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Parametric vibration analysis of single-walled carbon nanotubes based on Sanders shell theory

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Abstract. This paper based on Sanders theory aims to investigate the vibration of SWCNTs considering the clamped-simply supported, clamped-free, clamped-clamped and simply supported-simply supported end conditions. After developing the governing equation of the objective system, the Rayleigh-Ritz technique is implemented for the purpose of obtaining the frequency equation in the eigen form. In addition, the applicability of this model for the analysis of vibration of CNTs is examined with the effect of length and ratio of height-to-radius. A detailed description of different types of SWCNTs with different indices is provided in the theoretical methodology. The effect of extended length is stimulated with increasing the radii and the model is effective because it also predicts the effect of thickness on vibration of SWCNTs. For different boundary conditions, the present results are verified with earlier literature.

Keywords: SWCNTs; Sanders theory; boundary conditions; material parameters; vibration analysis

1. Introduction

Carbon nanotube as one of the most applicable miniature structure attracts many researchers in order to analytically and experimentally probes its dynamical properties using the nonlocal beam theory. Several researchers studied linear and nonlinear vibrations of the nanostructures utilizing the Eringen's nonlocal elasticity theory (Eringen 2002). They mostly focused on the free vibrational analysis of the nano-structure, specially, carbon nanotubes.

In addition, nano-structures can be mentioned as the important types of devices which have wide applications in a variety of technological and scientific fields. The nonlinear forced vibration of carbon nanotubes has seldom been observed. However, this issue is very crucial due to the widespread application of the forced nonlinear vibration carbon nanotubes in many practical instruments. The nonlocal elasticity introduced by Eringen (2002) becomes a turning point as small scale effect was inculcated in to fundamental equations as simply material parameter. Akbaş (2017a) investigated the free vibration analysis of edge cracked cantilever microscale beams composed of Functionally Graded Material (FGM) based on the Modified Couple Stress Theory (MCST). The material properties of the beam are assumed to change in the height direction according to the exponential distribution. The cracked beam is modeled as a modification of the classical cracked-beam theory consisting of two sub-beams connected by a massless elastic rotational spring.

Therefore, scientific community now propose to apply nonlocal continuum models to investigate nano-structured materials (Sudak 2003, Wang and Varadan 2006, Pradhan and Phadikar 2009, Ansari *et al*. 2013). Flügge (1962) have been two substantial shell theories practiced extensively in study of static and dynamic characteristics of CNTs. Flügge shell theory takes promising place to generate remarkably accurate developments to examine the CNTs. The existence of long range interactions in materials is the basic reason of application of nonlocal theory. The first ever work presented on use of nonlocal elasticity was by Peddieson *et al*. (2003). Prominent computational competence and accuracy makes nonlocal models an attractive choice for further advancements in field. Manevitch *et al*. (2017) demonstrated the new specific phenomenon of the long-time resonant energy exchange in the Carbon Nanotubes (CNTs) in the two optical branches-the Circumferential Flexure Mode (CFM) and Radial Breathing Mode (RBM).

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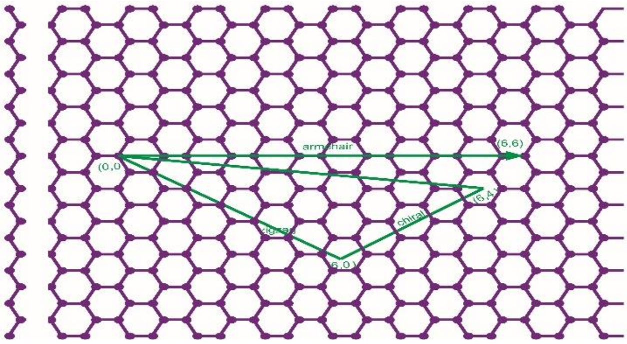
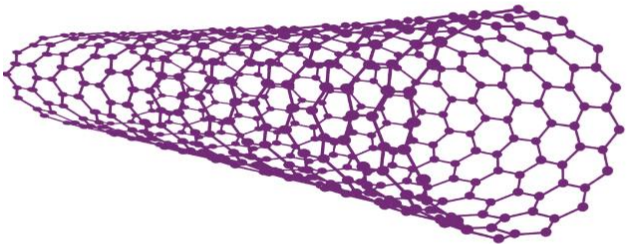
It is shown that the modified nonlinear Schrödinger equation, obtained in the framework of nonlinear elastic thin shell theory, allows to describe the CNT nonlinear dynamics connected with considered frequency bands. Comparative analysis of the oscillations of the CFM. Wang and Varadan (2006) introduced new modeling for vibration of CNTs and to find the critical buckling strain and tube thickness. Natsuki *et al*. (2007) carried out the vibration analysis of nested CNTs in elastic matrix. Strozzi and Pellicano (2018) investigated the the linear vibrations of Triple-Walled Carbon Nanotubes (TWNTs). A multiple elastic thin shell model is applied. The TWNT dynamics is studied in the framework of the Sanders-Koiter shell theory. The van der Waals interaction between any two layers of the TWNT is modelled by a radius-dependent function. The shell deformation is described in terms of longitudinal, tangential and radial displacements. Simply supported, clamped and free boundary conditions are applied. Flügge shell theory again had been engaged to establish administrative shell equations while proposed method was wave propagation. Natsuki *et al.* (2007) investigated single and double-walled CNTs filled with fluids by adopting wave propagation approach. Flügge shell theory was proposed to form governing equations of motion for CNTs. Lee and Chang (2008) analyzed the vibration mode shape and frequency of fluid-filled SWCNTs. It is found that mode shape and frequency are influenced significantly by the nonlocal parameters. Strozzi *et al*. (2018) analysed the nonlinear vibrations and energy exchange of Single-Walled Carbon Nanotubes (SWNTs). The Sanders-Koiter shell theory is used to model the nonlinear dynamics of the system in the case of finite amplitude of vibration. The SWNT deformation is described in terms of longitudinal, circumferential and radial displacement fields. Simply supported, clamped and free boundary conditions are applied. The resonant interaction between radial breathing (axisymmetric) modes (RBMs) is analysed.

Ke *et al*. (2009) investigated free nonlinear vibrations of double-walled CNT and applied differential quadrature technique to derive frequency equations. On the other side, for length scale coefficient and soft elastic medium with embedded carbon nanotube, the nonlocal frequencies are comparatively lower. It is also found that the frequencies of the nonlocal model at different stages of temperature are higher than the nonlocal with same temperature. Eringen nonlocal theory and Von-Karman geometry were fully studied by Yang *et al*. (2010). Akbaş (2019) presented the axially forced vibration of a cracked nanorod under the harmonic external dynamically load. In the constitutive equation of problem, the nonlocal elasticity theory, the crack is modelled as an axial spring in the crack section. In the axial spring model, the non-rod separates two sub-nanorods and the flexibility of the axial spring represents the effect of the crack. Strozzi and Pellicano (2019) presented the dynamical properties of Single-Walled Carbon Nanotubes (SWCNTs) and nonlinear modal interaction and energy exchange are analysed in detail. Resonance interactions between two conjugate Circum-ferential Flexural Modes (CFMs) are investigated. The nanotubes are analysed through a continuous shell model, and a thin shell theory is used to model the dynamics of the system; free-free boundary conditions are considered.

Rouhi *et al*. (2012) executed the axial buckling of double-walled CNT subject to various layer-wise conditions by using Rayleigh-Ritz based upon nonlocal Flügge shell theory. Their study showed that the number of different layer-wise boundary conditions dominates the choice of values for nonlocal parameter. Ansari and Rouhi (2013) summarized the effect of small scale, geometrical parameter and layer-wise end conditions of double-walled CNT by adopting Flügge Shell Model (FSM). They depicted that the continuum model considering the nonlocal effect compels the short double-walled CNT more flexible. Akbaş (2017b) presented the forced vibration responses of Functionally Graded (FG) nanobeams are presented for MCST with damping effect. The FG nanobeam is excited by a transverse triangular force impulse modulated by a harmonic motion. Mechanical properties of FG beam depend on the position. The Kelvin-Voigt model is considered in the damping effect. In solution of the dynamic problem. Moreover, Benguediab *et al*. (2014) explored the features of zigzag double-walled CNT. A comprehensive research presented by Brischotto (2015) to analyze the vibration characteristic of double-walled CNT by considering shell continuum model. The findings of article were evolved around effects of van der Waals interaction in terms of frequency ratio. Akbaş (2018a) conducted the forced vibration responses of a cantilever nanobeam using modified couple stress theory with damping effect. The crack is modeled with a rotational spring. The Kelvin-Voigt model is considered in the damping effect. In solution of the dynamic problem, finite element method is used within Timoshenko beam theory in the time domain. Influences of the geometry, crack and material parameters on forced vibration responses of cracked nanobeams are examined and discussed. Arani and Kolahchi (2016) used the nonlinear buckling of SWCNTs and the mixture rule was employed for buckling analysis of embedded CNTs with Euler and Timoshenko beam model. The influence of geometrical parameter and elastic foundation with different boundary conditions was investigated. Bilouei *et al*. (2016) and Zamanian *et al*. (2017) studied the buckling behavior of concrete columns with nanofiber reinforced polymer and SiO_2 nano-particles. By using the strain-displacements, Hamilton's principles and Mori-Tanka approach, the governing equation was derived. Numerical results were presented with the variation of elastic foundations. Akbas (2018b) investigated the forced vibration analysis of a cracked functionally graded microbeam using modified couple stress theory with damping effect. Mechanical properties of the functionally graded beam change vary along the thickness direction. The crack is modelled with a rotational spring. The Kelvin-Voigt model is considered in the damping effect. Strozzi *et al*. (2020) studied the nonlinear resonance interaction and energy exchange between bending and circumferential flexure modes in single-walled carbon nanotubes. First, the results of an analytical model of the resonance interaction between the considered nonlinear normal modes previously



(a)



(b)

Fig. 1 (a) Graphene sheet with indices (b) SWCNT

developed are reported. This approach was based on a reduced form of the Sanders-Koiter thin shell theory obtained by using simplifying hypotheses on the shell deformations.

Yazid *et al*. (2018) presented the new refined plate theory by employing nonlocal small effects. By using the principle of virtual displacements, the nonlocal relation for equation of motion was obtained. The results presented here may provide a useful design for nanostructures. In another study the viscoelastic effects of the medium were also studied using Kelvin model for the medium surrounding microtubules (MTs) but for the MTs the same classical orthotropic elastic shell model was used (Safeer *et al*. 2019). Akbaş (2018c) explored the static bending of an edge cracked cantilever nanobeam composed of FGM subjected to transversal point load at the free end of the beam based on modified couple stress theory. Material properties of the beam change in the height direction according to exponential distributions.

Many researchers directly used the classical theory for the structure of CNTs (He *et al*. 2005, Hu *et al*. 2008, Gibson *et al*. 2007, Ghavanloo *et al*. 2010, Yoon *et al*. 2002). The use of wave propagation approach is important for the study of nanostructures to develop a new formulism with different theories. In this approach, eigenvalue form is developed with the help of axial modal function in matrix representation. With the help of computer software MATLAB, frequencies of SWCNTs are extracted. The formulation of WPA is given by Zhang *et al*. (2001), a brief yet simple explanation first time. Due to the exactness of this approach, some researchers have been used for the vibration of CNT/shells (Simsek 2010, Narendar 2011, Hussain *et al*. 2017).

In the present analysis, effects of length and thickness to radius ratio are also examined using Sanders shell theory. An eigenvalue problem is framed by employing Rayleigh-Ritz formulation. Axial modal fields are measured by characteristics beam functions derived from the beam equation for various boundary conditions. The present CNTs problem is solved by adopting Rayleigh-Ritz approach. Axial modal deformation is simulated by employing characteristics beam functions. On substituting these functions, we get ordinary differential equations in three unknowns, which are functions of axial space variable. These functions have property to satisfy end conditions prescribed at edge of CNTs. For present CNT problem, vibration characteristics are analyzed for clamped –clamped (C–C), simply supported–simply supported (SS–SS) and clamped–free (C–F) edge conditions. The frequencies of three different types of SWCNTs are calculated.

2. Mathematical formulation

The lattice interpretation indices (m, n) for chiral nanotubes can be denoted as (m, n) for $n \neq m$, for zigzag as (m, 0) for $n = 0$ and armchair, nanotubes can be denoted as (m, m) for $n = m$ as shown in Fig. 1. Due to these indices, a large number of likely parameter arrangements can occur. In our present case, we have performed calculations, using Sanders shell theory based on Rayleigh-Ritz technique, for eigen-frequencies of different indices (7, 2), (9, 1), (13, 4), (13, 0), (15, 0), (17, 0), for chiral and zigzag SWCNTs, respectively and also indices for armchair SWCNTs are as (4, 4), (6, 6), (8, 8).

Suppose R and L represent radius and length of the tube respectively. h stands for tubel thickness and a_i denotes position of ith ring support in longitudinal direction of tube. An orthogonal coordinate system (x, θ, z) is supposed to be placed at tube mid surface. x, θ, z represent axial, circumferential and radial coordinates of tube respectively. The functions u, v, w yield the deformation displacements in axial, circumferential and radial directions respectively.

There are a number of shell theories found in open literature on shell/ tube problem. For present case, the formulas for reference surface strains and curvatures are adopted from Budiansky and Sanders (1963) first-order linear thin shell theory for simplicity. The strain components of strain vector $\{e\}$ are written in terms of the thickness variable $z$with linear expressions

$$e_x = e_1 + z\kappa_1, \qquad e_\theta = e_2 + z\kappa_2 \\ e_{x\theta} = \gamma + 2z\tau \tag{1}$$

The surface strains and curvatures are given as

$$\{e_1, e_2, \gamma\} = \left\{\frac{\partial u}{\partial x}, \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right), \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right\} \tag{2}$$

$$\{\kappa_1, \kappa_2, \tau\} = \left\{\begin{matrix} -\dfrac{\partial^2 w}{\partial x^2}, -\dfrac{1}{R^2}\left(\dfrac{\partial^2 w}{\partial \theta^2} - \dfrac{\partial v}{\partial \theta}\right), \\ -\dfrac{1}{R}\left(\dfrac{\partial^2 w}{\partial x \partial \theta} - \dfrac{\partial v}{\partial x}\right) \end{matrix}\right\} \tag{3}$$

The tube strain energy U is given by

$$U = \frac{R}{2}\int_0^L \int_0^{2\pi} [A_{11}e_1^2 + A_{22}e_2^2 + 2A_{12}e_1e_2 + A_{66}\gamma^2 + 2B_{11}e_1\kappa_1 + 2B_{12}e_1\kappa_2 + 2B_{12}e_2\kappa_1 + 2B_{22}e_2\kappa_2 + 4B_{66}\gamma\tau + D_{11}\kappa_1^2 + D_{22}\kappa_2^2 + 2D_{12}\kappa_1\kappa_2 + 4D_{66}\tau^2]\, d\theta dx \quad (4)$$

The following form of strain energy (U) in terms of u, v and w as

$$U = \frac{R}{2}\int_0^L \int_0^{2\pi} \left[A_{11}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{A_{22}}{R^2}\left(\frac{\partial v}{\partial \theta} + w\right)^2 + \frac{2A_{12}}{R}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial \theta} + w\right) + A_{66}\left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right)^2 - 2B_{11}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) - \frac{2B_{12}}{R^2}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) - \frac{2B_{12}}{R}\left(\frac{\partial v}{\partial \theta} + w\right)\left(\frac{\partial^2 w}{\partial x^2}\right) - \frac{2B_{22}}{R^3}\left(\frac{\partial v}{\partial \theta} + w\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) - \frac{4B_{66}}{R}\left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right)\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right) + D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{D_{22}}{R^4}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right)^2 + \frac{2D_{12}}{R^2}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) + \frac{4D_{66}}{R^2}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right)^2\right] \quad (5)$$

Energy of a system consists of two basic forms due to its position and motion, i.e., strain and kinetic energies respectively. Strain energy of a tube has been described in the above relation Eq. (5), where its counterpart, i.e., kinetic energy, symbolized by T, involved three physical basic quantities, mass, velocity and time is given as

$$T = \frac{R}{2}\int_0^L \int_0^{2\pi} \rho_T \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] d\theta\, dx \quad (6)$$

where ρ_T is mass density found in unit length and defined as

$$\rho_T = \int_{-h/2}^{h/2} \rho dz$$

The Lagrangian energy functional Π of a circular cylindrical tube is expressed as

$$\Pi = T - U \quad (7)$$

i.e., the difference between kinetic and strain energies of a tube is the Lagrangian energy functional.

The Rayleigh-Ritz method is considered as a variational approach. This technique has been used by several researchers to investigate the vibration characteristics of tube. In present work, the Rayleigh-Ritz technique is employed to analyze vibration characteristics of SWCNTs.

For linear systems of partial differential equations, the method of separation of variables is generally employed to such systems. For the present situation, the partial differential equation system involves three unknown functions u, v, w, which denote the tube deformation in axial, circumferential and radial directions respectively. Their modal displacement shapes are taken to be as follows

$$\begin{aligned} u(x,\theta,t) &= A_m \frac{d\varphi}{dx} \cos n\,\theta \sin \omega\, t \\ v(x,\theta,t) &= B_m \varphi(x) \sin n\,\theta \sin \omega\, t \\ w(x,\theta,t) &= C_m \varphi(x) \cos n\,\theta \sin \omega\, t \end{aligned} \quad (8)$$

where A_m, B_m and C_m are constants which denote the vibration amplitudes in the x, θ and z directions respectively. The axial function $\varphi(x)$ represents axial modal displacement shapes and satisfies the geometric boundary conditions, n denotes the circumferential wave number and ω is circular vibration frequency.

This functional is minimized with regard to the vibration amplitudes A_m, B_m and C_m as

$$\frac{\partial \Pi}{\partial A_m} = \frac{\partial \Pi}{\partial B_m} = \frac{\partial \Pi}{\partial C_m} = 0 \quad (9)$$

After solving the equations, the algebraic equations can be written in matrix notation and are converted into eigenvalue problem.

$$\begin{bmatrix} \dagger_{11} & \dagger_{12} & \dagger_{13} \\ \dagger_{12} & \dagger_{22} & \dagger_{23} \\ \dagger_{13} & \dagger_{23} & \dagger_{33} \end{bmatrix} \begin{bmatrix} A_m \\ B_m \\ C_m \end{bmatrix} = \rho_t \omega^2 \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_9 & 0 \\ 0 & 0 & I_{11} \end{bmatrix} \quad (10)$$

MATLAB package is used to solve the eigenvalue Eq. (10) for investigating tube vibrations. MATLAB provides an interactive working environment to carry out the quite complex computational tasks with a few commands. With the help of simple command, one gets both the eigenvalues representing CNTs natural frequencies and eigenvectors correspond to mode shapes.

Table 1 Comparison of natural frequencies (Hz) of a simply supported isotropic CNTs

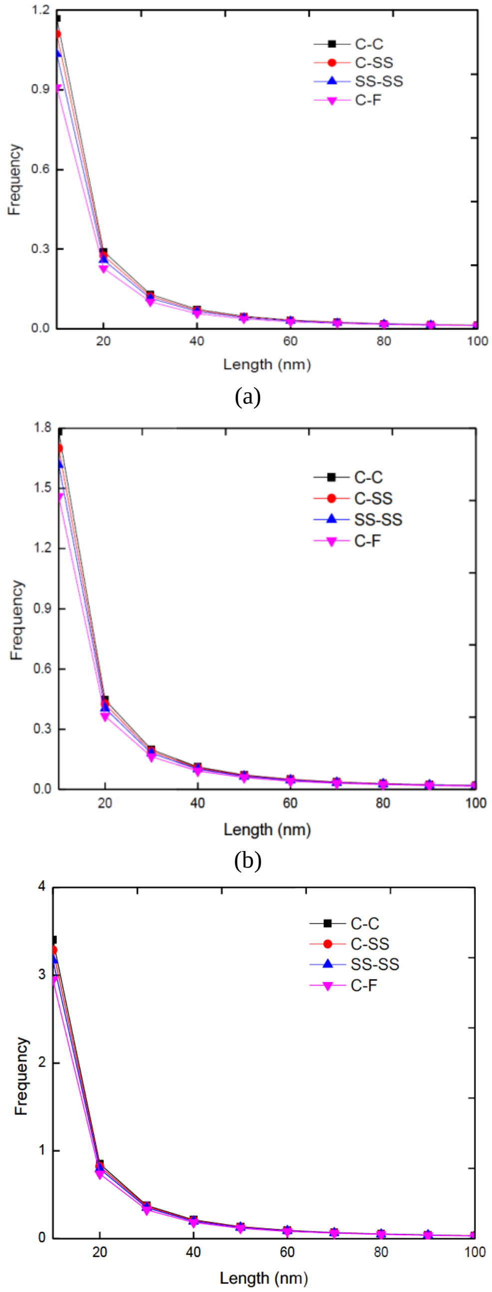
n	Naeem and Sharma (2000)	Present
4	287.59	287.58
5	201.85	201.84
6	166.59	166.56
7	166.22	166.20
8	189.29	189.27

3. Results and discussions

With the increasingly use of carbon nanotubes in various fields of engineering and technology, their vibration characteristics are analyzed numerically before their practical applications. The numerical analysis being a progressing field of research in applied mathematics, approximate techniques is developed to study a tube problem. A fast and vigorous method is favoured to obtain accurate results. The Rayleigh-Ritz method is employed to perform the present tube vibration problem. A few

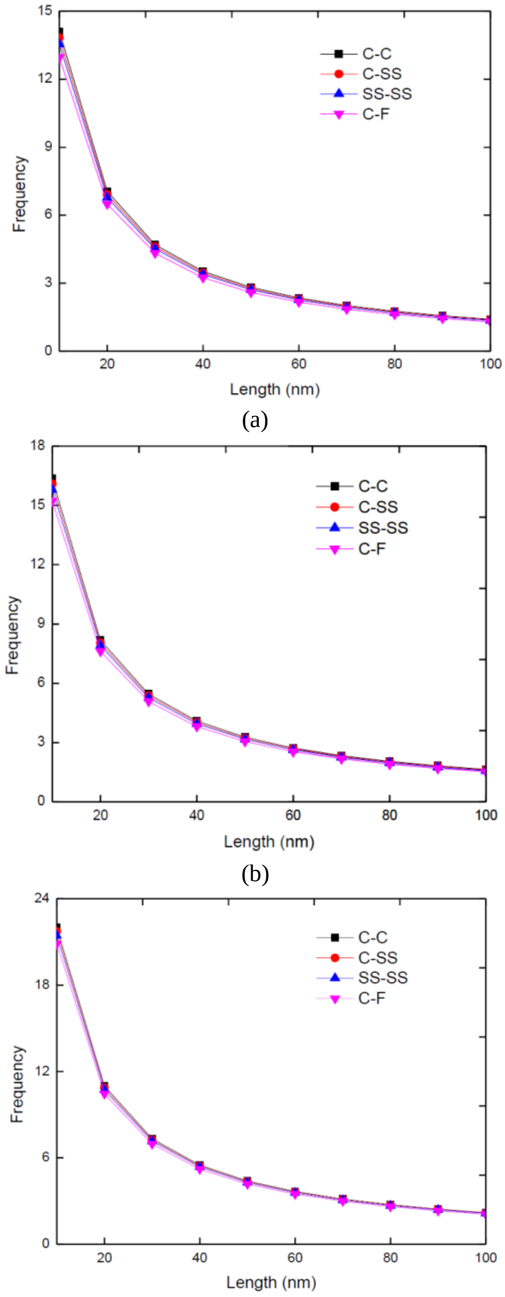
Table 2 Comparison of frequency parameter $\Omega = \omega R\sqrt{(1-\upsilon^2)\rho/E}$ of an isotropic CNTs with clamped-clamped conditions (m = 1, L/R = 20, h/R = 0.01, v = 0.3)

n	Zhang *et al.* (2001)	Present
1	0.03487	0.034395
2	0.014052	0.014256
3	0.022725	0.022713
4	0.042271	0.042216
5	0.068116	0.068051



(c)

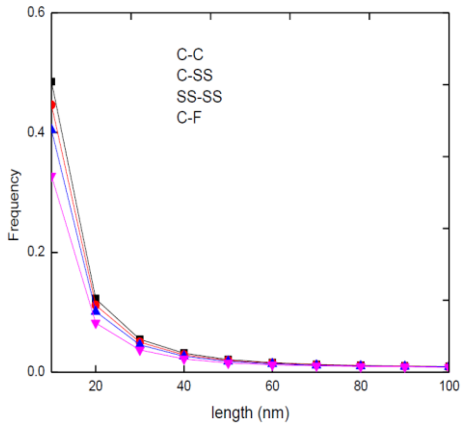
Fig. 2 Frequency distribution through length variation of chiral SWCNTs (a) (7, 2); (b) (9, 1); (c) (13, 4) for various edge conditions



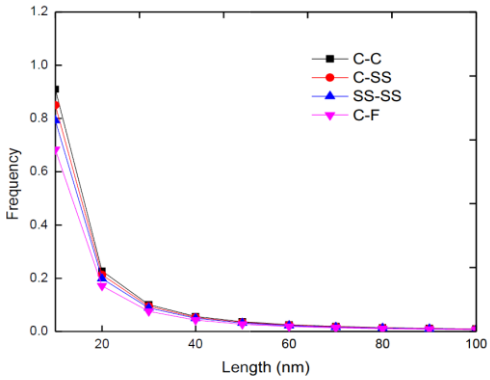
(c)

Fig. 3 Frequency distribution through length variation of zigzag SWCNTs (a) (13, 0); (b) (15, 0); (c) (17, 0) for various edge conditions

comparisons of numerical results are done to check the validity, accuracy and robustness of numerical procedure for different boundary conditions. For this purpose, vibration frequencies of SWCNTs for isotropic tube are compared with those results found in the open literature. Table 1 shows a comparison of natural frequencies for an isotropic tube with those results determined by Naeem and Sharma (2000). The simply supported edge conditions are described on both tube ends. The circumferential wave numbers n are considered to be from 4 to 8 for half axial wave mode, m = 1. It is observed that the two sets of results agreed very well with each other. In Table 2, the frequency



(a)



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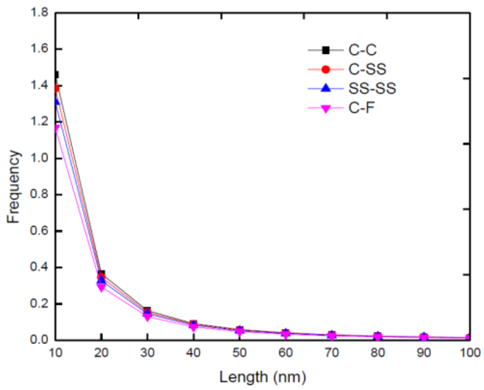
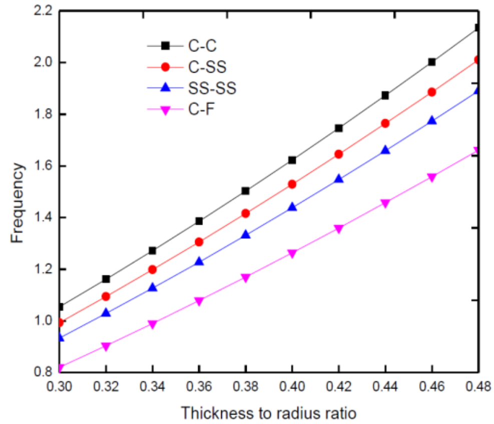
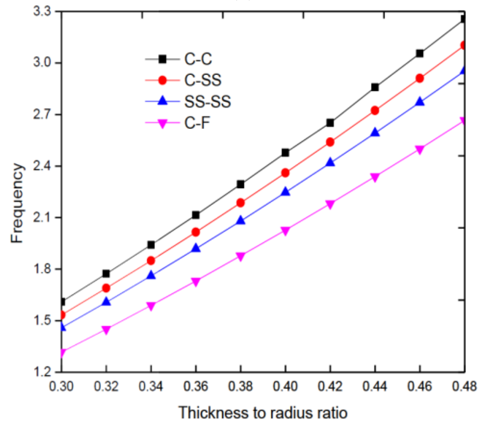


(c)

Fig. 4 Frequency distribution through length variation of armchair SWCNTs (a) (4, 4); (b) (6, 6); (c) (8, 8) for various edge conditions



(a)



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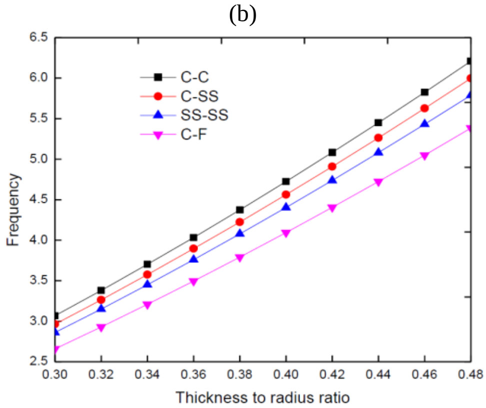

(c)

Fig. 5 Frequency distribution through thickness to radius ratio (a) (7, 2); (b) (9, 1); (c) (13, 4) for various edge conditions

parameters $\Omega = \omega R\sqrt{(1-\upsilon^2)\rho/E}$ of isotropic tube with clamped ends are compared with those calculated by Zhang *et al*. (2001). The comparison shows that an excellent agreement between two sets of frequency parameters is observed. Figs. 2(a)-(c) show the variation of frequencies versus length of chiral (7, 2), (9, 1), (13, 4), (8, 3), (10, 2) and (14, 5) with various boundary conditions. For all boundary conditions, fist frequencies fall down, then parallel and after seems linear. It is observed that there is minute difference with different boundary conditions. The frequencies are more visible as compared to zigzag case. It is can be seen from these figures that the FNF values of chiral (13, 4), are higher than those of (7, 2) and (9, 1). Figs. 3(a)-(c) show the variation of frequencies versus length of zigzag (13, 0), (15, 0), (17, 0) with various boundary conditions. It shows that the natural frequencies decreased as L is enhanced by these boundary conditions. It is observed that there is minute difference with different conditions. The frequencies of CF are lower than other boundary conditions. It is can be seen from these figures that the FNFs values of zigzag (13, 0) are lower than those of FNFs of (15, 0) and (17, 0). Variation of frequencies versus length for indices (4, 4), (6, 6) and (8, 8) with BCs has been plotted in graph of Figs. 4(a)-(c). The value of fundamental frequency decreases on increasing the length

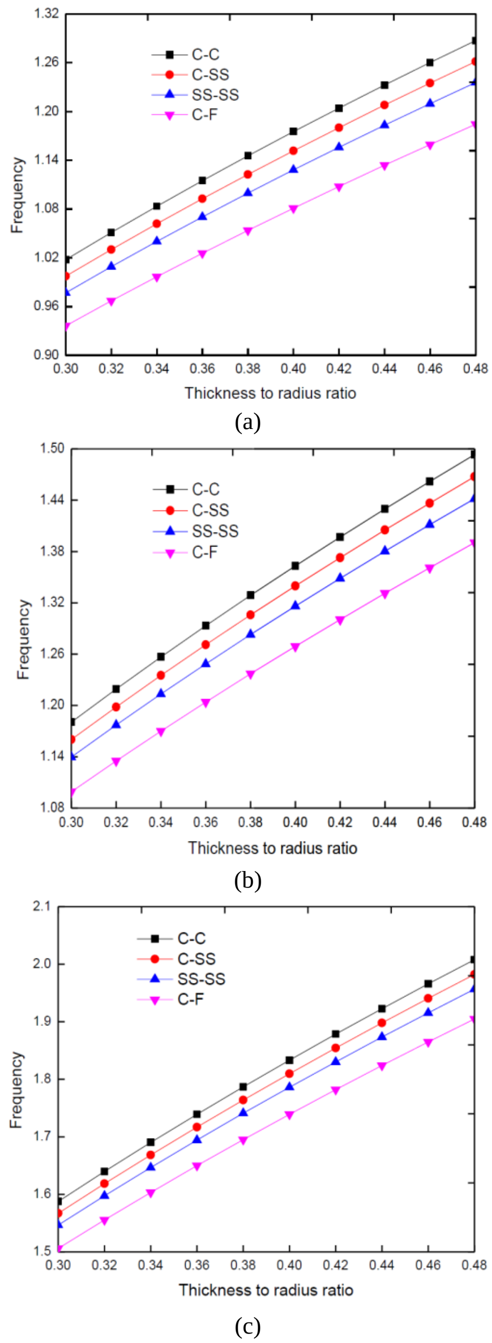

Fig. 6 Frequency distribution through thickness to radius ratio of zigzag SWCNTs (a) (13, 0); (b) (15, 0); (c) (17, 0) for various edge conditions

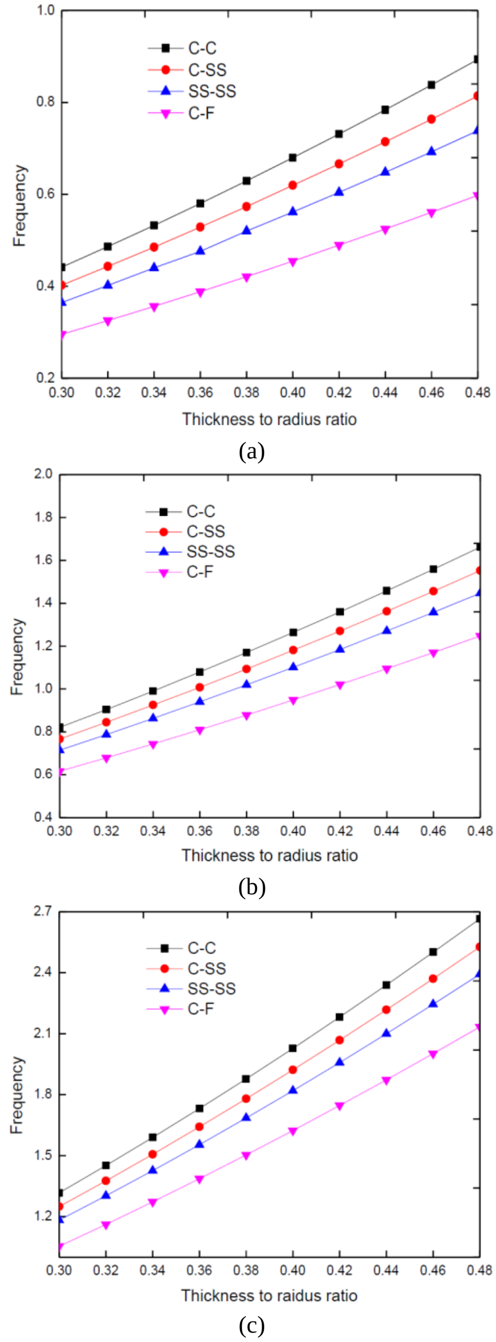

Fig. 7 Frequency distribution through thickness to radius ratio of armchairs SWCNTs (a) (4, 4); (b) (6, 6); (c) (8, 8) for various edge conditions

of the tube. The graphical behavior of frequencies versus aspect ratio/length-to-diameter ratio with different system's boundary conditions. The convergence and validity are achieved between two studies. In addition, when the tube length increases from 10 nm to 20 nm, the frequency decreases rapidly, while for the length (L = 20-30 nm), the frequency is gently parallel. In present result, the frequencies are insignificant at length (L = 70 nm) and these frequencies meet at length (L = 80 nm). Again, these results show a good coincidence. It shows that natural frequencies decrease as L is increased, for these boundary conditions. For long SWCNTs, it can be seen that the effect of BCs is insignificant and more prominent at shorter length (L = 10-40 nm) and moderately negligible at length (L = 80-100 nm). The results are given only for different eigen-frequencies for radius 678 nm, 1356 nm and 2034 nm for armchair, zigzag and chiral with length, and also ratio of thickness-radius for armchair, zigzag and chiral.

Figs. 5(a)-(c) show the variation of frequencies versus ratio of height-to-radius of chiral (7, 2), (9, 1) and (13, 4) with various boundary conditions. The frequencies go up as h/R is enhanced for these boundary conditions. At 0.30 to 0.48 all the frequencies are parallel. In these figures, it can be seen that the CS, SS-SS are sandwich between CC and

CF boundary conditions. The FNFs of (13, 4) is higher than those of (7, 2) and (9, 1). Figs. 6(a)-(c) depict the frequency effects of various boundary conditions for zigzag indices (13, 0), (15, 0), (17, 0), against thickness to radius ratio. The frequencies enlarge on increasing the height-to-radius ratio. All the frequencies are parallel for different boundary conditions. The frequency pattern of C-F condition is less than all other conditions. It is also observed that the FNFs of zigzag (14, 0) lies between the frequencies of (14, 0) and (19, 0). Figs. 7(a)-(c) shows the variation of frequencies versus ratio of height-to-radius of chiral (12, 0), (14, 0) and (19, 0) with various boundary conditions. As *h/R* is enhanced, the frequencies go up for these boundary conditions. At 0.30 to 0.48 all the frequencies are parallel. In these figures, it observed that the gap between the CC, CS, SS-SS is small than that of CF boundary condition. It is also observed that the FNFs of zigzag (14, 0) lies between the frequencies of (14, 0) and (19, 0).

4. Conclusions

In this paper, the governing equation of motion using Rayleigh-Ritz technique is written in the form of eigen value to extract the frequencies of CNTs. The effects of different physical and material parameters on the fundamental frequencies are investigated for armchair, zigzag and chiral SWCNTs invoking Sanders shell theory. However, for the analysis of structures at higher frequencies of length and ratio of thickness-to-radius under different boundary conditions is analyzed. The fundamental natural frequency spectra for three forms of SWCNTs are calculated and obtained for numerous material parameters like ratio of length and thickness to radius ratio. Computer software MATLAB is utilized for the frequencies of SWCNTs and present results show excellent stability across a wide range of parameters. The frequency response is investigated for different indices of chiral, zigzag and armchair SWCNTs (7, 2), (9, 1), (13, 4) and (13, 0), (15, 0), (17, 0) and (4, 4), (6, 6), (8, 8), respectively. The frequencies increase on decreasing the length and decreases on decreasing the height to radius ration. The frequency behavior of length and height to radius ratio is counterpart. Using various nano-material parameters, the vibration results are given in tabular and graphical form. In all frequency curves, it can be seen that the CS, SS-SS are sandwich between CC and CF boundary conditions. It is thus desirable to produce more precise estimations of the vibrational shapes and frequencies of CNTs. For future predictions, this model is used for the vibration behavior of nanoplates.

Declaration of conflicting of interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article

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