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Nonlocal free vibration analysis of porous FG nanobeams using hyperbolic shear deformation beam theory

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Abstract. This paper presents a new nonlocal Hyperbolic Shear Deformation Beam Theory (HSDBT) for the free vibration of porous Functionally Graded (FG) nanobeams. A new displacement field containing integrals is proposed which involves only three variables. The present model incorporates the length scale parameter (nonlocal parameter) which can capture the small scale effect and its account for shear deformation by a hyperbolic variation of all displacements through the thickness without using the shear correction factor. It has been observed that during the manufacture of Functionally Graded Materials (FGMs), micro-voids and porosities can occur inside the material. Thus, in this work, the investigation of the free vibration analysis of FG beams taking into account the influence of these imperfections is established. Four different porosity types are considered for FG nanobeam. Material characteristics of the FG beam are supposed to vary continuously within thickness direction according to a power-law scheme which is modified to approximate material characteristics for considering the influence of porosities. Based on the nonlocal differential constitutive relations of Eringen, the equations of motion of the nanobeam are derived using Hamilton's principle. The effects of nonlocal parameter, aspect ratio, and the porosity types on the dynamic responses of the nanobeam are discussed.

Keywords: free vibration; nonlocal theory; FGMs; porosity; nanobeam; HSDBT

1. Introduction

Nowadays, with the increasing demands of human being day by day, interest in nanomaterials is increasing as a result of their excellent mechanical, chemical, electronic, electrical, and optical properties. Carbon nanotubes are well-known nanomaterials and their discovery by (Iijima 1991) has initiated a new era in nano-technology. And then, nano-sized structures such as nanowires, nanotubes, nanobeams and nanoplates whose basis are nanomaterials being widely used in nanotechnology. Among these nanobeams are one-dimensional nanostructures and have applications in Micro-Electromechanical Systems (MEMS) and Nano-Electromechanical Systems (NEMS) some examples are Atomic Force Microscopy (AFM), energy harvesting, generators, micro/nano switches, photovoltaic cell, resonators, sensors, transistors (Elishakoff and Pentaras 2009, Murmu and Adhikari 2010, Danesh *et al*. 2012, Karlicic *et al*. 2015). However, in nanoscales, size effects become very significant in the mechanical behaviors of nanostructures whose dimensions are small and comparable to molecular distances. Therefore, understanding the mechanical behaviors of nanostructures is great of interest. For this aim, experimental, computer simulation methods such as Molecular Dynamics (MD) modeling and classical continuum mechanics could be used. Among these methods, classical continuum mechanics is the widely used one by researchers because it serves computational simplicity and effectiveness, as well as the experimental studies, which are difficult and time-consuming in nanoscale, and the cost of MD simulations is so high. However, classical continuum mechanics fails to describe size effects because its constitutive models do not include material length parameters. Therefore, higher order continuum mechanics theories which can capture size effects have been developed for mechanical behaviors of nanostructures, such as the strain gradient elasticity theory (Akgöz and Civalek 2013, 2017, Arani *et al*. 2015, Xu *et al*. 2017, Apuzzo *et al*. 2018, Bensaid *et al*. 2018, Liu *et al*. 2019, Şimşek 2019), the modified couple stress theory (Akgöz and Civalek 2012, Karami *et al*. 2019, Khorshidi and Shariati 2016, Togun and Bağdatli 2016, Akbaş 2017a, b, Civalek *et al*. 2018), non-local elasticity theory (Reddy 2007, Aydogdu 2009, Thai 2012, Berrabah *et al*. 2013, Demir and Civalek 2013, 2017, Kheroubi *et al*. 2016, Bensaid 2017, Demir *et al*. 2018, Numanoğlu *et al*. 2018, Uzun and Civalek 2019, Belmahi *et al*. 2019, Boutaleb *et al*. 2019, Asghar *et al*. 2020, Civalek *et al*. 2020, Khosravi *et al*. 2020) and surface elasticity theory (Wang and Feng 2007, Gheshlaghi and Hasheminejad 2011, Yan and Jiang 2011, Ansari *et al*. 2015, Ebrahimi *et al*. 2019, Eltaher *et al*. 2019, Shanab *et al*. 2020). Between said theories, the non-local elasticity theory of Eringen (1972, 1983) the most

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commonly used one because it includes only one small scale parameter (e_0a) to describe the atomic forces and the internal length scale of nanostructure (Thai *et al*. 2017). Therefore, the nonlocal elasticity theory of Eringen (1972, 1983) is adopted in this study.

Composite materials have superior features such as extremely high strength and stiffness combined with a very low density, resistance to chemicals, thermal and electrical insulation therefore they have a wide-ranging application variety in modern engineering. However, sharp discontinuity exists between the material properties at the interface of two different types of material that may cause strong failures in connection with stress concentrations, in composites. Therefore, a new class of composite materials called Functionally Graded Materials (FGMs) which can which eliminate the stress concentration at interfaces with the continuity and gradual variation of material properties, has been found (Koizumi 1993). In FGMs, the volume fraction of two materials (generally ceramic and metal) changes as a function of location throughout certain dimensions of the structure to realize the desired functions. FGMs have comprehensive applications in the fields of aerospace, medicine, defense, energy, optoelectronics, automotive, biotechnology, aviation, civil and mechanical engineering structures (Koizumi 1997, Birman and Byrd 2007, Chakraverty and Pradhan 2016, Civalek 2017, Avcar and Mohammed 2018, Noori *et al*. 2018, Temel and Noori 2019, Alsaid-Alwan and Avcar 2020, Bendenia *et al*. 2020, Civalek and Avcar 2020, Khadimallah *et al*. 2020, Menasria *et al*. 2020, Temel and Noori 2020).

Because of some technical problems occur during the production of FGMs such as the major difference in solidification temperatures between material constituents during sintering and production of FGM samples using multi-step sequential infiltration technique porosities may arise in FGMs and this results reduction of the density and strength of materials, and so it is important to examine the porosity effect on the mechanical behavior of FGM structures (Zhu *et al*. 2001, Wattanasakulpong *et al*. 2012, Wattanasakulpong and Ungbhakorn 2014, Ait Atmane *et al*. 2015, Hadji and Adda Bedia, 2015, Al Rjoub and Hamad 2017, Avcar 2019, Bekki *et al*. 2019, Bourada *et al*. 2019, Arefi *et al*. 2020, Kaddari *et al*. 2020, Noori *et al*. 2021, Zine *et al*. 2020).

With the development of modern technology FGMs and nanostructures are started to use together for benefitting superiorities of them. Therefore, the analysis of mechanical behaviors of FG nanobeams with and without porosity has become the hot topic of the open literature in recent years. Eltaher *et al*. (2012) presented the free vibration analysis of Functionally Graded (FG) size-dependent nanobeams using the finite element method on the basis of the nonlocal continuum model. Chaht *et al*. (2015) addressed the bending and buckling behaviors of size-dependent nanobeams made of FGMs including the thickness stretching effect. Ebrahimi and Barati (2016) investigated the thermal vibration behavior of FG nanobeams exposed to various kinds of thermo-mechanical loading including uniform, linear and non-linear temperature rise embedded in a two-parameter elastic foundation based on third-order shear deformation beam theory. Barretta *et al*. (2016) proposed the gradient nonlocal model of bending for Timoshenko functionally graded nanobeams based on the Eringen model. Ehyaei *et al*. (2016) investigated the classical and non-classical boundary conditions effect on free vibration characteristics of FG size-dependent nanobeams using a semi-analytical differential transform method. Barati (2017) concerned with the forced vibration analysis of imperfect FG nanobeams resting on three-parameter Kerr foundation under hygro-thermal loading employing a higher-order refined beam theory. Ebrahimi and Barati (2017) investigated the free vibration characteristics of FG nanobeams embedded on the elastic medium based on third-order shear deformation beam theory by presenting a Navier type solution. Ebrahimi and Daman (2017) examined the free vibration of the curved FG piezoelectric nanosize beam in the thermal environment using nonlocal elasticity theory. Ebrahimi *et al*. (2017) investigated the thermo-mechanical vibration characteristics of an FG porous microbeam subjected to various types of thermal loadings based on modified couple stress theory and the exact position of the neutral axis. Ehyaei *et al*. (2017) studied the vibration behavior of rotating FG nanobeam based on Eringen’s nonlocal theory and Euler-Bernoulli beam model. Mirjavadi *et al*. (2017) reported the thermo-mechanical vibration behavior of two-dimensional FG porous nanobeam. Shafiei *et al*. (2017) analyzed the vibration behavior of the two-dimensional FG nano and microbeams which are made of two kinds of porous materials based on the Timoshenko beam theory. Eltaher e*t al*. (2018) studied the mechanical bending and vibration of porous FG nanobeams using finite elements according to Euler beam theory. Hadi *et al*. (2018) presented an investigation on the free vibration of three-directional functionally graded material Euler-Bernoulli nano-beam, with small scale effects. Jouneghani *et al*. (2018) investigated the bending behavior of FG nanobeams with internal porosity and subjected to a hygro-thermo-mechanical loading. Rahmani *et al*. (2018) carried out the vibration analysis of deep curved FG nano-beam based on the modified couple stress theory. Arani *et al*. (2019) dealt with wave propagation of the FG nanobeams based on the nonlocal elasticity theory considering surface and flexoelectric effects. Aria *et al*. (2019) proposed a nonlocal finite element model to analyze the thermo-elastic behavior of imperfect FG porous nanobeams based on nonlocal elasticity theory and employing a double-parameter elastic foundation. Berghouti *et al*. (2019) studied the dynamic behavior of FG porous nanobeams based on nonlocal nth-order shear deformation theory which takes into the effect of shear deformation without considering shear correction factors. Ebrahimi-Nejad *et al*. (2019) investigated the vibrational behavior of a two-directional FG porous nanobeam under hygro-thermo-mechanical loading, Jalaei and Civalek (2019) examined the dynamic instability of viscoelastic porous FG nanobeam embedded on visco-Pasternak medium subjected to an axially oscillating loading as well as the magnetic field. Gafour *et al*. (2020) focused on the behavior of non-local shear deformation beam theory for the vibration of FG nanobeams with

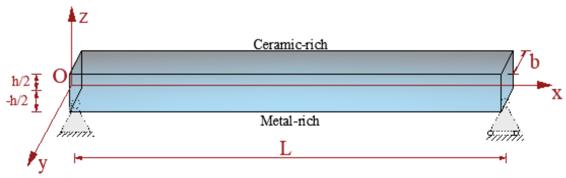


Fig. 1 The geometry of the FG porous nanobeam

porosities using the nonlocal differential constitutive relations of Eringen. Matouk (2020) studied the free vibrational behavior of the FG nanobeams integrated into the hygro-thermal environment and resting on the elastic foundation is using a novel integral Timoshenko beam theory. Rahmani *et al*. (2020) presented the vibration analysis of bi-directional FG rotating nanobeams with porosities is studied. She *et al*. (2020) examined the forced resonance vibration of porous FG curved nanobeam.

As a result of the literature research, it is seen that the effect of different porosity distributions on the free vibration of FG nanobeams considering the nonlocal HSDBT has not dealt with yet. Therefore, in the present study, an attempt is made to address this problem. For this aim, this paper presents a new nonlocal HSDBT for the dynamic behavior of porous FG nanobeams. A new displacement field containing integrals is proposed which involves only three variables. The present model incorporates the length scale parameter (nonlocal parameter) which can capture the small scale effect and its account for shear deformation by a hyperbolic variation of all displacements through the thickness without using the shear correction factor. It has been observed that during the manufacture of FGMs, micro-voids and porosities can occur inside the material. Thus, in this work, the investigation of the dynamic analysis of FG beams taking into account the influence of these imperfections is established. Four different porosity types are considered for FG nanobeam. Material characteristics of the FG beam are supposed to vary continuously within thickness direction according to a power-law scheme which is modified to approximate material characteristics for considering the influence of porosities. Based on the nonlocal differential constitutive relations of Eringen, the equations of motion of the nanobeam are derived using Hamilton's principle. The effects of nonlocal parameter, aspect ratio and the porosity types on the dynamic responses of the nanobeam are discussed.

2. Formulation of the problem

In this research, a straight FG porous nanobeam of thickness h, width b, length L having a rectangular cross-section is considered. The Cartesian coordinate system O (x, y, z) is placed on the left edge of the central axis of the beam, where x-, y-, and z-axes are taken along the length, width, and depth of the beam, as shown in Fig. 1. Namely, the following limit intervals apply.

$$\frac{-h}{2} \leq z \leq \frac{h}{2}, \qquad \frac{-b}{2} \leq y \leq \frac{b}{2}, \qquad 0 \leq x \leq L \tag{1}$$

2.1 Porous FG nanobeam

In this study, a porous FG nanobeam with a volume fraction of porosity ξ (ξ « 1) having different forms of distribution between the metal and the ceramic. The modified mixture rule proposed by Wattanasakulpong and Ungbhakorn (2014) is

$$P(z) = P_m\left(V_m - \frac{\xi}{2}\right) + P_c\left(V_c - \frac{\xi}{2}\right) \tag{2}$$

The power law of the volume fraction of the ceramic is assumed as

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^k \tag{3}$$

The modified mixture rule becomes

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_m - (P_c + P_m)\frac{\xi}{2} \tag{4}$$

where k is the power-law index that takes values greater than or equals to zero. FG nanobeam becomes a fully ceramic beam when k is set to zero and a fully metal one for the large value of k.

The mechanical properties of porous FG nanobeam such as Young's modulus E and mass density ρ of the can be written as a function of thickness coordinate z (middle surface), as follows (Ait Atmane *et al*. 2015, Hadji and Adda Bedia 2015).

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_m - (E_c + E_m)\frac{\xi}{2} \tag{5a}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_m - (\rho_c + \rho_m)\frac{\xi}{2} \tag{5b}$$

The material properties of a perfect FG nanobeam can be obtained as the volume fraction of porosity ξ is set to zero. Due to the small variations of the Poisson ratio ν, it is assumed to be constant. Table 1 shows the several porosity volume fraction distributions which have been studied in the present work.

The material properties used in this work are abstracted in Table 2.

2.2 Nonlocal hyperbolic shear deformation theory

Based on the higher-order shear deformation beam theory, the displacement fields of the beam are presented as follows (Zaoui *et al*. 2019, Chikr *et al*. 2020)

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,t)\,dx \tag{6a}$$

$$w(x,z,t) = w_0(x,t) \tag{6b}$$

where, u_0, w_0 and θ are the unknown displacement of the mid-plane of the beam.

In Eq. (7), $f(z)$ is defined according to the higher-order

Table 1 Different distribution forms of porosity volume fraction

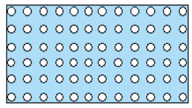
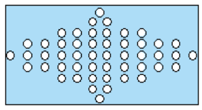
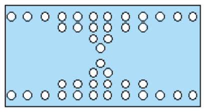
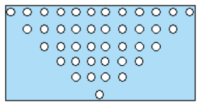
Distribution forms of porosity	Elasticity modulus expression	Type
Homogeneous	$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k+E_m-(E_c+E_m)\frac{\xi}{2}$	
Form ''O''	$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k+E_m-(E_c+E_m)\frac{\xi}{2}\left(1-2\frac{\lvert z\rvert}{h}\right)$	
Form ''X''	$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k+E_m-(E_c+E_m)\frac{\xi}{2}\left(2\frac{z}{h}\right)$	
Form ''V''	$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k+E_m-(E_c+E_m)\frac{\xi}{2}\left(\frac{1}{2}+\frac{z}{h}\right)$	

Table 2 Material properties of alumina (Al_2O_3) and steel

Material	E(GPa)	ρ (kg/m^3)	ν
Alumina	390	3960	0.3
Steel	210	7800	

shear deformation beam theory as follows (Akavci 2010, Bekki *et al*. 2019).

$$f(z)=\frac{3}{2}\pi h\tanh\left(\frac{z}{h}\right)-\frac{3}{2}\pi z\,\mathrm{sech}\left(\frac{1}{2}\right)^2 \tag{7}$$

The theory used in Eq. (7) accounts for the higher-order variation of transverse shear strain through the depth of the nanobeam and satisfies the zero traction boundary conditions on the surfaces of the nanobeam without using shear correction factors.

The strains associated with the displacements in Eq. (6) are

$$\varepsilon_x=\varepsilon_x^0+z\,k_x^b+f(z)\,k_x^s \tag{8a}$$

$$\gamma_{xz}=g(z)\,\gamma_{xz}^0 \tag{8b}$$

where

$$\varepsilon_x^0=\frac{\partial u_0}{\partial x},\qquad k_x^b=-\frac{\partial^2 w_0}{\partial x^2},\qquad k_x^s=k_1\theta \\ \gamma_{xz}^0=k_1\int\theta\,dx \tag{8c}$$

and

$$g(z)=\frac{df(z)}{dz} \tag{8d}$$

The integral defined in the above equations shall be resolved by a Navier type method and can be written as follow

$$\int\theta\,dx=A'\frac{\partial\theta}{\partial x} \tag{9}$$

where the coefficient A' is expressed according to the type of solution used, in this case via Navier. Therefore, A' and k_1 are expressed as follows

$$A'=-\frac{1}{\alpha^2},\qquad k_1=\alpha^2 \tag{10}$$

where α is defined in Eq. (23).

2.3 Constitutive relations

The response of materials at the nanoscale is different from those of their bulk counterparts. In the theory of nonlocal elasticity, the stress at a reference point x is considered to be a function of the strain field at every point in the body. For example, in the nonlocal elasticity, the uniaxial constitutive law is expressed as

$$\sigma_x-\mu\frac{d^2\sigma_x}{dx^2}=Q_{11}\varepsilon_x \tag{11a}$$

$$\tau_{xz}-\mu\frac{d^2\tau_{xz}}{dx^2}=Q_{55}\gamma_{xz} \tag{11b}$$

where

$$\mu=(e_0a)^2 \tag{12}$$

where e_0 and a are constant (depend on the material) and internal characteristic length. For Single-Walled Carbon Nanotube (SWCNT), the nonlocal parameter e_0a is estimated to be smaller than 2 nm (Tounsi *et al*. 2013a, Semmah et al. 2014).

The Q_{ij} expressions in terms of engineering constants are

$$Q_{11}=E,\qquad Q_{55}=G=\frac{E}{2(1+\nu)} \tag{13}$$

where E is the elasticity modulus, G is the shear modulus, and v the Poisson's ratio.

2.4 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in the analytical form as (Reddy 2002)

$$\delta \int_{t_1}^{t_2} (U - T) dt = 0 \tag{14}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy; and δT is the virtual variation of the kinetic energy.

In the present investigation, Hamilton's principle can be written as the function of the stresses and strains as follows (Mahi *et al*. 2015, Bourada *et al*. 2019).

$$\begin{aligned} &\int_0^L \int_A (\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz})\, dA dx \\ &- \int_0^L \int_A \rho(z)[\dot{u}\delta\, \dot{u} + \dot{w}\delta\, \dot{w}]\ dA dx \end{aligned} \tag{15}$$

By replacing Eqs. (5) and (8) into Eq. (15) and performing the integration by part and collecting the coefficients δu_0, δw_0 and $\delta\theta$ yields the three following equations of motion

$$\delta\, u_0{:}\quad \frac{dN}{dx} = I_0\ddot{u}_0 - I_1\frac{\partial \ddot{w}_0}{\partial x} + k_1 A' J_1 \frac{\partial \ddot{\theta}}{\partial x} \tag{16a}$$

$$\begin{aligned} &\delta\, w_0{:}\ \ \frac{d^2 M_b}{dx^2} + q \\ &= I_0\ddot{w}_0 + I_1\frac{\partial \ddot{u}_0}{\partial x} - I_2\frac{\partial^2 \ddot{w}_0}{\partial x^2} + J_2 k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} \end{aligned} \tag{16b}$$

$$\begin{aligned} &\delta\, \theta{:}\ \ -k_1 M_x^s + k_1 A' \frac{\partial Q_{xz}}{\partial x} \\ &= -J_1 k_1 A' \frac{\partial \ddot{u}_0}{\partial x} - K_2 (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + J_2 k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} \end{aligned} \tag{16c}$$

where the resultants stress and moments N, M_b, M_s and Q are defined as follow

$$(N, M_b, M_s) = \int_A (1, z, f)\, \sigma_x dA \ \text{ and } \ Q_{xz} = \int_A g \tau_{xz} dA \tag{17}$$

and

$$(I_0, I_1, I_2) = \int_A (1, z, z^2) \rho(z) dA \tag{18a}$$

$$(J_1, J_2, K_2) = \int_A (f, z\, f, f^2) \rho(z) dA \tag{18b}$$

By replacing the strain field of Eqs. (7) and (8) into non-local constitutive relation of Eq. (11) and the obtained results into Eq. (17), the stress resultants of the non-local FG beam can be obtained in the following form

$$N - \mu\frac{d^2 N}{dx^2} = A\frac{du_0}{dx} - B\frac{d^2 w_0}{dx^2} - B^s k_1 A' \frac{d\theta}{dx} \tag{19a}$$

$$M_b - \mu\frac{d^2 M_b}{dx^2} = B\frac{du_0}{dx} - D\frac{d^2 w_0}{dx^2} + D^s k_1 A' \frac{d\theta}{dx} \tag{19b}$$

$$M_s - \mu\frac{d^2 M_s}{dx^2} = B^s\frac{du_0}{dx} - D^s\frac{d^2 w_0}{dx^2} + H^s k_1 A' \frac{d\theta}{dx} \tag{19c}$$

$$Q_{xz} - \mu\frac{d^2 Q_{xz}}{dx^2} = A_s k_1 A' \frac{\partial\theta}{\partial x} \tag{19d}$$

where A, B, D, B_s, D_s, H_s and A_s are the stiffness components as follows.

$$(A, B, D, B_s, D_s, H_s) = \int_A E(z)(1, z, z^2, f, zf, f^2)\, dA \tag{20a}$$

$$A_{55}^s = \int_A G(z) g^2 dA \tag{20b}$$

By replacing the stress resultants of Eq. (19) into equations of motion of Eq. (16), the present nonlocal equations of motion can be obtained in terms of displacements u_0, w_0, θ as

$$\begin{aligned} &A\frac{d^2 u_0}{dx^2} - B\frac{d^3 w_0}{dx^3} - B^s k_1 A' \frac{d^2\theta}{dx^2} \\ &= I_0\left(\ddot{u}_0 - \mu\frac{d^2\ddot{u}_0}{dx^2}\right) - I_1\left(\frac{d\ddot{w}_o}{dx} - \mu\frac{d^3\ddot{w}_0}{dx^3}\right) \\ &+ k_1 A' J_1 \left(\frac{d\ddot{\theta}}{dx} - \mu\frac{d^3\ddot{\theta}}{dx^3}\right) \end{aligned} \tag{21a}$$

$$\begin{aligned} &B\frac{d^3 u_0}{dx^3} - D\frac{d^4 w_0}{dx^4} + D^s k_1 A' \frac{d^3\theta}{dx^3} \\ &= I_0\left(\ddot{w}_0 - \mu\frac{\partial^2\ddot{w}_0}{\partial x^2}\right) + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} - \mu\frac{\partial^3\ddot{u}_0}{\partial x^3}\right) \\ &- I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2} - \mu\frac{\partial^4\ddot{w}_0}{\partial x^4}\right) + J_2 k_1 A' \left(\frac{\partial^2\ddot{\theta}}{\partial x^2} - \mu\frac{\partial^4\ddot{\theta}}{\partial x^4}\right) \end{aligned} \tag{21b}$$

$$\begin{aligned} &B^s\frac{d^3 u_0}{dx^3} - D^s\frac{d^4 w_0}{dx^4} + H^s k_1 A' \frac{d^3\theta}{dx^3} + A_s k_1 A' \frac{\partial^2\theta}{dx^3} = \\ &= -J_1 k_1 A' \left(\frac{\partial\ddot{u}_0}{\partial x} - \mu\frac{\partial^3\ddot{u}_0}{\partial x^3}\right) - K_2 (k_1 A')^2 \left(\frac{\partial^2\ddot{\theta}}{\partial x^2} - \mu\frac{\partial^4\ddot{\theta}}{\partial x^4}\right) \\ &+ J_2 k_1 A' \left(\frac{\partial^2\ddot{w}_0}{\partial x^2} - \mu\frac{\partial^4\ddot{w}_0}{\partial x^4}\right) \end{aligned} \tag{21c}$$

To obtain the equations of motion of local beam theory, just put $\mu = 0$ into Eq. (21).

3. Solution procedure of porous FG-nanobeam

The Navier's procedure is used in this study to solve the previous nonlocal equations of motion for vibrational analysis of non-local FG beam. The Navier's procedure can be presented in the following form (Tagrara *et al*. 2015, Bourada *et al*. 2018).

Table 3 Dimensionless fundamental frequencies versus varying nonlocal parameter and span-to-depth ratio

L/h	μ (nm^2)	EBT Reddy (2007)	TBT Reddy (2007)	RBT Reddy (2007)	SSDBT Tounsi *et al*. (2013b)	Present model
5	0	9.7112	9.2740	9.2745	9.2752	9.2749
	1	9.2647	8.8477	8.8482	8.8488	8.8485
	2	8.8747	8.4752	8.4757	8.4763	8.4759
	3	8.5301	8.1461	8.1466	8.1472	8.1469
	4	8.2228	7.8526	7.8530	7.8536	7.8534
10	0	9.8293	9.7075	9.7075	9.7077	9.7076
	1	9.3774	9.2612	9.2612	9.2614	9.2613
	2	8.9826	8.8713	8.8714	8.8715	8.8714
	3	8.6338	8.5269	8.5269	8.5271	8.5269
	4	8.3228	8.2196	8.2197	8.2198	8.2197
20	0	9.8595	9.8281	9.8281	9.8282	9.8282
	1	9.4062	9.3763	9.3763	9.3764	9.3763
	2	9.0102	8.9816	8.9816	8.9816	8.9816
	3	8.6604	8.6328	8.6328	8.6329	8.6329
	4	8.3483	8.3218	8.3218	8.3218	8.3218
100	0	9.8692	9.8679	9.8679	9.8679	9.8679
	1	9.4155	9.4143	9.4143	9.4143	9.4143
	2	9.0191	9.0180	9.0180	9.0180	9.0179
	3	8.6689	8.6678	8.6678	8.6678	8.6678
	4	8.3566	8.3555	8.3555	8.3555	8.3555

Table 4 Dimensionless the first three frequencies versus varying nonlocal parameter and the number of mode (L/h = 5)

m	μ (nm^2)	EBT Reddy (2007)	TBT Reddy (2007)	RBT Reddy (2007)	SSDBT Tounsi *et al*. (2013b)	Present model
1	0	9.7112	9.2740	9.2745	9.2752	9.2749
	1	9.2647	8.8477	8.8482	8.8488	8.8485
	2	8.8747	8.4752	8.4757	8.4763	8.4759
	3	8.5301	8.1461	8.1466	8.1472	8.1469
	4	8.2228	7.8526	7.8530	7.8536	7.8534
2	0	37.1120	32.1665	32.1847	32.1948	32.1905
	1	31.4239	27.2364	27.2519	27.2604	27.2568
	2	27.7422	24.0453	24.0589	24.0664	24.0632
	3	25.1104	21.7642	21.7765	21.7833	21.7804
	4	23.1088	20.0293	20.0407	20.0470	20.0443
3	0	78.0234	61.4581	61.5746	61.6192	61.6019
	1	56.7798	44.7247	44.8095	44.8420	44.8294
	2	46.8246	36.8831	36.9531	36.9798	36.9695
	3	40.7568	32.1036	32.1645	32.1878	32.1788
	4	36.5657	28.8023	28.8569	28.8778	28.8697

$$\begin{Bmatrix} u_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\alpha\, x)\, e^{i\,\omega\, t} \\ W_m \sin(\alpha\, x)\, e^{i\,\omega\, t} \\ X_m \sin(\alpha\, x)\, e^{i\,\omega\, t} \end{Bmatrix} \tag{22}$$

with

$$\alpha = m\pi/L \tag{23}$$

where U_m, W_m and X_m are arbitrary parameters to be determined, ω is the frequency of the free vibration of FG nano-beam and $\sqrt{i} = -1$ the imaginary unit.

Table 5 Dimensionless fundamental frequencies of porous FG nanobeam versus varying nonlocal parameter and the power-law index

e_0a	k	ξ			
		0.05	0.1	0.2	0.3
0	0	9.4524	9.6568	10.1753	10.9237
	0.5	7.3166	7.3515	7.4344	7.5410
	1	6.5638	6.5509	6.5174	6.4681
	5	5.4994	5.4354	5.2825	5.0833
	10	5.2454	5.1725	5.0007	4.7814
0.5	0	9.0179	9.2128	9.7075	10.4215
	0.5	6.9803	7.0136	7.0927	7.1943
	1	6.2621	6.2498	6.2178	6.1708
	5	5.2466	5.1855	5.0396	4.8496
	10	5.0042	4.9347	4.7708	4.5616
1	0	8.0037	8.1767	8.6157	9.2494
	0.5	6.1952	6.2248	6.2949	6.3852
	1	5.5578	5.5469	5.5185	5.4768
	5	4.6565	4.6023	4.4729	4.3042
	10	4.4414	4.3797	4.2343	4.0485
1.5	0	6.8788	7.0275	7.4048	7.9495
	0.5	5.3245	5.3499	5.4102	5.4878
	1	4.7767	4.7673	4.7429	4.7070
	5	4.0021	3.9555	3.8442	3.6993
	10	3.8172	3.7642	3.6392	3.4795
2	0	5.8858	6.0130	6.3359	6.8019
	0.5	4.5559	4.5776	4.6293	4.6956
	1	4.0871	4.0791	4.0582	4.0276
	5	3.4244	3.3845	3.2893	3.1653
	10	3.2662	3.2208	3.1138	2.9773

Replacing the functions u_0, w_0, θ of Eq. (22) into the equation of motion of Eqs. (21), the analytical solutions can be obtained in the following matrix form

$$([K]-\lambda\omega^2[M])\{\Delta\}=\{0\} \tag{24}$$

where $[K]$, $[M]$ and $\{\Delta\}$ are the stiffness matrix, mass matrix and the displacement vector, respectively and are defined as follows

$$[K]=\begin{bmatrix} A\alpha^2 & -B\alpha^3 & -B_s\alpha k_1 \\ \text{Sym.} & D\alpha^4 & D_s\alpha^2 k_1 \\ \text{Sym.} & \text{Sym.} & H_s k_1^2 + A_s(k_1A')^2\alpha^2 \end{bmatrix} \tag{25a}$$

$$[M]=\begin{bmatrix} I_0 & -I_1\alpha & J_1\alpha k_1 A' \\ \text{Sym.} & I_0+I_2\alpha^2 & -J_2\alpha^2 k_1 A' \\ \text{Sym.} & \text{Sym.} & K_2\alpha^2(k_1A')^2 \end{bmatrix} \tag{25b}$$

$$\{\Delta\}^T=\{U_m, W_m, X_m\} \tag{25c}$$

$$\lambda = 1+\mu\alpha^2 \tag{26}$$

4. Results and discussion

In this part, several numerical examples are presented and discussed to verify the accuracy of the present model in predicting the fundamental frequencies of simply supported porous FG nanobeams.

4.1 Comparison studies

Tables 3 and 4 present the comparison of dimensionless fundamental frequencies of perfect FG nanobeam versus varying nonlocal parameter (μ), span-to-depth ratio, (L/h), and the number of the modes (m), respectively with the results of Reddy (2007) and Tounsi *et al*. (2013b). Here, EBBT, TBT, RBT and SSBT denote Euler-Bernoulli beam theory, Timoshenko beam theory, Reddy's beam theory, and sinusoidal shear deformation beam theory, respectively. For all calculations, the shear correction factor and Poisson's ratio are taken as 5/6 and 0.3, respectively. A conservative estimate of the nonlocal parameter $0 \leq e_0a \leq 2$ nm for SWCNTs is proposed by Wang (2005). For all presented results, the following non-dimensional fundamental frequency is utilized.

$$\overline{\omega}=\omega L^2\sqrt{\frac{\rho_t A}{E_t I}} \tag{28}$$

As seen from Tables 3 and 4, the present HSDBT results are in good agreement with the previously published ones adopting EBBT, TBT, RBT and SSBT.

4.2 Parametrical studies

Study 1: Table 5 shows the change in the values of dimensionless fundamental frequencies of porous FG nanobeam versus varying nonlocal parameter (e_0a), the power-law index (k), and porosity volume fraction (ξ). Here, homogenous porosity distribution and the span-to-depth ratio, L/h = 5, are considered. The results demonstrated that the values of dimensionless fundamental frequencies decrease with the increment of the nonlocal parameter. Similarly, the values of dimensionless fundamental frequencies decrease with the increase of the power-law index due to the percentage of metal phases which are weaker than ceramic phases become more prominent. Furthermore, the porosity volume fraction has the highest effect on the values of dimensionless fundamental frequencies for homogenous case (k = 0) while the related effect becomes more noticeable as the values of the power-law index is larger than one (k > 1) for the inhomogeneous case. The values of dimensionless fundamental frequencies decrease with the increase of the porosity volume fraction for higher values of the power-law index, while the values of dimensionless fundamental frequencies increase with the increase of the porosity volume fraction for lower values of the power-law index. It should be noted that all of the said effects are independent of the variation of the nonlocal parameter.

Study 2: Table 6 displays the change in the values of dimensionless fundamental frequencies of porous FG

Table 6 Dimensionless fundamental frequencies of porous FG nanobeam versus varying span-to-depth ratio, porosity volume fraction, and power-law index

L/h	ξ	k = 0	0.5	1	5	10
5	0.05	8.0037	6.1952	5.5578	4.6565	4.4414
	0.1	8.1767	6.2248	5.5469	4.6023	4.3797
	0.2	8.6157	6.2949	5.5185	4.4729	4.2343
	0.3	9.2494	6.3852	5.4768	4.3042	4.0485
10	0.05	9.4386	7.3018	6.5542	5.5127	5.2557
	0.1	9.6426	7.3366	6.5413	5.4500	5.1842
	0.2	10.1604	7.4192	6.5078	5.3005	5.0157
	0.3	10.9077	7.5256	6.4586	5.1055	4.8008
20	0.05	9.8949	7.6536	6.8711	5.7857	5.5153
	0.1	10.1089	7.6900	6.8576	5.7204	5.4407
	0.2	10.6516	7.7766	6.8224	5.5646	5.2650
	0.3	11.4351	7.8882	6.7709	5.3615	5.0411
50	0.05	10.0319	7.7592	6.9662	5.8677	5.5933
	0.1	10.2488	7.7961	6.9525	5.8017	5.5178
	0.2	10.7991	7.8839	6.9169	5.6439	5.3399
	0.3	11.5934	7.9970	6.8646	5.4384	5.1133
100	0.05	10.0519	7.7746	6.9800	5.8796	5.6047
	0.1	10.2692	7.8116	6.9663	5.8135	5.5290
	0.2	10.8206	7.8995	6.9307	5.6555	5.3508
	0.3	11.6164	8.0129	6.8783	5.4496	5.1238

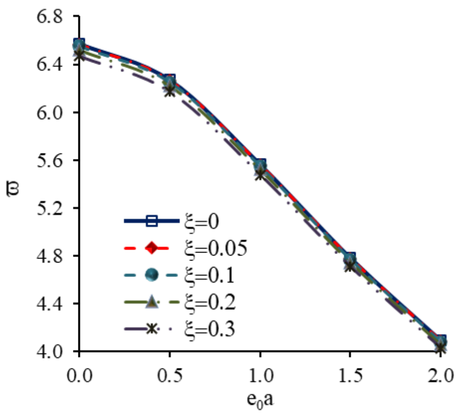


Fig. 2 Dimensionless fundamental frequencies of porous FG nanobeam versus varying nonlocal parameter

nanobeam versus varying span-to-depth ratio (L/h), porosity volume fraction (ξ), and power-law index (k). Here, homogenous porosity distribution and the nonlocal parameter, (e_0a = 1 nm), are considered. The results revealed that the values of dimensionless fundamental frequencies increase with the increment of the span to depth ratio. The changes in the power-law index and the effect of porosity volume fraction on the values of dimensionless fundamental frequencies have the same effects as mentioned in study 1. It should be noted that the change in

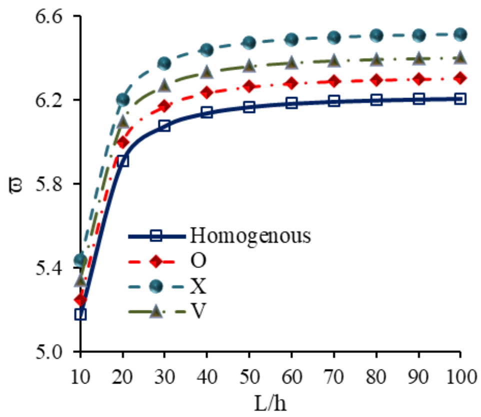


Fig. 3 Dimensionless fundamental frequencies of porous FG nanobeam versus varying span to depth ratio

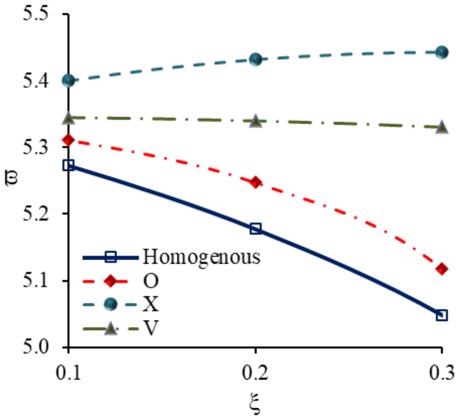


Fig. 4 Dimensionless fundamental frequencies of porous FG nanobeam versus varying porosity volume fraction

the dimensionless fundamental frequency versus porosity volume fraction as well as power-law index almost stay constant for the varying span to depth ratio.

Study 3: Fig. 2 illustrates the change in the values of dimensionless fundamental frequencies of porous FG nanobeam versus varying nonlocal parameter (e_0a). Here, the power law index and span to depth ratio are taken to be k = 1 and L/h = 5 respectively. Homogenous porosity distribution and the nonlocal parameter, (e_0a = 1 nm), are considered. The results indicate that the values of dimensionless fundamental frequencies decrease with the increase of the nonlocal parameter and porosity volume fraction. It is also observed that the nonlocal parameter has a noteworthy effect on the values of dimensionless fundamental frequencies of porous FG nanobeam, however, it is independent of the varying porosity volume fraction.

Study 4: Fig. 3 illuminates the change in the values of dimensionless fundamental frequencies of porous FG nanobeam versus varying span to depth ratio (L/h). Here, the power-law index and porosity volume fraction, and nonlocal parameter are taken to be k = 2, ξ = 0.2 and e_0a = 2 nm, respectively. The results indicate that the values of dimensionless fundamental frequencies increase with the increase of the span to depth ratio. It is also found that type of porosity volume fraction distribution has a significant effect on the values of dimensionless fundamental frequencies of porous FG nanobeam, however, it is independent of the varying span to depth ratio.

Study 5: Fig. 4 shows the change in the values of dimensionless fundamental frequencies of porous FG nanobeam versus varying porosity volume fraction (ξ). Here, the power-law index, span to depth ratio, and nonlocal parameter are taken to be k = 2, L/h=10, and e_0a = 2 nm, respectively. The results point out that the values of dimensionless fundamental frequencies decrease with the increase of the porosity volume fraction. It is also seen that the effect of porosity volume fraction on the fundamental frequencies of porous FG nanobeam depends on the type of porosity volume fraction distribution, considerably and it becomes more pronounced with the increment of porosity volume fraction.

5. Conclusions

In the present work, the dynamic behavior of porous FG nanobeams is examined utilizing a new nonlocal HSDBT. A new displacement field containing integrals is proposed which involves only three variables. The present model incorporates the length scale parameter which can capture the small scale effect and its account for shear deformation by a hyperbolic variation of all displacements through the thickness without using the shear correction factor. Material characteristics of the FG beam are supposed to vary continuously within thickness direction according to a power-law scheme which is modified to approximate material characteristics for considering the influence of porosities.

The following results are found for porous FG nanobeams:

• The values of dimensionless fundamental frequencies decrease with the increment of the nonlocal parameter

• The values of dimensionless fundamental frequencies decrease with the increase of the power-law index

• The porosity volume fraction has the highest effect on the values of dimensionless fundamental frequencies for homogenous case

• The values of dimensionless fundamental frequencies increase with the increment of the span to depth ratio

• The values of dimensionless fundamental frequencies decrease with the increase of the porosity volume fraction for higher values of the power-law index, while the values of dimensionless fundamental frequencies increase with the increase of the porosity volume fraction for lower values of the power-law index

• The values of dimensionless fundamental frequencies considerably affected by the type of porosity volume fraction distribution and the said effect become more pronounced with the increment of porosity volume fraction.

Finally, it is concluded that the types of porosity volume fraction and its distribution, power-law index, span to depth ratio, and nonlocal parameter have major effects on the vibration frequencies of the nanobeams. Besides, the proposed nonlocal HSDBT not only satisfactorily handled the present problem and yielded successful results but also it has provided ease in the nonlocal free vibration analysis of porous FG nanobeams. In future studies, the presented solution procedure will be extended for mechanical behaviors of other types of structures composed of different materials with macro/micro dimensions.

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