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Elastic wave phenomenon of nanobeams including thickness stretching effect

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Abstract. The present work deals with an investigation on longitudinal wave propagation in nanobeams made of graphene sheets, for the first time. The nanobeam is modelled via a higher-order shear deformation theory accounts for both higher-order and thickness stretching terms. The general nonlocal strain gradient theory including nonlocality and strain gradient characteristics of size-dependency in order is used to examine the small-scale effects. This model has three-small scale coefficients in which two of them are for nonlocality and one of them applied for gradient effects. Hamilton supposition is applied to obtain the governing motion equation which is solved using a harmonic solution procedure. It is indicated that the longitudinal wave characteristics of the nanobeams are significantly influenced by the nonlocal parameters and strain gradient parameter. It is shown that higher nonlocal parameter is more efficient than lower nonlocal parameter to change longitudinal phase velocities, while the strain gradient parameter is the determining factor for their efficiency on the results.

Keywords: wave propagation; homogeneous materials; bi-Helmholtz nonlocal strain gradient theory; thickness stretching effect

1. Introduction

Although there is a wide use of classical continuum theories for modeling the mechanical characteristics of composites structures (Gao *et al*. 2018b, Gao and Zhang 2019, Huang *et al*. 2020, Safaei 2020), many other problems related to micro/nano structures cannot be accurately examined via the pioneer classical approaches (Chen *et al*. 2018, Guo *et al*. 2019, Ashraf *et al*. 2020, Luo *et al*. 2020, Yan *et al*. 2020, Yu *et al*. 2020). Hence, several enriched models of which contain additional small-scale parameters have been developed to capture the size effect in the structures so far. As a popular model, Eringen and Edelen (1972) presented the Nonlocal Elasticity Theory (NET) for considering the softening-stiffness mechanism in which the stress field at a point is a function of strain field at all points in that continuum body. Up to now, a considerable effort has been made to study the behavior of nanostructures using NET (Shahsavari and Janghorban 2017, Balubaid *et al*. 2019, Berghouti *et al*. 2019, Fattahi *et al*. 2019a, Hussain *et al*. 2019, 2020, Karami and Karami 2019, Semmah *et al*. 2019, Asghar *et al*. 2020, Bellal *et al*. 2020, Fan *et al*. 2020, Khosravi *et al*. 2020, Matouk *et al*. 2020). Thai *et al*. (2017) investigated the static analysis of nanoscale beam via NET. Ghadiri *et al*. (2017) investigated the influence of surface effects on vibration behaviors of rotary functionally graded nanobeam using this theory. Fernández-Sáez and Zaera (2017) studied the vibration response of Euler nanobeam on the basis of local/nonlocal elasticity theory. NET was applied to study the wave dispersion analysis of size-dependent rotating inhomogeneous nanobeams by Ebrahimi and Barati (2017a). Shahsavari *et al*. (2017) employed the classical plate theory in conjunction with NET to investigate the dynamic analysis of viscoelastic orthotropic nanoplates under moving load. Eringen nonlocal model application was determined for vibrational behavior of FGM nano-size plate by Fattahi *et al*. (2019b).

On the other hand, Eringen's elasticity theory is not capable to consider the softening-stiffness mechanism which is crucial in exact analysis of micro/nanostructures Fan *et al*. (2021). To tackle this weakness, Askes and Aifantis (2009) and Lim *et al*. (2015) introduced the Nonlocal Strain Gradient elasticity Theory (NSGT) for considering both softening-stiffness as well as hardening-stiffness mechanisms simultaneously. Therefore, due to the effeteness of NSGT to report more accurate results when compared with experimental investigations, many works have been applied this theory in recent years (Askes and Aifantis 2009, Lim *et al*. 2015, Karami *et al*. 2017, 2018b, 2019b, 2020, Bensaid *et al*. 2018, Heydari 2018, Gao *et al*. 2019d, e, Karami and Janghorban 2019, Karami and Karami 2019, Eyvazian *et al*. 2020, Mirjavadi *et al*. 2020, Noroozi *et al*. 2020, Rabczuk 2020, Shariati *et al*. 2020, Wu and Lin 2020). Nami and Janghorban (2014) studied

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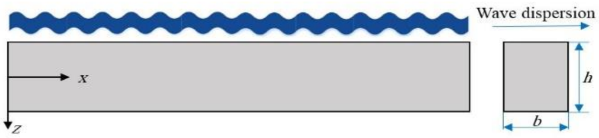


Fig. 1 Geometry of a homogeneous nanobeam

resonance behavior of functionally graded rectangular micro/nano plate basis on nonlocal elasticity theory enriched by strain gradient theory with one gradient constant. Li *et al*. (2015) studied the flexural wave dispersion in small-scaled functionally graded beams, while Mohammadimehr *et al*. (2015) investigated vibration analysis of viscoelastic tapered micro-rod via strain gradient theory. Li and Hu (2015) applied this theory to investigate the buckling analysis of size-dependent nonlinear beams. Ebrahimi *et al*. (2016) studied the dispersion of elastic waves analysis in temperature-dependent functionally graded nanoplates. Ebrahimi and Barati (2017a) examined flexural wave propagation analysis of embedded S-FGM nanobeams under longitudinal magnetic field considering size effects. More recently, an enriched and also more accurate model of nonlocal strain gradient theory called has been introduced including one additional small-scale parameter which experimental observation confirmed that this novel theory is more efficient, especially for the analysis of wave propagation in high wave number; wave propagation is observable in phenomena related to geophysics, blood flow, non-destructive evaluation, acoustics, hydrodynamics, and also there are other extra applications like structural health monitoring as well as ultrasonic inspection techniques in which waves are moving in a demanding direction. Now, in this article, the longitudinal direction is considered for its propagation.

There are so many beam theories which have been used to investigate the buckling, static, vibration and wave dispersion responses of continuous structures (Gao *et al*. 2019a, Al-Furjan *et al*. 2020a, b, Chikr *et al*. 2020, Khadimallah *et al*. 2020, Mou *et al*. 2020). The simplest one may be Classical Beam Theory (CBT) in which the shear deformation effect has been neglected. Since the application of materials is increasing, the requirement of more accurate beam theory is vital. Higher-order Shear Deformation Theories (HSDTs) are novel and accurate theories in which the shear deformation has been considered while in spite of First order Shear Deformation Theory models (FSDTs) Bourada *et al*. (2020), Bousahla *et al*. (2020) do not need any shear correction factor with multiple applications in analyzing beam, plates and shell structures (Karami *et al*. 2018c, Allam *et al*. 2020, Belbachir *et al*. 2020, Bendenia *et al*. 2020, Boussoula *et al*. 2020, Chikr *et al*. 2020, Khiloun *et al*. 2020, Menasria *et al*. 2020, Rabhi *et al*. 2020, Rahmani *et al*. 2020, Refrafi *et al*. 2020, Tounsi *et al*. 2020, Zine *et al*. 2020). Recently, higher-order shear deformable beam/plate models containing thickness stretching effect has been developed and used to investigate the mechanics of such structures (Shahsavari *et al*. 2018, Boutaleb *et al*. 2019, Kaddari *et al*. 2020). A size-dependent quasi-3D model was used to examined the transverse wave behavior of FG nanoplates by Karami *et al*. (2018) while similar investigation was performed for nanobeams in Ref. Karami *et al*. (2019b).

Having said that it can be seen that there are few published on investigation of the wave propagation of nanobeams. However, there is not any research on longitudinal wave propagation of isotropic nanobeam via a higher-order shear deformation beam theory containing thickness stretching effect in conjunction with bi-Helmholtz nonlocal strain gradient theory. This present work investigates the dispersive behavior of the longitudinal wave. The governing equations are derived using Hamilton's principle and afterwards solved via an analytical method. Obtained results show the effects of stiffness-softening mechanism as well as stiffness-hardening mechanism and wave number on longitudinal wave characteristics (wave frequency and phase velocity) of isotropic nanobeams. The obtained results may be beneficial in multi-conceptualization physical phenomena of beams (Gao *et al*. 2019b, c) in which wave are traveling inside longitudinal direction; these are including drug delivery (Su *et al*. 2019), geophysics (Liu *et al*. 2019, 2020b), biomedical (Hu *et al*. 2019, Lin *et al*. 2020), acoustics (Gao *et al*. 2016, 2018c, Chen *et al*. 2020, Huang *et al*. 2020), non-destructive evaluation (Li *et al*. 2014, Gao *et al*. 2018a, d), hydrodynamics (Cai *et al*. 2020a), solar cells (Zhu *et al*. 2020), thermodynamic (Cai *et al*. 2020b), and nanomechanics (Guo *et al*. 2020, Liu *et al*. 2020a).

2. Preliminary concepts and definitions

Consider a nanobeam made of isotropic material, referred to Cartesian coordinate (x, y, z), shown in Fig. 1. Let the width of the nanobeam b and the thickness h along z-direction.

2.1 Kinematic relations

The current work utilizes a higher-order model of beams to approximate the displacements in the plane as given below (Polit *et al*. 2016, Ganapathi and Polit 2017)

$$\begin{aligned} u_1(x,z,t) &= u_0(x,t) - z\frac{\partial w_0(x,t)}{\partial x} + f(z)\gamma_0(x,t) \\ u_3(x,z,t) &= w_0(x,t) + z w_1(x,t) + z^2 w_2(x,t) \end{aligned} \tag{1}$$

where $\gamma_0(x,t) = \theta(x,t) + \frac{\partial w_0(x,t)}{\partial x}$; u_0 and w_0 indicate the displacements corresponding to the mid-line point of beam; w_1 and w_2 denotes the stretching effect; moreover, θ indicated the rotation of the section; $f(z)$ represents the function of shape and is given as below.

$$f(z) = \frac{h}{\pi}\sin\frac{\pi z}{h} \tag{2}$$

Considering the displacement fields, the non-zero strain-displacement relations can be calculated by following equations.

$$\varepsilon_{xx} = \frac{\partial u_1}{\partial x}, \qquad \gamma_{xz} = \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}, \qquad \varepsilon_{zz} = \frac{\partial u_3}{\partial z} \tag{3}$$

Having Eq. (1) into the previous relations, the non-zero strains in the terms of displacements could be found as below.

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \gamma_0}{\partial x} \\ \gamma_{xz} &= z\frac{\partial w_1}{\partial x} + z^2\frac{\partial w_2}{\partial x} + \frac{\partial f(z)}{\partial z}\gamma_0 \\ \varepsilon_{zz} &= w_1 + 2zw_2 \end{aligned} \tag{4}$$

The equations of equilibrium according to the current model with fourfold coupled of axial, rotation, transverse, and stretching effects can be simply found as (Polit *et al*. 2016)

$$\frac{\partial N_{xx}}{\partial x} = -I_0\frac{\partial^2 u_0}{\partial t^2} - I_1\frac{\partial^3 w_0}{\partial x \partial t^2} - I_5\frac{\partial^2 \theta}{\partial t^2} \tag{5}$$

$$\begin{aligned} &-\left(\frac{\partial^2 M_{xx}}{\partial x^2} - \frac{\partial^2 \widetilde{M}_{xx}}{\partial x^2}\right) - \frac{\partial \tilde{Q}_{xz}}{\partial x} = I_1\frac{\partial^3 u_0}{\partial x \partial t^2} + I_2\frac{\partial^4 w_0}{\partial x^2 \partial t^2} \\ &+ I_6\frac{\partial^3 \theta}{\partial x \partial t^2} - I_0\frac{\partial^2 w_0}{\partial t^2} - I_1\frac{\partial^2 w_1}{\partial t^2} - I_2\frac{\partial^2 w_2}{\partial t^2} \end{aligned} \tag{6}$$

$$-\frac{\partial \widetilde{M}_{xx}}{\partial x} + \tilde{Q}_{xz} = -I_5\frac{\partial^2 u_0}{\partial t^2} - I_6\frac{\partial^3 w_0}{\partial x \partial t^2} - I_7\frac{\partial^2 \theta}{\partial t^2} \tag{7}$$

$$-\frac{\partial Q_{xz}}{\partial x} + N_{zz} = -I_1\frac{\partial^2 w_0}{\partial t^2} - I_2\frac{\partial^2 w_1}{\partial t^2} - I_3\frac{\partial^2 w_2}{\partial t^2} \tag{8}$$

$$-\frac{\partial \bar{Q}_{xz}}{\partial x} + 2M_{zz} = -I_2\frac{\partial^2 w_0}{\partial t^2} - I_3\frac{\partial^2 w_1}{\partial t^2} - I_4\frac{\partial^2 w_2}{\partial t^2} \tag{9}$$

where

$$\begin{aligned} [I_0, I_1, I_2, I_3, I_4] &= \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, z^2, z^3, z^4)\rho dz \\ [I_5, I_6, I_7] &= \int_{-\frac{h}{2}}^{\frac{h}{2}} (f(z), zf(z), f(z)^2)\rho dz \end{aligned} \tag{10}$$

And the moments are

$$\begin{aligned} \left[N_{xx}, M_{xx}, \bar{M}_{xx}, \widetilde{M}_{xx}\right] &= \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, z^2, f(z))\sigma_{xx} dz \\ [N_{zz}, M_{zz}] &= \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z)\sigma_{zz} dz \\ \left[Q_{xz}, \bar{Q}_{xz}, \tilde{Q}_{xz}\right] &= \int_{-\frac{h}{2}}^{\frac{h}{2}} (z, z^2, \frac{\partial f(z)}{\partial z})\tau_{xz} dz \end{aligned} \tag{11}$$

2.2 Higher order nonlocal strain gradient theory

It's well understood that there is long-range interaction between atoms when scale of structures tends to nanoscale (Wang *et al*. 2015). The nonlocal strain gradient theory (Lim *et al*. 2015) includes the nonlocality of stress field and also strain gradients by considering the stress field in the following form

$$\sigma_{xx} = \sigma_{xx}^{(0)} - \frac{\partial \sigma_{xx}^{(1)}}{\partial x} \tag{12}$$

in which $\sigma_{xx}^{(0)}$ and $\sigma_{xx}^{(1)}$ are given as below

$$\sigma_{ij}^{(0)} = \int_0^L C_{ijkl}\alpha_0(x, x', e_0 a)\varepsilon'_{kl}(x')dx' \tag{13}$$

$$\sigma_{ij}^{(1)} = \lambda^2 \int_0^L C_{ijkl}\alpha_1(x, x', e_1 a)\nabla\varepsilon'_{kl,x}(x')dx' \tag{14}$$

herein C_{ijkl} indicates the elastic coefficients; the magnitudes of the nonlocal stress field are estimated by $e_i a$; the effect of higher-order strain gradient stress field is represented by λ. According to Lim *et al*. (2015), the general higher-order nonlocal strain gradient model is founded as below

$$\begin{aligned} &[1 - (e_1 a)^2\nabla^2][1 - (e_0 a)^2\nabla^2]\sigma_{ij} = \\ &C_{ijkl}[1 - (e_1 a)^2\nabla^2]\varepsilon_{kl} - C_{ijkl}\lambda^2[1 - (e_0 a)^2\nabla^2]\nabla^2\varepsilon_{kl} \end{aligned} \tag{15}$$

where $\nabla^2 = \frac{\partial}{\partial x^2}$. For simplicity, we assume that $\mu_0 = e_0 a$, $\mu_1 = e_1 a$ in which a represents internal length's characteristic; e_i is a constant. Furthermore, $e_i a$ is an important coefficient to convince the accuracy of nonlocal models. The coefficient introduced above will estimate by matching the propagation phenomenon on the basis of the atomic models. It is interesting to note that λ represents a characteristic of the material that is often founded by particle spacing. Recently, it has counseled to find the gradient coefficient on the basis of the size of representative volume elements (RVEs). The relation of a size-dependent nano-size beam is given as follows

$$\begin{aligned} &(1 - \mu_0^2\nabla^2)(1 - \mu_1^2\nabla^2)\begin{Bmatrix}\sigma_{xx}\\ \sigma_{zz}\\ \tau_{xz}\end{Bmatrix} = \\ &(1 - \mu_1^2\nabla^2 - \lambda^2(1 - \mu_0^2\nabla^2)\nabla^2)\begin{bmatrix}C_{11} & C_{13} & 0\\ C_{31} & C_{33} & 0\\ 0 & 0 & C_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{zz}\\ \gamma_{xz}\end{Bmatrix} \end{aligned} \tag{16}$$

where

$$\begin{aligned} C_{11} &= C_{33} = \frac{E}{1 - \nu^2} \\ C_{13} &= C_{31} = \frac{\nu E}{1 - \nu^2} \\ C_{55} &= \frac{E}{2 + 2\nu} \end{aligned} \tag{17}$$

By replacing Eq. (16) into the resultants presented in Eq. (11) gives the following resultants

$$\begin{aligned} &(1 - \mu_0^2\nabla^2)(1 - \mu_1^2\nabla^2)N_{xx} = \\ &(1 - \mu_1^2\nabla^2 - \lambda^2(1 - \mu_0^2\nabla^2)\nabla^2)(A_{11}\frac{\partial u_0}{\partial x} - B_{11}\frac{\partial^2 w_0}{\partial x^2} \\ &+ E_{11}\frac{\partial \gamma_0}{\partial x} + A_{13}w_1 + 2B_{13}w_2) \end{aligned} \tag{18}$$

$$(1 - \mu_0^2\nabla^2)(1 - \mu_1^2\nabla^2)M_{xx} = \tag{19}$$

$$(1-\mu_1^2\nabla^2-\lambda^2(1-\mu_0^2\nabla^2)\nabla^2)(B_{11}\frac{\partial u_0}{\partial x}-D_{11}\frac{\partial^2 w_0}{\partial x^2} \\ +G_{11}\frac{\partial \gamma_0}{\partial x}+B_{13}w_1+2D_{13}w_2) \tag{19}$$

$$(1-\mu_0^2\nabla^2)(1-\mu_1^2\nabla^2)\bar{M}_{xx}= \\ (1-\mu_1^2\nabla^2-\lambda^2(1-\mu_0^2\nabla^2)\nabla^2)(D_{11}\frac{\partial u_0}{\partial x}-F_{11}\frac{\partial^2 w_0}{\partial x^2} \\ +J_{11}\frac{\partial \gamma_0}{\partial x}+D_{13}w_1+2F_{13}w_2) \tag{20}$$

$$(1-\mu_0^2\nabla^2)(1-\mu_1^2\nabla^2)\tilde{M}_{xx}= \\ (1-\mu_1^2\nabla^2-\lambda^2(1-\mu_0^2\nabla^2)\nabla^2)(D_{11}\frac{\partial u_0}{\partial x}-G_{11}\frac{\partial^2 w_0}{\partial x^2} \\ +K_{11}\frac{\partial \gamma_0}{\partial x}+G_{13}w_1+2J_{13}w_2) \tag{21}$$

$$(1-\mu_0^2\nabla^2)(1-\mu_1^2\nabla^2)N_{zz}= \\ (1-\mu_1^2\nabla^2-\lambda^2(1-\mu_0^2\nabla^2)\nabla^2)(A_{31}\frac{\partial u_0}{\partial x}-B_{31}\frac{\partial^2 w_0}{\partial x^2} \\ +E_{31}\frac{\partial \gamma_0}{\partial x}+A_{33}w_1+2B_{33}w_2) \tag{22}$$

$$(1-\mu_0^2\nabla^2)(1-\mu_1^2\nabla^2)M_{zz}= \\ (1-\mu_1^2\nabla^2-\lambda^2(1-\mu_0^2\nabla^2)\nabla^2)(B_{31}\frac{\partial u_0}{\partial x}-D_{31}\frac{\partial^2 w_0}{\partial x^2} \\ +G_{31}\frac{\partial \gamma_0}{\partial x}+B_{33}w_1+2D_{33}w_2) \tag{23}$$

$$(1-\mu_0^2\nabla^2)(1-\mu_1^2\nabla^2)Q_{xz}=(1-\mu_1^2\nabla^2-\lambda^2\times \\ (1-\mu_0^2\nabla^2)\nabla^2)(D_{55}\frac{\partial w_1}{\partial x}+F_{55}\frac{\partial w_2}{\partial x}+L_{55}\gamma_0) \tag{24}$$

$$(1-\mu_0^2\nabla^2)(1-\mu_1^2\nabla^2)\bar{Q}_{xz}=(1-\mu_1^2\nabla^2-\lambda^2 \\ (1-\mu_0^2\nabla^2)\nabla^2)(F_{55}\frac{\partial w_1}{\partial x}+H_{55}\frac{\partial w_2}{\partial x}+M_{55}\gamma_0) \tag{25}$$

$$(1-\mu_0^2\nabla^2)(1-\mu_1^2\nabla^2)\tilde{Q}_{xz}=(1-\mu_1^2\nabla^2-\lambda^2 \\ (1-\mu_0^2\nabla^2)\nabla^2)(L_{55}\frac{\partial w_1}{\partial x}+M_{55}\frac{\partial w_2}{\partial x}+N_{55}\gamma_0) \tag{26}$$

where

$$\begin{bmatrix} A_{ij} & B_{ij} & D_{ij} & E_{ij} \\ F_{ij} & G_{ij} & H_{ij} & J_{ij} \\ K_{ij} & L_{ij} & M_{ij} & N_{ij} \end{bmatrix} = \\ \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} 1 & z & z^2 & f(z) \\ z^3 & zf(z) & z^4 & z^2 f(z) \\ f(z)^2 & z\frac{\partial f(z)}{\partial z} & z^2\frac{\partial f(z)}{\partial z} & \frac{\partial f(z)}{\partial z}\frac{\partial f(z)}{\partial z} \end{bmatrix} C_{ij}dz \tag{27}$$

The equations of motion in the displacement's term for a nano-sized beam made of isotropic materials including

Table 1 Material properties of isotropic nanobeam

E (TPa)	v	ρ (kg/m^3)	h (nm)
1.06	0.25	2250	0.34

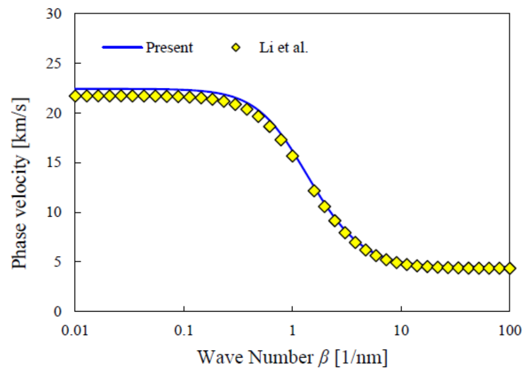


Fig. 2 Longitudinal phase velocity of isotropic nanobeam vs. wave number, μ = 1, λ = 0.2

three-small scale coefficients might be derived by placing Eqs. (18)-(26), into Eqs. (5)-(9) which are neglected here.

3. Analytical wave propagation solution

A harmonic series are applied to solve the governing equations of wave motion in isotropic nanobeam including thickness stretching. The present article assumed that the wave length is relative short and the wave cannot reach the boundary of the nanobeams. The following series are introduced to approximate the equation of wave motion as below

$$\begin{aligned} u_0 &= A_1 \exp[i(\beta x-\omega t())\,[]] \\ w_0 &= A_2 \exp[i(\beta x-\omega t())\,[]] \\ \theta &= A_3 \exp[i(\beta x-\omega t())\,[]] \\ w_1 &= A_4\, exp[i(\beta x-\omega t())\,[]] \\ w_2 &= A_5\, exp[i(\beta x-\omega t())\,[]] \end{aligned} \tag{28}$$

where A_i denote the wave amplitudes; β is the number of waves via the x-direction; ω indicates the circular frequency. To get the phase velocity, the following relation is given as below.

$$c=\frac{\omega}{\beta} \tag{29}$$

4. Numerical results

The current section develops to study the wave phenomenon in nano-sized beams by using a size-dependent beam model including thickness stretching effect. Furthermore, the non-classical model includes three-small scale coefficients. Table 1 shows the properties of used material in the current paper.

Initially, the validity and accuracy of the current solution procedure is studied. Fig. 2 illustrates a comparison between the current model with those reported by Li *et al*. (2015) using the nonlocal strain gradient Euler-Bernoulli beam model. It is indicated that the present beam model and the presented solving procedure can accurately predict wave behaviors of isotropic nanobeams.

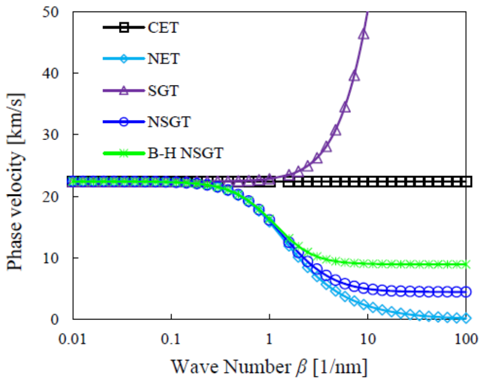
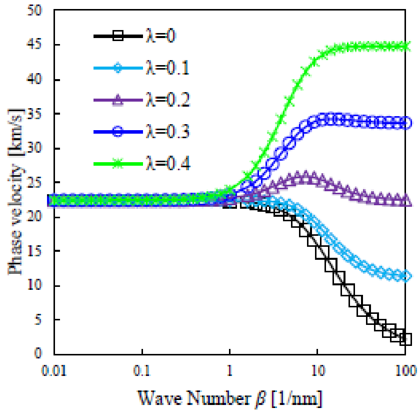


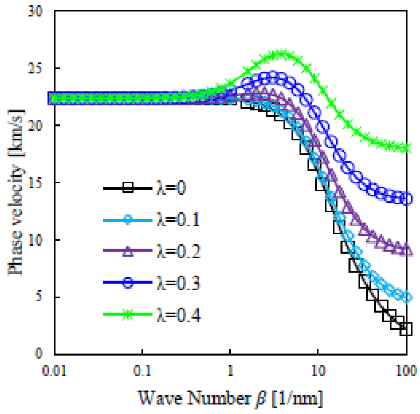
Fig. 3 Longitudinal desperation relation between phase velocity as well as wave number for various elasticity theories μ_0 = 1.0 nm, μ_1 = 0.5 nm and λ = 0.2 nm

In order to examine the different continuum theories, the phase velocity curves in relation to wave number is shown in Fig. 3. As it can be seen, the longitudinal phase velocities of different continuum theories are almost unchanged for low wave number. It may be interesting to note that the phase velocities variations are almost equal for different theories in the low wave number β < 0.1 1/nm. Hence, it can be concluded that the different continuum theories in the low wave number can provide good results. The phenomenon is also found the longitudinal waves in an axial bar (Papargyri-Beskou *et al*. 2009). It also concludes from this diagram that the phase velocities of Strain Gradient Theory (SGT) and Classical Elasticity Theory (CET) are larger than that bi-Helmholtz Nonlocal Strain Gradient Theory (B-H NSGT), while the phase velocities of NET and NSGT are smaller than those of B-H NSGT. Also, the longitudinal phase velocity of NSGT, and B-H NSGT almost unchanged when β < 0.1 1/nm, then reduces with rising wave number when 0.1 < β < 20 1/nm, and finally remains almost unchanged at about its asymptotic phase velocity when β > 20 1/nm. In addition, as can be demonstrated in Fig. 3 the longitudinal phase velocity is close to zero for NET in high wave numbers. Although, the longitudinal phase velocity for SGT always rises with increasing in the wave number.

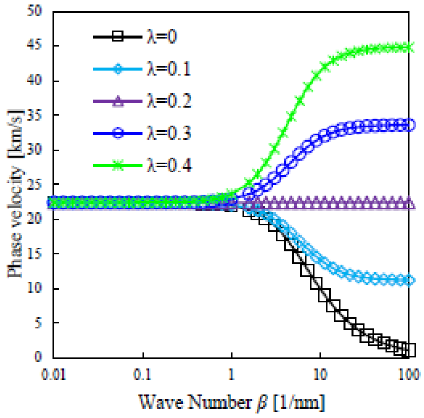
As another study to examine the effect of nonlocal parameters and strain gradient parameter, the longitudinal phase velocity variation versus wave number β is plotted in Fig. 4. As a general concept in nanostructures, the phase velocity rises with an increment in value of wave number until a recognized value of such number. In more precise, the behavior of longitudinal phase velocity after that recognized wave number relies on nonlocal parameters and strain gradient parameter impact. It can be seen that strain gradient parameter λ does not have any considerable effect on longitudinal phase velocity behavior at low wave numbers, while phase velocities have influenced by any small-scale parameters at high level of wave numbers. The phase velocity of the nanobeam decreases by increasing the wave number when both nonlocal coefficients are higher than the strain gradient one (μ_0, μ_1 > λ), while the phase velocity increased by increasing the wave number for



(a) μ_0 = 0.1, μ_1 = 0.2



(b) μ_0 = 0.1, μ_1 = 0.5



(a) μ_0 = 0.2, μ_1 = 0.2

Fig. 4 Longitudinal phase velocity of isotropic nanobeam versus wave number for multiple nonlocal and strain gradient parameters

reverse conditions (λ > μ_0, μ_1). It is because of the strain gradient coefficient magnitude in enhancing the rigidity of the system. Furthermore, it has been observed that the higher-order nonlocal coefficient is more effective in decreasing the results compared to the lower-order one. Thus, having both nonlocal coefficients and a strain gradient parameter is more crucial in analyzing the wave phenomena.

The propagation of elastic bulk waves in a nanobeam is studied for both frequency and phase velocity considering the magnitude of lower-order nonlocal parameters and the

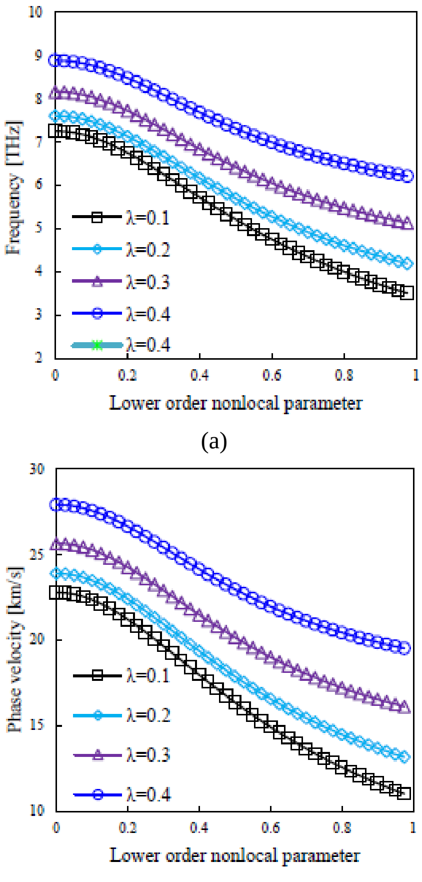


Fig. 5 Frequency (a) and phase velocity (b) variations of isotropic nanobeam versus lower order nonlocal parameter for different strain gradient parameter (β = 2 1/nm, μ_1 =0.2 nm)

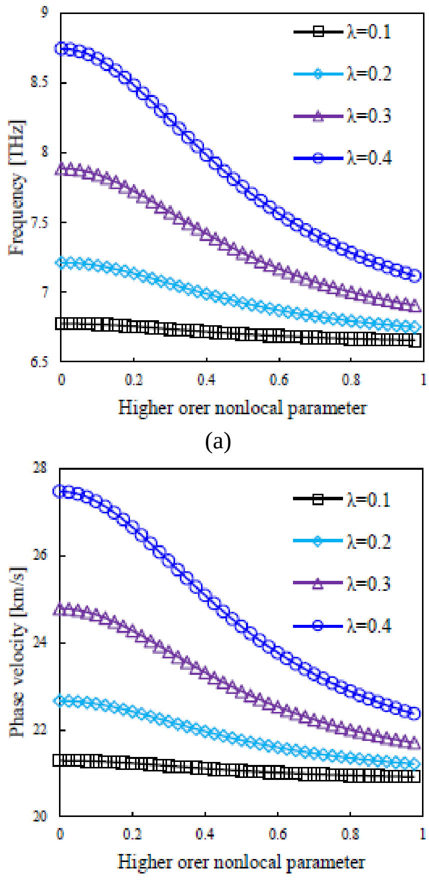


Fig. 6 Frequency (a) and phase velocity (b) variations of isotropic nanobeam versus higher order nonlocal parameter for different values of strain gradient parameter at β = 2 1/nm when μ_0 = 0.2 nm

result are shows in Fig. 5. Also, the influences of different strain gradient coefficients are captured. A constant wave number β = 2 1/nm is utilized. It has been observed that the wave frequency as well as phase velocity are decreased for all strain gradient coefficients by increasing the lower-order nonlocal coefficient. However, the wave response which is demonstrated by frequency and phase velocity increases by rising the strain gradient coefficient.

Fig. 6 shows the influence of the higher-order nonlocal coefficient on the wave dispersion of the nanobeam. Again, the effect of different strain gradient coefficient is considered. As reported for the lower-order nonlocal coefficient in the above paragraph, the higher-order one has the same effect on the elastic bulk wave response. In another world, increasing the higher-order nonlocal coefficient decreases the frequency and phase velocity for all reported strain gradient coefficients. By a comparison between Figs. 5 and 6, it can be concluded that both nonlocal coefficients have same effects on the response in trend.

5. Conclusions

Elastic waves in a controlled domain was studied using a higher-order shear deformation theory considering thickness stretching effect. From the best author's knowledge, it was the first time that a combination of the general nonlocal strain gradient theory and the current beam model is utilized to model the nano-size beams. In the current work was shown that the small-scale coefficients play an important role in reported results after a specific wave number, while both of nonlocality and strain gradient size-dependency were ignorable parameters at small wave numbers. The longitudinal phase velocities based on high wave numbers close to zero and infinity for NET and SGT, respectively. Furthermore, the magnetite of strain gradient parameter deepens on nonlocal parameters. Also, the stiffness-softening effect induced by the higher order nonlocal parameter is physically more effective than the effect of the lower order nonlocal one on longitudinal phase velocities. These results can be used in design of nanostructures which are dealing with traveling of waves in longitudinal direction.

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