh number and buoyancy-ratio parameter. Buoyancy-ratio parameter helps to decrease the fluid velocity at far field

and enhance the momentum boundary layer thickness for suction/injection, while Rayleigh number is the increasing function of velocity profile. These parameter does not affect the velocity profile near the filed.

Fig. 12 presents the magnetic field parameter effects on temperature profile. Magnetic field parameter enhances the Lorentz force, which works to amplify the friction drag between the fluid layers, this act reduces the rate of heat

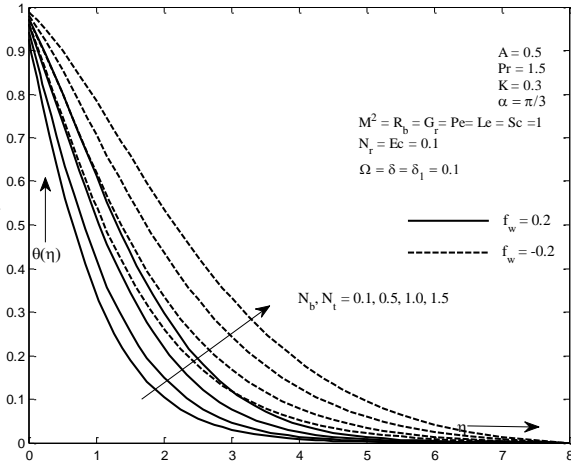


Fig. 14 Influence of  $N_b$  and  $N_t$  on temperature profile

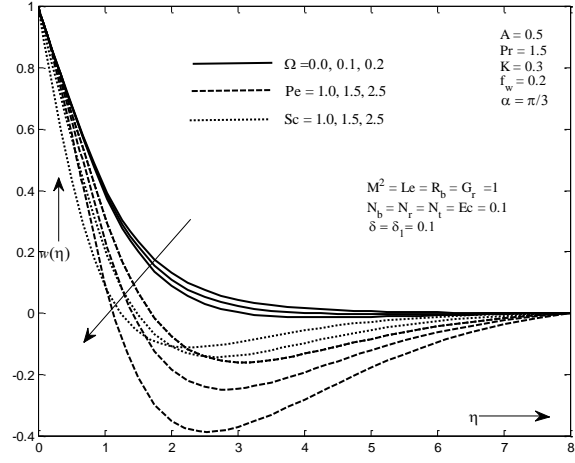


Fig. 16 Influence of  $\Omega$ ,  $Pe$  and  $Sc$  on microorganism conservation profile

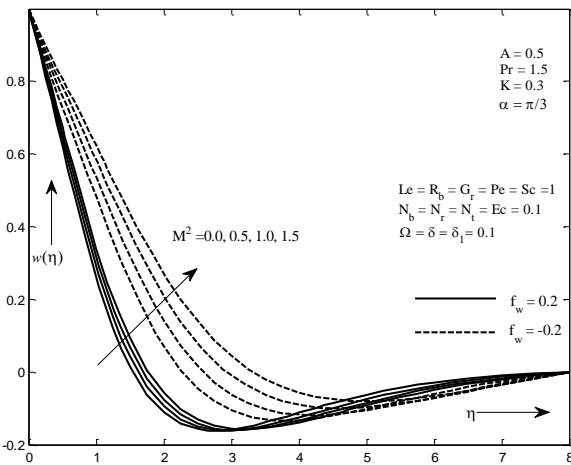


Fig. 15 Influence of  $M^2$  on microorganism conservation profile

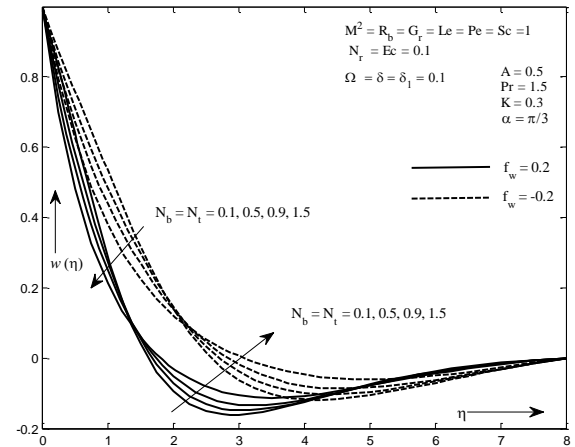


Fig. 17 Influence of  $N_b$  and  $N_t$  on microorganism conservation profile

transfer; as an outcome the thermal boundary layer thickness increases. It can be noticed that for injection, the fluid temperature higher than suction. This is the fact that due to injection, fluid is dragged out from the wall, hence minimum fluid on the wall results higher temperature.

Fig. 13 exhibits permeability parameter effects on thermal boundary layer in the presence of suction/injection. As increasing value of permeability parameter, thermal boundary layer thickness increases. This is because enhancement of permeability parameter gives rooms to the fluid near the wall, hence fluid temperature increases. These results arise in both condition, when suction or injection applied.

Fig. 14 describes the effects of Brownian motion parameter, thermophoresis parameter on temperature profile. Temperature profile shows enhancement with these parameters. Physically, Brownian motion parameter and thermophoresis parameter gives rise to the temperature of the fluid.

Figs. 15 and 16 present the effects of different parameter on microorganism conservation profile. Fig. 15 represents the density of motile microorganism profile for different values of magnetic field parameter. Lorentz force

works as an incentive for the gyrotactic microorganism. Due to Lorentz force the self-drive microorganism is involved and get a tendency to swim quickly in the boundary region, hence enhanced the microorganism profile. Fig. 16 reveals the effects of Peclet number, Schmidt number and microorganism concentration difference parameter on microorganism conservation profile. Peclet number is the proportion of the rate of advection to the rate of diffusion. So the increment in bioconvection peclet number, microorganism conservation profile shows reduction. Similar results have been obtained for Schmidt number and microorganism concentration difference parameter. Fig. 17 plotted to know the influence of Brownian motion parameter and thermophoresis parameter on microorganism conservation profile. Presence of nanoparticle in the base fluid are responsible for increase the thermal conductivity, this phenomenon helps to increase the microorganism conservation profile.

Table 2 describes the numerical values of skin friction coefficient, Nusselt number, Sherwood number and density number of motile microorganism when  $G_r=1$ ,  $N_r=0.1$ ,  $R_b=1$ ,  $Pr=1.5$ ,  $N_b=N_t=0.1$ ,  $A=0.5$ ,  $Sc=1$ ,  $\alpha=60^\circ$ ,  $\Omega=1$  and  $f_w=0.2$ .

Table 2 Numerical values of Skin friction coefficient, Nusselt number, Sherwood number and Density number of motile microorganism

$M^2$	$K$	$\delta$	$\delta_1$	$Pe$	$Ec$	$Cr$	$Nu$	$Sh$	$Nn$
1	0.3	0.1	0.1	1	0.1	-0.434546507	0.731618847	-0.048998864	0.671845780
2						-0.395378807	0.692166476	-0.033701361	0.618659407
	0					-0.450822464	0.747503893	-0.060570924	0.703298448
	0.2					-0.439955965	0.736736850	-0.052587548	0.681864166
		0.2				-0.451365563	0.680215543	-0.088928036	0.742121557
		0.3				-0.466056751	0.635446039	-0.123974104	0.803455353
			0.2			-0.435747962	0.731708716	-0.046333115	0.667025793
			0.3			-0.436818054	0.731788407	-0.043942760	0.662741232
				2		-0.326021562	0.733513023	-0.047082054	0.701859206
				3		-0.237178945	0.733648059	-0.046704367	0.739853842
					0.3	-0.419754816	0.658726347	-0.116012824	0.800268379
					0.4	-0.412390511	0.622727332	-0.149423829	0.863650429

## 5. Conclusions

The paper offers the effects of temperature jump and concentration slip on inclined MHD bioconvection past a vertical porous plate via porous media.

Following outcomes has been set up from this investigation

- Velocity profile is a decreasing function of suction/injection parameter, magnetic field parameter, inclination angle and porous medium parameter.

- Velocity profile at far field increases with suction/injection parameter, Brownian motion parameter, thermophoresis parameter, Eckert number, Rayleigh number and microorganism concentration difference parameter, however it is decreasing with magnetic field parameter, permeability parameter, inclination angle, buoyancy ratio parameter, Grashof number, thermal and concentration slip.

- Temperature profile decreases with thermal slip parameter, while it enhances with magnetic field parameter, permeability parameter, Brownian motion parameter, thermophoresis parameter and Eckert number.

- Concentration profile is the increasing function of magnetic field parameter, permeability parameter, Brownian motion parameter, thermophoresis parameter but on the other hand concentration profile reduces with concentration slip parameter and Lewis number.

- Motile microorganism density profile enhances with magnetic field parameter, concentration slip, Brownian motion parameter, thermophoresis parameter, Eckert number and permeability parameter but it decreases with Pecklet number, temperature slip, Lewis number, Schmidt number and microorganism concentration difference parameter.

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