Smart Structures and Systems, *Vol. 36, No. 1 (2025) 39-50*
https://doi.org/10.12989/sss.2025.36.1.039

Adaptive time-delay compensation method combined with sliding mode control: Theory and verification

Yunhai Zeng [1], Huimeng Zhou [2], Yucai Chen [1], Yu Guo [3], Ping Tan [*1] and Zhen Wang [**3]

[1] *School of Civil Engineering and Transportation, Guangzhou University, Guangzhou 510006, China*
[2] *Earthquake Engineering Research & Test Center, Guangzhou University, Guangzhou 510006, China*
[3] *School of Civil Engineering and Architecture, Wuhan University of Technology, Wuhan 430070, China*

(*Received October 10, 2024, Revised June 29, 2025, Accepted August 3, 2025*)

Abstract. Delay compensation presents a key component to ensure an accurate and robust real-time hybrid test (RTHT). As RTHT continues to evolve, the scale and degrees of freedom of studied structures increase, requiring greater attention to the responses of higher-order modes. High-frequency components in structural responses at the boundaries of substructures must also be accurately imposed onto the specimen. Robust delay compensation for high-frequency signals loading in RTHT needs to be explored. This paper proposes an adaptive time delay compensation based discrete model strategy combined with sliding mode control method (ADM-SMC). In this method, combining with the robust sliding mode control (SMC), the least squares method is used to update the parameters of the discrete model of the SMC-loading system in real time, which provides a robust compensation method for the wide-band and high-frequency excitation loading. A delay compensation test and a RTHT were conducted to verify the feasibility of this method. The results indicate that the proposed method can compensate varying time delay precisely with robustness for different high-frequency excitation loading.

Keywords: adaptive delay compensation; high-frequency signal; real-time hybrid testing; sliding mode control

1. Introduction

Real-time hybrid testing (RTHT) is a reliable structural dynamic testing method developed by Nakashima and Masaoka (1999). Unlike quasi-static tests or shaking table test, RTHT divides a structure into experimental and numerical substructures: The parts of the structure exhibiting complex non-linear behaviors are designated as the experimental substructure and are evaluated through real-time physical tests, while the remaining parts are treated as the numerical substructure and simulated via computational models. This unique approach enables RTHT to support large-scale, even full-scale, dynamic tests, making it more favored among researchers. Igarashi *et al.* (2009) used RTHT to assess the performance of laminated rubber dampers. Calabrese *et al.* (2014) investigated the feasibility of fiber-reinforced bearings as a cost-effective base isolation system using RTHT. Riascos *et al.* (2022) examined the influence of rocking effects on seismically isolated structures through RTHT. The combination of RTHT with shaking table technology has led to the development of Real-time Hybrid Shaking Table Testing (RTHSTT). Schellenberg *et al.* (2016) first applied RTHSTT as a mature testing application, successfully evaluating the performance of a seismic isolation structure. Avci and Christenson (2018) developed a hybrid shaking table test in which the superstructure of a mid-story isolation system was tested on a shaking table. Yoshida *et al.* (2018) evaluated a semi-actively controlled layered-isolated building through RTHSTT. To date, RTHT has been widely employed in various structural tests and evaluations, and it is increasingly regarded as a promising testing methodology in structural engineering.

Servo-hydraulic loading systems are widely used in RTHT due to their exceptional dynamic loading capacity. However, their dynamic characteristics inevitably introduce time delays. Additionally, the interaction between the loading system and the specimen, coupled with the inherent non-linearity of both components and the relationship between time delay and signal frequency, further contribute to variations in time delay. In slow loading tests or shaking table tests, these delays can be corrected in time due to the lower loading rate. However, in RTHT, errors cannot be corrected promptly during the loading process, leading to cumulative errors. This accumulation negatively impacts the stability and robustness of the test.

Based on control theory, delay compensation strategies were implemented to address time delays throughout the testing process. Horiuchi and Konno introduced the concept of time delay accumulation in servo-hydraulic loading systems. They proposed a fixed time delay compensation method (Horiuchi *et al.* 1999), predicting the target displacement with a fixed time delay and compensating for it using a third-order polynomial extrapolation algorithm.

As control theories developed, more control strategies were proposed for better RTHT performance. Nakashima

*Corresponding author, Ph.D., Professor,
E-mail: ptan@gzhu.edu

**Co-corresponding author, Ph.D., Professor,
E-mail: wang_zhen@whut.edu.cn

http://www.techno-press.org/?journal=sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

and Masaoka (1999) employed a Lagrange polynomial for extrapolation and polynomial for interpolation method to design RTHT, and ensure smooth loading throughout the test. Neild *et al*. (2005) proposed applied minimal controller synthesis for real-time hybrid shaking table test, reducing the oil-column resonance in the hydraulic actuator. Jung *et al*. (2007) developed a real-time pseudodynamic test system that adopts an implicit time integration method for solving the governing equations of motion for a test structure. Chen (2007) proposed an inverse model compensation method based on the discrete transfer function of actuators, where the response of the actual test substructure is employed to estimate the time delay of the test substructure, and to incorporate this estimated value to adjust the control commands of the test substructure. An improved forward extrapolation scheme used model-based response prediction was proposed by Carrion and Spencer (2008), which incorporates loading and structural properties into a model-based approach. A Smith predictor was proposed by Shao and Reinhorn (2012), which delay can be compensated by the prediction of the control action from the Smith predictor and then correcting the error. Günay and Mosalam (2015) developed a three-variable control for a hybrid simulation system. The method can improve the acceleration response that can improve the accuracy of the RTHT. Stehman and Nakata (2014) developed an infinite-impulse-response compensator, which enables a compensation for the time delay in shaking table and interference of the control-structural-interaction. Verma and Sivaselvan (2019) developed an impedance matching method for the benchmark control problem in RTHT. Different from conventional controller, the aim of this method is to make the impendence of the transfer system closer to the numerical and physical substructures, which simplified the control design process and shows robustness. Peiris *et al.* (2022) proposed a normalized passively control that using normalized measures of power transfer to replace the conventional measurements. The proposed method makes it possible to adequate the different test conditions by a single tuning.

Among the various time delay compensation methods, the sliding mode control (SMC) approach is regarded as a reliable solution for enhancing the robustness and accuracy of RTHT platforms due to its exceptional resistance to external disturbances. Compared to traditional linear control algorithms, sliding mode controller is more effective at managing nonlinear systems, compensating for model uncertainties and disturbances through their inherent robustness. This advantage has been demonstrated in several applications of the method. For example, Maghareh *et al*. (2020) developed a Self-tuning Robust Control System. The author implemented the SMC as the first layer and a bounded adaptation controller as the second layer. The robustness and accuracy of this method were verified through RTHT. Yao *et al*. (2023) proposed an acceleration-based SMC method, which utilizes acceleration-based SMC as an outer-loop controller to achieve time delay compensation in shaking table tests. Compared to traditional SMC methods, the acceleration-based SMC method computes control forces for the shake table without requiring precise information about the forces exerted on the structure by the shake table or accurately modeling complex nonlinear forces that not included in the model.

However, due to the nonlinearity of the servo loading system and the specimen, as well as the dynamic interactions between them, the time delay varies throughout the test. As a result, the fixed time delay compensation method can lead to either under-compensation or over-compensation, causing errors to accumulate. Adaptive control strategies are considered as a reliable solution in compensate time varying delay. Based on adaptive control theory, Wagg and Stoten (2001) designed an outer loop control method by using an adaptive minimal control synthesis method and proved its applicability in continuous time. Darby *et al*. (2002) proposed an adaptive delay compensation method that allows an estimation of actuator delay and adjusts the delay through the stiffness of the substructure. Chen *et al*. (2015) proposed adaptive model-based tracking control that can accommodate specimen nonlinearity and improve the tracking performance of the actuator. Chae *et al*. (2017) proposed an adaptive time-delay compensation method based on time series in real-time control RTHT and expand it to force control scenario. The method uses parameters update in real-time tests through the standard least squares method, adapting based on the actuator's response to minimize the error between the desired displacement and the actual actuator displacement. Li *et al*. (2021) employed an adaptive feedforward delay compensator based on displacement prediction to perform a RTHT on a three-story Y-eccentric braced frame model, and verified the feasibility of this method for RTHT on large frame structures. Tao and Mercan (2022) designed an adaptive discrete feedforward controller and employed it in multi-degree of freedom RTHT, which makes it possible to employ more realistic model in RTHT.

While these compensation methods have been researched and applied in various cases, they are often limited to structures with few degrees of freedom or specific scenarios. As compensation methods evolve, target structures are becoming more complex, and the number of degrees of freedom increases. Large-scale and multi-degree of freedom RTHT (Friedman *et al*. 2015, Sarebanha *et al*. 2019) attracted attention from researcher. Accurately replicating response in high-order mode in structures is crucial for advancing the study of seismic responses in complex systems. Therefore, it is essential to develop a robust compensation strategy for high-frequency signal RTHT.

To address the aforementioned issues, this paper proposes a sliding mode combined with adaptive time delay compensation method based discrete model (ADM-SMC). A sliding mode controller, integrated with a Kalman filter, is used to enhance the robustness of the loading system and compensate for time delays. Additionally, an adaptive compensation method based discrete model is incorporated to further address variable time delay. This method reliably compensates for varying time delay under high-frequency signals while maintaining robustness. Section 2 introduces the fundamental concept of ADM-SMC. In Section 3, a delay compensation test is detailed to verify the basic performance of the method. Section 4 presents a Real-Time Hybrid Test (RTHT) to further validate the ADM-SMC approach. Finally, the conclusions are summarized in Section 5.

2. Adaptive time delay compensation method combined with sliding mode control (ADM-SMC)

The ADM-SMC method consists of a sliding mode controller (SMC), a state observer, and an adaptive delay compensation method based discrete model (ADM). The SMC compensates for fixed time delays while enhancing system robustness. The state observer filters noise from the transducer signals, and the ADM method further compensates time delays. By combining the high robustness of the SMC with the precision of the ADM, the ADM-SMC method effectively addresses time delay compensation for high-frequency signals, ultimately improving the robustness and accuracy of Real-Time Hybrid Testing (RTHT). The principle of the ADM-SMC method is illustrated in Fig. 1. In Fig. 1, y_a represents the desired displacement, y_{ac} is the input signal for the SMC, and y_c and y_m represent the command and response of the servo loading system and the specimen, respectively. A detailed introduction of the ADM-SMC method will be discussed in Sections 2.1 to 2.3.

2.1 Design of the sliding mode controller

The sliding mode controller consists of three components: the SMC method, the state observer, and the servo loading system and the specimen. Before determining the parameters of the SMC method, a numerical model of the servo loading system and the specimen must first be established using a band-limited white noise excitation test. A state-space model is then used to simulate the dynamic characteristics of the controlled components, which can be expressed as Eq. (1)

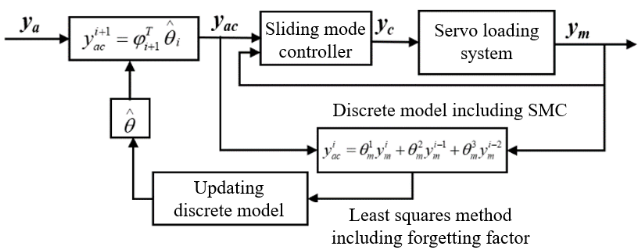


Fig. 1 The principle of ADM-SMC

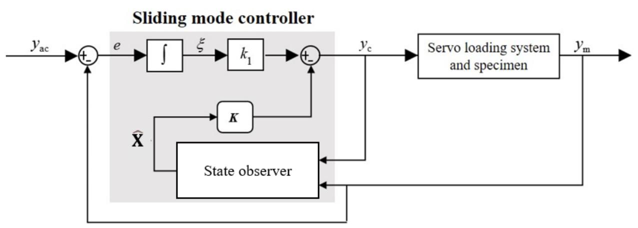


Fig. 2 Block diagram of the sliding mode controller

$$\begin{aligned} \dot{\boldsymbol{X}} &= \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}y_c \\ y_m &= \boldsymbol{C}\boldsymbol{X} \end{aligned} \tag{1}$$

where $\boldsymbol{X} = [x_1, x_2 \cdots x_n]^T$ represents the system state vector, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ represent three state-space matrixes, respectively.

The design of the sliding mode controller is shown in Fig. 2. The command y_c sending to the servo loading system and specimen can be expressed as

$$y_c(t) = k_1\xi - \boldsymbol{K}\boldsymbol{X}(t) \tag{2}$$

where $\xi = \int_0^t (y_{ac} - y_m)\, d\tau$, k_1 and $\boldsymbol{K}$ are the gain factors and state feedback gain matrix determined by SMC, respectively.

The state vector $\boldsymbol{X}$ contains higher order derivatives of the measured signal, which will be affected by the noise, which can be determined as a stochastic system. Therefore, the Kalman filter is employed to estimate the state vector to eliminate noise interference and obtain a more accurate state signal in the state observer. The corresponding mathematical model of the observer $\dot{\hat{\boldsymbol{X}}}$ can be expressed by the state-space equation

$$\dot{\hat{\boldsymbol{X}}} = \boldsymbol{A}\hat{\boldsymbol{X}} + \boldsymbol{B}y_c + \boldsymbol{K}_e\left(y_m - \boldsymbol{C}\hat{\boldsymbol{X}}\right) \tag{3}$$

where $\dot{\hat{\boldsymbol{X}}}$ is the observed state vector and $\boldsymbol{K}_e$ is the state observation gain matrix, respectively. Therefore, the command of the servo loading system and the specimen after Kalman filtering $y_c(t)$ can be expressed as Eq. (4)

$$y_c(t) = k_1\xi - \boldsymbol{K}\hat{\boldsymbol{X}}(t) \tag{4}$$

To obtain the SMC gain factors k_1 and the state feedback gain matrix $\boldsymbol{K}$, the sliding surface should be determined first. The state-space equation containing the error term can be assumed as

$$\dot{\boldsymbol{Y}}(t) = \boldsymbol{A}^*\boldsymbol{Y}(t) + \boldsymbol{B}^* y_c(t) \tag{5}$$

where $\boldsymbol{Y} = [\xi, \boldsymbol{X}^T]^T$, $\boldsymbol{A}^* = [0, \boldsymbol{C}; 0_{n\times1}, \boldsymbol{A}]$, $\boldsymbol{B}^* = [0, \boldsymbol{B}^T]^T$.

Assuming that the sliding surface equation can be expressed as

$$\boldsymbol{S} = \boldsymbol{P}\boldsymbol{Y} = 0 \tag{6}$$

where $\boldsymbol{P}$ is the matrix need to be determined. The state space of the system can be linearly trans-formed to

$$\eta = \boldsymbol{\Gamma}\boldsymbol{Y} \tag{7}$$

where $\boldsymbol{\Gamma}$ is the state transition matrix. In this step, it can be partition that $\boldsymbol{B}^* = \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{bmatrix}$, where the number of the elements in $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ is *n* and 1, respectively. Then $\boldsymbol{\Gamma}$ can be expressed as

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{I}_n & -\boldsymbol{B}\boldsymbol{B}_2^{-1} \\ 0 & \boldsymbol{I}_p \end{bmatrix} \tag{8}$$

Substitute Eq. (7) and Eq. (8) into Eq. (5) and Eq. (6) respectively, the state-space equation of the system and the sliding surface equation can be expressed as Eqs. (9)-(10)

$$\dot{\eta} = \widehat{\boldsymbol{A}}\eta + \widehat{\boldsymbol{B}}y_c \tag{9}$$

$$\boldsymbol{S} = \widehat{\boldsymbol{P}} = 0 \tag{10}$$

where $\widehat{\boldsymbol{A}} = \boldsymbol{\Gamma}\boldsymbol{A}^*\boldsymbol{\Gamma}^{-1}$, $\widehat{\boldsymbol{B}} = [0, {\boldsymbol{B}_2}^T]^T$, $\widehat{\boldsymbol{P}} = \boldsymbol{P}\boldsymbol{\Gamma}^{-1}$.

In Eqs. (9)-(10)

$$\eta = \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} \tag{11}$$

$$\widehat{\boldsymbol{A}} = \begin{bmatrix}\widehat{\boldsymbol{A}}_{11} & \widehat{\boldsymbol{A}}_{12}\\ \widehat{\boldsymbol{A}}_{21} & \widehat{\boldsymbol{A}}_{22}\end{bmatrix} \tag{12}$$

$$\widehat{\boldsymbol{P}} = [\widehat{\boldsymbol{P}}_1 \quad \widehat{\boldsymbol{P}}_2] \tag{13}$$

Therefore, Eqs. (9)-(10) can be written as

$$\dot{\eta}_1 = \widehat{\boldsymbol{A}}_{11}\eta_1 + \widehat{\boldsymbol{A}}_{12}\eta_2 \tag{14}$$

$$\boldsymbol{S} = \widehat{\boldsymbol{P}}_1\eta_1 + \widehat{\boldsymbol{P}}_2\eta_2 = 0 \tag{15}$$

Assume $\widehat{\boldsymbol{P}}_2 = \boldsymbol{I}$ is the identity matrix, $\eta_2 = -\widehat{\boldsymbol{P}}_1\eta_1$. The equation of the system when moving on a sliding surface can be written as

$$\dot{\eta}_1 = (\widehat{\boldsymbol{A}}_{11} - \widehat{\boldsymbol{A}}_{12}\widehat{\boldsymbol{P}}_1)\eta_1 \tag{16}$$

To ensure the stability of the system moving on the sliding surface, $\widehat{\boldsymbol{P}}$ can be determined by using LQR method if $\widehat{\boldsymbol{P}}_1$ is determined. The sliding surface can be determined as

$$\boldsymbol{P} = [\widehat{\boldsymbol{P}} \quad \boldsymbol{I}]\boldsymbol{\Gamma} \tag{17}$$

Besides, the Lyapunov's direct method can be used to design the SMC. The Lyapunov function in SMC can be written as

$$v = 0.5\boldsymbol{S}^T\boldsymbol{S} = 0.5\boldsymbol{Y}^T\boldsymbol{P}^T\boldsymbol{P}\boldsymbol{Y} \tag{18}$$

Assume $v \geq 0$ and $t \to \infty$, the sufficient condition to ensure that the system motion on the sliding surface asymptotically stable is $\dot{v} = \boldsymbol{S}^T\dot{\boldsymbol{S}} \leq 0$. By substituting Eqs. (4)-(5) into Eq. (18), $\dot{v}$ can be expressed as

$$\begin{aligned}\dot{v} &= \boldsymbol{S}^T\boldsymbol{P}\dot{\boldsymbol{Y}} = \boldsymbol{S}^T\boldsymbol{P}(\boldsymbol{A}^*\boldsymbol{Y} + \boldsymbol{B}^*y_c)\\ &= (\boldsymbol{P}\boldsymbol{Y})^T\boldsymbol{P}\boldsymbol{B}^*((\boldsymbol{P}\boldsymbol{B}^*)^{-1}\boldsymbol{P}\boldsymbol{A}^*\boldsymbol{Y} + y_c)\\ &= \boldsymbol{Y}^T\boldsymbol{\chi}(y_c - \boldsymbol{G}_s\boldsymbol{Y})\end{aligned} \tag{19}$$

where $\boldsymbol{\chi} = \boldsymbol{P}^T\boldsymbol{P}\boldsymbol{B}^*$, $\boldsymbol{G}_s = -(\boldsymbol{P}\boldsymbol{B}^*)^{-1}\boldsymbol{P}\boldsymbol{A}^*$.

To ensure the stability of the system, the continuous control can be designed as

$$y_c = (\boldsymbol{G}_s - \delta\boldsymbol{\chi}^T)\boldsymbol{Y} = [-k_1, -\boldsymbol{K}]\boldsymbol{Y} \tag{20}$$

where δ is a given number called sliding margin. $\dot{v} = -\boldsymbol{Y}^T\boldsymbol{\chi}\delta(\boldsymbol{Y}^T\boldsymbol{\chi})^T \leq 0$ is always true throughout the control process. At this step, the SMC gain factors k_1 and state feedback gain matrix $\boldsymbol{K}$ can be determined. The design scheme of SMC can be expressed as Fig. 3, including three main steps: First, Transform the transfer function model of the servo loading system and specimen to the state-space matrix. Second, determine the gain factors k_1 and state feedback gain matrix and $\boldsymbol{K}$, respectively. Third, determine the state observation gain matrix $\boldsymbol{K}_e$.

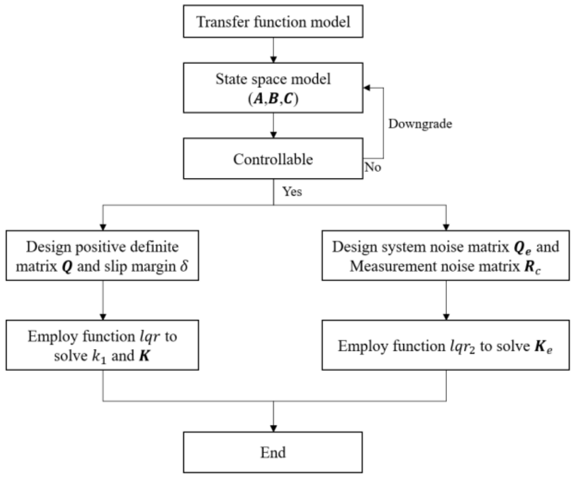


Fig. 3 Flow chart of the designation of SMC

2.2 Adaptive time delay compensation method based discrete model

The sliding mode controller described in Section 2.1 reduces phase lag in the loading system while enhancing its robustness. In this section, the adaptive compensation method based discrete model (ADM) is employed to compensate for the remaining time delay. The discrete model of the servo system and specimen can be expressed as follows

$$y_{ac}^i = \boldsymbol{\varphi}_i^T\boldsymbol{\theta} \tag{21}$$

with

$$\boldsymbol{\varphi}_i^T = \left[y_m^i, y_m^{i-1}, y_m^{i-2}, \dots, y_m^{i-q+1} \dots\right] \tag{22}$$

$$\boldsymbol{\theta} = \left[\theta_m^1, \theta_m^2, \theta_m^3, \dots, \theta_m^q, \dots\right] \tag{23}$$

In Eqs. (21)-(23), y_m^i represent the response from the servo loading system and specimen in i-th step, q represent the number of parameters. After the test system simplified into a discrete model, parameter $\boldsymbol{\theta}$ need to be estimate during the test to track changes of system characteristics in real time. In this paper, the parameter $\boldsymbol{\theta}$ are estimated by using the recursive least squares algorithm with a forgetting factor. With smaller storage capacity and simpler calculations, this method can effectively overcome the data saturation. The recursive least squares algorithm with forgetting factor can be expressed as

$$\widehat{\boldsymbol{\theta}}_i = \widehat{\boldsymbol{\theta}}_{i-1} + \frac{\boldsymbol{P}_{i-1}\boldsymbol{\varphi}_i}{\lambda + \boldsymbol{\varphi}_i^T\boldsymbol{P}_{i-1}}\left(y_{ac}^i - \boldsymbol{\varphi}_i^T\widehat{\boldsymbol{\theta}}_{i-1}\right) \tag{24}$$

$$\boldsymbol{P}_i = \frac{1}{\lambda}\left(\boldsymbol{I} - \frac{\boldsymbol{P}_{i-1}\boldsymbol{\varphi}_i}{\lambda + \boldsymbol{\varphi}_i^T\boldsymbol{P}_{i-1}}\boldsymbol{\varphi}_i^T\right)\boldsymbol{P}_{i-1} \tag{25}$$

where λ is the forgetting factor, $\boldsymbol{I}$ is the identity matrix.

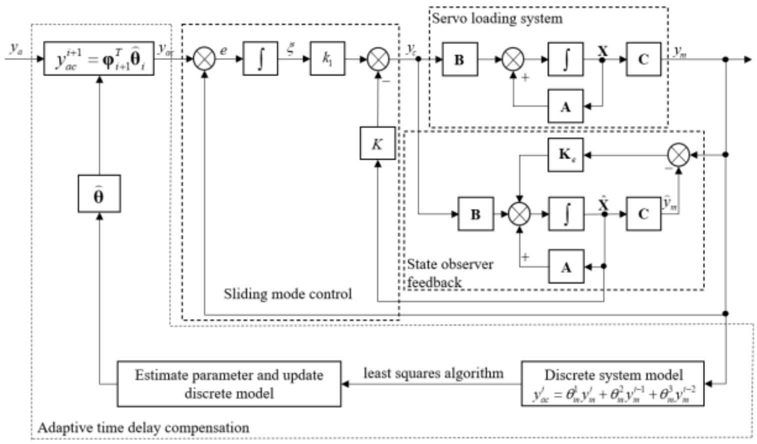


Fig. 4 Overview of ADM-SMC method

Initial parameter $\boldsymbol{P}_0$ and $\boldsymbol{\theta}_0$ can be calculated from a standard least squares method (Ding *et al.* 2006), which related data can be found in preliminary offline tests. The formula of the least squares method can be expressed as

$$\widehat{\boldsymbol{\theta}}_0 = (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\boldsymbol{U} \tag{26}$$

$$\boldsymbol{P}_0 = (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1} \tag{27}$$

where $\boldsymbol{\Phi} = [\varphi_1, \varphi_2, \ldots, \varphi_L]^T$, $\boldsymbol{U} = [y_{ac}^1, y_{ac}^2, \ldots, y_{ac}^L]^T$, L is the number of datasets. Accurate estimation of the initial value requires sufficient frequency components from the early offline signal; otherwise, errors in the estimated initial value may occur, impacting the overall accuracy of the test.

The parameters estimated by Eq. (11) can reflect the current state of the system containing time-delay characteristics. The aim of delay compensation strategy is to make measured displacement y_m as close as possible to the desired displacement y_a, and replace y_a to y_m. Assume that the model parameters have little change during the time interval $t_i \sim t_{i+1}$, which means the model parameter estimated in i-th step ($\boldsymbol{\theta}_{i+1}$) can be used to approximate the model parameter in $i+1$-th step ($\widehat{\boldsymbol{\theta}}_i$), that is, the compensation strategy can be expressed as follow

$$y_{ac}^{i+1} = \boldsymbol{\varphi}_{i+1}^T\boldsymbol{\theta}_{i+1} \approx \boldsymbol{\varphi}_{i+1}^T\widehat{\boldsymbol{\theta}}_i \tag{28}$$

where $\boldsymbol{\varphi}_{i+1}^T = \left[y_a^{i+1}, y_a^i, y_a^{i-1}, \ldots, y_m^{i-q+2}, \ldots\right]$.

After analysis, it can be found that in the ADM-SMC method, for the loading system containing the sliding mode controller, the following three-parameter or four-parameter system discrete model has better time-delay compensation performance. The following formula takes three parameters as an example

$$y_{ac}^i = \theta_m^1 y_m^i + \theta_m^2 y_m^{i-1} + \theta_m^3 y_m^{i-2} \tag{29}$$

Correspondingly, the compensation method is

$$y_{ac}^i = \boldsymbol{\varphi}_i^T\,\boldsymbol{\theta} = \boldsymbol{\varphi}_i^T\widehat{\boldsymbol{\theta}}_{i-1} \tag{30}$$

where

$$\boldsymbol{\varphi}_i^T = [y_m^i, y_m^{i-1}, y_m^{i-2}] \tag{31}$$

$$\boldsymbol{\theta} = [\theta_m^1, \theta_m^2, \theta_m^3] \tag{32}$$

An overview of the ADM-SMC method is presented in Fig. 4. In summary, the ADM method assumes the system follows a differential equation model, predicting and updating parameters based on the actuator's displacement response in the initial steps. These updated parameters, along with the system's expected displacement, are then used to predict the next displacement command for compensation.

3. Delay compensation tests

In Section 2, the principles of the ADM-SMC method were introduced. In this section, delay compensation tests were conducted to preliminarily verify the performance of the proposed ADM-SMC method. The test platform, results, and analysis are discussed in detail.

3.1 Test platform

The test was conducted at the Key Laboratory of Earthquake Resistance, Earthquake Mitigation, and Structural Safety, Ministry of Education, Guangzhou University. The MTS-Speedgoat real-time hybrid test platform was used for the delay compensation tests in this section and for RTHT in Section 4. Details of the test platform are shown in Fig. 5. The setup consists of a host computer for sending commands, a monitoring computer for data tracking and recording, a Speedgoat real-time hybrid test target machine, a Scramnet shared memory system for data transmission, an MTS controller, and an MTS hydraulic servo actuator for real-time loading. The Speedgoat test machine, developed by the Swiss company Speedgoat, features a range of digital and analog input/output interfaces and integrates seamlessly with mainstream development and simulation software. Its high-performance real-time processing capabilities allow for complex control and simulation tasks with minimal delay, making it ideal for real-time hybrid testing. The MTS 244.41S actuator used in the test supports a maximum dynamic load of 500 kN and a maximum dynamic stroke of 750 mm. The Scramnet shared memory network connects

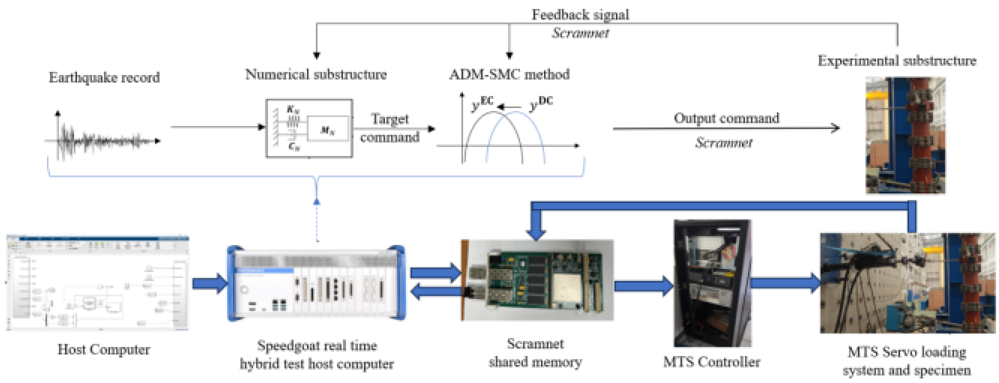


Fig. 5 Test platform

all devices via optical fibers, ensuring real-time communication throughout the MTS-Speedgoat hybrid test platform.

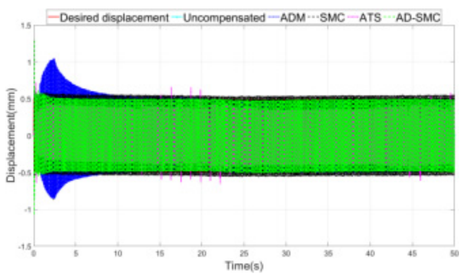
3.2 Test results

To compare the compensation performance of the ADM-SMC method, four compensation methods were employed in the delay compensation tests, including the SMC method, the adaptive delay compensation method (ADM), the adaptive time series method (ATS), and the ADM-SMC method. The desired displacement is set as a 15 hz 1 mm sine wave, a 0.1~20 Hz 1 mm sweep signal, and 20 Hz 1 mm band-limited white noise, respectively. The test results are shown in Figs. 6-8.

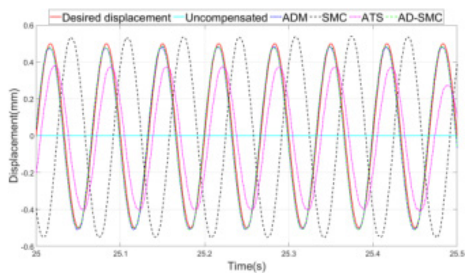
In this study, the performance of four compensation methods was evaluated under three different signal excitation loadings. The first group used a 15 Hz, 1 mm sine wave signal lasting 50 seconds. The results are shown in Fig. 6(a) for the overall time history and Fig. 6(b) for the enlarged time history from 25 s to 25.5 s. The ADM method exhibited some tracking errors during the first 10 seconds due to the need for parameter adjustments; the SMC method performed well in terms of amplitude but showed a consistent time delay throughout the test; and the ATS method had inadequate amplitude compensation along with a noticeable time delay. The ADM-SMC method demonstrated the smallest tracking error among all four methods.

Fig. 7 presents the results of the second group, which employed a 0-20 Hz, 1 mm sweep signal. The ADM-SMC method performed well in both amplitude and time delay, as seen in Fig. 7(c), which illustrates the tracking error for each method. The ATS method struggled during the first 10 seconds before adjusting, but its error increased as the frequency rose. The SMC method exhibited consistent time delays, while the ADM method initially had issues with both delay and amplitude but performed well after parameters are adjusted.

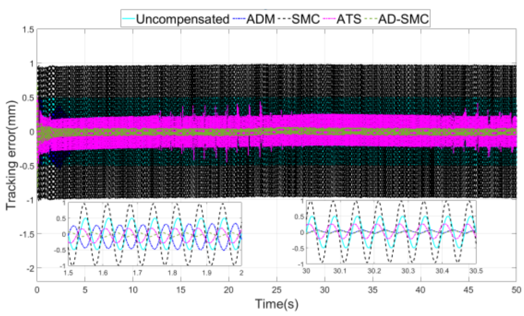
Fig. 8 shows the compensation results of a 1 mm, 15 Hz



(a) Overall view

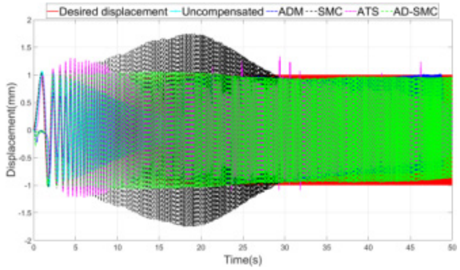


(b) Close up view

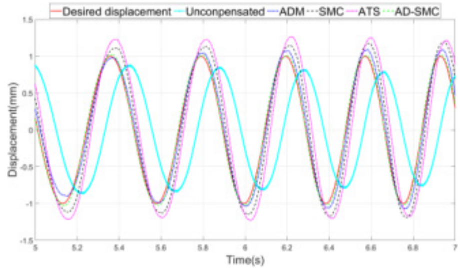


(c) Tracking error

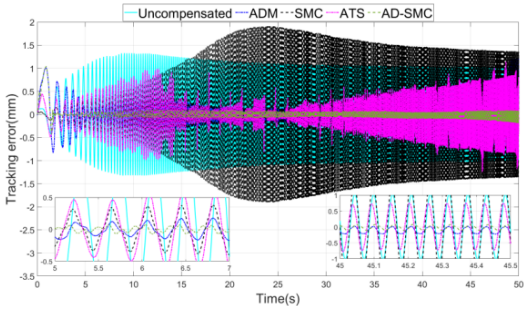
Fig. 6 Time history of 15 Hz 1 mm sine wave signal



(a) Overall view

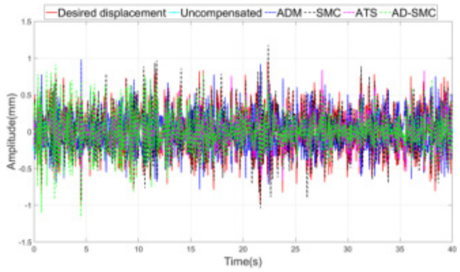


(b) Close up view

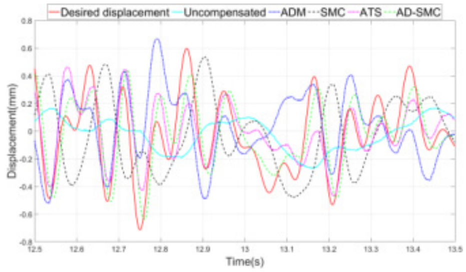


(c) Tracking error

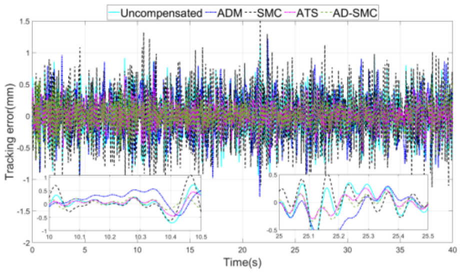
Fig. 7 Time history of 15 Hz 1 mm sweep excitation loading



(a) Overall view



(b) Close up view



(c) Tracking error

Fig. 8 Time history of 20 Hz 1 mm band-limited white noise excitation loading

band-limited white noise signal excitation. All methods experienced some time delay and amplitude errors due to the high-frequency and irregular changes. The tracking error results indicate that the ADM-SMC method provided the best compensation performance. The ATS and ADM methods performed adequately in delay compensation but had larger amplitude errors, whereas the SMC method showed a significant time delay.

3.3 Result discussion

To evaluate the compensation performance of the above control methods, the root means square error (RMSE) and the relative peak error (PE) are employed to evaluate the performance of each test. The definition of RMSE and PE can be written as

$$RMSE = \sqrt{\frac{1}{n}\sum_{1}^{n}(y_{de} - y)^2} \tag{33}$$

$$PE = \left(\frac{y - y_{de}}{y_{de}}\right) \tag{34}$$

Table 1 RMSE of different control methods (mm)

	Uncompensated	ADM	SMC	ATS	ADM-SMC
15 Hz Sin wave	0.141	0.071	0.153	0.097	0.042
0 ~ 20 Hz sweep	0.767	0.237	0.963	0.379	0.127
15 Hz band limited white noise	0.298	0.312	0.383	0.179	0.185

Table 2 PE of different control methods (mm)

	Uncompensated	ADM	SMC	ATS	ADM-SMC
15 Hz Sin wave	0.200	0.215	0.229	0.291	0.190
0 ~ 20 Hz sweep	1.326	1.211	1.913	1.246	1.045
15 Hz band limited white noise	1.112	1.279	1.510	0.702	0.718

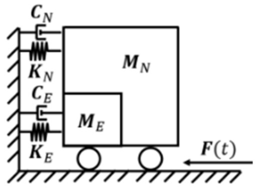
where y_{de} is the desired displacement, y is the displacement after compensation. The results of RMSE and PE are listed in Tables 1-2, respectively.

It can be observed from Tables 1 and 2 that compared to other four methods, the ADM-SMC method reaches 0.118 of the RMSE and 0.651 mm of the PE at average, reduced the average RMSE by 43.0-70.2% and the average PE by 12.7-41.7% compared to the uncompensated condition. These results demonstrate the promising compensation ability of the ADM-SMC method. However, it should be noted that in the 15 Hz band-limited white noise test, the ADM-SMC method was inferior to the ATS method, with approximately 3.2% higher RMSE and 2.2% higher PE. This issue is considered due to the initial parameters being held overtime, resulting in some errors.

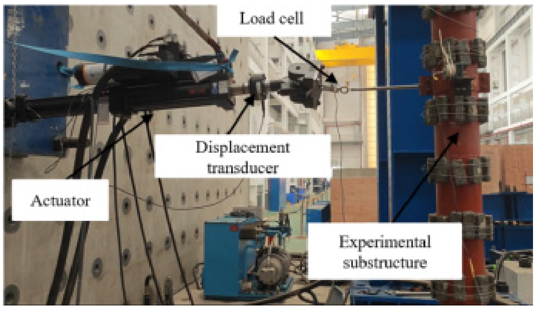
In conclusion, all four compensation methods demonstrated improvement compared to the case without compensation. The SMC method accurately reproduced the signal shape, but persistent time delays resulted in noticeable errors in both PE and RMSE. The ADM method showed strong compensation performance, although the adjustment of initial parameters introduced startup errors. The ADM-SMC method enhanced both accuracy and robustness by using the ADM to further compensate for the time delay inherent in the SMC, making it well-suited for wideband and high-frequency excitation loadings. The ATS method, however, exhibited some errors due to its reliance on old data through a windowing approach in the least squares method. While stable and resistant to divergence, it was less responsive to high-frequency signal changes. Additionally, the filtering used in the ATS method to reduce noise introduced further time delay, diminishing its ability to compensate the signal with high-frequency. Overall, the proposed ADM-SMC method demonstrated reliable compensation performance, with excellent results across various cases proving its robustness and effectiveness in compensating for high-frequency signals.

4. RTHT verification

RTHT is a test method that divide the structure into experimental and numerical substructure, which the experimental substructure is tested in real condition while the numerical substructure is modelled and simulated in computer. RTHT is an efficient and reliable technique for achieving desirable seismic performances of engineering structures. In RTHT, time delay is an important issue that will affect the accuracy and robustly. A delay compensation strategy is necessary for RTHT. In this section, to further verify the compensation performance of the proposed ADM-SMC method, a RTHS was carried out.



(a) Overall



(b) Experimental substructure

Fig. 9 Division of substructure

4.1 Test platform

The test was divided into two groups: In group 1, the ADM-SMC method was employed to recurrent the structural response under the El-Centro earthquake, 20 Hz band limited white noise and 10 Hz sweep excitation loading, respectively. In group 2, three compensation method (SMC, ATS, ADM-SMC) was employed to recurrent the structural response of a 2 mm band limited white noise. The test platform employed the platform mentioned in section 3. The object structure is set as a single degree of freedom structure and the division of substructure is shown in Fig. 9, where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ represent the mass, damping and stiffness matrix, respectively. Subscribe N and E represent the numerical and experimental substructure, respectively. The numerical substructure is set as a state-space matrix. The servo loading system employs the MTS 244.22 actuator, which maximum dynamic load can reach 100 kN, and the maximum dynamic stroke is 500 mm.

4.2 Test results

4.2.1 Group 1: Analysis under different cases

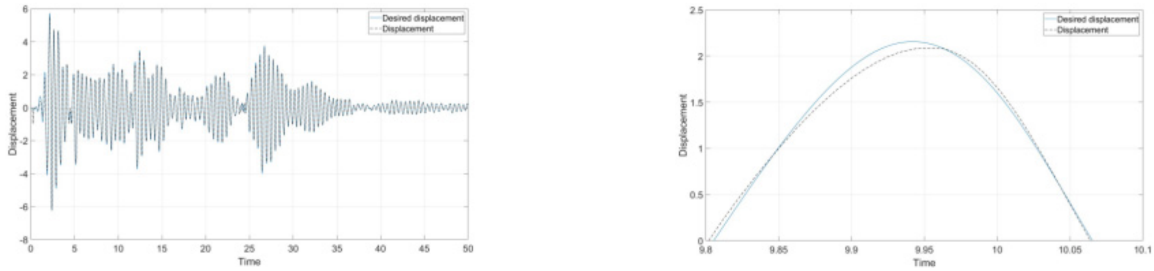
To verify the robustness of the ADM-SMC method under RTHS, its compensation performance was tested in Group 1 using three different cases: the El-Centro earthquake, 0.5 mm 20 Hz band-limited white noise, and a 10 Hz sweep signal.

The comparison between the desired displacement and the actuator displacement is shown in Figs. 10-12. The results demonstrate that the ADM-SMC method effectively compensates for time delays across different cases. Even under high-frequency excitation loadings, the remaining time delay and relative peak errors did not become significantly more pronounced. A closer view of a 1/4 period reveals that the time delay is nearly constant, and the peak errors remain relatively small. Thus, it can be concluded that the ADM-SMC method can robustly compensate for time delays under high-frequency excitation loadings.

It should be noted that in the first 0-1 seconds of the 0.5 mm 20 Hz band-limited white noise test, an error occurred due to the prolonged duration of the initial parameter adjustment in the ADM. Shortening this duration or reducing the weight of the ADM method is expected to resolve this issue.

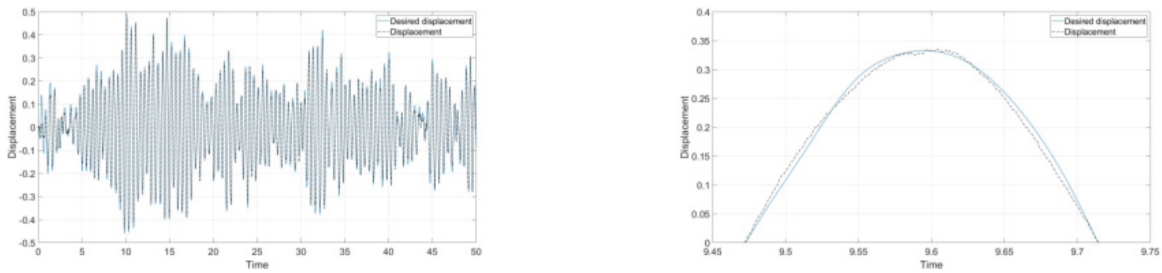
4.2.2 Group 2: Comparison of the ADM-SMC method and other methods

To further validate the compensation performance of the ADM-SMC method, it was compared with two other methods: the sliding mode control method (SMC) and adaptive time series compensation methods. The desired displacement was set as a 2 mm band-limited white noise



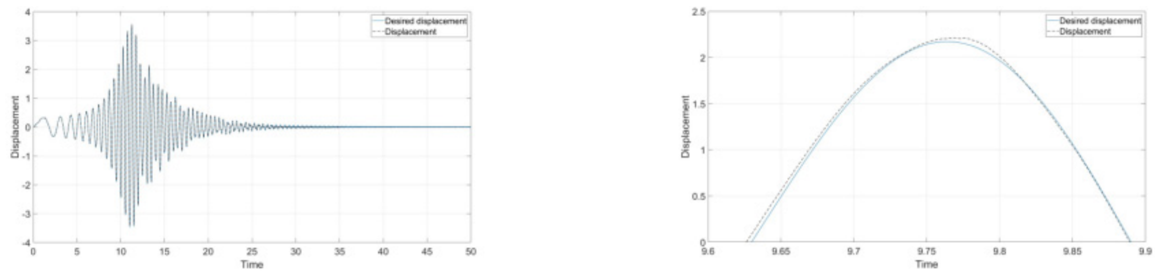
(a) Overall view (b) Close-up view

Fig. 10 Time history of structural response under El-Centro earthquake



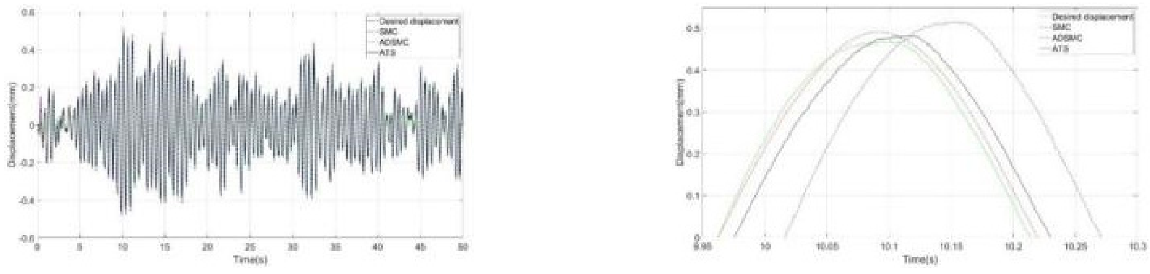
(a) Overall view (b) Close-up view

Fig. 11 Time history of structural response under band limited white noise



(a) Overall view (b) Close-up view

Fig. 12 Time history of structural response under sweep



(a) Overall view (b) Close-up view

Fig. 13 Time history of Group 2 RTHS

signal, and the test results are shown in Fig. 13.

The results demonstrate that the ADM-SMC method offers promising compensation performance compared to the other methods. Similar to the delay compensation tests, the SMC method exhibited a noticeable time delay, though its amplitude was close to the desired command. The ATS method showed a smaller time delay and performed consistently well throughout the test. The ADM-SMC method outperformed both alternatives. A closer view at the 1/4 period time history confirms this: compared to the other methods, the ADM-SMC method significantly reduced time delay, resulting in better RMSE outcomes. Additionally, the ADM-SMC method performed well in terms of amplitude, second only to the ATS method. Overall, the ADM-SMC method demonstrated superior performance under high-frequency excitation loadings compared to the other methods.

4.3 Result discussion

In this section, numerical analysis is employed to further evaluate the test results, including RMSE, PE in group 2, and additionally, remaining time delay in group 3. The results are shown in Tables 3-4.

From Table 3, it can be found that among three different cases, the average RMSE is 0.042, and the average PE is 0.372 mm, which shows that the promising compensation performance and robustness of the ADM-SMC method in different conditions. In case of the sweep, the compensation effect of the ADM-SMC method performed excellent throughout the process. While it should be noted that in Figs. 10-11, in case of the band-limited white noise and El-Centro earthquake, due to the initial parameters in the ADM method didn't change significantly, the compensation performed not well in 0~1 s. This problem can be corrected after adjusting the initial parameters. In conclusion, the ADM-SMC method shows great compensation ability in wide-band frequency and demonstrated promising robustness throughout the test. In the beginning few seconds it shows some error, but it can be partly fixed by the sliding mode controller and corrected by adjustment of the initial parameters of the ADM-SMC method in follow-up process.

Group 2 employs RMSE, PE and remain time delay to evaluate the performance of the three compensation methods. When compared to the other methods, the ADM-SMC method reaches 0.008 in RMSE and 0.040 in

Table 3 Test result of Group 1 RTHS

	El-Centro	Band limited white noise	Sweep
RMSE	0.085	0.019	0.021
PE	0.852	0.138	0.128

Table 4 Test result of Group 2 RTHS

	SMC	ATS	ADM-SMC
RMSE	0.1145	0.0284	0.0199
PE (mm)	0.3391	0.0792	0.1384
Time delay (ms)	53.71	0.1367	0.097

remaining time delay lower than the second performed method (ATS), and the PE is higher than the ATS method, by 0.059 mm. It can be found that the RMSE indicates the ADM-SMC method performs better than the other methods, achieving 82.6% of the SMC method and 29.9% of the ATS method. The ATS method performs better than the ADM-SMC method in the PE, achieving 42.8%. The ADM-SMC method has the smallest time delay among three methods, which is 0.097 s. In conclusion, compared to other compensation, the ADM-SMC method shows great promise in compensate the time delay. Additionally, the adaptive ability and great robustness of the ADM-SMC method can be better competent in high-frequency area and other complex occasions, which can better recurrent the performance of the complex structures under earthquake.

The result of the RTHT indicates that the ADM-SMC method exhibits promising compensation capabilities and robustness under high-frequency excitation. The ADM-SMC method demonstrates its versatility by reliably compensating for various types of excitations, showcasing strong robustness. Compared to other compensation methods, the ADM-SMC method achieved superior compensation performance. Overall, the ADM-SMC method offers a reliable and robust approach for accurately replicating the behavior of complex structures in RTHT.

5. Conclusions

To achieve accurate, robust loading with high-frequency signals in real time hybrid test (RTHT), this paper proposes a composite time delay compensation strategy. The effectiveness of this method was validated through delay compensation tests and RTHT on a single degree-of-freedom structure. The key findings are summarized as follows:

- An adaptive delay compensation strategy based discrete model combined with the sliding mode control method (ADM-SMC) was proposed. This composite compensation strategy integrates a sliding mode controller, a Kalman filter, and an adaptive compensation method based on a discrete model. Compared to previous approaches, the proposed method can accurately and robustly compensate varying time delay under high-frequency excitations.
- The compensation performance of the ADM-SMC method was basically evaluated through delay compensation tests. By implementing the ADM-SMC method, the time delay and relative peak error of the system were reduced under different excitations, demonstrating effective compensation for varying time delays and enhancement of robustness of the system.
- In RTHT, the ADM-SMC method effectively compensates for the time delay of the servo loading system, demonstrating strong robustness and accuracy. In comparative tests, the ADM-SMC method outperformed other methods in terms of root mean square error and remaining time delay. These findings suggest that the ADM-SMC method provides reliable delay compensation for high-frequency excitation loadings in RTHT.
- The results imply that the proposed ADM-SMC method can compensate for wide-band frequency signals with reliable robustness. This robustness makes this approach possible for loading wide-band frequency and complex excitation, and to employ actuators with larger loading capacities in RTHT.

Declaration of interest statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by the National Key Research and Development Program (2022YFC3801201), the National Natural Science Foundation of China (52378292, 51878630, 52078398), and the Natural Science Foundation of Guangdong Province (2022A1515010500).

Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

References

Avci, M. and Christenson, R. (2018), “Real-time hybrid substructuring shake table test of a seismically excited base isolated building”, *Dyn. Coupl. Struct.*, **4**, 71-78. https://doi.org/10.1007/978-3-319-74654-8_6

Calabrese, A., Strano, S. and Terzo, M. (2014), “Real-time hybrid simulations vs shaking table tests: Case study of a fiber-reinforced bearings isolated building under seismic loading”, *Struct. Control Health Monitor.*, **22**, 535-556. https://doi.org/10.1002/stc.1687

Carrion, J.E. and Spencer, B.F. (2008), “Real-time hybrid testing using model-based delay compensation”, *Smart Struct. Syste., Int. J.*, **4**(6), 809-828. https://doi.org/10.12989/sss.2008.4.6.809

Chae, Y., Rabiee, R., Dursun, A. and Kim, C.Y. (2017), “Real-time force control for servo-hydraulic actuator systems using adaptive time series compensator and compliance springs”, *Earthq. Eng. Struct. Dyn.*, **47**(4), 854-871. https://doi.org/10.1002/eqe.2994

Chen, C. (2007), “Development and numerical simulation of hybrid effective force testing method”, Ph.D. Dissertation; Lehigh University, Bethlehem, PA, USA.

Chen, P.C., Chang, C.M., Spencer Jr, B.F. and Tsai, K.C. (2015), “Adaptive model-based tracking control for real-time hybrid simulation”, *Bull. Earthq. Eng.*, **13**(6), 1633-1653. https://10.1007/s10518-014-9681-2

Darby, A.P., Williams, M.S. and Blakeborough, A. (2002), “Stability and delay compensation for real-time substructure testing”, *J. Eng. Mech.*, **128**(12), 1276-1284. https://10.1061/(ASCE)0733-9399(2002)128:12(1276)

Ding, F., Shi, Y. and Chen, T. (2006), “Performance analysis of estimation algorithms of non-stationary ARMA processes”, *IEEE Transact. Signal Process.*, **54**(3) 1041-1053. https://10.1109/TSP.2005.862845

Friedman, A., Dyke, S.J., Phillips, B., Ahn, R., Dong, B., Chae, Y., Castaneda, N., Jiang, Z., Zhang, J., Cha, Y. and Ozdagli, A.I. (2015), “Large-scale real-time hybrid simulation for evaluation of advanced damping system performance”, *J. Struct. Eng.*, **141**(6). https://doi.org/10.1061/(ASCE)ST.1943-541X.000109

Günay, S. and Mosalam, K.M. (2015), “Enhancement of real-time hybrid simulation on a shaking table configuration with implementation of an advanced control method”, *Earthq. Eng. Struct. Dyn.*, **44**(5), 657-675. https://doi.org/10.1002/eqe.2477

Horiuchi, T., Inoue, M., Konno, T. and Namita, Y. (1999), “Real-time hybrid experimental system with actuator delay compensation and its application to a piping system with energy absorber”, *Earthq. Eng. Struct. Dyn.*, **28**(10), 1121-1141. https://doi.org/10.1002/(SICI)1096-9845(199910)28:10<1121::AID-EQE858>3.0.CO;2-O

Igarashi, A., Sanchez-Flores, F., Iemura, H., Fujii, K. and Toyooka, A. (2009), “Real-time hybrid testing of laminated rubber dampers for seismic retrofit of bridges”, In: *The 3rd International Conference on Advances in Experimental Structural Engineering*, San Francisco, CA, USA.

Jung, R.Y., Benson Shing, P., Stauffer, E. and Thoen, B. (2007), “Performance of a real-time pseudodynamic test system considering nonlinear structural response”, *Earthq. Eng. Struct. Dyn.*, **36**(12),1785-1809. https://doi.org/10.1002/eqe.722

Li, T., Su, M., Sui, Y. and Ma, L. (2021), “Real-time hybrid simulation of a space substructure based on high-strength steel composite Y-eccentrically braced frames”, *Struct. Control Health Monitor.*, p. e277L. https://doi.org/10.1002/stc.2771

Maghareh, A., Dyke, S.J. and Silva, C.E. (2020), “A self-tuning robust control system for nonlinear real-time hybrid simulation”, *Earthq. Eng. Struct. Dyn.*, **49**(7), 695-715. https://doi.org/10.1002/eqe.3260

Nakashima, M. and Masaoka, N. (1999), “Real-time on-line test for MDOF systems", *Earthq. Eng. Struct. Dyn.*, **28**(4), 393-420. https://doi.org/10.1002/(SICI)1096-9845(199904)28:4<393::AID-EQE823>3.0.CO;2-C

Neild, S.A., Stoten, D.P., Drury, D. and Wagg, D.J. (2005), "Control issues relating to real-time substructuring experiments using a shaking table", *Earthq. Eng. Struct. Dyn.*, **34**(11), 1171-1192. https://doi.org/10.1002/eqe.473

Peiris, L.D.H., Plummer, A.R. and du Bois, J.L. (2022), "Normalized passivity control for robust tuning in real-time hybrid tests", *Int. J. Robust Nonlinear Control*, **32**(7), 4355-4375. https://doi.org/10.1002/rnc.6029

Riascos, C., Klopov, S., Losanno, D., Marulanda, J. and Thomson, P. (2022), "Real-time hybrid simulations including rocking effects: The case of a frame-structure with unbounded elastomeric isolators", *Struct. Control Health Monitor.*, **29**, p. e2984. https://doi.org/10.1002/stc.2984

Sarebanha, A., Schellenberg, A.H., Schoettler, M.J., Mosqueda, G. and Mahin, S.A. (2019), "Real-time hybrid simulation of seismically isolated structures with full-scale bearings and large computational models", *Comput. Model. Eng. Sci.*, **120**(3), 693-717. https://doi.org/10.32604/cmes.2019.04846

Schellenberg, A.H., Becker, T.C. and Mahin, S.A. (2017), "Hybrid shake table testing method: Theory, implementation and application to midlevel isolation", *Struct. Control Health Monitor.*, **24**, p. e1915. https://doi.org/10.1002/stc.1915

Shao, X. and Reinhorn, A.M. (2012), "Development of a controller platform for force-based real-time hybrid simulation", *J. Earthq. Eng.*, **16**(2), 274-295. https://10.1080/13632469.2011.597487

Stehman, M. and Nakata, N. (2014), "IIR compensation in real-time hybrid simulation using shake tables with complex control-structure-interaction", *J. Earthq. Eng.*, **20**(4), 633-653. https://doi.org/10.1080/13632469.2015.11047453

Tao, J. and Mercan, O. (2022), "Adaptive discrete feedforward controller for tracking error compensation of servo-hydraulic actuators in real-time hybrid simulation", *Struct. Control Health Monitor.*, **29**(3), p. e2895. https://doi.org/10.1002/stc.2895

Verma, M. and Sivaselvan, M.V. (2019), "Impedance matching control design for the benchmark problem in real-time hybrid simulation", *Mech. Syst. Signal Process.*, **134**, p. 106343. https://doi.org/10.1016/j.ymssp.2019.106343

Wagg, D.J. and Stoten, D.P. (2001), "Substructuring of dynamical systems via the adaptive minimal control synthesis algorithm", *Earthq. Eng. Struct. Dyn.*, **30**, 865-877. https://doi.org/10.1002/eqe.44

Yao, H., Tan, P., Yang, T.Y. and Zhou, F. (2023), "Shake table real-time hybrid testing for shear buildings based on sliding mode acceleration control method", *Structures*, **52**, 230-240. https://doi.org/10.1016/j.istruc.2023.03.140

Yoshida, S., Fujitani, H., Mukai, Y. and Ito, M. (2018), "Real-time hybrid simulation of semi-active control using shaking table: Proposal and verification of a testing method for mid-story isolated buildings", *Japan Archit. Rev.*, **1**, 221-234. https://doi.org/10.1002/2475-8876.12034