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The effects of ring and fraction laws: Vibration of rotating isotropic cylindrical shell

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Abstract. This paper deals with the specific influence of three different fraction laws for vibrational analysis of rotating cylindrical shells. The rotating cylindrical shells are stabilized by ring-stiffeners to increase the stiffness and strength. Isotopic materials are the constituents of these rings. The frequencies are investigated versus circumferential wave number, length- and height-to- radius ratios using three volume fraction laws. Moreover, the effect of rotation speed is investigated. It is examined that the backward and forward frequencies increase and decrease on increasing the ratio of height- and length-to-radius ratio. When the position of ring supports increases, the backward and forward frequency first increases and obtains its maximum value at the shell mid length position and then decreases and get a bell shape with clamped-clamped and clamped-free conditions. The assessment of present model is judged with the comparison of non-rotating and rotating results with former exploration.

Keywords: clamped condition; fraction laws; ring; rotating

1. Introduction

Vibration investigation of static and rotating cylindrical shells is a significant discipline in theoretical and applied mechanics. These shells have wide applications in engineering science and technology. For example their uses are observed in civil, mechanical, electrical, nuclear engineering, aerodynamics and missile technology etc. Sewall and Naumann (1968) considered the vibration analysis of CSs based on analytical and experimental methods. The shells were strengthened with longitudinal stiffeners. The shell motion equations were used for rotating cylindrical shell by different researchers (Saito and Endo 1996, Sivadas and Ganesan 1994, Chen *et al*. 1993). Amabili *et al*. (1999) used Donnell's shallow-shell model with the quiescent, inviscid and incompressible fluid. Also, the dense fluid was studied for the influence of both the internal and external side of the shell. In the external side of the shell, the fluid was considered as an unbounded domain in the radial direction, while internally, the shell was considered as filled completely. Zohar and Aboudi (1973) studied vibrations of rotating cylindrical shells having finite length and matrix approach was used to derive the shell vibration. Lam and Loy (1994) investigated the vibrations of rotating composite and sandwich cylindrical shells. The vibration frequencies of composited rotating cylindrical shells were evaluated by applying different shell theories. Sharma *et al*. (2019) studied the functionally graded material using distribution law under hygrothermal effect. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Ergin and Temarel (2002) did a vibration study of cylindrical shells. The shells lied in a horizontal direction and contained fluid and submerged in it. Bryan (1890) is considered to be the primer research worker who examined studied vibrations of rotating cylindrical shells. The free vibrations of a rotating ring were related with those of these shells. Sharma *et al*. (1998) determined frequencies of composite cylindrical shells containing fluid. They estimated the axial modal deformations by trigonometric functions. Wang and Chen (1974) performed frequencies of rotating cylindrical shells based on energy variational approach.

Boutaleb *et al*. (2019) conducted a novel study for variation of aspect ratios with different parameters. A new refined 2D and 3D deformation theory for free vibration of FG plated with elastic foundations was used. Wang and Wu (2017) focused on performing a free vibration analysis of a functionally graded (FG) porous cylindrical shell subject to different sets of immovable boundary conditions. It is assumed that the modulus of elasticity of the porous

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composite is graded in the thickness direction. The open-cell metal foam provides a typical mechanical feature to determine the relationship between coefficients of density and porosity. A sinusoidal shear deformation theory (SSDT) in conjunction with the Rayleigh–Ritz method is employed to derive the governing equations associated with the free vibration of the circular cylindrical shell.

Penzes and Kraus (1972) applied generalized end conditions to analyze vibrations of rotating cylindrical shells. The analysis of rotating shells was confined to some special cases owing to need of approximate approach and calculation process. With powerful numerical methodologies, shell vibration analysis has completely revolutionized by advanced computers. Wang *et al*. (2020) investigated the nonlinear static behaviors of polymer-based nanocomposite circular microarches reinforced by a low content of graphene oxide (GO) and with inclusions of von Kármán nonlinearity. The weight fractions of the GO nanofillers are assumed to gradually vary along the thickness directions of the arches according to a given law. The GO nanofillers are considered to be circular reinforcements dispersed within a polymer matrix. Khiloun *et al*. (2019) presented the analytical modeling for vibration of composite plates with quasi 3D HSD. The governing equation was derived with the help of Hamilton's principle. Padovan (1975) did analysis of pre-stress influence on buckling and vibration aspects of rotating cylindrical shells. Fox and Hardie (1985) examined vibrations of rotating cylindrical shells. They used shell theory due to Flugge for shell motion equations. Aliyu and Li (2020) explored the Hirota–Satsuma–Ito (HSI) equation in (2+ 1)-dimensions which possess lump solutions. We first used the concept of Bell's polynomials to derive the bilinear form of the equation. Then, we proceed to derive a quadratic function solution of the bilinear form and then expand it as the sums of squares of linear functions satisfying some conditions.

Li and Lam (1998) studied influence of edge conditions vibration frequencies and modes of rotating composite CSs. Kaddari *et al*. (2020) investigated the free vibration of FG plates using quasi-3D hyperbolic shear theory. The rule of mixture with additional term of thickness variable was used. Di Taranto and Lessen (1964) investigated the vibrations of thin isotropic and infinite long rotating cylindrical shells. Najafizadeh and Isvandzibaei (2007) applied ring supports to CSs for vibration analysis of along the tangential direction and founded their research on angular deformation theory of higher order. The angular deformation was used for shell equations and determined the effects of constituent volume fractions and shell configurations on the shell vibrations. Wang *et al*. (2021) developed an accurate modified couple stress-based cylindrical micro-panel model in the framework of sinusoidal shear deformation theory. The classical strain tensors and curvature tensor components in a modified couple stress-based model are given with consideration of the geometric curve of cylindrical panels. In conjunction with a general Lagrange procedure, the proposed model is implemented to derive a system of Mathieu-Hill equations of governing dynamic stability behaviors of the cylindrical micro-panels. The Navier-type solution and Bolotin's method are applied to predict the principle unstable regions of the micro-panels with simply-supported boundary condition.

Wang *et al*. (2021) investigated the nonlinear functionally graded curved beam model including the von Kármán geometric nonlinearity on the basis of the third-order shear deformation theory. Due to incorporating the trapezoidal shape factor in the proposed model, the errors caused by geometric curvatures are eliminated. The governing equations of motions related to the dynamics of curved beams are derived by Lagrange method.

Sharma (1974) analyzed the vibration frequencies circular cylinder with using the Rayleigh - Ritz formulation and made comparisons of his results with some experimental ones. Srinivasan and Lauterbach (1971) conducted the research on isotropic long rotating cylindrical shells including influence of coriolis actions on their travelling modes. Moreover, Boulefrakh *et al*. (2019) employed the bending behavior of FG plates using shear deformation model based on visco-Pasternak foundations. The stretching effect with considering the four unknowns was clarified. Chung *et al*. (1981) studied the frequency response of fluid-filled CSs and presented an analysis of experimental and analytical investigation. Recently some researcher used different methods for nonlinear modeling (Eltaher *et al*. 2019, Ebrahimi *et al*. 2019, Safaei *et al*. 2019, Shahsavari *et al*. 2019, Benmansour *et al*. 2019). Recently some researcher used different methods for nonlinear modeling (Eltaher *et al*. 2019, Ebrahimi *et al*. 2019, Safaei *et al*. 2019, Shahsavari *et al*. 2019, Benmansour *et al*. 2019).

In this study, Galerkin's method is used for isotropic cylindrical shells vibration based on clamped-clamped and clamped free edge condition. The tube material is taken as isotropic material and their material quantity is located by the polynomial, exponential and trigonometric volume fraction law. The effects of various parameters for length-to-radius ratio and height-to-radius ratio and rotating speed for isotropic cylindrical shell have been fully inspected. For motion of a static cylindrical shell, a stationary wave is generated due to vibration. Moreover, on increasing the rotating speed, the backward frequencies increases and forward frequencies decreases. The suggested method to investigate the solution of fundamental eigen relations of rotating cylindrical shell, which is a well-known and efficient technique to develop the fundamental frequency equations.

2. Fraction laws

Vibrations of rotating isotropic cylindrical shells are inspected for three volume fraction laws (polynomial, exponential and trigonometric). These laws control the thickness in the shell radius direction. where stands for the power law exponent and its value is always between zero and infinity.

Polynomial Law (Vf - Polynomial)

$$V_f = \left(\frac{z}{h} + 0.5\right)^{\Lambda} \tag{1}$$

Exponential Law (Vf - Exponential)

$$V_f = 1 - e^{-\left(\frac{2}{h}+0.5\right)^{\Lambda}} \quad (2)$$

Trigonometric Law (Vf - Trigonometric)

$$V_{f1} = \sin^2\left[\left(\frac{2}{h}+0.5\right)^{\Lambda}\right] \quad (3)$$

$$V_{f2} = \cos^2\left[\left(\frac{2}{h}+0.5\right)^{\Lambda}\right] \quad (4)$$

3. Theoretical formulations

The geometrical parameters L, h, R denotes as length, thickness and radius for CSs with its coordinate system (x, θ, z) as shown in Fig. 1. The equation of rotating shell (Xiang *et al*. 2012) is given in the form of differential operator from Sander shell theory as

$$\begin{aligned} & R^2 A_{11}\frac{\partial^2 u}{\partial x^2} + \left(A_{66} - \frac{B_{66}}{R} + \frac{D_{66}}{4R^2}\right)\frac{\partial^2 u}{\partial \theta^2} \\ & + R\left(A_{12} + A_{66} + \frac{B_{12}+B_{66}}{R} - \frac{3D_{66}}{4R^2}\right)\frac{\partial^2 v}{\partial x \partial \theta} \\ & + RA_{12}\frac{\partial w}{\partial x} - \left(B_{12} + 2B_{66} - \frac{1}{R}D_{66}\right)\frac{\partial^3 w}{\partial x \partial \theta^2} \\ & - R^2 B_{11}\frac{\partial^2 w}{\partial x^3} = R^2 \rho_t \frac{\partial^2 v}{\partial t^2} \end{aligned} \quad (5a)$$

$$\begin{aligned} & R\left(A_{12} + A_{66} + \frac{B_{12} + 2B_{66}}{R} - 3\frac{D_{66}}{R}\right)\frac{\partial^3 w}{\partial x \partial \theta^2} \\ & + \left(A_{22} + \frac{2B_{22}}{R} + \frac{D_{22}}{R^2}\right)\frac{\partial^2 v}{\partial \theta^2} + R^2\left(A_{66} + 9\frac{D_{66}}{4R^2}\right)\frac{\partial^2 v}{\partial x^2} \\ & + \left(A_{22} + \frac{B_{22}}{R}\right)\frac{\partial w}{\partial \theta} - \left(\frac{B_{22}}{R} + \frac{D_{22}}{R}\right)\frac{\partial^3 w}{\partial v^3} \\ & - \left(B_{12} + 2B_{66}\frac{D_{12} + 3D_{66}}{R}\right)\frac{\partial^3 w}{\partial x^2 \partial \theta} \\ & = R^2 \rho_t\left(\frac{\partial^2 v}{\partial t^2} + \aleph\frac{\partial w}{\partial t} - \aleph^2 v\right) \end{aligned} \quad (5b)$$

$$\begin{aligned} & -RA_{12}\frac{\partial u}{\partial x} + \left(B_{12} + 2B_{66} - \frac{D_{66}}{R}\right)\frac{\partial^3 u}{\partial x \partial \theta^2} + R^2 B_{11}\frac{\partial^3 u}{\partial x^3} \\ & - \left(A_{22} + \frac{B_{22}}{R}\right)\frac{\partial v}{\partial \theta} + \left(\frac{B_{22}}{R} + \frac{D_{22}}{R}\right)\frac{\partial^3 v}{\partial \theta^3} \\ & + R\left(B_{12} + 2B_{66} + \frac{D_{12} + 3D_{66}}{R}\right)\frac{\partial^3 v}{\partial x^2 \partial \theta} - A_{22} w \\ & + \left(\frac{2B_{22}}{R}\frac{\partial^2 w}{\partial \theta^2} - \frac{D_{22}}{R^2}\frac{\partial^4 w}{\partial \theta^4} + 2RB_{12}\frac{\partial^2 w}{\partial x^2}\right) \\ & = R^2 \rho_t\left(\frac{\partial^2 w}{\partial t^2} - \aleph\frac{\partial v}{\partial t} - \aleph^2 w\right) \end{aligned} \quad (5c)$$

For a linear thin walled shell theory the strain mechanism are linear function of the shell thickness variable

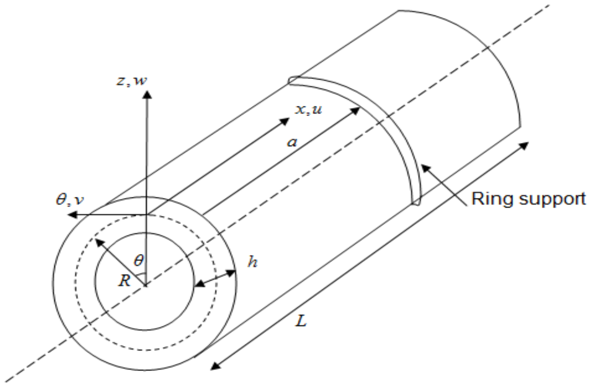

Fig. 1 Geometrical sketch of rotating CS

zand the strain vector [e] is written as

$$e_x = e_{11} + z\kappa_{11} \quad (6a)$$

$$e_\theta = e_{22} + z\kappa_{22} \quad (6b)$$

$$e_{x\theta} = e_{12} + zk_{12} \quad (6c)$$

where as k_{11}, k_{22} and k_{12} exhibit for the surface curvatures while e_{11}, e_{22} and e_{12} are the reference surface strains. For these surface strains and curvatures, the relationships with the deformation displacement functions curvatures from Sander's shell theory (1963) are presented as

$$[e_{11} \quad e_{22} \quad e_{12}] = \left[\frac{\partial u}{\partial x} \quad \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) \quad \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right] \quad (7a)$$

$$\begin{aligned} & [k_{11} \quad k_{22} \quad k_{12}] \\ & = \left[-\frac{\partial^2 w}{\partial x^2} \quad -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) \quad -\frac{1}{R}\left(\frac{\partial^2 w}{\partial x \, \partial \theta} - \frac{\partial v}{\partial x}\right)\right] \end{aligned} \quad (7b)$$

Here, ρ_t denotes density and expressed as

$$\rho_t = \int_{\frac{-h}{2}}^{\frac{h}{2}} \rho \, \mathrm{d}z \quad (8)$$

The modal displacement fields of shell can be written as

$$\begin{aligned} u &= \dot{\hbar}_m \psi(x)\sin(n\theta)\sin(\varepsilon t) \\ v &= \ddot{\hbar}_m \psi(x)\cos(n\theta)\sin(\varepsilon t) \\ w &= \dddot{\hbar}_m \psi(x)\prod_{i=1}^{\Diamond}(x - c_i)^{\alpha_i}\sin(n\theta)s \end{aligned} \quad (9)$$

The coefficients $\dot{\hbar}_m, \ddot{\hbar}_m, \dddot{\hbar}_m$ represent the vibration amplitudes in the x, θ and z directions correspondingly. The angular frequency and circumferential wave number are represented by ϵ and n respectively. $x = c_i$ designated for the position of i^{th} ring support along the longitudinal direction. The term $\Diamond$, α_i describes the number and existence of ring supports. When there is no ring on the shell the condition is $\alpha_i = 0$ and for ring supports is $\alpha_i = 1$.

Substituting the expressions for *u*, *v*, *w* and their respective partial derivatives in the equations into the Eq. (1) and We attained an equation after integrating *x* from 0 to *L*:

For the eigenvalue form, the arrangement of terms is framed as

$$\{\kappa_1\varepsilon^2 + \kappa_2\varepsilon + \kappa_3\}[x] = 0 \tag{10}$$

Here κ_1, κ_2, κ_3 denote square matrices of order three. Entries of the matrix κ_3 implicate the shell parameters.

Solutions of the above problem are evaluated by utilizing MATLAB software. This is a very expedient and powerful package to manage numerical problems.

4. Results and duscussions

Some numerical results are evaluated for isotropic stationary, nonrotating and rotating cylindrical shells for comparing with existing results found in the literature. Table 1 demonstrates values of frequency comparison of simply supported cylindrical shell. A comparison of the frequency parameters is made with those results observed by Loy *et al*. (1999) and Naeem and Sharma (2000) in Tables 1, 2 and 3. Table 4 shows a judgment with those frequencies obtained by Zhang *et al*. (2006). The shell has been supported on ends with simply supported end conditions. Table 4 shows the comparison of rotating frequencies with Chen *et al*. (1993). It can be seen that there is minute difference between two studies. Two sets of results have been got by wave propagation technique. There is minute difference between them. This difference may be due to the software package to get the results. An excellent consistency between the natural frequencies has been noticed.

Application of this programming provides both eigenvalues and eigenvector very fast. The solution of the shell frequency equation consists of six roots. From these six roots, the minimum absolute real values are selected. When a shell sets into rotation, two eigen-solution occurs, i.e., one is positive (backward, $\aleph$ > 0) and other is negative (forward, $\aleph$ < 0). These two frequencies are similar when the shell vibrates but static. From numerical values of the frequencies, the forward frequencies are lower than that of backward wave. Table 6 shows the rotating frequencies (backward & forward) versus *n* (wave number) with ring support *c* = 0.2 L for different rotating speeds $\aleph$ = 0, 1, 5, 10. The frequencies for backward and forward waves increase indefinitely as *n* and also on increasing the $\aleph$. It can be seen that the frequencies of τ_b is greater than that of τ_f and when $\aleph = 0$ the cylinder is stationary, in this case the backward and forward frequencies are same, because there is no rotation in the cylinder. When cylinder rotates at the speed $\aleph$ = 0, 1, 5, 10, the backward and forward frequencies enhances and so for as the forward frequencies are less than backward frequencies. Table 7 shows the frequency value w.r.t *L/R* with rotating speed $\aleph$ = 1.5 and wring support *c* = 0.2L. The frequencies increase

Table 1 A non-rotating comparison study between analytical method and present simulation (Loy *et al*. 1999)

n	Loy *et al*. (1999)	Present
1	2.106	2.107
2	1.344	1.345
3	0.9588	0.9590
4	0.7494	0.7495
5	0.6422	0.6424
6	0.6137	0.6138

Table 2 Comparison of frequency parameter for a clamped – clamped cylindrical shell (m = 1, L/R = 20, h/R = 0.01, E = 68.95x10^9 N/m^2, v = 0.3, p = 2.7145x10^3 Kg/m^3)

n	Loy *et al*. (1999)	Present
1	0.032885	0.032889
2	0.013932	0.013935
3	0.022672	0.022674
4	0.042208	0.042210
5	0.068046	0.068049
6	0.099748	0.099750
7	0.137249	0.137251
8	0.180535	0.180538
9	0.229599	0.229599
10	0.2844391	0.284439

Table 3 A non-rotating comparison study between analytical method and present simulation (Naeem and Sharma, 2000)

n	Naeem and Sharma (2000)	Present
1	0.140641	0.140643
2	0.054323	0.054321
3	0.027074	0.027075
4	0.017776	0.017768
5	0.017088	0.017086
6	0.021303	0.021302
7	0.028089	0.028090
8	0.036469	0.036470
9	0.046174	0.046172
10	0.057088	0.057084

Table 4 A non-rotating comparison study between analytical method and present simulation (Zhang *et al*. 2006)

n	Zhang *et al*. (2006)	Present
1	0.016102	0.016103
2	0.039271	0.039272
3	0.109811	0.109813
4	0.210277	0.210278
5	0.339877	0.339879

Table 5. A rotating comparison study between analytical method and present simulation (Chen *et al*. 1993)

ℵ (rps)	n	Chen *et al*.(1993) τ_b	Chen *et al*.(1993) τ_f	Present τ_b	Present τ_f
0.05	2	0.00155697	0.0015415	0.00155697	0.00154153
0.05	3	0.00438753	0.0043760	0.00438764	0.00437601
0.1	3	0.0043942	0.00437024	0.00439361	0.00437047
0.1	4	0.0084110	0.00839262	0.00841112	0.00839289

Table 6 Natural frequencies (Hz) of different speed of rotating cylinder for C-C model

n	ℵ=0 τ_b	ℵ=0 τ_f	ℵ=1 τ_b	ℵ=1 τ_f	ℵ=5 τ_b	ℵ=5 τ_f	ℵ=10 τ_b	ℵ=10 τ_f
1	12.643	12.643	12.800	12.486	13.429	11.855	14.214	11.066
2	7.384	7.384	7.514	7.260	8.107	6.833	8.997	6.451
3	17.414	17.414	17.514	17.323	18.010	17.055	18.840	16.929
4	33.163	33.163	33.243	33.093	33.665	32.915	34.418	32.919
5	53.578	53.578	53.644	53.522	54.015	53.402	54.713	53.488
6	78.569	78.569	78.626	78.522	78.961	78.444	79.618	78.585
7	108.119	108.119	108.169	108.080	108.477	108.031	109.104	108.212
8	142.223	142.223	142.268	142.189	142.555	142.163	143.158	142.374
9	180.879	180.879	180.920	180.850	181.190	180.840	181.774	181.074
10	224.084	224.084	224.122	224.059	224.379	224.063	224.948	224.316

Table 7 Natural frequencies (Hz) of rotating cylinder against *L/R* for C-C and C-F model (m = 1, h = 0.02, n = 1, ℵ = 1.5 rps, c = 0.2 L)

L/R	C-C τ_b	C-C τ_f	C-F τ_b	C-F τ_f
1	694.7134	694.6403	694.7190	694.6459
2	469.3511	469.2006	469.3432	469.1928
3	305.3423	305.1883	305.3347	305.1807
4	210.5238	210.3717	210.5183	210.3663
5	152.7815	152.6300	152.7776	152.6262

on decreasing the ratio of length-to-radius. The τ_b and τ_f frequencies of shell for C-F is lower than C-C.

Figs. 2 and 3 show the frequency response of isotropic cylindrical shells versus *h/R* with parameter A = 0.7, and *L/R* = 0.003, c = 0.2 L. Th frequencies of with different laws are calculated for C-C and C-F conditions. It is found from these figures that the frequency outcomes of C-C frequencies are higher than those of C-F for varying *h/R*. The fundamental natural frequencies seem parallel and increasing for all calculated values of ratio *h/R* = 0.001~ 0.010. The sketched τ_b frequency curves at *h/R* = 0.001 for polynomial law is higher than τ_f frequency curves. The same pattern is observed for all values of *h/R* and other two laws. The trigonometric τ_b and τ_f frequency curves are inferior than polynomial and exponential. The symmetric of frequency for BCs as C-C > C-F for all laws and the

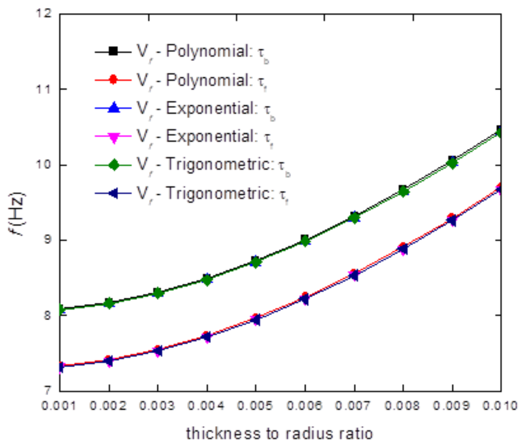

Fig. 2 Frequency distribution of C-C volume fractions through thickness-radius ratio (n = 3, m = 1, h = 0.003, R = 1 m, ℵ = 1.5 rps, c = 0.2 L)

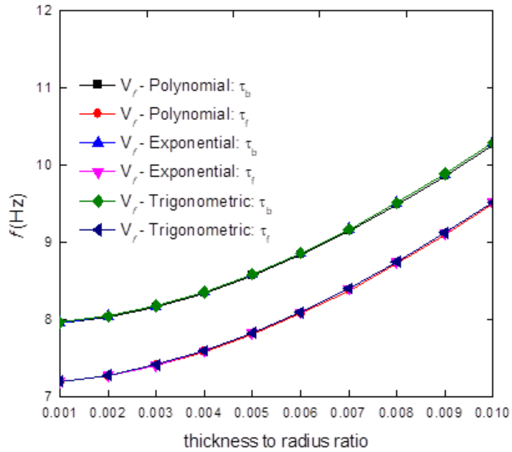

Fig. 3 Frequency distribution of C-F through thickness-radius ratio (n = 3, m = 1, h = 0.003, R = 1 m, ℵ =1.5 rps, c = 0.2 L)

frequency continuation can be elaborated mathematically as V_f-Polynomial > V_f-exponential > V_f-trigonometric.

Table 8 indicates that the rotating speed ℵ frequency values with ring support c = 0.2 L. It is observed that the frequencies are C-C highly visible and are higher than those for C-F ones. As ℵ grows, frequencies for cylindrical shells boost indefinitely. When shell starts rotation from ℵ = 0, the τ_b and τ_f frequencies constant for both C-C and C-F end conditions. The τ_b frequencies of shell increases and τ_f frequencies decrease on increasing the rotation speed.

Figs. 4-6 show the frequency variations versus ring support for Categories-I and-II cylindrical shells. These variations of frequencies are drawn with three volume fraction laws with clamped-clamped and clamped-free end conditions. As *c/L* is enhanced for these boundary conditions, the frequencies go up. At *c/L* (= 0.5) all the frequencies are higher and at *c/L* (= 0.6~0.9), the frequencies decrease. The frequencies are same at *c/L* = 0, 1 and rust itself a bell shape. In Fig. 4, due to polynomial law the frequencies of forward and backward formed as

Table 8 The C-C and C-F frequencies with angular speed for model (m = 1, n = 1, L/R = 2, h/R = 0.001, c = 0.2L)

$\aleph$	C-C		C-F	
	τ_b	τ_f	τ_b	τ_f
0	469.1597	469.1597	469.1593	469.1593
0.5	469.2350	469.0845	469.2346	469.0841
1	469.3101	469.0093	469.3098	469.0089
1.5	469.3854	468.9341	469.3850	468.9337
2	469.4606	468.8588	469.4602	468.8584
2.5	469.5358	468.7836	469.5354	468.7832
3	469.6110	468.7084	469.6106	468.7080
3.5	469.6862	468.6331	469.6858	468.6327
4	469.7614	468.5579	469.7610	468.5575
4.5	469.8366	468.4827	469.8362	468.4823
5	469.9118	468.4074	469.9114	468.4070

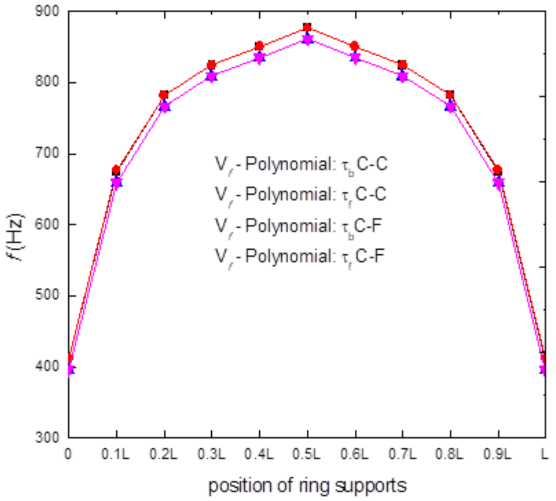

Fig. 4 Frequency distribution of polynomial volume fractions through C-C and C-F ring supports, c (n = 3, m = 1, L = 5 m, h = 0.003 m, R = 1 m, $\aleph$ = 1.5 rps)

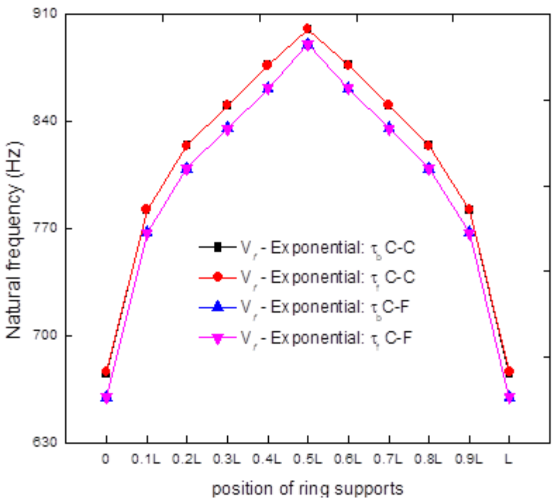

Fig. 5 Frequency distribution of exponential volume fractions through C-C and C-F ring supports, c (n = 3, m = 1, L = 5 m, h = 0.003 m, R = 1 m, $\aleph$ = 1.5 rps)

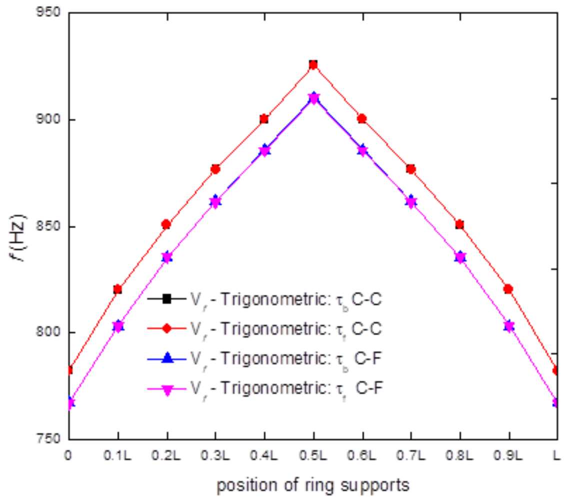

Fig. 6 Frequency distribution of trigonometric volume fractions through C-C and C-F ring supports, c (n = 3, m = 1, L = 5 m, h = 0.003 m, R = 1 m, $\aleph$ = 1.5 rps)

symmetrical which in Figs. 5-6, the frequencies forms as sharp corner, it is due to the exponential and trigonometric law. Because the mathematical formulation of these laws differs in comparison to each other. In these Figs., the frequencies of exponential law is composed between other two laws. The frequencies of C-C are greater than C-F frequencies for all three laws. These frequencies have a great impact on the vibration of isotropic CSs. It is inferred this frequency behavior with position of the ring supports has paramount influence on the vibrations of isotropic CSs.

5. Concluding remarks

In this present paper, the detailed vibrational analysis of FG porous nanoplate has been carried out using exact i.e., Navier method and approximate i.e., Finite element method. The non-polynomial based refined trigonometric higher-order shear deformation theory has been developed in conjunction with nonlocal theory to capture the small-scale effects. In the analysis, it has been found that the incorporation of the nonlocal parameter and porosity volume fraction has a significant effect on nanoplate.

The influence of various parameters like side to thickness ratio (a/h), aspect ratios, volume fraction index (P), Nonlocal parameter (μ) with existing and newly developed generalized porosity models have been observed and reported. Various observations from the above analysis are as follows.

- The reduction in fundamental frequency has been observed with the increase in side to thickness ratio and volume fraction index. Around 12% reduction in frequency values have been observed in thick plates and around 7% in thin plates as the plate changes from nonporous to porous plate.
- With the incorporation of small-scale effects, the percentage reduction in fundamental frequency is more pronounced. Around 30% reduction in frequency values has been observed as the nonlocal parameter incorporated.

• The behavior of all three porosity models has also been reported with varying nonlocal parameter for thick and thin nanoplate at certain volume fraction index.

• The newly developed generalized porosity model has been analyzed for varying nonlocal parameters, volume fraction index, etc. It has been found that the values of natural frequency increase for uneven porous behavior and decrease the values of nonlocal parameter, volume fraction index, and side to thickness ratio increases.

• Higher modes of vibration have also been reported in the present study. It has been shown that the natural frequency of the nanoplate is increased with the increases in modes of vibration. At higher modes of vibration, the percentage is a difference in porosity models is quite high in comparison at lower modes.

• Effects of varying non-local parameters, Volume fraction index and porosity volume fractions on frequency parameters have been analyzed using the finite element method with conventional and unconventional boundary conditions.

So, in the present paper, a detailed account of FG porous nanoplate has been presented. The results produced in this study will provide a holistic view to the Design engineer.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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