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Boundary conditions effect for buckling analysis of porous functionally graded nanobeam

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Abstract. This paper is concerned with the buckling behavior of 2D and quasi-3D problem of functionally graded nanobeam founded on high order shear deformation beams theory and made by two different types of porous distribution materials in Nano- and micro-scales. The used Quasi-3D formulation takes into account the transverse shear effect and uses only three variables. Both formulations do not include the correction factor that is required in the first shear deformation theory proposed by Timoshenko. Governing equations are derived using the principle of virtual work. Analytical resolutions for buckling of FG nanobeam are introduced under tow different boundary conditions, and the results obtained are compared to those proposed in literatures.

Keywords: buckling; functionally graded material; porous materials; nanobeam

1. Introduction

For the analysis of micro and nano-rods/tubes (such as nano-probes made from carbon nanotubes), it goes back to applying classical beam theories where one distinguishes the theory of Euler-Bernoulli or simple beam in 18th century then the Timoshenko theory beam that allows the effect of transverse shear deformation was formulated in the early 20th century.

Further, many numerical, analytical and 3D elasticity method are used to study static and dynamic behavior of functionally graded carbon nanotube reinforced composite structures such as (Murmu and Pradhan 2009, Formica *et al.* 2010, Janghorban and Zare 2011, Ke *et al.* 2012, Lei *et al.* 2013, Rokni *et al.* 2015, Mehar *et al.* 2016, 2017, 2018a, b, 2019, 2020, Mehar and Panda 2016, 2017, 2018, 2020, Mahapatra *et al.* 2017, Khiloun *et al.* 2020, Khadimallah *et al.* 2020, Rabhi *et al.* 2020, Al-Maliki *et al.* 2020, Al-Furjan *et al.* 2021).

Shown invalidity the classical theories of beams to determine the decrease in phase velocities of wave propagation in a carbon nanotube when the number of waves is large as the microstructure has a significant influence on wave dispersion.

Since 1972, the concept of non-local elasticity for the analysis of different bending, buckling and vibration stresses of microbeams/rods/tubes (Wang and Hu 2005, Peddieson *et al.* 2003, Zhang *et al.* 2005, Wang and Varadan 2006, Lu *et al.* 2006), has been applied through research following Eringen's pioneering work (Eringen 1972, 1983, Eringen and Edelen 1972), strain gradient theory (Lam *et al.* 2003) and the like, which could predict size effect by considering material length scale parameters.

On the other hand, the small-scale effect in mechanical properties was not taken up by these theories, those proved inadequate, being at the free scale. For example, Wang and Hu (2005) have shown invalidity the classical theories of beams to determine the decrease in phase velocities of wave propagation in a carbon nanotube when the number of waves is large as the microstructure has a significant influence on wave dispersion.

Since 1972, the concept of non-local elasticity for the analysis of different bending, buckling and vibration stresses of microbeams/rods/tubes (Wang and Hu 2005, Peddieson *et al.* 2003, Zhang *et al.* 2005, Wang and Varadan 2006, Lu *et al.* 2006), has been applied through research following Eringen's pioneering work (Eringen 1972, 1983, Eringen and Edelen 1972), strain gradient theory (Lam *et al.* 2003) and the like, which could predict size effect by considering material length scale parameters.

Many researchers have applied the nonlocal elasticity concept for bending, buckling and vibration analyses of nano-sized structures such as beams (Aranda-Ruiz *et al.* 2012, Aydogdu 2009, De Sciarra 2014, Ghannadpour *et al.* 2013, Loya *et al.* 2009, Lu 2007, Miandoab *et al.* 2014, Murmu and Pradhan 2009, Reddy and El-Borgi 2014, Wang *et al.* 2006, Yang *et al.* 2010, Semmah *et al.* 2019, Hussain *et al.* 2019, Matouk *et al.* 2020, Asghar *et al.* 2020, Taj *et al.* 2020, Gafour *et al.* 2020), rods (Murmu and Pradhan 2009, Khosravi *et al.* 2020) and plates (Ansari *et al.* 2016, Daneshmehr *et al.* 2015, Ke *et al.* 2015, Boutaleb *et al.* 2019, Balubaid *et al.* 2019, Bellal *et al.* 2020, Abdulrazzaq et al. 2020).

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The post-buckling of nano-beams by nonlocal elasticity has been a topical topic in recent years. Thongyothee and Chucheepsakul (2015) carried out a post-buckling analysis of the perfect nano-beam, which took into account the effects of non-local and surface stress based on classical beam theory. Li and Hu (2017) performed a post-buckling analysis of functionally graduated nano-beam, which took on non-local effects and deformation gradients. Mohammadi *et al.* (2014) analysed the post-buckling behaviour of homogeneous nano-beams with geometric imperfection based on a classical beam model. Nateghi *et al.* (2012) examined the buckling of FG micro beams using modified couple stress theory. Nejad *et al.* (2016) analyzed the buckling analysis of the nano-beams made of two-directional functionally graded materials based on nonlocal theory. Recently, Aria and Friswell (2019) examine the free vibration and buckling behavior of functionally graded (FG) nanobeams using nonlocal finite element model. Sahmani and Safaei (2020) study the influence of homogenization models on size-dependent nonlinear bending and post-buckling of bi-directional functionally graded micro/nano-beams. Bensaid *et al.* (2020) investigate the free vibration and buckling behaviors of size-dependent functionally graded sandwich nanobeams. Fang *et al.* (2020) used a new nonlocal model for free Vibration and thermal buckling analysis of rotating nonlocal functionally graded nanobeams in thermal environment.

Pores in porous materials can be filled with liquid or oils. Since porous materials are important in the field of materials engineering in the future, knowledge of the behavior of micro-beams made of porous materials, is very interesting on the constitution of the microstructure of innovative materials and their applications. Several researches are dedicated for theorical analysis of functionally graded and carbon nanotube reinforced composite structuresunder several distrubution of porosity such as those presented by (Jabbari *et al.* 2014, Chen *et al.* 2015, 2016, Akbaş 2017, Fouda *et al.* 2017, Batou *et al.* 2019, Bourada *et al.* 2019, Medani *et al.* 2019, Addou *et al.* 2019, Ramteke *et al.* 2019, 2020a, b and c, Kaddari *et al.* 2020, Thanh *et al.* 2020, Cuong-Le *et al.* 2020).

However, there are some research papers on analysis of mechanical behaviors of porous FG nanostructures based on modified power-law function. Berghouti *et al.* (2019) studied vibrational characteristics of pouros FG nanobeams using a higher ordre refined plate theory. Barati (2017) explored forced vibration behavior of FG nanobeams with porosities under dynamic loads and resting on an elastic foundation. More recently, many researches are established on the mechanical behavior of structures made of porous material such as (Mojahedin *et al.* 2016, Yang *et al.* 2016, Li *et al.* 2016, Zadpoor and Hedayati 2016, Avcar 2019, Fenjan *et al.* 2020, Zine *et al.* 2020, Ebrahimi *et al.* 2020).

This research is concerned with the analysis is aimed to study the size dependent bucklingcharacteristics of 2D and quasi-3D FG porous micro- and nano-beams according to the Timoshenko beam theory of different conditions of beam support. Two types of porosity dispersion are adopted. Eringen's elasticity theory and the modified couple stress theory are respectively served to study the nano and micro beams.

2. Nano-beam with different porosity distributions

Material composition is varying along z direction with the **FG** index $\boldsymbol{k}$. The mechanical properties of the Nano-beam such as Young's modulus "***E***", Poisson's ratio "***υ***", shear modulus "***G***" change as the material composition change. The porosity is considered to be of two different types: even (**type-I**) and uneven (**type-II**) distribution of pores. The porosity volume fraction, which defines the density of the pores, is $\boldsymbol{\beta}$ ($\boldsymbol{\beta} \ll \mathbf{1}$). The modified rule of mixture for even (**type-I**) and uneven (**type-II**) is proposed (Shafiei and Kazemi 2017, Wattanasakulpong and Chaikittiratana 2015) as:

For even porosity (**type-I**)

$$P(z) = P_2 + (P_1 - P_2)V - \frac{\beta}{2}(P_1 + P_2) \tag{1}$$

For uneven (**type-II**)

$$P(z) = P_2 + (P_1 - P_2)V - \frac{\beta}{2}(P_1 + P_2)\left(1 - \frac{2|z|}{h}\right) \tag{2}$$

Where **P** is the effective material property. $\mathbf{P_1}$ and $\mathbf{P_2}$ are the properties of the upper and lower faces of beam respectively.

3. Theoretical formulation

In this work, two different displacement fields are used based on the conventional HSDT given by

$$\begin{aligned} u(x,z) &= u_0(x) - z\frac{\partial w_0}{\partial x} + f(z)\phi_x(x) \\ w(x,z) &= w_0(x) \end{aligned} \tag{3}$$

w_0, ϕ_x, are the two unknowns displacement of the mid-plane of the beam. By considering that $\phi_x = \int \theta(x)dx$ for the first assumption, we will not take into account stretching effect, the displacement field will have (Allam *et al.* 2020, Menasria *et al.* 2020, Chikr *et al.* 2020, Refrafi *et al.* 2020, Rahmani *et al.* 2020, Tounsi *et al.* 2020)

$$\begin{aligned} u(x,z) &= -z\frac{\partial w_0(x)}{\partial x} + k_1 f(z)\int \theta(x)\,dx \\ w(x,z) &= w_0(x) \end{aligned} \tag{4}$$

Secondly, we take into account the stretching effect, the displacement field will have

$$\begin{aligned} u(x,z) &= -z\frac{\partial w_0(x)}{\partial x} + k_1 f(z)\int \theta(x)\,dx \\ w(x,z) &= w_0(x) + \phi_{st}(x) \end{aligned} \tag{5}$$

$$\phi_{st}(x) = g(z)\varphi(x) \tag{6}$$

Where $w_0(x,y)$, $\theta(x,y)$ and $\varphi(x)$ are the three unknowns displacement functions of middle surface of the beam. Note that the integrals do not have limits. In the present work is considered terms with integrals instead of terms with derivatives. The constants k_1 and k_2 depends on the geometry.

The shear strain shape function is given by

$$\begin{aligned} f(z) &= \frac{3\pi z}{25}\left[\pi - \sqrt[3]{0.135}cosh\left(\frac{\pi z}{h}\right)\right] \\ g(z) &= \frac{df(z)}{dz} \end{aligned} \tag{7}$$

The kinematic relations can be obtained as follows

$$\varepsilon_x = zk_x^b + f(z)k_x^s, \quad \{\gamma_{xz}\} = f'(z)\{\gamma_{xz}^0\} + c * g(z)\varphi_{,x} \tag{8}$$

Where

$$\begin{Bmatrix} k_x^b \\ k_x^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ k_1\theta \end{Bmatrix}, \quad \{\gamma_{xz}^0\} = \left\{k_1 \int \theta\, dx\right\} \tag{9}$$

Here: $c = 0$ for 2D formulation case and $c = 1$ for quasi-3D formulation.

The integral used in the above equations shall be resolved by a Navier type method and can be given as follows

$$\int \theta\, dx = A' \frac{\partial\theta}{\partial x} \tag{10}$$

Where the coefficient A' is expressed according to the type of solution used, in this case via Navier. Therefore, $A'$and k_1 are expressed as follows

$$A' = -\frac{1}{\lambda^2}, \quad k_1 = \lambda^2, \quad \lambda = m\pi/a \tag{11}$$

4. Constitutive relations

The linear constitutive relations of a FG nanobeam can be expressed as:

For the first assumption (without stretching effect) as

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} \tag{12}$$

The C_{ij} $(i,j = 1{,}5.)$ expressions in terms of engineering constants are given below

$$C_{11} = E(z), \quad C_{55} = \frac{E(z)}{2(1+\nu)}. \tag{13}$$

For the second assumption (with stretching effect) as

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} \tag{14}$$

The C_{ij} $(i,j = 1{,}5.)$ expressions in terms of engineering constants are given below

$$\begin{aligned} &C_{ii} = \frac{E(z)}{(1-\nu^2)}, \quad C_{ij} = \frac{E(z)}{(1-\nu^2)} \quad (i,j = 1{,}3), \\ &C_{jj} = \frac{E(z)}{2(1+\nu)} \quad (j = 5). \end{aligned} \tag{15}$$

The nonlocal constitutive relation for the macroscopic stress takes the following special relations Eringen (1972)

$$\begin{aligned} &\sigma_x + \mu\frac{d^2\sigma_x}{dx^2} = C_{11}\varepsilon_x, \qquad \sigma_z + \mu\frac{d^2\sigma_z}{dz^2} = C_{33}\varepsilon_z, \\ &\tau_{xz} + \mu\frac{d^2\tau_{xz}}{dx^2} = C_{55}\gamma_{xz} \end{aligned} \tag{16}$$

Where $\boldsymbol{E}$ and $\boldsymbol{G}$ are Young's modulus and shear modulus, respectively. Moreover, $\boldsymbol{e_0}$ is a material constant, and "$\boldsymbol{a}$" is the internal characteristic length. Once the nonlocal parameter $\mu = (e_0 a)^2$ is equal to zero, we obtain the constitutive relations of the local theories.

5. Stability equations

The principle of virtual work is here utilized to determine the equations of stability (Karami *et al.* 2019a, b, Bendenia *et al.* 2020, Al-Furjan *et al.* 2020, Bourada *et al.* 2020, Bousahla *et al.* 2020a, b).

$$\delta U + \delta V = 0 \tag{17}$$

The variation of strain energy of the Nano-beam is calculated by

$$\begin{aligned} \delta U = \int_A \int_0^L &[-M_x^b \delta w_{0,xx} + k_1 A' M_x^s \delta\theta_{,xx} \\ &+ k_1 Q_{xz}\delta\theta_{,x}]dAdx \end{aligned} \tag{18}$$

With stretching effect

$$\begin{aligned} \delta U = \int_A \int_0^L &[-M_x^b \delta w_{0,xx} + k_1 A' M_x^s \delta\theta_{,xx} + N_z\varphi \\ &+ Q_{xz}(k_1 A'\delta\theta_{,x} + \delta\varphi_{,x})]dAdx \end{aligned} \tag{19}$$

Where N, M, Q, are stress resultants and they are defined by

$$\begin{aligned} \begin{Bmatrix} M_x^b \\ M_x^s \end{Bmatrix} &= \int_A \sigma_x \begin{Bmatrix} z \\ f(z) \end{Bmatrix} dA \\ Q_{xz} &= \int_A \tau_{xz}\, f'(z)\, dA \\ N_z &= \int_A g'(z)\sigma_z\, dA \end{aligned} \tag{20}$$

The variation of potential energy of the applied loads can be expressed:

For the first assumption (without stretching effect) as

$$\delta V = -\int_0^L N_0 \frac{dw_0}{dx}\frac{d\delta w_0}{dx} dx \tag{21}$$

For the second assumption (with stretching effect) as

$$\delta V = -\int_0^L N_0 \frac{d(w_0 + g(z)\varphi)}{dx}\frac{d\delta(w_0 + g(z)\varphi)}{dx}dx \tag{22}$$

Substituting Eqs. (17) and (20) into (16), we got

$$\begin{aligned}\delta U + \delta V = \int_A\int_0^L [-M_x^b \delta w_{0,xx} + k_1 A' M_x^s \delta\theta_{,xx} \\ +k_1 Q_{xz}\delta\theta_{,x}]dAdx \\ -\int_0^L N_0 \frac{dw_0}{dx}\frac{d\delta w_0}{dx}dx = 0\end{aligned} \tag{23a}$$

With stretching effect

$$\begin{aligned}\delta U + \delta V = \int_A\int_0^L [-M_x^b \delta w_{0,xx} + k_1 A' M_x^s \delta\theta_{,xx} + N_z\varphi \\ +Q_{xz}(k_1 A'\delta\theta_{,x} + \delta\varphi_{,x})]dAdx \\ -\int_0^L N_0 \frac{d(w_0 + g(z)\varphi)}{dx}\frac{d\delta(w_0 + g(z)\varphi)}{dx}dx = 0\end{aligned} \tag{23b}$$

Integrating by parts, and collecting the coefficients of δw_0, $\delta\theta$ and $\delta\varphi$, and the following equations of stability are obtained:

Without stretching effect

$$\delta w_0: \quad \frac{\partial^2 M_x^b}{\partial x^2} - N_0\frac{\partial^2 w_0}{\partial x^2} = 0 \tag{24a}$$

$$\delta\theta: \quad \frac{\partial^2 M_x^s}{\partial x^2} - \frac{\partial Q_{xz}}{\partial x} = 0 \tag{24b}$$

With stretching effect

$$\delta w_0: \quad \frac{\partial^2 M_x^b}{\partial x^2} - N_0\left(\frac{\partial^2 w_0}{\partial x^2} + g(z)\frac{\partial^2\varphi}{\partial x^2}\right) = 0 \tag{25a}$$

$$\delta\theta: \quad \frac{\partial^2 M_x^s}{\partial x^2} - \frac{\partial\varphi}{\partial x} = 0 \tag{25b}$$

$$\delta\varphi: \; -N_z + \frac{\partial Q_{xz}}{\partial x} - N_0 g(z)\left(\frac{\partial^2 w_0}{\partial x^2} + g(z)\frac{\partial^2\varphi}{\partial x^2}\right) = 0 \tag{25c}$$

The stability equations can be expressed in terms of displacements (δw_0, $\delta\theta$, $\delta\phi$) as:

Without stretching effect

$$\begin{aligned}\delta w_0: \quad D\frac{\partial^4 w_0}{\partial x^4} + k_1 A^{'D^s\frac{\partial^4\theta}{\partial x^4}} \\ +N_0\left(\frac{\partial^2 w_0}{\partial x^2} - \mu\frac{\partial^4 w_0}{\partial x^4}\right) = 0\end{aligned} \tag{26a}$$

$$\delta\theta: \; -D^s\frac{\partial^4 w_0}{\partial x^4} + k_1 A' H^s\frac{\partial^4\theta}{\partial x^4} + k_1 A' A^s\frac{\partial^2\theta}{\partial x^2} = 0 \tag{26b}$$

Where

$$\{D \quad D^s \quad H^s\} = \int_{-h/2}^{h/2} C_{11}\,[z^2 \quad zf(z) \quad f^2(z)]dz \tag{27a}$$

$$A^s = \int_{-h/2}^{h/2} C_{55} f'^2(z)\,dz \tag{27b}$$

With stretching effect

$$\begin{aligned}\delta w_0: \; -D\frac{\partial^4 w_0}{\partial x^4} + k_1 A^{'D^s\frac{\partial^4\theta}{\partial x^4}} + L^a\frac{\partial^2\varphi}{\partial x^2} \\ -N_0\left(\frac{\partial^2 w_0}{\partial x^2} - \mu\frac{\partial^4 w_0}{\partial x^4}\right. \\ \left. + g(z)\left(\frac{\partial^2\varphi}{\partial x^2} - \mu\frac{\partial^4\varphi}{\partial x^4}\right)\right) \\ = 0\end{aligned} \tag{28a}$$

$$\begin{aligned}\delta\theta: \; -D^s\frac{\partial^4 w_0}{\partial x^4} + k_1 A' H^s\frac{\partial^4\theta}{\partial x^4} + R\frac{\partial^2\varphi}{\partial x^2} \\ -k_1 A' A^s\frac{\partial^2\theta}{\partial x^2} - A^s\frac{\partial^2\varphi}{\partial x^2} = 0\end{aligned} \tag{28b}$$

$$\begin{aligned}\delta\varphi: \; -R_a\varphi + L^a\frac{\partial^2 w_0}{\partial x^2} - k_1 A'R\frac{\partial^2\theta}{\partial x^2} + k_1 A^{'A^s\frac{\partial^2\theta}{\partial x^2}} \\ + A^s\frac{\partial^2\varphi}{\partial x^2} - N_0 g(z)\left(\begin{array}{c}(\frac{\partial^2 w_0}{\partial x^2} - \mu\frac{\partial^2 w_0}{\partial x^2}) \\ +g(z)\left(\frac{\partial^2\varphi}{\partial x^2} - \mu\frac{\partial^4\varphi}{\partial x^4}\right)\end{array}\right) \\ = 0\end{aligned} \tag{28c}$$

Where

$$\{D \quad D^s \quad H^s\} = \int_{-h/2}^{h/2} C_{11}\,[z^2 \quad zf(z) \quad f^2(z)]dz \tag{29a}$$

$$A^s = \int_{-h/2}^{h/2} C_{55} f'^2(z)\,dz \tag{29b}$$

$$\begin{aligned}\{L \quad L^a \quad R\} \\ = \int_{-h/2}^{h/2} C_{13}[g'(z) \quad zg'(z) \quad f(z)g'(z)]\,dz\end{aligned} \tag{29c}$$

$$R^a = \int_{-h/2}^{h/2} C_{33} g'^2(z)\,dz \tag{29d}$$

6. Exact solution for nano-beam

The exact solution of Eqs. (25) and (27) for the FG Nanobeam under various boundary conditions can be constructed. The boundary conditions for an arbitrary edge with simply supported and clamped edge conditions are assumed as (Bouhadra *et al.* 2015, Liu *et al.* 2018)

6.1 Clamped (CC)

$$\begin{aligned}w_0 = \theta = \varphi = \partial w_0/\partial x = \partial\theta/\partial x = \partial\varphi/\partial x = 0 \\ \text{at} \quad x = 0, l\end{aligned} \tag{30}$$

6.2 Simply supported (SS)

$$w_0 = \theta = \varphi = M_x^b = M_x^s = 0 \quad \text{at} \quad x = 0, l \tag{31}$$

The admissible function represented in Table 1 is used to satisfy the above boundary conditions in the case of our problem

Table 1 The admissible function $X_m(x)$

	Boundary conditions	X_m	
	Pour $x = 0, l$	$X_m(x)$	
S-S	$X_m(0) = X_m^{''}(0) = 0$ $X_m(a) = X_m^{''}(a) = 0$ $X_m(a) = X_m^{'}(a) = 0$	$sin(\lambda x)$	
		$X_m(x)$	$\chi_m(x)$
C-C	$X_m(0) = X_m^{'}(0) = 0$ $X_m(a) = X_m^{'}(a) = 0$ $X_m^{''}(a) = X_m^{'''}(a) = 0$	$sin^2(\lambda x)$	$sin(\lambda x) cos(\lambda x)$

$$\begin{Bmatrix} w_0 \\ \theta \\ \varphi \end{Bmatrix} = \begin{Bmatrix} W_m X_m \\ \psi_m X_m \\ \vartheta_m X_m \end{Bmatrix}, for\ SS$$
$$\begin{Bmatrix} w_0 \\ \theta \\ \varphi \end{Bmatrix} = \begin{Bmatrix} W_m X_m \\ \psi_m \chi_m \\ \vartheta_m X_m \end{Bmatrix}, for\ CC \tag{32}$$

Where W_m, ψ_m and ϑ_m are arbitrary parameters to be determined and λ is defined as

$$\lambda = m\pi/a \tag{33}$$

The analytical solutions can be obtained from

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{Bmatrix} W_m \\ \psi_m \\ \vartheta_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{34}$$

In which

$$\begin{aligned}
&a_{11} = D\left(\frac{\partial^4 X_m}{\partial x^4}\right) + N_0\left(\frac{\partial^2 X_m}{\partial x^2} - \mu\frac{\partial^4 X_m}{\partial x^4}\right),\\
&a_{22} = H^s\left(\frac{\partial^4 \chi_m}{\partial x^4}\right) - A^s\left(\frac{\partial^2 \chi_m}{\partial^2 x}\right),\\
&a_{12} = D^s\left(\frac{\partial^4 \chi_m}{\partial x^4}\right), \qquad a_{21} = D^s\left(\frac{\partial^4 X_m}{\partial x^4}\right),\\
&a_{13} = -La\left(\frac{\partial^2 \chi_m}{\partial^2 x}\right), \qquad a_{31} = -La\left(\frac{\partial^2 X_m}{\partial^2 x}\right)\\
&a_{23} = -(R - A^s)\left(\frac{\partial^2 \chi_m}{\partial^2 x}\right),\\
&a_{32} = -(R - A^s)\left(\frac{\partial^2 \chi_m}{\partial^2 x}\right),\\
&a_{33} = R\chi_m(x) - A^s\left(\frac{\partial^2 \chi_m}{\partial^2 x}\right).
\end{aligned} \tag{35}$$

Eq. (34) has a nontrivial solution only for discrete values of the critical buckling load. For a nontrivial solution, the determinant of the matrix $[a_{ij}]$ must equal zero.

7. Numerical results and discussion

In this research, analysis of critical buckling behavior of porous materials nanobeams performed based on a nonlocal shear deformation beam theory with various boundary conditions and various size scale parameters. In the porous materials nanobeam, two different types: even (**type-I**) and uneven (**type-II**) distribution of pores are used.

Table 2 Material properties used in the FG nanobeam

	Materials	$E(GPa)$	υ
Type 1	Ceramic	69	0.3
	Metal	339	0.3
Type 2	Ceramic (Al2O3)	349.55	0.24
	Metal (SUS304)	201.04	0.3262

Non-dimensional parameters are used herein

$$\bar{N} = N_{cr}\frac{L^2}{EI} \tag{36}$$

$$\alpha = \frac{e_0 a}{l} = \frac{\mu^{0.5}}{l} \tag{37}$$

In the Table 3, the results are presented for 2D and quasi-3D of non-dimensional critical buckling load parameters for Nano-beams with two type's boundary conditions and various size scale parameters. The values of **α** (0, 0.2 and 1) are selected in such a way that make possible comparison with other references (Ghannadpour *et al.* 2013, Pradhan and Phadikar 2009, Nejad *et al.* 2016). Also comparing the results the non-dimensional critical buckling load of S-S nanobeam with the results of Reddy (2007) and Shafiei and Kazemi (2017) in Table. 4. It can be seen that is an excellent agreement between the obtained

Table 3 Non-dimensional critical buckling load for nanobeam with various boundary conditions and various size scale parameters

α	theory	Boundary conditions	
		Simply supported S-S	Clamped C-C
0	Present ($\varepsilon_z \neq 0$)	9.8885	39.5303
	Present ($\varepsilon_z = 0$)	9.8671	39.4381
	Nejad *et al.* (2016)	9.8696	39.4784
	Ghannadpour *et al.* (2013)	9.8696	39.4784
	Pradhan and Phadikar (2009)	9.8696	39.4784
0.2	Present ($\varepsilon_z \neq 0$)	7.089	15.3270
	Present ($\varepsilon_z = 0$)	7.074	15.2911
	Nejad *et al.* (2016)	7.076	15.3068
	Ghannadpour *et al.* (2013)	7.076	15.3068
	Pradhan and Phadikar (2009)	7.076	15.3068
1	Present ($\varepsilon_z \neq 0$)	0.910	0.9766
	Present ($\varepsilon_z = 0$)	0.908	0.9743
	Nejad *et al.* (2016)	0.908	0.9753
	Ghannadpour *et al.* (2013)	0.908	0.9753
	Pradhan and Phadikar (2009)	0.908	0.9753

results in this paper and those reported in (Reddy 2007, Pradhan and Phadikar 2009, Ghannadpour *et al.* 2013, Nejad *et al.* 2016, Shafiei and Kazemi 2017). It is clear that the increasing of scale-parameter's result the decrease of critical buckling load for both boundary conditions (C-C and S-S), and this because the scale parameters decreasing the stiffness of the Nano-beam. The length of the beam gives more than the critical buckling load, so more than the scale parameter's increase the length of the beam decrease and difference between the critical buckling load of the both conditions (C-C and S-S) are convergent.

Table 5 shows the effects of the power law index "*k*" on non-dimensional critical buckling load parameters of nanobeam with various boundary conditions, various size scale parameters "α". The results are obtained for 2D and quasi-3D (with stretching effect), where critical buckling load is rapidly decreasing with the increase of the size scale parameters "α". In addition, it is increasing with the increase of the power law index "*k*" (ceramic to metal).

Table 6 displays the effects of the size scale parameters "α" on the non-dimensional critical buckling load of nanobeam with various boundary conditions for different values of aspect ratio "*l*/*h*". The results are obtained for 2D

Table 4 Comparison of the critical buckling load of simply supported (SS) nanobeam

$(e_0a/L)^2$	0	0.5	1	1.5	2	3	4	5
Present ($\varepsilon_z \neq 0$)	9.8885	9.4159	8.993	8.606	8.252	7.623	7.084	6.616
Present ($\varepsilon_z = 0$)	9.8671	9.4031	8.981	8.5948	8.2405	7.6130	7.0743	6.607
Shafiei and Kazemi (2017)	9.8696	9.4055	8.983	8.5969	8.2426	7.6149	7.0761	6.608
Reddy (2007)	9.8696	9.4055	8.983	8.5969	8.2426	7.6149	7.0761	6.608

Table 5 Non-dimensional critical buckling load parameters for nanobeam with various boundary conditions, various size scale parameters and various material index

α	theory	Boundary conditions							
		S-S				C-C			
		$k=0$	$k=2$	$k=10$	$k=\infty$	$k=0$	$k=2$	$k=10$	$k=\infty$
0	Present ($\varepsilon_z \neq 0$)	9.888	33.109	41.009	48.583	39.530	132.365	163.954	194.214
	Present ($\varepsilon_z = 0$)	9.867	33.034	40.918	48.477	39.438	132.046	163.564	193.760
0.2	Present ($\varepsilon_z \neq 0$)	7.089	23.737	29.402	34.832	15.327	51.321	63.569	75.302
	Present ($\varepsilon_z = 0$)	7.074	23.684	29.337	34.756	15.291	51.198	63.418	75.126
0.5	Present ($\varepsilon_z \neq 0$)	2.852	9.549	11.827	14.011	3.637	12.178	15.084	17.868
	Present ($\varepsilon_z = 0$)	2.846	9.527	11.801	13.981	3.628	12.148	15.048	17.826
0.7	Present ($\varepsilon_z \neq 0$)	1.694	5.673	7.027	8.324	1.943	6.506	8.059	9.546
	Present ($\varepsilon_z = 0$)	1.691	5.660	7.011	8.306	1.939	6.490	8.040	9.524
1	Present ($\varepsilon_z \neq 0$)	0.910	3.046	3.773	4.470	0.977	3.270	4.050	4.798
	Present ($\varepsilon_z = 0$)	0.908	3.039	3.764	4.460	0.974	3.262	4.041	4.787

Table 6 Non-dimensional critical buckling load parameters for nanobeam with various boundary conditions, various size scale parameters and various aspect ratio

l/*h*	Theory	Boundary conditions							
		S-S				C-C			
		$\alpha=0$	$\alpha=0.2$	$\alpha=0.5$	$\alpha=1$	$\alpha=0$	$\alpha=0.2$	$\alpha=0.5$	$\alpha=1$
5	Present ($\varepsilon_z \neq 0$)	9,146	6.557	2.638	0.841	29,480	11.430	2.712	0.728
	Present ($\varepsilon_z = 0$)	8,953	6.419	2.582	0.824	28,035	10.870	2.579	0.692
10	Present ($\varepsilon_z \neq 0$)	9,696	6.951	2.796	0.892	36,583	14.184	3.366	0.904
	Present ($\varepsilon_z = 0$)	9,623	6.899	2.775	0.885	35,814	13.886	3.295	0.885
20	Present ($\varepsilon_z \neq 0$)	9,841	7.056	2.838	0.905	38,782	15.037	3.568	0.958
	Present ($\varepsilon_z = 0$)	9,807	7.031	2.828	0.902	38,493	14.925	3.541	0.951
50	Present ($\varepsilon_z \neq 0$)	9,883	7.085	2.850	0.909	39,435	15.290	3.628	0.974
	Present ($\varepsilon_z = 0$)	9,860	7.069	2.843	0.907	39,317	15.244	3.617	0.971

Table 7 Non-dimensional critical buckling load parameters for nanobeam with various boundary conditions, various size scale parameters and various types of porosity

α	Theory	Boundary conditions					
		S-S			C-C		
		Perfect	Even	Uneven	Perfect	Even	Uneven
0	Present ($\varepsilon_z \neq 0$)	6.778	6.389	6.663	21.520	20.265	21.025
	Present ($\varepsilon_z = 0$)	6.635	6.250	6.521	20.541	19.310	20.069
0.2	Present ($\varepsilon_z \neq 0$)	4.860	4.581	4.777	8.344	7.857	8.1519
	Present ($\varepsilon_z = 0$)	4.757	4.481	4.676	7.964	7.487	7.781
0.5	Present ($\varepsilon_z \neq 0$)	1.955	1.843	1.922	1.980	1.864	1.934
	Present ($\varepsilon_z = 0$)	1.913	1.803	1.881	1.890	1.776	1.846
1	Present ($\varepsilon_z \neq 0$)	0.624	0.588	0.613	0.532	0.501	0.519
	Present ($\varepsilon_z = 0$)	0.610	0.575	0.599	0.507	0.477	0.496

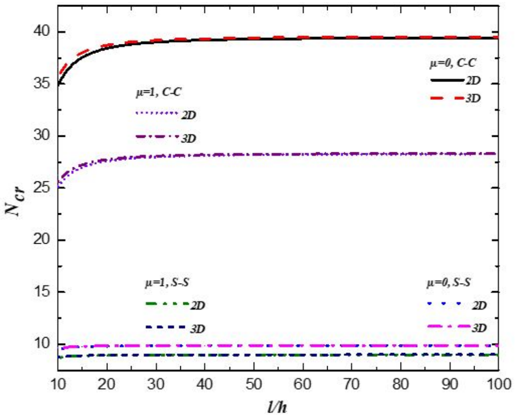

Fig. 1 Non-dimensional critical buckling load versus aspect ratio for clamped and simply supported beam ($k = 0$, $\mu = 0, 1$)

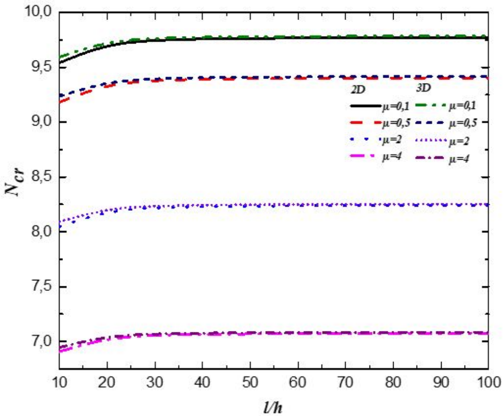

Fig. 2 Non-dimensional critical buckling load of simply supported edges of FG Nano-beam versus aspect ratio under different values of size scale parameter ($k = 0$)

and quasi-3D (with stretching effect) where critical buckling load is rapidly decreasing with the increase of the aspect ratio "*l*/*h*". In addition, it is increasing with the increase of the size scale parameters "α".

Table 7 shows the impact of the pores distribution in the material type 2 on the non-dimensional critical buckling load N_{cr} of the nanobeam for both cases of boundary conditions using 2D and quasi-3D (with stretching effect) beam theory. It can be observed that the non-dimensional critical buckling load of uneven porous is more than even porous and lower than perfect nanobeam. The results obtained for both boundary conditions, the non-dimensional critical buckling load N_{cr} is decreasing with the increase of size scale parameters "α" for each type of porous materials.

Fig. 1 shows the variation of non-dimensional critical buckling load N_{cr} for various boundary conditions versus aspect ratio "*l*/*h*". The gradient index is assumed constant ($k = 0$), where using the material type I. In this example, the nonlocal parameter ($\mu = 0$) and ($\mu = 1$). It is seen from this figure that the nondimensional critical buckling load predicted by clamped support are larger than those of simple support for both values of nonlocal parameter "μ" whereas the critical buckling load of nonlocal elasticity theory is smaller than one obtained by local elasticity theory due to the small scale effects. This result indicates that the effect of the nonlocal parameter softens the nanobeam. An interesting result deduced from Fig. 1 is that critical buckling load is independent of the aspect ratio.

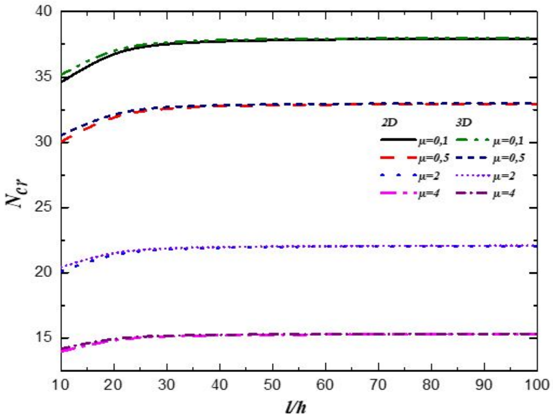

Fig. 3 Non-dimensional critical buckling load of clamped edges of FG Nano-beam versus aspect ratio under different values of size scale parameter ($k = 0$)

The effect of aspect ratio "*l*/*h*" for boundary conditions

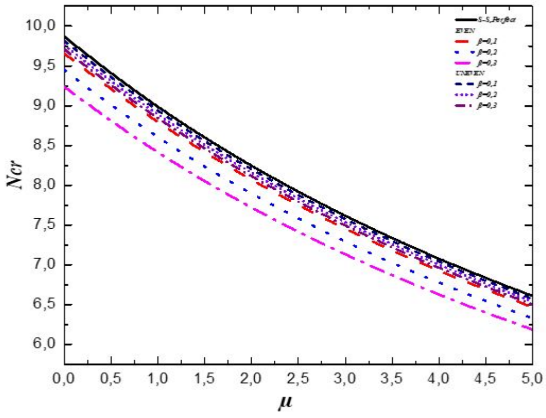

Fig. 4 Comparison between even and uneven porosity for present 2D with pined boundary conditions

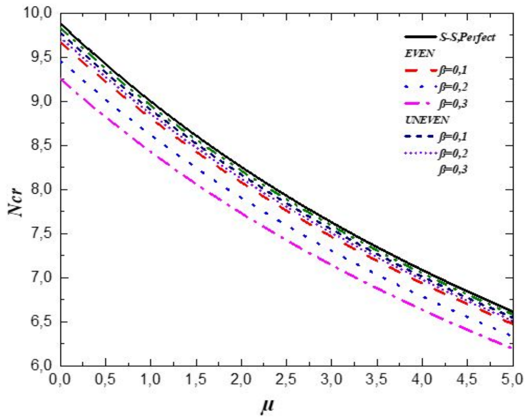

Fig. 6 Comparison between even and uneven porosity for present 3D with pined boundary conditions

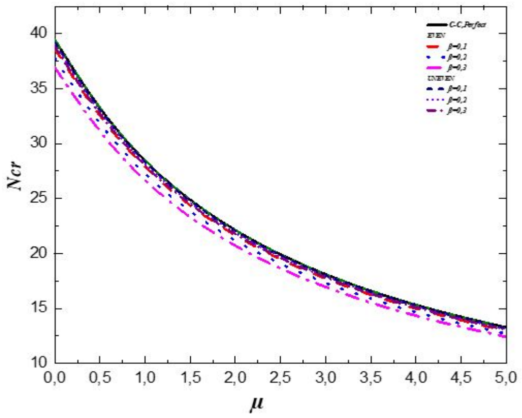

Fig. 5 Comparison between even and uneven porosity for present 2D with clamped boundary conditions

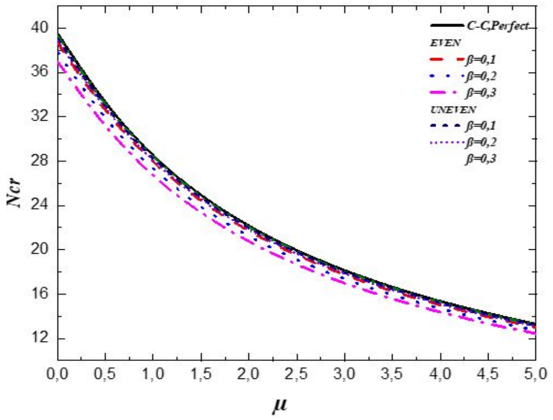

Fig. 7 Comparison between even and uneven porosity for present 3D with clamped boundary conditions

and critical buckling load N_{cr} of FG nanobeams demonstrated in Figs. 2 and 3 for simply supported and clamped beam successively. The results in this figure are obtained by using the constant value of k = 0 and the various values of the nonlocal parameter ($\boldsymbol{\mu}$ = 0.1, 0.2, 2, 4), 2D and quasi-3D formulation for both figures. Those figures show a critical buckling load for seen by the nonlocal parameter less value is larger than those more values of nonlocal parameter "$\boldsymbol{\mu}$" whereas the critical buckling load of simple support beam of Fig. 2 is smaller than the critical buckling load due to the clamped beam Fig. 3. Moreover, interesting result deduced from is that critical buckling load is more important values with small values of the nonlocal parameters and critical buckling load is increasing with the increase of aspect ratio for both figures. Those figures show a critical buckling load for seen by the nonlocal parameter less value is larger than those more values of nonlocal parameter "$\boldsymbol{\mu}$" whereas the critical buckling load of simple support beam of Fig. 2 is smaller than the critical buckling load due to the clamped beam Fig. 3. Moreover, interesting result deduced from is that critical buckling load is more important values with small values of the nonlocal parameters and critical buckling load is increasing with the increase of aspect ratio for both nanobeam theory.

Figs. 4 to 7 illustrate the variation of the critical buckling load N_{cr} for various nonlocal parameter "$\boldsymbol{\mu}$" of porous material type II, the porosity volume fraction "$\boldsymbol{\beta}$" is varied for values (0.1, 0.2 and 0.3) for even, uneven porosity and perfect material using 2D and quasi 3D nanobeam theory in both boundary conditions.

From these figures, it can be seen that as the nonlocal parameter increases, the critical buckling load decreases. Moreover, the behavior of the beam is more important for the perfect material than the uneven porous material then the even porous material according to the increase of the porosity volume fraction, using either the 2D nanobeam theory (Figs. 4. and 5) or the quasi-3D nanobeam theory (Figs. 6. and 7).

8. Conclusions

The buckling analysis is performed on the 2D and quasi-3D FG porous nanobeam based on the height order shear deformation beam theory with various boundary conditions and various size scale parameters. Two types of porosity dispersion were adopted. The results of the presented formulations for buckling of nanobeam using porous materials under different boundary conditions were computed and compared. The obtained results were in an excellent agreement with those reported in the literature. Some results obtained from this study as follows:

- An excellent agreement between the obtained results in this paper and those in literature.
- Clamped beams present height values of critical buckling load than the simply supported beams.
- Critical buckling load decreasing with the increase of the size scale parameters "α" and increasing with the increase of the power law index "k" (ceramic to metal).
- The critical buckling load of uneven porous is more than even porous and lower than perfect nanobeam.
- Critical buckling load is independent of the aspect ratio "l/h".

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